ベクトルの公式

1)

$$\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A} \tag{1.1}$$

2)

$$\begin{aligned}&\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})=\mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})\\&=\begin{vmatrix}A_x & A_y & A_z\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}=[\mathbf{ABC}]\end{aligned} \tag{1.2}$$

3)

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\cdot\mathbf{C})\mathbf{B}-(\mathbf{A}\cdot\mathbf{B})\mathbf{C} \tag{1.3}$$

4)

$$\nabla(\varphi\psi)=\psi\nabla\phi+\phi\nabla\psi \tag{1.4}$$

5)

$$\nabla\cdot(\varphi\mathbf{A})=\nabla\varphi\cdot\mathbf{A}+\varphi\nabla\cdot\mathbf{A} \tag{1.5}$$

6)

$$\nabla\times(\varphi\mathbf{A})=\nabla\varphi\times\mathbf{A}+\varphi\nabla\times\mathbf{A} \tag{1.6}$$

7)

$$\nabla\cdot(\mathbf{A}\times\mathbf{B})=\mathbf{B}\cdot(\nabla\times\mathbf{A})-\mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{1.7}$$

8)

$$\nabla\times(\mathbf{A}\times\mathbf{B})=(\mathbf{B}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{B}+\mathbf{A}\nabla\cdot\mathbf{B}-\mathbf{B}\nabla\cdot\mathbf{A} \tag{1.8}$$

9)

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times\nabla\times\mathbf{B} + \mathbf{B}\times\nabla\times\mathbf{A} \qquad (1.9)$$

10)

$$\nabla\cdot(\nabla\varphi) = \nabla^2\varphi \qquad (1.10)$$

11)

$$\nabla\times\nabla\varphi = 0 \qquad (1.11)$$

12)

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

13) (1.12)

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - (\nabla\cdot\nabla)\mathbf{A}$$

直角座標系のとき (1.13)

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$