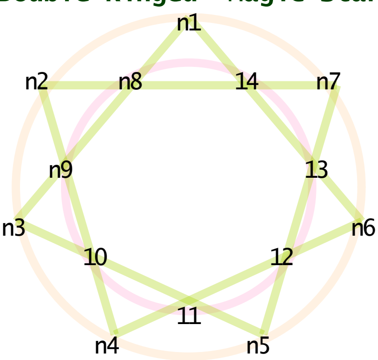
Part 2: Basic Studies of Magic Stars I: Kanji Setsuda

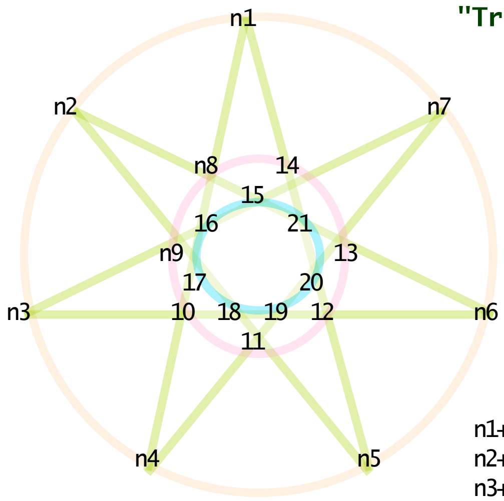
Chapter 1: Introduction: What are Magic Stars?

"Double-Ringed" Magic Star 7



n1+n8+n9+n3=K
n2+n9+n10+n4=K
n3+n10+n11+n5=K
n4+n11+n12+n6=K
n5+n12+n13+n7=K
n6+n13+n14+n1=K
n7+n14+n8+n2=K

"Triple-Ringed" MS7



n1+n8+n16+n17+n10+n4=K2
n2+n9+n17+n18+n11+n5=K2
n3+n10+n18+n19+n12+n6=K2
n4+n11+n19+n20+n13+n7=K2
n5+n12+n20+n21+n14+n1=K2
n6+n13+n21+n15+n8+n2=K2
n7+n14+n15+n16+n9+n3=K2

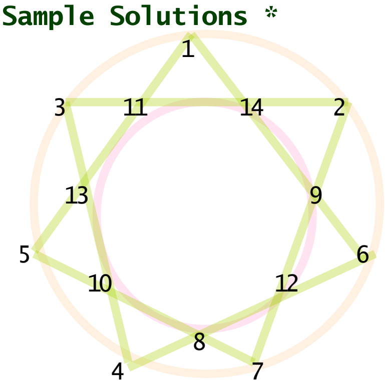
You may probably know well about 'Magic Stars', but let me explain a little about them in my own way and report about the result of my recent study here.

What do the magic stars look like? I would like you to give your kind look at the representative figures above about (1) "Double-Ringed" Magic Star of Order 7 and (2) "Triple-Ringed" Magic Star of Order 7.

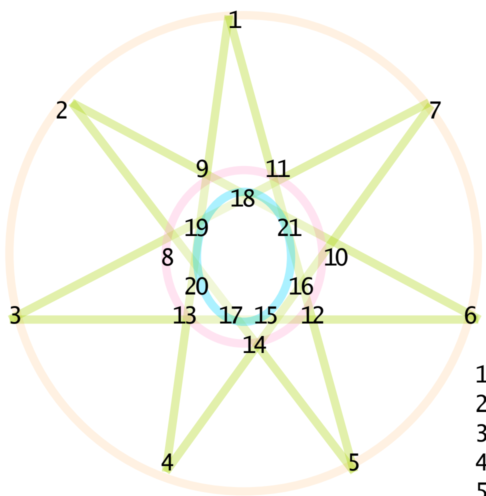
The former type of MS7 is made up of the 2x7 natural numbers {1, 2, 3, 4, 5, ..., 12, 13, 14}, while the latter one is made up of the 3x7 natural numbers {1, 2, 3, 4, 5, ..., 25, 26, 27}, each number of which must be used strictly once and neither duplication nor lack of any one is allowed.

(1) Every 4 numbers on each of 7 diagonals must add up to the characteristic magic constant 30 for the Double-Ringed type of Magic Stars 7. (2) Every 6 numbers on each diagonal must add up to its own magic constant 66 for the Triple-Ringed MS7.

*** Sample Solutions ***



1+11+13+5=30; 3+13+10+4=30;
5+10+8+7=30; 4+8+12+6=30;
7+12+9+2=30; 6+9+14+1=30;
2+14+11+3=30;

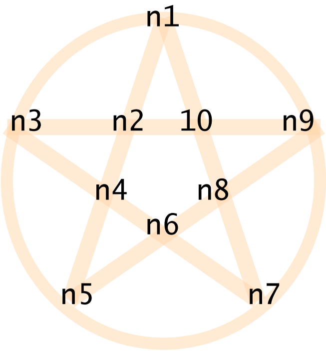


1+9+19+20+13+4=66;
2+8+20+17+14+5=66;
3+13+17+15+12+6=66;
4+14+15+16+10+7=66;
5+12+16+21+11+1=66;
6+10+21+18+9+2=66;
7+11+18+19+8+3=66;

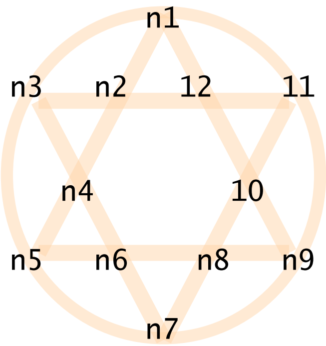
Let's study for a while about the Double-Ringed Magic Stars of various orders.

Please read the next list of various definition diagrams as follows, will you?
As you see, they all have the same structure in common, so simple and impressive.

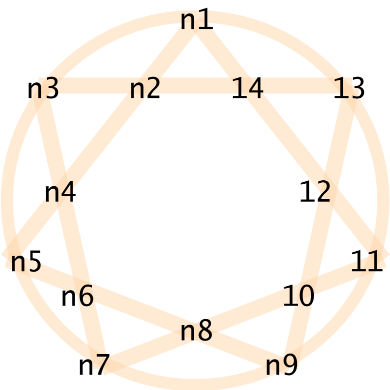
* Basic Forms and Definitions for "Double-Ringed" Magic Stars *



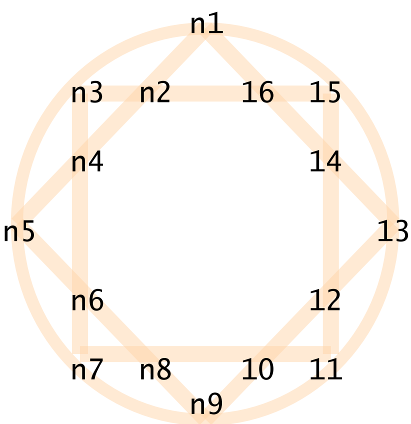

```
* Basic Definition for Order 5 *
 n1+n2+n4+n5=C       ... 11
 n3+n4+n6+n7=C       ... 12
 n5+n6+n8+n9=C       ... 13
 n7+n8+n10+n1=C      ... 14
 n9+n10+n2+n3=C      ... 15
 -----------------------------
 2*(n1+n2+n3+...+n9+n10)=5*C
          2*55=5*C
 Therefore  C=22     ... r0
```




```
* Basic Definition for Order 6 *
 n1+n2+n4+n5=C     ... 11
 n3+n4+n6+n7=C     ... 12
 n5+n6+n8+n9=C     ... 13
 n7+n8+n10+n11=C   ... 14
 n9+n10+n12+n1=C   ... 15
 n11+n12+n2+n3=C   ... 16
 -----------------------------
 2*(n1+n2+n3+...+n11+n12)=6*C
          2*78=6*C
 Therefore  C=26   ... r0
```

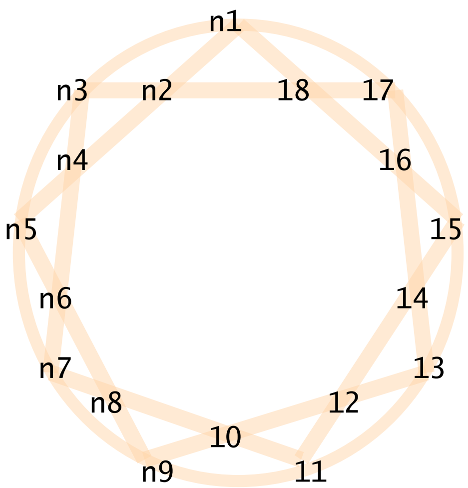



```
* Basic Definition for Order 7 *
 n1+n2+n4+n5=C     ... 11
 n3+n4+n6+n7=C     ... 12
 n5+n6+n8+n9=C     ... 13
 n7+n8+n10+n11=C   ... 14
 n9+n10+n12+n13=C ... 15
 n11+n12+n14+n1=C ... 16
 n13+n14+n2+n3=C   ... 17
 -----------------------------
 2*(n1+n2+n3+...+n13+n14)=7*C
          2*105=7*C
 Therefore  C=30   ... r0
```

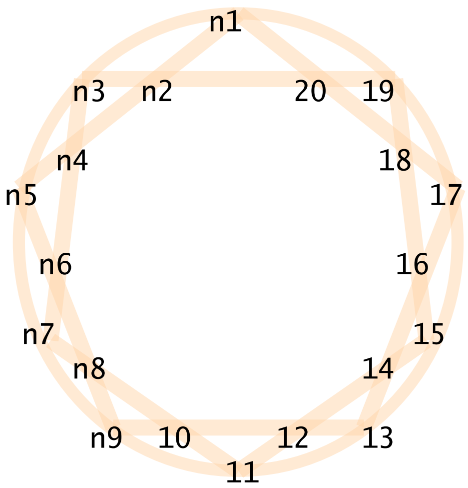



```
* Basic Definition for Order 8 *
 n1+n2+n4+n5=C       ... 11
 n3+n4+n6+n7=C       ... 12
 n5+n6+n8+n9=C       ... 13
 n7+n8+n10+n11=C     ... 14
 n9+n10+n12+n13=C    ... 15
 n11+n12+n14+n15=C   ... 16
 n13+n14+n16+n1=C    ... 17
 n15+n16+n2+n3=C     ... 18
 -----------------------------
 2*(n1+n2+n3+...+n15+n16)=8*C
          2*136=8*C
 Therefore  C=34     ... r0
```

Since all numbers look like making a picture of any brightest star, we used to call those objects as 'Magic Stars'. However, they will look quite different in higher orders.

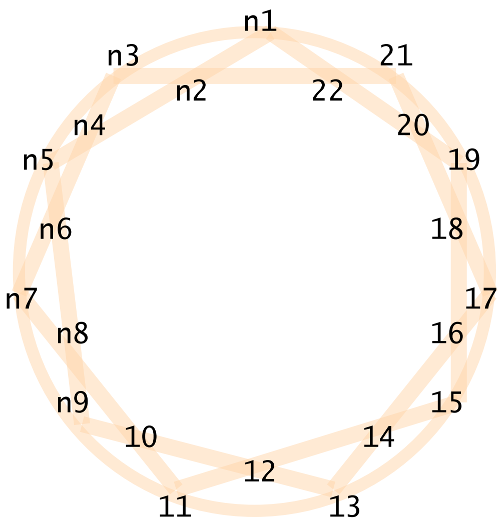



```
* Basic Definition for Order 9 *
  n1+n2+n4+n5=C       ... l1
  n3+n4+n6+n7=C       ... l2
  n5+n6+n8+n9=C       ... l3
  n7+n8+n10+n11=C     ... l4
  n9+n10+n12+n13=C    ... l5
  n11+n12+n14+n15=C   ... l6
  n13+n14+n16+n17=C   ... l7
  n15+n16+n18+n1=C    ... l8
  n17+n18+n2+n3=C     ... l9
  ------------------------------
  2*(n1+n2+n3+...+n17+n18)=9*C
            2*171=9*C
  Therefore   C=38    ... r0
```



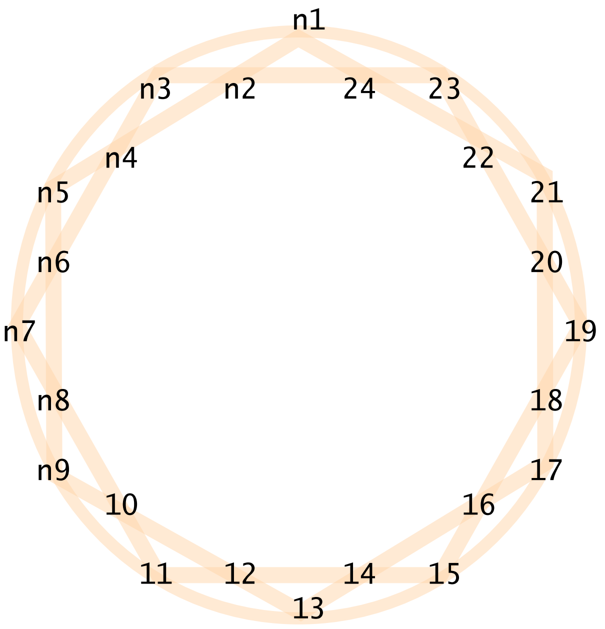

```
* Basic Definition for Order 10 *
  n1+n2+n4+n5=C       ... l1
  n3+n4+n6+n7=C       ... l2
  n5+n6+n8+n9=C       ... l3
  n7+n8+n10+n11=C     ... l4
  n9+n10+n12+n13=C    ... l5
  n11+n12+n14+n15=C   ... l6
  n13+n14+n16+n17=C   ... l7
  n15+n16+n18+n19=C   ... l8
  n17+n18+n20+n1=C    ... l9
  n19+n20+n2+n3=C     ... l10
  ------------------------------
  2*(n1+n2+n3+...+n19+n20)=10*C
            2*210=10*C
  Therefore   C=42    ... r0
```

They indeed look like pictures of a bracelet or a royal crown in higher orders rather than a brightest star, don't they? The higher orders, the closer they are almost getting to the outer frame of a circle.




```
* Basic Definition for Order 11 *
  n1+n2+n4+n5=C       ... l1
  n3+n4+n6+n7=C       ... l2
  n5+n6+n8+n9=C       ... l3
  n7+n8+n10+n11=C     ... l4
  n9+n10+n12+n13=C    ... l5
  n11+n12+n14+n15=C   ... l6
  n13+n14+n16+n17=C   ... l7
  n15+n16+n18+n19=C   ... l8
  n17+n18+n20+n21=C   ... l9
  n19+n20+n22+n1=C    ... l10
  n21+n22+n2+n3=C     ... l11
  ------------------------------
  2*(n1+n2+n3+...+n21+n22)=11*C
            2*253=11*C
  Therefore   C=46    ... r0
```

'Double ring' and 'net structure' are the essential properties of magic stars, as you see. The outer ring only consists of variables named with odd numbers, while the inner ring consists of even variables. Both rings share the two common numbers among four in each diagonal, making the 'net structure' between themselves.




```
Basic Definition for Order 12
  n1+n2+n4+n5=C         ... l1
  n3+n4+n6+n7=C         ... l2
  n5+n6+n8+n9=C         ... l3
  n7+n8+n10+n11=C       ... l4
  n9+n10+n12+n13=C      ... l5
  n11+n12+n14+n15=C     ... l6
  n13+n14+n16+n17=C     ... l7
  n15+n16+n18+n19=C     ... l8
  n17+n18+n20+n21=C     ... l9
  n19+n20+n22+n23=C     ... l10
  n21+n22+n24+n1=C      ... l11
  n23+n24+n2+n3=C       ... l12
-------------------------------
 2*(n1+n2+n3+...+n23+n24)=12*C
           2*300=12*C
  Therefore   C=50      ... r0
```

Chapter 2: Algebraic Study of Double-Ringed Magic Stars 5

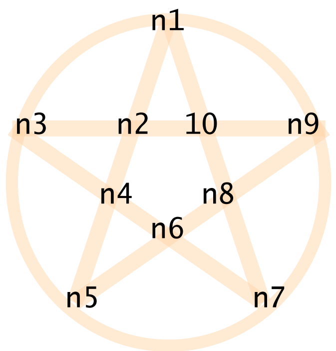
Let's study about various magic stars precisely, order by order.

We know we cannot compose any magic stars at all in lower orders than 5, since we cannot draw any double ring with the net structure. It is in the order 5 that we seem to be able to compose the smallest type of magic stars for the first time.

Let's make any actual magic stars of order 5.
However, whenever we try to compose any of them, we would always fail in getting the result. It seems to be impossible for us to make it up.
But why? Can we prove its impossibility? And how?

We have to make some algebraic study on our object before all.

*** Basic Form and Basic Definition for Magic Stars of Order 5 ***




```
* Basic Definition *
  n1+n2+n4+n5=C       ... l1
  n3+n4+n6+n7=C       ... l2
  n5+n6+n8+n9=C       ... l3
  n7+n8+n10+n1=C      ... l4
  n9+n10+n2+n3=C      ... l5
-----------------------------
  2*(n1+n2+n3+...+n9+n10)=5*C
             2*55=5*C
  Therefore     C=22    ... r1
```

Of course, we have to use each of the natural numbers {1, 2, 3, 4, 5, 6, 7, 8, 9 and

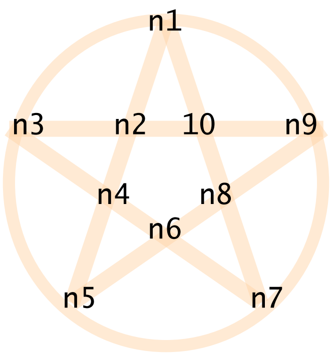
10} strictly once in a figure. We must not use any certain number twice or more often, and we must not neglect any number and un-use it, either.

```
l1+l4+l5
  n1+n2+n4+n5+n7+n8+n10+n1+n9+n10+n2+n3=3*C
  (n1+n2+n10)+(n1+n2+n3+n4+n5+n7+n8+n9+n10)=3*22
Add n6 to both sides.
  (n1+n2+n10)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10)=66+n6
  (n1+n2+n10)+     55    =66+n6
Therefore  n1+n2+n10=11+n6  or  n1=11-n2+n6-n10  ... r2
l1+l2+l5
  n1+n2+n4+n5+n3+n4+n6+n7+n9+n10+n2+n3=3*C
  (n2+n3+n4)+(n1+n2+n3+n4+n5+n6+n7+n9+n10)=66
Add n8 to both sides.
  (n2+n3+n4)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10)=66+n8
Therefore  n2+n3+n4=11+n8   or  n3=11-n2-n4+n8   ... r3
l1+l2+l3
  n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9=3*C
  (n4+n5+n6)+(n1+n2+n3+n4+n5+n6+n7+n8+n9)=66
Add n10 to both sides.
  (n4+n5+n6)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10)=66+n10
Therefore  n4+n5+n6=11+n10  or  n5=11-n4-n6+n10  ... r4
Similarly  n6+n7+n8=11+n2   or  n7=11+n2-n6-n8   ... r5
           n8+n9+n10=11+n4  or  n9=11+n4-n8-n10  ... r6

l1+l3+l5
  n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n2+n3=3*C
  (n2+n5+n9)+(n1+n2+n3+n4+n5+n6+n8+n9+n10)=66
Add n7 to both sides.
  (n2+n5+n9)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10)=66+n7
Therefore  n2+n5+n9=11+n7   or  n2=11-n5+n7-n9   ... r7
l1+l2+l4
  n1+n2+n4+n5+n3+n4+n6+n7+n7+n8+n10+n1=3*C
  (n1+n4+n7)+(n1+n2+n3+n4+n5+n6+n7+n8+n10)=66
Add n9 to both sides.
  (n1+n4+n7)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10)=66+n9
Therefore  n1+n4+n7=11+n9   or  n4=11-n1-n7+n9   ... r8
Similarly  n3+n6+n9=11+n1   or  n6=11+n1-n3-n9   ... r9
           n1+n5+n8=11+n3   or  n8=11+n3-n5-n1   ... r10
           n3+n7+n10=11+n5  or  n10=11-n3+n5-n7  ... r11

l1+l2+l3+l4-l5
  n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1-n9-n10-n2-n3=3*C
  2*(n1+n4+n5+n6+n7+n8)=66
Therefore  n1+n4+n5+n6+n7+n8=33  or  n1=33-n4-n5-n6-n7-n8  ... r12
-l1+l2+l3+l4+l5
  -n1-n2-n4-n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3=3*C
  2*(n3+n6+n7+n8+n9+n10)=66
Therefore  n3+n6+n7+n8+n9+n10=33 or  n3=33-n6-n7-n8-n9-n10 ... r13
l1-l2+l3+l4+l5
  n1+n2+n4+n5-n3-n4-n6-n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3=3*C
  2*(n1+n2+n5+n8+n9+n10)=66
Therefore  n1+n2+n5+n8+n9+n10=33 or  n5=33-n1-n2-n8-n9-n10 ... r14
l1+l2-l3+l4+l5
```

```
  n1+n2+n4+n5+n3+n4+n6+n7-n5-n6-n8-n9+n7+n8+n10+n1+n9+n10+n2+n3=3*C
  2*(n1+n2+n3+n4+n7+n10)=66
Therefore  n1+n2+n3+n4+n7+n10=33 or  n7=33-n1-n2-n3-n4-n10 ... r15
l1+l2+l3-l4+l5
  n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9-n7-n8-n10-n1+n9+n10+n2+n3=3*C
  2*(n2+n3+n4+n5+n6+n9)=66
Therefore  n2+n3+n4+n5+n6+n9=33  or  n9=33-n2-n3-n4-n5-n6  ... r16
```




```
* Basic Definition *
  n1+n2+n4+n5=C    ... l1
  n3+n4+n6+n7=C    ... l2
  n5+n6+n8+n9=C    ... l3
  n7+n8+n10+n1=C   ... l4
  n9+n10+n2+n3=C   ... l5
```

```
* Summary: List of Relations: *
  n1+n2+n10=11+n6  or  n1=11-n2+n6-n10       ... r2
  n2+n3+n4=11+n8   or  n3=11-n2-n4+n8        ... r3
  n4+n5+n6=11+n10  or  n5=11-n4-n6+n10       ... r4
  n6+n7+n8=11+n2   or  n7=11+n2-n6-n8        ... r5
  n8+n9+n10=11+n4  or  n9=11+n4-n8-n10       ... r6
  n2+n5+n9=11+n7   or  n2=11-n5+n7-n9        ... r7
  n1+n4+n7=11+n9   or  n4=11-n1-n7+n9        ... r8
  n3+n6+n9=11+n1   or  n6=11+n1-n3-n9        ... r9
  n1+n5+n8=11+n3   or  n8=11+n3-n5-n1        ... r10
  n3+n7+n10=11+n5  or  n10=11-n3+n5-n7       ... r11
  n1+n4+n5+n6+n7+n8=33   or  n1=33-n4-n5-n6-n7-n8  ... r12
  n3+n6+n7+n8+n9+n10=33 or  n3=33-n6-n7-n8-n9-n10 ... r13
  n1+n2+n5+n8+n9+n10=33 or  n5=33-n1-n2-n8-n9-n10 ... r14
  n1+n2+n3+n4+n7+n10=33 or  n7=33-n1-n2-n3-n4-n10 ... r15
  n2+n3+n4+n5+n6+n9=33   or  n9=33-n2-n3-n4-n5-n6  ... r16
```

We could not find any logical contradictions among all the relations listed above.

We now have to make any 'value test' to the same relations for the next step. We want to know the reason why no solution could be found.

Let's take the value 1 for n1 at first and then n2, because both of them act as a representative position for some: n1 for the 'outer ring' and n2 for the 'inner one'.

The solution whose n3 takes the value 1 means the same thing to that one whose n1=1, because we can transform them into each other by some simple rotation and mirror reflection if necessary. Any solution whose n5, n7 or n9=1 means the same to the one whose n1=1 for the same reason.

The solution whose n4, n6, n8 or n10 takes the value 1 also means the same thing to that one whose n2=1, because we can transform them into each other by some simple rotation and mirror reflection without changing anything essential.

```
** Value Tests **
If n1=1 then {n2+n10-n6=10; n4+n7-n9=10; n5+n8-n3=10; n3+n6+n9=12;}
Therefore  (n2+n10-n6)+(n4+n7-n9)+(n5+n8-n3)+(n3+n6+n9)=42;
  n2+n4+n5+n7+n8+n10=42;
```

```
If n2=1 then {n1+n10-n6=10; n3+n4-n8=10; n5+n9-n7=10; n6+n7+n8=12;}
Therefore   n1+n10-n6+n3+n4-n8+n5+n9-n7+n6+n7+n8=42;
  n1+n3+n4+n5+n9+n10=42;

** Impossible Magic Star of Order 5 **
** Possible Combinations of Values: **
-------------------------------------------
* When n1=1: *
 n2+n4+n5+n7+n8+n10=42; n3+n6+n9=10;
  n2+n10-n6=10; n4+n7-n9=10; n5+n8-n3=10;
-------------------------------------------
[ 1]  (2,6,7,8,9,10);    (3,4,5);
 {(6,7,3), (6,8,4), (6,9,5), (7,8,5), ()}
[ 2]  (3,5,7,8,9,10);    (2,4,6);
 {(3,9,2), (5,7,2), (5,9,4), (7,9,6), ()}
[ 3]  (4,5,6,8,9,10);    (2,3,7);
 {(4,8,2), (4,9,3), (5,8,3), (8,9,7), ()}
-------------------------------------------
* When n2=1: *
 n1+n3+n4+n5+n9+n10=42; n6+n7+n8=12;
  n1+n10-n6=10; n3+n4-n8=10; n5+n9-n7=10;
-------------------------------------------
[ 1]  (2,6,7,8,9,10);    (3,4,5);
 {(6,7,3), (6,8,4), (6,9,5), (7,8,5), ()}
[ 2]  (3,5,7,8,9,10);    (2,4,6);
 {(3,9,2), (5,7,2), (5,9,4), (7,9,6), ()}
[ 3]  (4,5,6,8,9,10);    (2,3,7);
 {(4,8,2), (4,9,3), (5,8,3), (8,9,7), ()}
-------------------------------------------
```

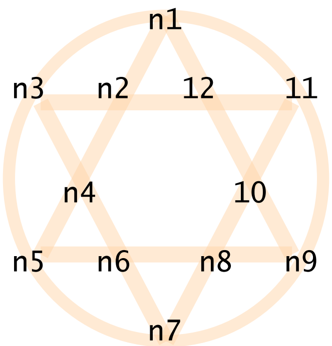
We could not really satisfy all the equations above with the serial natural numbers {1, 2, 3, 4, 5, ... , 9 and 10}, each one of which must be used strictly once in a figure.

We could not avoid repeating usage of any certain numbers for some equations.

We could not really make any solution whose n1=1. That means we could no longer make any one whose n3, n5, n7 or n9=1. The fact we could not make any one whose n2=1 also means we could no longer make any one whose n4, n6, n8 or n10=1.

As a consequence we now know we cannot practically make any magic star of order 5 only with the limited usage of natural number elements.

Chapter 3: Algebraic Study of Double-Ringed Magic Stars 6




```
* Basic Definition for Order 6 *
  n1+n2+n4+n5=C     ... l1
  n3+n4+n6+n7=C     ... l2
  n5+n6+n8+n9=C     ... l3
  n7+n8+n10+n11=C   ... l4
  n9+n10+n12+n1=C   ... l5
  n11+n12+n2+n3=C   ... l6
  -----------------------------
  2*(n1+n2+n3+...+n11+n12)=6*C
            2*78=6*C
  Therefore   C=26  ... r0
```

Let's make any real Magic Stars of order 6 next.
How many solutions can we compose for Magic Stars in all?

```
/** Composing Magic Stars of Order 6 **/
/** 'MStar6.c' made by Kanji Setsuda **/
/**  on Nov.25, 2003; Nov.16, 2005  **/
/**  Worked on MacOSX and Xcode 1.5 **/
/**/
#include <stdio.h>
/**/
short cnt, cnt2;
short LSM;
short nm[13], uflg[13];
short anm[5][13];
/**/
/* Main Program: Reset all and Go! */
int main(){
 short n;
  printf("\n** List of Standard Magic Stars of Order 6 **\n");
  for(n=0;n<13;n++){nm[n]=0; uflg[n]=0;}
  LSM=26;
  cnt=0; cnt2=0;
  stp01();/* Begin the Calculations */
  printf("\n [Count = %d]\n",cnt);
  printf("  OK!\n");
 return 0;
}
/**/
/* Begin the Calculations */
/* Set N1 */
void stp01(){
 short a;
  for(a=1;a<8;a++){
    if(uflg[a]==0){
      nm[1]=a; uflg[a]=1;
      stp02();
      uflg[a]=0;
  }}
}
/* Set N5(>n1) */
void stp02(){
 short a;
  for(a=nm[1]+1;a<13;a++){
    if(uflg[a]==0){
      nm[5]=a; uflg[a]=1;
      stp03();
      uflg[a]=0;
  }}
}
/* Set N2 */
void stp03(){
 short a;
  for(a=12;a>0;a--){
    if(uflg[a]==0){
      nm[2]=a; uflg[a]=1;
      stp04();
      uflg[a]=0;
  }}
```

```
}
/* Set n4=30-n1-n2-n5 */
void stp04(){
 short a;
  a=LSM-nm[1]-nm[2]-nm[5];
  if((0<a)&&(a<13)){
    if(uflg[a]==0){
      nm[4]=a; uflg[a]=1;
      stp05();
      uflg[a]=0;
  }}
}
/* Set N3(>n1) */
void stp05(){
 short a;
  for(a=nm[1]+1;a<13;a++){
    if(uflg[a]==0){
      nm[3]=a; uflg[a]=1;
      stp06();
      uflg[a]=0;
  }}
}
/* Set N7(>n1) */
void stp06(){
 short a;
  for(a=nm[1]+1;a<13;a++){
    if(uflg[a]==0){
      nm[7]=a; uflg[a]=1;
      stp07();
      uflg[a]=0;
  }}
}
/* Set n6=30-n3-n4-n7 */
void stp07(){
 short a;
  a=LSM-nm[3]-nm[4]-nm[7];
  if((0<a)&&(a<13)){
    if(uflg[a]==0){
      nm[6]=a; uflg[a]=1;
      stp08();
      uflg[a]=0;
  }}
}
/* Set N9(>n5) */
void stp08(){
 short a;
  for(a=nm[5]+1;a<13;a++){
    if(uflg[a]==0){
      nm[9]=a; uflg[a]=1;
      stp09();
      uflg[a]=0;
  }}
}
/* Set n8=30-n5-n6-n9 */
void stp09(){
 short a;
  a=LSM-nm[5]-nm[6]-nm[9];
  if((0<a)&&(a<13)){
    if(uflg[a]==0){
```

```
        nm[8]=a; uflg[a]=1;
        stp10();
        uflg[a]=0;
    }}
}
/* Set N11(>n1) */
void stp10(){
 short a;
  for(a=nm[1]+1;a<13;a++){
    if(uflg[a]==0){
        nm[11]=a; uflg[a]=1;
        stp11();
        uflg[a]=0;
    }}
}
/* Set n10=30-n7-n8-n11 */
void stp11(){
 short a;
  a=LSM-nm[7]-nm[8]-nm[11];
  if((0<a)&&(a<13)){
    if(uflg[a]==0){
        nm[10]=a; uflg[a]=1;
        stp12();
        uflg[a]=0;
    }}
}
/* Set n12=30-n9-n10-n1 */
void stp12(){
 short a,b;
  a=LSM-nm[9]-nm[10]-nm[1];
  b=LSM-nm[11]-nm[2]-nm[3];
  if((a==b)&&(0<a)&&(a<13)){
    if(uflg[a]==0){
        nm[12]=a; uflg[a]=1;
        ansprint();
        uflg[a]=0;
    }}
}
/* Print The Answers */
void ansprint(){
 short n;
  cnt++;
  anm[cnt2][0]=cnt; anm[cnt2+1][0]=0;
  for(n=1;n<13;n++){anm[cnt2][n]=nm[n]; anm[cnt2+1][n]=0;}
  cnt2++;
  if(cnt2==4){pr4ans(); cnt2=0;}
}
/**/
/* Print 4 Answers */
void pr4ans(){
 short m;
  for(m=0;m<4;m++){
    printf("[%2d]             ",anm[m][0]);}
  printf("\n");
  for(m=0;m<4;m++){
    printf("%9d        ",anm[m][1]);}
  printf("\n");
  printf("      / \\             / \\             / \\             / \\\n");
  for(m=0;m<4;m++){
    printf("%3d--%2d--%2d--%2d   ",anm[m][3],anm[m][2],anm[m][12],anm[m][11]);}
```

```
  printf("\n");
  printf("   \\ /     \\ /       \\ /     \\ /       \\ /     \\ /       \\ /     \\ /\n");
  for(m=0;m<4;m++){
    printf("%5d%8d       ",anm[m][4],anm[m][10]);}
  printf("\n");
  printf("   / \\     / \\       / \\     / \\       / \\     / \\       / \\     / \\\n");
  for(m=0;m<4;m++){
    printf("%3d--%2d--%2d--%2d   ",anm[m][5],anm[m][6],anm[m][8],anm[m][9]);}
  printf("\n");
  printf("      \\ /                 \\ /                 \\ /                 \\ /\n");
  for(m=0;m<4;m++){
    printf("%9d           ",anm[m][7]);}
  printf("\n");
}
/**/
```

The next list shows the result of my recent computation by that program above. It contains all the 80 standard solutions of Magic Stars of order 6.

**** List of Standard Magic Stars of Order 6 ****

```
[ 1]                [ 2]                [ 3]                [ 4]
       1                   1                   1                   1
      / \                 / \                 / \                 / \
 3--12-- 6-- 5       4--12-- 7-- 3       3--11-- 8-- 4       4--11-- 6-- 5
  \ /     \ /         \ /     \ /         \ /     \ /         \ /     \ /
  11      10          11       8          12       7          12      10
  / \     / \         / \     / \         / \     / \         / \     / \
 2-- 8-- 7-- 9       2-- 5-- 9--10       2-- 5-- 9--10       2-- 7-- 8-- 9
      \ /                 \ /                 \ /                 \ /
       4                   6                   6                   3
[ 5]                [ 6]                [ 7]                [ 8]
       1                   1                   1                   1
      / \                 / \                 / \                 / \
 2--12-- 5-- 7       2--12-- 8-- 4       4--10-- 5-- 7       5--11-- 7-- 3
  \ /     \ /         \ /     \ /         \ /     \ /         \ /     \ /
  10       9          10       6          12      11          10       6
  / \     / \         / \     / \         / \     / \         / \     / \
 3-- 8-- 4--11       3-- 5-- 7--11       3-- 8-- 6-- 9       4-- 2-- 8--12
      \ /                 \ /                 \ /                 \ /
       6                   9                   2                   9
[ 9]                [10]                [11]                [12]
       1                   1                   1                   1
      / \                 / \                 / \                 / \
 2--10-- 5-- 9       5-- 9--10-- 2       3--12-- 4-- 7       3--12-- 9-- 2
  \ /     \ /         \ /     \ /         \ /     \ /         \ /     \ /
  11       8          12       7           8      11           8       6
  / \     / \         / \     / \         / \     / \         / \     / \
 4-- 7-- 3--12       4-- 3--11-- 8       5-- 9-- 2--10       5-- 4-- 7--10
      \ /                 \ /                 \ /                 \ /
       6                   6                   6                  11
[13]                [14]                [15]                [16]
       1                   1                   1                   1
      / \                 / \                 / \                 / \
 4--11-- 3-- 8       3-- 9-- 6-- 8       3-- 9--12-- 2       6-- 9-- 3-- 8
  \ /     \ /         \ /     \ /         \ /     \ /         \ /     \ /
   9      10          11      12          11       6          11      12
  / \     / \         / \     / \         / \     / \         / \     / \
 5-- 7-- 2--12       5--10-- 4-- 7       5-- 4--10-- 7       5-- 7-- 4--10
      \ /                 \ /                 \ /                 \ /
       6                   2                   8                   2
```

[17]
```
        1
  2 -- 8 -- 10 -- 6
    12          4
  5 -- 3 -- 7 -- 11
        9
```

[18]
```
        1
  7 -- 8 -- 9 -- 2
    12          10
  5 -- 4 -- 11 -- 6
        3
```

[19]
```
        1
  7 -- 9 -- 2 -- 8
    10          11
  6 -- 5 -- 3 -- 12
        4
```

[20]
```
        1
  8 -- 9 -- 2 -- 7
    10          12
  6 -- 5 -- 4 -- 11
        3
```

[21]
```
        1
  2 -- 8 -- 9 -- 7
    11          4
  6 -- 3 -- 5 -- 12
        10
```

[22]
```
        1
  3 -- 7 -- 11 -- 5
    12          4
  6 -- 2 -- 8 -- 10
        9
```

[23]
```
        1
  4 -- 7 -- 5 -- 10
    12          11
  6 -- 8 -- 3 -- 9
        2
```

[24]
```
        1
  8 -- 12 -- 4 -- 2
    6           10
  7 -- 3 -- 5 -- 11
        9
```

[25]
```
        1
  2 -- 10 -- 11 -- 3
    8           5
  7 -- 4 -- 6 -- 9
        12
```

[26]
```
        1
  4 -- 10 -- 9 -- 3
    8           5
  7 -- 2 -- 6 -- 11
        12
```

[27]
```
        1
  9 -- 10 -- 2 -- 5
    8           11
  7 -- 3 -- 4 -- 12
        6
```

[28]
```
        1
  3 -- 8 -- 9 -- 6
    10          4
  7 -- 2 -- 5 -- 12
        11
```

[29]
```
        1
  2 -- 6 -- 10 -- 8
    12          4
  7 -- 3 -- 5 -- 11
        9
```

[30]
```
        1
  7 -- 11 -- 3 -- 5
    6           10
  8 -- 4 -- 2 -- 12
        9
```

[31]
```
        1
  9 -- 10 -- 2 -- 5
    7           12
  8 -- 4 -- 3 -- 11
        6
```

[32]
```
        1
  2 -- 7 -- 12 -- 5
    10          4
  8 -- 3 -- 6 -- 9
        11
```

[33]
```
        1
  3 -- 6 -- 12 -- 5
    11          4
  8 -- 2 -- 7 -- 9
        10
```

[34]
```
        1
  3 -- 5 -- 11 -- 7
    12          4
  8 -- 2 -- 6 -- 10
        9
```

[35]
```
        1
  4 -- 5 -- 6 -- 11
    12          10
  8 -- 7 -- 2 -- 9
        3
```

[36]
```
        1
  6 -- 11 -- 7 -- 2
    5           8
  9 -- 3 -- 4 -- 10
        12
```

[37]
```
        1
  7 -- 10 -- 5 -- 4
    6           8
  9 -- 2 -- 3 -- 12
        11
```

[38]
```
        1
  4 -- 5 -- 7 -- 10
    11          6
  9 -- 3 -- 2 -- 12
        8
```

[39]
```
        1
  3 -- 4 -- 8 -- 11
    12          7
  9 -- 5 -- 2 -- 10
        6
```

[40]
```
        1
  4 -- 7 -- 9 -- 6
    8           5
  10 -- 2 -- 3 -- 11
        12
```

[41]

```
       2
 6   8   9   3
  12       5
 4   1  11  10
       7
```

[42]

```
       2
 7  10   1   8
   9      11
 5   6   3  12
       4
```

[43]

```
       2
 8  10   1   7
   9      12
 5   6   4  11
       3
```

[44]

```
       2
 4  11   3   8
   7      12
 6  10   1   9
       5
```

[45]

```
       2
 4   8   3  11
  10       9
 6   7   1  12
       5
```

[46]

```
       2
 4   8   9   5
  10       3
 6   1   7  12
      11
```

[47]

```
       2
10   9   1   6
   8      11
 7   3   4  12
       5
```

[48]

```
       2
 3   6   5  12
  11       9
 7   8   1  10
       4
```

[49]

```
       2
 5   6  12   3
  11       4
 7   1  10   8
       9
```

[50]

```
       2
 4   5  11   6
  12       3
 7   1   8  10
       9
```

[51]

```
       2
 6  12   3   5
   4      11
 8   7   1  10
       9
```

[52]

```
       2
10   9   1   6
   7      12
 8   4   3  11
       5
```

[53]

```
       2
 4   7  10   5
   9       3
 8   1   6  11
      12
```

[54]

```
       2
 4   6   9   7
  10       3
 8   1   5  12
      11
```

[55]

```
       2
12   7   3   4
   8      11
 9   1   6  10
       5
```

[56]

```
       2
 7   9   4   6
   5       8
10   3   1  12
      11
```

[57]

```
       2
 8   9   6   3
   5       7
10   1   4  11
      12
```

[58]

```
       2
 4   6   7   9
   8       5
10   3   1  12
      11
```

[59]

```
       3
 4  12   2   8
   6      10
 5   9   1  11
       7
```

[60]

```
       3
 6  11   1   8
   7      12
 5   9   2  10
       4
```

[61]

```
       3
 4   7   9   6
  11       2
 5   1   8  12
      10
```

[62]

```
       3
 4   5  10   7
  12       2
 6   1   8  11
       9
```

[63]

```
       3
 4   5   9   8
  11       2
 7   1   6  12
      10
```

[64]

```
       3
 7  11   2   6
   4       9
 8   5   1  12
      10
```

[65]
```
       3
      / \
 9--11-- 1-- 5
  \ /     \ /
   4      12
  / \     / \
 8-- 6-- 2--10
      \ /
       7
```
[66]
```
       3
      / \
11-- 9-- 1-- 5
  \ /     \ /
   6      10
  / \     / \
 8-- 2-- 4--12
      \ /
       7
```
[67]
```
       3
      / \
 4-- 6--11-- 5
  \ /     \ /
   9       2
  / \     / \
 8-- 1-- 7--10
      \ /
      12
```
[68]
```
       3
      / \
12-- 8-- 2-- 4
  \ /     \ /
   6      10
  / \     / \
 9-- 1-- 5--11
      \ /
       7
```
[69]
```
       3
      / \
 6-- 2-- 8--10
  \ /     \ /
  12       4
  / \     / \
 9-- 1-- 5--11
      \ /
       7
```
[70]
```
       3
      / \
 7-- 4-- 5--10
  \ /     \ /
   8       6
  / \     / \
11-- 2-- 1--12
      \ /
       9
```
[71]
```
       3
      / \
 7-- 4-- 6-- 9
  \ /     \ /
   8       5
  / \     / \
11-- 1-- 2--12
      \ /
      10
```
[72]
```
       4
      / \
 8--12-- 1-- 5
  \ /     \ /
   3      10
  / \     / \
 7-- 6-- 2--11
      \ /
       9
```
[73]
```
       4
      / \
 6--12-- 3-- 5
  \ /     \ /
   2       9
  / \     / \
 8-- 7-- 1--10
      \ /
      11
```
[74]
```
       4
      / \
11-- 7-- 2-- 6
  \ /     \ /
   5       8
  / \     / \
10-- 1-- 3--12
      \ /
       9
```
[75]
```
       4
      / \
 9-- 6-- 3-- 8
  \ /     \ /
   5       7
  / \     / \
11-- 2-- 1--12
      \ /
      10
```
[76]
```
       4
      / \
10-- 5-- 3-- 8
  \ /     \ /
   6       7
  / \     / \
11-- 1-- 2--12
      \ /
       9
```
[77]
```
       5
      / \
 7-- 2-- 6--11
  \ /     \ /
  10       3
  / \     / \
 9-- 1-- 4--12
      \ /
       8
```
[78]
```
       5
      / \
 8-- 9-- 3-- 6
  \ /     \ /
   2       7
  / \     / \
10-- 4-- 1--11
      \ /
      12
```
[79]
```
       5
      / \
12-- 7-- 1-- 6
  \ /     \ /
   4       9
  / \     / \
10-- 2-- 3--11
      \ /
       8
```
[80]
```
       6
      / \
 8-- 1-- 5--12
  \ /     \ /
   9       4
  / \     / \
10-- 2-- 3--11
      \ /
       7
```
[Count = 80]   OK!

As you see, we could get the 40 solutions whose n1=1 and the 40 others. In the latter half any one of {n2, n4, n6, n8 or n10} takes the value 1. It means the former 40 belong to the 'outer ring type' and the latter 40 belong to the 'inner ring type'.

It is only in the order 6 that we could have the same count of solutions for both of those two types. It seems to imply their possibility of transforming into each other.

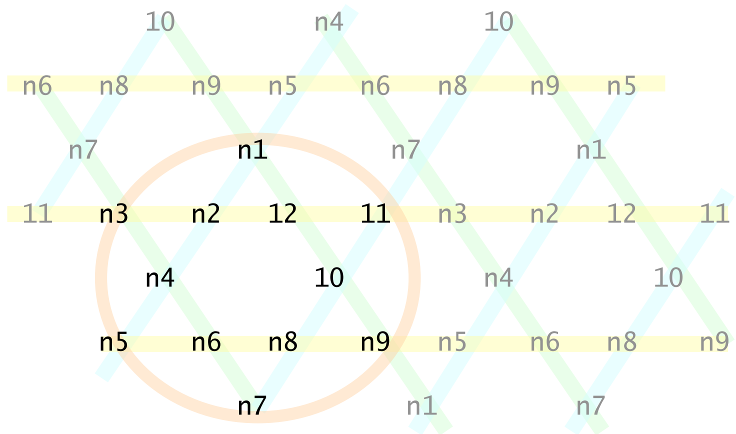
Let's extend the space for magic star here by duplicating several lines in the outer space, just in the same way as we often made for various pandiagonal magic squares.

We could do that job successfully as shown above, and could really read another 3 different solutions out of the extended space.

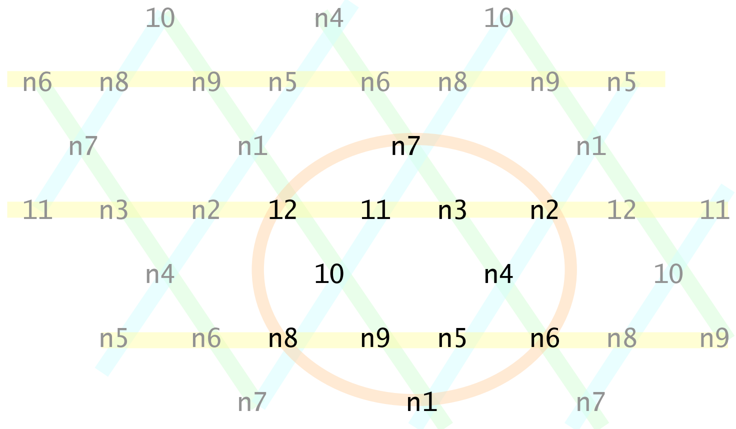
They were already written in the space, where we can just find and read them out to have those 4 transformations including the original itself.

*** Extended Space for Magic Star of Order 6**

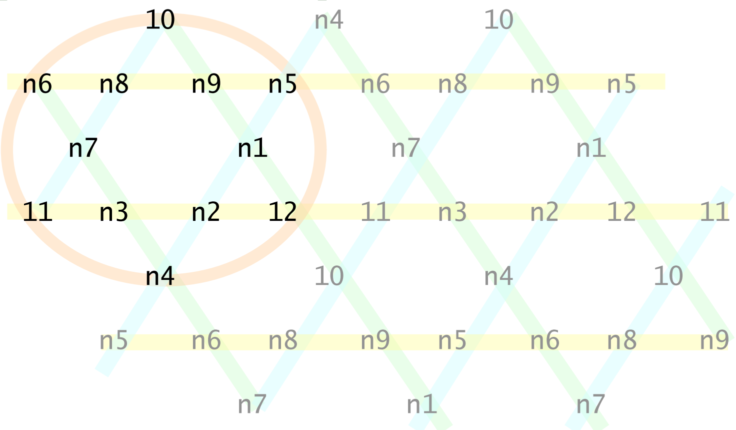
[Original Form and the Extended Space]



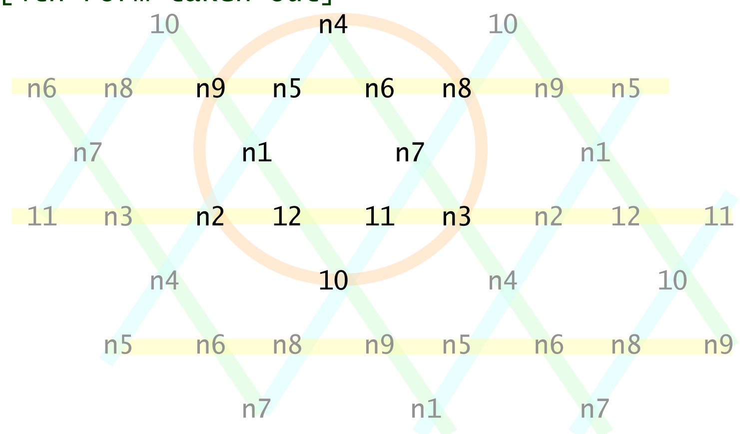
[2nd Form taken out of the Extended Space]



[3rd Form taken out]



[4th Form taken out]



Let's take those 4 solutions in a group and assume some transformation system for them. Take your careful look at the next conceptual diagrams below.

```
/* Concept of 6 Transformations */
 Org#               T1#                T2#                T3#
        n1                 n1                 n5                 n2
        / \                / \                / \                / \
 n3---n2---12---11  n6---n5---n9---n8  12---n1---n9---10  n9---n1---12---10
   \ /       \ /      \ /       \ /      \ /       \ /      \ /       \ /
   n4        10       n4        10       n2        n8       n5        11
   / \       / \      / \       / \      / \       / \      / \       / \
 n5---n6---n8---n9  n2---n3---11---12  n4---n3---n7---n6  n4---n6---n7---n3
        \ /                \ /                \ /                \ /
        n7                 n7                 11                 n8

 Org#               Ref#               Rot1#              Rot2#
        n1                 n1                 n3                 n5
        / \                / \                / \                / \
 n3---n2---12---11  11---12---n2---n3  n5---n4---n2---n1  n7---n6---n4---n3
   \ /       \ /      \ /       \ /      \ /       \ /      \ /       \ /
   n4        10       10        n4       n6        12        8        n2
   / \       / \      / \       / \      / \       / \      / \       / \
 n5---n6---n8---n9  n9---n8---n6---n5  n7---n8---10---11  n9---10---12---n1
        \ /                \ /                \ /                \ /
        n7                 n7                  9                 11
```

If you have both the 20 fundamental solutions and those transformations above, you can make each fundamental into three different solutions automatically. If you also assume another set of transformations such as 'mirror reflection' or 'rotation by 60 degrees', you can easily make the smart list of 80 standard solutions at last.

The next list shows the result of my recent classification. It gives you all the standard solutions by the 20 groups of four transformations. In order to get the 20 fundamental solutions I took the list-forming inequality conditions as follows:

`n1=1; n1<n3; n1<n5; n1<n7; n1<n9; n1<n11; n2>n5; and n4<n10;`

```
** List of Standard Solution: Magic Stars of Order 6 **
** The Fundamental 20 and 3 Transformations for Each **

 1/ 1#              34#                42#                71#
         1                  1                  2                  3
        / \                / \                / \                / \
  3---11--- 8--- 4   9---10--- 2--- 5   7---10--- 1--- 8   7--- 4--- 6--- 9
   \ /       \ /      \ /       \ /      \ /       \ /      \ /       \ /
   12         7        7        12        9        11        8         5
   / \       / \      / \       / \      / \       / \      / \       / \
  2--- 5--- 9---10   8--- 4--- 3---11   5--- 6--- 3---12  11--- 1--- 2---12
        \ /                \ /                \ /                \ /
         6                  6                  4                 10

 2/ 2#              23#                50#                75#
         1                  1                  2                  4
        / \                / \                / \                / \
  4---11--- 6--- 5   8--- 9--- 2--- 7  10--- 9--- 1--- 6  10--- 5--- 3--- 8
   \ /       \ /      \ /       \ /      \ /       \ /      \ /       \ /
   12        10       10        12        8        11        6         7
   / \       / \      / \       / \      / \       / \      / \       / \
  2--- 7--- 8--- 9   6--- 5--- 4---11   7--- 3--- 4---12  11--- 1--- 2---12
        \ /                \ /                \ /                \ /
         3                  3                  5                  9
```

3/ 3#

```
        1
       / \
 3---12--- 6--- 5
  \ /       \ /
  11        10
  / \       / \
 2--- 8--- 7--- 9
       \ /
        4
```

22#

```
        1
       / \
 7--- 9--- 2--- 8
  \ /       \ /
  10        11
  / \       / \
 6--- 5--- 3---12
       \ /
        4
```

53#

```
        2
       / \
10--- 9--- 1--- 6
  \ /       \ /
   7        12
  / \       / \
 8--- 4--- 3---11
       \ /
        5
```

70#

```
        3
       / \
 7--- 4--- 5---10
  \ /       \ /
   8         6
  / \       / \
11--- 2--- 1---12
       \ /
        9
```

4/ 4#

```
        1
       / \
 4---12--- 7--- 3
  \ /       \ /
  11         8
  / \       / \
 2--- 5--- 9---10
       \ /
        6
```

28#

```
        1
       / \
 9---10--- 2--- 5
  \ /       \ /
   8        11
  / \       / \
 7--- 3--- 4---12
       \ /
        6
```

43#

```
        2
       / \
 8---10--- 1--- 7
  \ /       \ /
   9        12
  / \       / \
 5--- 6--- 4---11
       \ /
        3
```

76#

```
        4
       / \
 9--- 6--- 3--- 8
  \ /       \ /
   5         7
  / \       / \
11--- 2--- 1---12
       \ /
       10
```

5/ 5#

```
        1
       / \
 4---10--- 5--- 7
  \ /       \ /
  12        11
  / \       / \
 3--- 8--- 6--- 9
       \ /
        2
```

15#

```
        1
       / \
 6--- 9--- 3--- 8
  \ /       \ /
  11        12
  / \       / \
 5--- 7--- 4---10
       \ /
        2
```

65#

```
        3
       / \
11--- 9--- 1--- 5
  \ /       \ /
   6        10
  / \       / \
 8--- 2--- 4---12
       \ /
        7
```

74#

```
        4
       / \
11--- 7--- 2--- 6
  \ /       \ /
   5         8
  / \       / \
10--- 1--- 3---12
       \ /
        9
```

6/ 6#

```
        1
       / \
 2---12--- 5--- 7
  \ /       \ /
  10         9
  / \       / \
 3--- 8--- 4---11
       \ /
        6
```

16#

```
        1
       / \
 4---11--- 3--- 8
  \ /       \ /
   9        10
  / \       / \
 5--- 7--- 2---12
       \ /
        6
```

67#

```
        3
       / \
 9---11--- 1--- 5
  \ /       \ /
   4        12
  / \       / \
 8--- 6--- 2---10
       \ /
        7
```

56#

```
        2
       / \
 4--- 6--- 7--- 9
  \ /       \ /
   8         5
  / \       / \
10--- 3--- 1---12
       \ /
       11
```

7/ 7#

```
        1
       / \
 2---12--- 8--- 4
  \ /       \ /
  10         6
  / \       / \
 3--- 5--- 7---11
       \ /
        9
```

35#

```
        1
       / \
 7---11--- 3--- 5
  \ /       \ /
   6        10
  / \       / \
 8--- 4--- 2---12
       \ /
        9
```

60#

```
        3
       / \
 6---11--- 1--- 8
  \ /       \ /
   7        12
  / \       / \
 5--- 9--- 2---10
       \ /
        4
```

57#

```
        2
       / \
 7--- 9--- 4--- 6
  \ /       \ /
   5         8
  / \       / \
10--- 3--- 1---12
       \ /
       11
```

8/ 8#

```
        1
       / \
 5--- 9---10--- 2
  \ /       \ /
  12         7
  / \       / \
 4--- 3---11--- 8
       \ /
        6
```

36#

```
        1
       / \
 3--- 4--- 8---11
  \ /       \ /
  12         7
  / \       / \
 9--- 5--- 2---10
       \ /
        6
```

48#

```
        2
       / \
 3--- 6--- 5---12
  \ /       \ /
  11         9
  / \       / \
 7--- 8--- 1---10
       \ /
        4
```

77#

```
        5
       / \
 7--- 2--- 6---11
  \ /       \ /
  10         3
  / \       / \
 9--- 1--- 4---12
       \ /
        8
```

9/ 9#

```
        1
       / \
 2---10--- 5--- 9
  \ /       \ /
  11         8
  / \       / \
 4--- 7--- 3---12
       \ /
        6
```

17#

```
        1
       / \
 3---12--- 4--- 7
  \ /       \ /
   8        11
  / \       / \
 5--- 9--- 2---10
       \ /
        6
```

72#

```
        4
       / \
 8---12--- 1--- 5
  \ /       \ /
   3        10
  / \       / \
 7--- 6--- 2---11
       \ /
        9
```

58#

```
        2
       / \
 8--- 9--- 6--- 3
  \ /       \ /
   5         7
  / \       / \
10--- 1--- 4---11
       \ /
       12
```

10/10#

```
        1
       / \
 5---11--- 7--- 3
  \ /       \ /
  10         6
  / \       / \
 4--- 2--- 8---12
       \ /
        9
```

29#

```
        1
       / \
 8---12--- 4--- 2
  \ /       \ /
   6        10
  / \       / \
 7--- 3--- 5---11
       \ /
        9
```

41#

```
        2
       / \
 6--- 8--- 9--- 3
  \ /       \ /
  12         5
  / \       / \
 4--- 1---11---10
       \ /
        7
```

79#

```
        5
       / \
 8--- 9--- 3--- 6
  \ /       \ /
   2         7
  / \       / \
10--- 4--- 1---11
       \ /
       12
```

11/11#

```
        1
       / \
 2--- 8---10--- 6
  \ /       \ /
  12         4
  / \       / \
 5--- 3--- 7---11
       \ /
        9
```

30#

```
        1
       / \
 3--- 5---11--- 7
  \ /       \ /
  12         4
  / \       / \
 8--- 2--- 6---10
       \ /
        9
```

59#

```
        3
       / \
 4--- 7--- 9--- 6
  \ /       \ /
  11         2
  / \       / \
 5--- 1--- 8---12
       \ /
       10
```

51#

```
        2
       / \
 4--- 6--- 9--- 7
  \ /       \ /
  10         3
  / \       / \
 8--- 1--- 5---12
       \ /
       11
```

12/12#

```
        1
       / \
 7--- 8--- 9--- 2
  \ /       \ /
  12        10
  / \       / \
 5--- 4---11--- 6
       \ /
        3
```

31#

```
        1
       / \
 4--- 5--- 6---11
  \ /       \ /
  12        10
  / \       / \
 8--- 7--- 2--- 9
       \ /
        3
```

55#

```
        2
       / \
12--- 7--- 3--- 4
  \ /       \ /
   8        11
  / \       / \
 9--- 1--- 6---10
       \ /
        5
```

80#

```
        6
       / \
 8--- 1--- 5---12
  \ /       \ /
   9         4
  / \       / \
10--- 2--- 3---11
       \ /
        7
```

13/14#

```
        1
       / \
 3--- 9---12--- 2
  \ /       \ /
  11         6
  / \       / \
 5--- 4---10--- 7
       \ /
        8
```

37#

```
        1
       / \
 4--- 5--- 7---10
  \ /       \ /
  11         6
  / \       / \
 9--- 3--- 2---12
       \ /
        8
```

44#

```
        2
       / \
 4--- 8--- 3---11
  \ /       \ /
  10         9
  / \       / \
 6--- 7--- 1---12
       \ /
        5
```

68#

```
        3
       / \
 6--- 2--- 8---10
  \ /       \ /
  12         4
  / \       / \
 9--- 1--- 5---11
       \ /
        7
```

14/18#

```
        1
       / \
 3---12--- 9--- 2
  \ /       \ /
   8         6
  / \       / \
 5--- 4--- 7---10
       \ /
       11
```

38#

```
        1
       / \
 7---10--- 5--- 4
  \ /       \ /
   6         8
  / \       / \
 9--- 2--- 3---12
       \ /
       11
```

46#

```
        2
       / \
 4---11--- 3--- 8
  \ /       \ /
   7        12
  / \       / \
 6---10--- 1--- 9
       \ /
        5
```

66#

```
        3
       / \
 7---11--- 2--- 6
  \ /       \ /
   4         9
  / \       / \
 8--- 5--- 1---12
       \ /
       10
```

15/19#

```
       1
      / \
 3--- 7---11--- 5
  \ /       \ /
  12         4
  / \       / \
 6--- 2--- 8---10
      \ /
       9
```

24#

```
       1
      / \
 2--- 6---10--- 8
  \ /       \ /
  12         4
  / \       / \
 7--- 3--- 5---11
      \ /
       9
```

45#

```
       2
      / \
 4--- 8--- 9--- 5
  \ /       \ /
  10         3
  / \       / \
 6--- 1--- 7---12
      \ /
      11
```

63#

```
       3
      / \
 4--- 5--- 9--- 8
  \ /       \ /
  11         2
  / \       / \
 7--- 1--- 6---12
      \ /
      10
```

16/20#

```
       1
      / \
 4--- 7--- 5---10
  \ /       \ /
  12        11
  / \       / \
 6--- 8--- 3--- 9
      \ /
       2
```

13#

```
       1
      / \
 3--- 9--- 6--- 8
  \ /       \ /
  11        12
  / \       / \
 5---10--- 4--- 7
      \ /
       2
```

78#

```
       5
      / \
12--- 7--- 1--- 6
  \ /       \ /
   4         9
  / \       / \
10--- 2--- 3---11
      \ /
       8
```

69#

```
       3
      / \
12--- 8--- 2--- 4
  \ /       \ /
   6        10
  / \       / \
 9--- 1--- 5---11
      \ /
       7
```

17/21#

```
       1
      / \
 2--- 8--- 9--- 7
  \ /       \ /
  11         4
  / \       / \
 6--- 3--- 5---12
      \ /
      10
```

32#

```
       1
      / \
 3--- 6---12--- 5
  \ /       \ /
  11         4
  / \       / \
 8--- 2--- 7--- 9
      \ /
      10
```

62#

```
       3
      / \
 4--- 5---10--- 7
  \ /       \ /
  12         2
  / \       / \
 6--- 1--- 8---11
      \ /
       9
```

52#

```
       2
      / \
 4--- 7---10--- 5
  \ /       \ /
   9         3
  / \       / \
 8--- 1--- 6---11
      \ /
      12
```

18/25#

```
       1
      / \
 3--- 8--- 9--- 6
  \ /       \ /
  10         4
  / \       / \
 7--- 2--- 5---12
      \ /
      11
```

33#

```
       1
      / \
 2--- 7---12--- 5
  \ /       \ /
  10         4
  / \       / \
 8--- 3--- 6--- 9
      \ /
      11
```

47#

```
       2
      / \
 4--- 5---11--- 6
  \ /       \ /
  12         3
  / \       / \
 7--- 1--- 8---10
      \ /
       9
```

64#

```
       3
      / \
 4--- 6---11--- 5
  \ /       \ /
   9         2
  / \       / \
 8--- 1--- 7---10
      \ /
      12
```

19/26#

```
       1
      / \
 2---10---11--- 3
  \ /       \ /
   8         5
  / \       / \
 7--- 4--- 6--- 9
      \ /
      12
```

40#

```
       1
      / \
 4--- 7--- 9--- 6
  \ /       \ /
   8         5
  / \       / \
10--- 2--- 3---11
      \ /
      12
```

61#

```
       3
      / \
 4---12--- 2--- 8
  \ /       \ /
   6        10
  / \       / \
 5--- 9--- 1---11
      \ /
       7
```

54#

```
       2
      / \
 6---12--- 3--- 5
  \ /       \ /
   4        11
  / \       / \
 8--- 7--- 1---10
      \ /
       9
```

20/27#

```
       1
      / \
 4---10--- 9--- 3
  \ /       \ /
   8         5
  / \       / \
 7--- 2--- 6---11
      \ /
      12
```

39#

```
       1
      / \
 6---11--- 7--- 2
  \ /       \ /
   5         8
  / \       / \
 9--- 3--- 4---10
      \ /
      12
```

49#

```
       2
      / \
 5--- 6---12--- 3
  \ /       \ /
  11         4
  / \       / \
 7--- 1---10--- 8
      \ /
       9
```

73#

```
       4
      / \
 6---12--- 3--- 5
  \ /       \ /
   2         9
  / \       / \
 8--- 7--- 1---10
      \ /
      11
```

```
 [Count = 20 x 4 = 80]
```

*** Duplication Monitor** of Solutions *

```
 ??: 0
  1: 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 21: 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 41: 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 61: 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
```

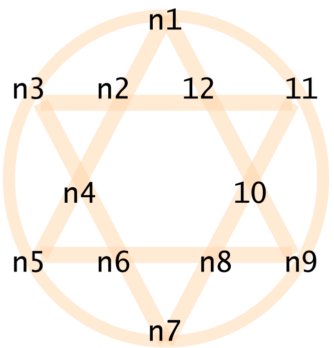
I tried to examine each solution by consulting it with the old solution list and getting the old solution number. I put it on the new list as N#.

I also made the monitor list of correspondence between the old list and new.

The beautiful result list above means our success. We could really reconstruct the whole set of 80 standard solutions of magic stars of order 6, using both the 20 fundamental solutions and the new transformation system. How wonderful it is!

But, what makes it possible? What conditions makes this miracle realized?

It is the time now to have some algebraic study for magic star of order 6.



* Basic Definition for **Order 6** *

```
n1+n2+n4+n5=C     ... l1
n3+n4+n6+n7=C     ... l2
n5+n6+n8+n9=C     ... l3
n7+n8+n10+n11=C   ... l4
n9+n10+n12+n1=C   ... l5
n11+n12+n2+n3=C   ... l6
-----------------------------
2*(n1+n2+n3+...+n11+n12)=6*C
         2*78=6*C
Therefore  C=26   ... r0
```

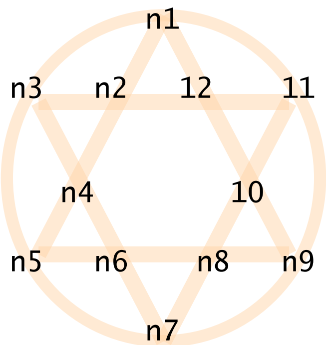
```
l1+l3+l5
  n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n12+n1=3*26
  2*(n1+n5+n9)+(n2+n4+n6+n8+n10+n12)=78  ... r1
l2+l4+l6
  n3+n4+n6+n7+n7+n8+n10+n11+n11+n12+n2+n3=3*26
  2*(n3+n7+n11)+(n2+n4+n6+n8+n10+n12)=78 ... r2
r1-r2
  n1+n5+n9=n3+n7+n11                     ... r3
l1+l2+l3
  n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9=3*26
  (n4+n5+n6)+n1+n2+n3+n4+n5+n6+n7+n8+n9=78
Add (n10+n11+n12) to both sides
  (n4+n5+n6)+n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12=78+n10+n11+n12
Therefore n4+n5+n6=n10+n11+n12           ... r4
l2+l3+l4
  n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n11=3*26
  (n6+n7+n8)+n3+n4+n5+n6+n7+n8+n9+n10+n11=78
Add (n1+n2+n12) to both sides
  (n6+n7+n8)+n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12=78+n1+n2+n12
Therefore n6+n7+n8=n1+n2+n12             ... r5
l3+l4+l5 and add (n2+n3+n4) to both sides
```

```
Similarly n8+n9+n10=n2+n3+n4                  ... r6

l1+l2+l4+l5
  n1+n2+n4+n5+n3+n4+n6+n7+n7+n8+n10+n11+n9+n10+n12+n1=4*C
  (n1+n4+n7+n10)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Therefore n1+n4+n7+n10=26                     ... r7
l2+l3+l5+l6
  n3+n4+n6+n7+n5+n6+n8+n9+n9+n10+n12+n1+n11+n12+n2+n3=4*C
  (n3+n6+n9+n12)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Therefore n3+n6+n9+n12=26                     ... r8
l1+l3+l4+l6
  n1+n2+n4+n5+n5+n6+n8+n9+n7+n8+n10+n11+n11+n12+n2+n3=4*C
  (n2+n5+n8+n11)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Therefore n2+n5+n8+n11=26                     ... r9

l2+l3+l4+l5
  n1+n3+n4+n5+n6+n6+n7+n7+n8+n8+n9+n9+n10+n10+n11+n12=4*C
  (n6+n7+n8+n9+n10)+(n1+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Add n2 to both sides
  (n6+n7+n8+n9+n10)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104+n2
Therefore n2=n6+n7+n8+n9+n10-26               ... r10
l3+l4+l5+l6
  n5+n6+n8+n9+n7+n8+n10+n11+n9+n10+n12+n1+n11+n12+n2+n3=4*C
  (n8+n9+n10+n11+n12)+(n1+n2+n3+n5+n6+n7+n8+n9+n10+n11+n12)=104
Add n4 to both sides
  (n8+n9+n10+n11+n12)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104+n4
Therefore n4=n8+n9+n10+n11+n12-26             ... r11
Similarly n6=n1+n2+n10+n11+n12-26             ... r12
          n8=n1+n2+n3+n4+n12-26               ... r13
          n10=n2+n3+n4+n5+n6-26               ... r14
          n12=n4+n5+n6+n7+n8-26               ... r15

l2+l3+l4+l6
  n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n11+n11+n12+n2+n3=4*C
  (n3+n6+n7+n8+n11)+(n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Add n1 to both sides
  (n3+n6+n7+n8+n11)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104+n1
Therefore n1=n3+n6+n7+n8+n11-26               ... r16
l1+l3+l4+l5
  n1+n2+n4+n5+n5+n6+n8+n9+n7+n8+n10+n11+n9+n10+n12+n1=4*C
  (n1+n5+n8+n9+n10)+(n1+n2+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104
Add n3 to both sides
  (n1+n5+n8+n9+n10)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12)=104+n3
```



* Basic Definition for **Order 6** *

```
n1+n2+n4+n5=C     ... l1
n3+n4+n6+n7=C     ... l2
n5+n6+n8+n9=C     ... l3
n7+n8+n10+n11=C   ... l4
n9+n10+n12+n1=C   ... l5
n11+n12+n2+n3=C   ... l6
```

```
Therefore n3=n1+n5+n8+n9+n10-26          ... r17
Similarly n5=n3+n7+n10+n11+n12-26        ... r18
          n7=n1+n2+n5+n9+n12-26          ... r19
          n9=n2+n3+n4+n7+n11-26          ... r20
          n11=n1+n4+n5+n6+n9-26          ... r21
```

**** Summary: List of Relations ****

```
 n1+n5+n9=n3+n7+n11          ... r3
 n4+n5+n6=n10+n11+n12        ... r4
 n6+n7+n8=n1+n2+n12          ... r5
 n8+n9+n10=n2+n3+n4          ... r6
 n1+n4+n7+n10=26             ... r7
 n3+n6+n9+n12=26             ... r8
 n2+n5+n8+n11=26             ... r9
 n2=n6+n7+n8+n9+n10-26       ... r10
 n4=n8+n9+n10+n11+n12-26     ... r11
 n6=n1+n2+n10+n11+n12-26     ... r12
 n8=n1+n2+n3+n4+n12-26       ... r13
 n10=n2+n3+n4+n5+n6-26       ... r14
 n12=n4+n5+n6+n7+n8-26       ... r15
 n1=n3+n6+n7+n8+n11-26       ... r16
 n3=n1+n5+n8+n9+n10-26       ... r17
 n5=n3+n7+n10+n11+n12-26     ... r18
 n7=n1+n2+n5+n9+n12-26       ... r19
 n9=n2+n3+n4+n7+n11-26       ... r20
 n11=n1+n4+n5+n6+n9-26       ... r21
```

Yes. All of them really make it possible. We are lucky we could find that miracle.

It is only for the order 6 that we could draw the extended space and find that beautiful transformation system. Nothing such as the Extended Space could be drawn for the other orders than 6, and no beautiful transformation system such as of order 6 could be found anywhere else, either. This miraculous phenomenon is realized only for magic stars of order 6, as well as the pandiagonal magic squares of order 4 or 8.

Study of magic stars of higher orders might get rather boring, I am afraid.

Chapter 4: Algebraic Study of Double Ringed Magic Stars 7

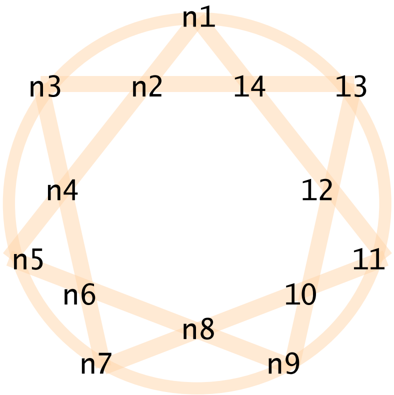
Let's study about Magic Stars of order 7 next.

Two more numerical elements and one more equation are added to the case of order 6. How do these new comers change the structure?

*** Algebra for Magic Star of Order 7 ***

```
Calculations:
l1+l3+l5+l7
 n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n12+n13+n13+n14+n2+n3=4*C
 (n2+n5+n9+n13)+(n1+n2+n3+n4+n5+n6+n8+n9+n10+n12+n13+n14)=120
Add n7+n11 to both sides.
 (n2+n5+n9+n13)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n7+n11
 (n2+n5+n9+n13)+       105        =120+n7+n11
Therefore
 n2=15-n5+n7-n9+n11-n13                          ... r1
```

```
l1+l2+l4+l6
 n1+n2+n4+n5+n3+n4+n6+n7+n7+n8+n10+n11+n11+n12+n14+n1=4*C
 (n1+n4+n7+n11)+(n1+n2+n3+n4+n5+n6+n7+n8+n10+n11+n12+n14)=120
Add n9+n13 to both sides.
 (n1+n4+n7+n11)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n9+n13
```




```
* Basic Definition *
 n1+n2+n4+n5=C    ... l1
 n3+n4+n6+n7=C    ... l2
 n5+n6+n8+n9=C    ... l3
 n7+n8+n10+n11=C  ... l4
 n9+n10+n12+n13=C ... l5
 n11+n12+n14+n1=C ... l6
 n13+n14+n2+n3=C  ... l7
 ----------------------------
 2*(n1+n2+n3+...+n13+n14)=7*C
           2*105=7*C
 Therefore  C=30  ... r0
```

```
Therefore
 n4=15-n1-n7+n9-n11+n13                     ... r2
Similarly
 n6=15+n1-n3-n9+n11-n13                     ... r3
 n8=15-n1+n3-n5-n11+n13                     ... r4
 n10=15+n1-n3+n5-n7-n13                     ... r5
 n12=15-n1+n3-n5+n7-n9                      ... r6
 n14=15-n3+n5-n7+n9-n11                     ... r7
l1+l2+l5+l6
 n1+n2+n4+n5+n3+n4+n6+n7+n9+n10+n12+n13+n11+n12+n14+n1=4*C
 (n1+n4+n12)+(n1+n2+n3+n4+n5+n6+n7+n9+n10+n11+n12+n13+n14)=120
Add n8 to both sides
 (n1+n4+n12)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n8
 n1+n4+n12+          105          =120+n8
Therefore n1=15-n4+n8-n12                   ... r8
l2+l3+l6+l7
 n3+n4+n6+n7+n5+n6+n8+n9+n11+n12+n14+n1+n13+n14+n2+n3=4*C
 (n3+n6+n14)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n11+n12+n13+n14)=120
Add n10 to both sides
 (n3+n6+n14)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n10
 n3+n6+n14+          105          =120+n10
Therefore  n3=15-n6+n10-n14                 ... r9
Similarly  n5=15-n2+n12-n8                  ... r10
           n7=15-n4+n14-n10                 ... r11
           n9=15+n2-n6-n12                  ... r12
           n11=15+n4-n8-n14                 ... r13
           n13=15-n2+n6-n10                 ... r14
```

We now know how the outer 'ring' of {n1, n3, n5, n7, n9, n11 and n13} relates to the inner one of {n2, n4, n6, n8, n10, n12 and n14} by those relational equations: r1~r14. However, they do not always mean their total exchangeability with each other.

The next list shows the result of my new calculation. It contains all the 72 standard solutions of magic star of order 7.

**** List of Standard Magic Stars of Order 7 ****

1#
1
3 14 8 5
13 12
2 9
10 6
11
4 7

2#
1
3 14 7 6
13 12
2 10
9 4
11
5 8

3#
1
4 14 9 3
13 12
2 8
7 5
11
6 10

4#
1
5 14 7 4
13 12
2 10
9 6
11
3 8

5#
1
4 14 10 2
12 13
3 6
9 8
11
5 7

6#
1
4 14 5 7
12 11
3 13
8 2
9
6 10

7#
1
4 14 7 5
12 13
3 9
6 2
11
8 10

8#
1
6 14 8 2
12 11
3 10
5 4
9
7 13

9#
1
6 13 8 3
12 7
4 14
10 9
5
2 11

10#
1
9 13 5 3
12 10
4 14
7 6
8
2 11

11#
1
3 11 14 2
13 9
5 6
10 12
8
4 7

12#
1
3 11 14 2
13 7
5 8
10 12
6
4 9

13#
1
7 14 6 3
10 12
5 11
4 2
8
9 13

14#
1
4 12 9 5
10 6
7 14
13 11
2
3 8

15#
1
11 12 2 5
10 13
7 14
6 4
9
3 8

16#
1
2 13 11 4
9 8
7 10
14 12
3
5 6

17#
1
5 13 8 4
9 11
7 10
14 12
6
2 3

18#
1
5 9 13 3
12 6
8 10
11 14
4
2 7

19#
1
2 10 14 4
11 6
8 9
12 13
3
5 7

20#
1
4 11 9 6
10 7
8 13
14 12
3
2 5

21#
1
3 12 5 10
9 11
8 13
14 7
6
4 2

22#
1
9 14 5 2
7 13
8 11
4 3
6
10 12

23#
1
5 7 12 6
13 3
9 14
8 10
2
4 11

24#
1
5 8 14 3
12 4
9 11
6 10
2
7 13

25#
1
6 12 2 10
8 13
9 14
11 4
7
5 3

26#
1
4 14 5 7
6 11
9 13
8 2
3
12 10

27#
1
4 7 5 14
12 11
10 13
8 2
9
6 3

28#
1
13 7 4 6
12 11
10 14
3 5
9
2 8

29#

1
13 8 3 6
11 12
10 14
4 5
9
2 7

30#

1
12 11 2 5
8 13
10 14
4 3
7
6 9

31#

1
4 5 12 9
13 3
11 14
7 8
2
6 10

32#

1
14 10 4 2
8 13
11 12
3 6
7
5 9

33#

1
7 4 9 10
13 6
12 14
2 3
5
8 11

34#

1
7 11 2 10
6 13
12 14
9 3
5
8 4

35#

1
4 6 8 12
10 7
13 14
5 2
3
11 9

36#

1
4 9 5 12
7 10
13 14
8 2
3
11 6

37#

1
7 10 4 9
6 11
13 14
12 8
3
5 2

38#

1
8 10 3 9
6 12
13 14
11 7
4
5 2

39#

2
5 13 4 8
12 10
3 14
7 1
9
6 11

40#

2
7 9 10 4
14 12
5 6
1 3
13
8 11

41#

2
8 9 10 3
14 12
5 6
1 4
13
7 11

42#

2
9 10 3 8
12 14
6 11
4 1
13
5 7

43#

2
8 13 5 4
9 11
6 12
3 1
7
10 14

44#

2
3 11 10 6
9 5
8 13
14 12
1
4 7

45#

2
12 11 1 6
9 14
8 13
5 3
10
4 7

46#

2
11 5 6 8
14 12
9 10
1 3
13
4 7

47#

2
4 11 12 3
8 6
9 10
13 14
1
5 7

48#

2
5 13 4 8
6 10
9 14
7 1
3
12 11

49#

2
6 7 13 4
11 3
10 12
5 9
1
8 14

50#

2
8 3 10 9
14 5
11 13
1 4
6
7 12

51#

2
13 8 4 5
9 10
11 14
1 3
6
7 12

52#

2
8 12 1 9
5 14
11 13
10 4
6
7 3

53#

2
8 6 11 5
10 4
12 13
3 7
1
9 14

54#

2
10 3 8 9
12 6
13 14
1 4
5
7 11

55#

3
5 9 12 4
11 2
7 13
8 10
1
6 14

56#

4
13 10 1 6
9 14
7 11
3 2
12
5 8

```
57#                 58#                 59#                 60#
         4                   4                   4                   5
   7  2    12  9       8  2    11  9       8 14     1  7       8  1     9 12
   14        3         14        3          2       12         14        3
 10           11     10           12     10           13     10           13
     1      5             1      5           11      5              2      4
         6                   6                   3                   7
      8    13             7    13             9     6             6    11
```

```
61#                 62#                 63#                 64#
         5                   5                   5                   5
   9  3     7 11       7 11     3  9       6 13     4  7       7  9     3 11
   12        4          4        8          2        9          4        8
 10           14     10           14     10           12     12           14
     1      2             6      1             8      3             6      1
         6                   2                   1                   2
      8    13            13    12            14    11            13    10
```

```
65#                 66#                 67#                 68#
         5                   5                   5                   6
   7 10     4  9       8  1     9 12      12  3     4 11       8  2     9 11
    3        8         11        2          9        7         12        1
 12           13     13           14     13           14     10           14
     6      2             4      6             1      2             3      5
         1                   3                   6                   4
     14    11             7    10             8    10             7    13
```

```
69#                 70#                 71#                 72#
         6                   6                   8                   8
  11  9     3  7       8  9     3 10      11  7     3  9      10  4     7  9
    5        8          4        7          5        6          6        2
 10           13     11           14     10           13     12           13
     2      1             5      1             2      1             3      5
         4                   2                   4                   1
     12    14            13    12            12    14            11    14
```

All the solutions 39#~72# have the value '1' in any position of the inner ring {n2, n4, n6, n8, n10, n12 and n14} and '1' cannot be found anywhere in the outer ring.

Therefore, you can rearrange and make the list smart for both types: the solutions whose n1=1 and the ones whose n2=1 as follows.

** Smart List of Standard Magic Stars of Order 7 **

[Type n1=1]

```
1#                  2#                  3#                  4#
         1                   1                   1                   1
   3 14     8  5       3 14     7  6       4 14     9  3       5 14     7  4
   13       12         13       12         13       12         13       12
  2            9      2           10      2            8      2           10
    10      6             9      4             7      5             9      6
        11                  11                  11                  11
      4     7             5     8             6    10             3     8
```

```
5#                  6#                  7#                  8#
         1                   1                   1                   1
   4 14    10  2       4 14     5  7       4 14     7  5       6 14     8  2
   12       13         12       11         12       13         12       11
  3            6      3           13      3            9      3           10
     9      8             8      2             6      2             5      4
        11                   9                  11                   9
      5     7             6    10             8    10             7    13
```

9#
1
6 13 8 3
12 7
4 14
10 9
5
2 11

10#
1
9 13 5 3
12 10
4 14
7 6
8
2 11

11#
1
3 11 14 2
13 9
5 6
10 12
8
4 7

12#
1
3 11 14 2
13 7
5 8
10 12
6
4 9

13#
1
7 14 6 3
10 12
5 11
4 2
8
9 13

14#
1
4 12 9 5
10 6
7 14
13 11
2
3 8

15#
1
11 12 2 5
10 13
7 14
6 4
9
3 8

16#
1
2 13 11 4
9 8
7 10
14 12
3
5 6

17#
1
5 13 8 4
9 11
7 10
14 12
6
2 3

18#
1
5 9 13 3
12 6
8 10
11 14
4
2 7

19#
1
2 10 14 4
11 6
8 9
12 13
3
5 7

20#
1
4 11 9 6
10 7
8 13
14 12
3
2 5

21#
1
3 12 5 10
9 11
8 13
14 7
6
4 2

22#
1
9 14 5 2
7 13
8 11
4 3
6
10 12

23#
1
5 7 12 6
13 3
9 14
8 10
2
4 11

24#
1
5 8 14 3
12 4
9 11
6 10
2
7 13

25#
1
6 12 2 10
8 13
9 14
11 4
7
5 3

26#
1
4 14 5 7
6 11
9 13
8 2
3
12 10

27#
1
4 7 5 14
12 11
10 13
8 2
9
6 3

28#
1
13 7 4 6
12 11
10 14
3 5
9
2 8

29#
1
13 8 3 6
11 12
10 14
4 5
9
2 7

30#
1
12 11 2 5
8 13
10 14
4 3
7
6 9

31#
1
4 5 12 9
13 3
11 14
7 8
2
6 10

32#
1
14 10 4 2
8 13
11 12
3 6
7
5 9

33#
1
7 4 9 10
13 6
12 14
2 3
5
8 11

34#
1
7 11 2 10
6 13
12 14
9 3
5
8 4

35#
1
4 6 8 12
10 7
13 14
5 2
3
11 9

36#
1
4 9 5 12
7 10
13 14
8 2
3
11 6

37#
```
       1
 7 10     4  9
  6        11
13            14
  12       8
       3
    5     2
```
38#
```
       1
 8 10     3  9
  6        12
13            14
  11       7
       4
    5     2
```
[Type n2=1]

39#
```
       14
 12  1     7 10
  2         3
13            6
   5       9
       4
   11     8
```
40#
```
       14
 10  1     8 11
  2         3
13            5
   6       9
       4
   12     7
```
41#
```
       14
 12  1     8  9
  2         3
13            5
   6       11
       4
   10     7
```
42#
```
       14
 11  1     6 12
  2         3
13            7
   8       10
       4
    9     5
```
43#
```
       14
 11  1     5 13
  3         2
12            9
   6       7
       4
   10     8
```
44#
```
       14
 11  1    10  8
  3         4
12            2
   7       13
       6
    9     5
```
45#
```
       14
 11  1     8 10
  3         2
12            6
   9       13
       4
    7     5
```
46#
```
       14
  9  1     7 13
  3         4
12            5
   10      11
       6
    8     2
```
47#
```
       13
 14  1     6  9
  4         3
12            8
   2       7
       5
   10     11
```
48#
```
       13
 14  1     8  7
  4         3
12            6
   2       9
       5
   10     11
```
49#
```
       11
 14  1     9  6
  5         2
13            8
   4       12
       3
    7     10
```
50#
```
       13
  7  1    12 10
  5         3
11            2
   4       8
       6
   14     9
```
51#
```
       14
  8  1     9 12
  5         3
10            4
  11       13
       7
    6     2
```
52#
```
       10
  8  1    12  9
  6         3
13            5
   2       7
       4
   14     11
```
53#
```
       10
  7  1    14  8
  6         2
13            4
   5       11
       3
   12     9
```
54#
```
       10
  8  1    14  7
  6         2
13            4
   5       12
       3
   11     9
```
55#
```
       11
  7  1    14  8
  6         3
12            2
   4       10
       5
   13     9
```
56#
```
       12
 14  1    11  4
  7         5
10            2
   3       13
       9
    6     8
```
57#
```
       14
  6  1    10 13
  8         2
 7            4
  11       12
       9
    5     3
```
58#
```
        7
 11  1     6 12
  9         3
13            14
   8       10
       4
    2     5
```
59#
```
        8
  5  1    14 10
  9         2
12            6
   3       7
       4
   13     11
```
60#
```
        8
 13  1     6 10
  9         2
12            14
   3       7
       4
    5     11
```
61#
```
       14
 11  1    10  8
  9         4
 6            2
   7       13
       12
    3     5
```
62#
```
        6
  4  1    14 11
 10         2
13            8
   9       12
       3
    7     5
```

```
63#
        5
   8 1     9 12
  11         2
 13          14
    4      6
        3
     7   10

64#
        6
   2 1    14 13
  11         3
 12            7
    9     10
        5
     8    4

65#
        4
   7 1    14  8
  12         2
 13          10
    5     11
        3
     6    9

66#
        4
   7 1    14  8
  12         9
 13            3
    5     11
       10
     6    2

67#
        7
   5 1    14 10
  13         6
  9            3
    8     12
       11
     4    2

68#
        2
   9 1    12  8
  14         5
 13          11
    4     10
        6
     3    7

69#
        4
   9 1    13  7
  14         3
 11          10
    5     12
        6
     2    8

70#
        7
  11 1    13  5
  14         4
  8            6
    3     12
       10
     2    9

71#
        7
   5 1    13 11
  14         4
  8            6
    9     12
       10
     2    3

72#
        8
   5 1    13 11
  14         3
  7            6
    9     12
       10
     2    4
```

[Count(Tn1+Tn2) = 38 + 34 = 72] OK!

Now I have nothing good to report of any smart classification and transformation system. I have not yet discovered what should be the fundamental solutions of magic stars of order 7 and how they could be transformed so as to reconstruct the complete set of standard solutions. It is still one of the problems to be solved in the future.

(Original: Written in Japanese first on August 12, 2000;
English version: Written first on Nov.23, 2005 by Kanji Setsuda;
Revised on July 3, 2008 working on MacOSX and Xcode 2.2)

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