

Part 4: "New Advanced Study of Magic Squares and Cubes"

Chapter 4: Commentary Articles No.2 by Kanji Setsuda:

"Various Arts and Tools for Studying Magic Squares"

Section 11: Various "Euler Squares" of Order 3, 5 and 7 made by "New Euler's Method" using PNS of Base 3, 5 and 7

0. Our purpose

This time I would like to examine our "New Euler's Method" using Positional Number System of Base 3, 5 and 7 by applying it to the cases of order 3, 5 and 7.

I once skipped applying it to the first and smallest case of magic squares simply because we cannot make any pandiagonal magic squares of order 3.

When I invented the composing method of "Knight's Movement" for order 5 and 7, I did not yet get any total idea of "New Euler's Method", though these two methods are essentially similar, I would say, almost the same.

Let's examine our newest method applying to these cases right here, shall we?

1. How to apply our New Euler's Method to Magic Square 3x3

We know we cannot make any pan-diagonal magic squares 3x3 but we can only make self-complementary one. This means we cannot make any 'Complete Euler Squares' of order 3. But it bothers us no longer. We now know what we have to do with it.

Please watch the next solution example of S-C MS33. Every complementary pair of 10 is located symmetrically with respect to the geometric center. The n5 always takes 5 just because it is placed on the center and never moves to anywhere else.

If you have the decomposed patterns of solution calculated under the Positional Number System of Base 3, you see n5 is always occupied by '1' on both layers high and low, as you see below. It is logically determined by the basic definitions.

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	[Mathematical Notations]			[Decomposed Patterns]																																			

Therefore, one of the primary diagonals must be made of {1,1,1} instead of {0,1,2}. Let me skip my explanation about the logical proof here. Since we accept every row and every column must be made of {0,1,2}, and '1' must be used three times anywhere in each layer, we cannot make any "Complete Euler Squares" of order 3 consequently.

Let's compose the Self-Complementary Euler Square of order 3 that has every row and every column made of only {0,1,2}.

Between those figures above, you see the following equations are active.

$$\begin{aligned}
 1 &= 0 \times 3 + 0 + 1; & 2 &= 0 \times 3 + 1 + 1; & 3 &= 0 \times 3 + 2 + 1; & 4 &= 1 \times 3 + 0 + 1; & 5 &= 1 \times 3 + 1 + 1; & 6 &= 1 \times 3 + 2 + 1; \\
 7 &= 2 \times 3 + 0 + 1; & 8 &= 2 \times 3 + 1 + 1; & 9 &= 2 \times 3 + 2 + 1; \\
 V_n &= A_n \times 3 + B_n + 1 \quad (n=1, 2, 3, \dots, 8, 9)
 \end{aligned}$$

Let me show you the following basic diagram and basic conditions I prepared for.

Let's try to compose its layer units with only '0', '1' and '2' at first.

**** Basic Positions and Basic Conditions ****

n1 n2 n3	$n1+n2+n3=C \dots rw1;$	$n1+n4+n7=C \dots cl 1;$
n4 n5 n6	$n4+n5+n6=C \dots rw2;$	$n2+n5+n8=C \dots cl 2;$
n7 n8 n9	$n7+n8+n9=C \dots rw3;$	$n3+n6+n0=C \dots cl 3;$

**** Self-Complementary Conditions ****

$n1+n9=CC;$ $n2+n8=CC;$ $n3+n7=CC;$ $n4+n6=CC;$ $n5+n5=CC;$
 $n6+n4=CC;$ $n7+n3=CC;$ $n8+n2=CC;$ $n9+n1=CC;$

The magic constant C and CC above must take the values: $C=3;$ $CC=2;$
I do not define anything explicit about the two primary diagonals:

$n1+n5+n9 \dots pd1;$ and $n3+n5+n7 \dots pb3;$

According to the Self-Complementary Conditions above,

$n5=CC/2;$ $n1+n9=CC;$ and $n3+n7=CC;$...

therefore $n1+n5+n9=n3+n5+n7=CC+CC/2=2+1=C;$

Both of them add up to the same sum, the magic constant. We cannot deny it at all.

I do not define any number pattern for them any longer. It may be {1,1,1}, or may be {0,1,2}, but I do surely define every row and every column must always have the same number pattern {0,1,2} taking each number strictly once, and set each flag for every row and every column so that we can precisely watch how often each number is used there.

The next program is dictated for our new experiment.

If you want to have its whole list, please click at the file end to get it.

```

/**/
/** Core Part of ' EulerS3SC.c' */
/**/
#include <stdio.h>
/**/
short int cnt, cnt3;
short CC, LSM;
short u1, u2;
short nm[10], flg[10];
short anm[5][12];
short tlu[5][10];
short mtc[5][5], cmtc[5][5], scmtc[5][5];
/**/
short rw1[3], cl 1[3];
short rw2[3], cl 2[3];
short rw3[3], cl 3[3];
/**/
/* Main Program */
main(){
short n;
printf("\n** Self-Complementary Magic Squares of Order 3 **\n");
printf("** by 'New Euler's Method' using PNS of Base 3 **\n");
for(n=0; n<10; n++){nm[n]=0;}
for(n=0; n<3; n++){
rw1[n]=0; cl 1[n]=0;

```

```

    rw2[n]=0; cl 2[n]=0;
    rw3[n]=0; cl 3[n]=0;
}
CC=2; LSM=3; cnt=0; cnt3=0;
stp01(); /* Make The Layer Units */
printf("\n [Layer Units]\n");
prlunit(); /* Print the Layer Units */
printf("\n [Composition of Primitive MS33]\n");
/* printf("\n [Composition of Standard MS33]\n"); : for Substitution */
cnt=0; cnt3=0;
cmbcmp(); /* Combine, Compose and Print */
printf(" [Count = %d]\n", cnt);
printf(" OK!\n");
return 0;
}
/* Make The Layer Units */
/* Set n1 & n9 */
void stp01(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((rw1[a]==0)&&(cl 1[a]==0)){
if((rw3[b]==0)&&(cl 3[b]==0)){
nm[1]=a; nm[9]=b;
rw1[a]=1; cl 1[a]=1;
rw3[b]=1; cl 3[b]=1;
stp02();
rw1[a]=0; cl 1[a]=0;
rw3[b]=0; cl 3[b]=0;
}}}}
}
/* Set n2 & n8 */
void stp02(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((rw1[a]==0)&&(cl 2[a]==0)){
if((rw3[b]==0)&&(cl 2[b]==0)){
nm[2]=a; nm[8]=b;
rw1[a]=1; cl 2[a]=1;
rw3[b]=1; cl 2[b]=1;
stp03();
rw1[a]=0; cl 2[a]=0;
rw3[b]=0; cl 2[b]=0;
}}}}
}
/* Set n3 & n7 */
void stp03(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((rw1[a]==0)&&(cl 3[a]==0)){
if((rw3[b]==0)&&(cl 1[b]==0)){
nm[3]=a; nm[7]=b;
rw1[a]=1; cl 3[a]=1;
rw3[b]=1; cl 1[b]=1;
stp04();
rw1[a]=0; cl 3[a]=0;
rw3[b]=0; cl 1[b]=0;
}}}}
}
/* Set n4 & n6 */

```

```

void stp04(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if((rw2[a]==0)&&(cl 1[a]==0)){
if((rw2[b]==0)&&(cl 3[b]==0)){
nm[4]=a; nm[6]=b;
rw2[a]=1; cl 1[a]=1;
rw2[b]=1; cl 3[b]=1;
stp05();
rw2[a]=0; cl 1[a]=0;
rw2[b]=0; cl 3[b]=0;
}}}
}
/* Set n5 */
void stp05(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if(a==b){
if((rw2[a]==0)&&(cl 2[a]==0)){
nm[5]=a;
rw2[a]=1; cl 2[a]=1;
lurecord();
rw2[a]=0; cl 2[a]=0;
}}}
}
/**/
/* Record the Layer Units */
void lurecord(){
short n;
tlu[cnt][0]=cnt+1;
for(n=1;n<10;n++){tlu[cnt][n]=nm[n];}
cnt++;
}
/**/

```

By this program we have got 4 layer units as follows:

** Self-Complementary Magic Squares of Order 3 **
 ** by 'New Euler's Method' using PNS of Base 3 **

[Layer Units]

1/	2/	3/	4/
0 2 1	1 0 2	1 2 0	2 0 1
2 1 0	2 1 0	0 1 2	0 1 2
1 0 2	0 2 1	2 0 1	1 2 0

[Count of Layer Units = 4]

[Composition of Primitive MS33]

1/	1/	1#	1/	2/	2#
0 2 1	0 2 1	1 9 5	0 2 1	1 0 2	2 7 6
2 1 0	2 1 0	9 5 1	2 1 0	2 1 0	9 5 1
1 0 2	1 0 2	5 1 9	1 0 2	0 2 1	4 3 8
1/	3/	3#	1/	4/	4#
0 2 1	1 2 0	2 9 4	0 2 1	2 0 1	3 7 5
2 1 0	0 1 2	7 5 3	2 1 0	0 1 2	7 5 3
1 0 2	2 0 1	6 1 8	1 0 2	1 2 0	5 3 7

2/	1/	5#	2/	2/	6#
1 0 2	0 2 1	4 3 8	1 0 2	1 0 2	5 1 9
2 1 0	2 1 0	9 5 1	2 1 0	2 1 0	9 5 1
0 2 1	1 0 2	2 7 6	0 2 1	0 2 1	1 9 5
2/	3/	7#	2/	4/	8#
1 0 2	1 2 0	5 3 7	1 0 2	2 0 1	6 1 8
2 1 0	0 1 2	7 5 3	2 1 0	0 1 2	7 5 3
0 2 1	2 0 1	3 7 5	0 2 1	1 2 0	2 9 4
3/	1/	9#	3/	2/	10#
1 2 0	0 2 1	4 9 2	1 2 0	1 0 2	5 7 3
0 1 2	2 1 0	3 5 7	0 1 2	2 1 0	3 5 7
2 0 1	1 0 2	8 1 6	2 0 1	0 2 1	7 3 5
3/	3/	11#	3/	4/	12#
1 2 0	1 2 0	5 9 1	1 2 0	2 0 1	6 7 2
0 1 2	0 1 2	1 5 9	0 1 2	0 1 2	1 5 9
2 0 1	2 0 1	9 1 5	2 0 1	1 2 0	8 3 4
4/	1/	13#	4/	2/	14#
2 0 1	0 2 1	7 3 5	2 0 1	1 0 2	8 1 6
0 1 2	2 1 0	3 5 7	0 1 2	2 1 0	3 5 7
1 2 0	1 0 2	5 7 3	1 2 0	0 2 1	4 9 2
4/	3/	15#	4/	4/	16#
2 0 1	1 2 0	8 3 4	2 0 1	2 0 1	9 1 5
0 1 2	0 1 2	1 5 9	0 1 2	0 1 2	1 5 9
1 2 0	2 0 1	6 7 2	1 2 0	1 2 0	5 9 1

[Count = 16]

Simple combinations of any two units make 4x4 compositions as shown above.

But they are not yet any final solutions at all. Among 16 there are 8 wrong answers against the definition (3). For instance, composition No.1 has three '1's, three '5's, three '9's and nothing else. No.4 has '3's, '5's and '7's three times for each and nothing else.

No.1 is made up of unit 1/ and the same 1/, but this kind of repeating usage of any one unit should make a wrong composition inevitably.

No.4 is made of unit 1/ and 4/, but this pair(1/, 4/) is a 'complementary unit' that should always make your composition wrong.

Between those units every value of corresponding positions always adds up to 2.

If $(H_n + L_n = 2)$ then we call (H_n, L_n) "Complementary Pair of 2," and if $(\text{all } H_n + L_n = 2)$ then we call $(H/, L/)$ "Complementary Units."

How can we stop these kinds of wrong answers?

I propose you making such a reference table for the best combinations as follows:

We need the 'Collective Table of S & C' for our references like the one on the right most below, where we can only select the appropriate pair units whose count are 3.

[Reference Table for the Best Pairs]

S	1	2	3	4	C	1	2	3	4	SC	1	2	3	4
1	9	3	3	3	1	3	3	3	9	1	9	3	3	9
2	3	9	3	3	2	3	3	9	3	2	3	9	9	3
3	3	3	9	3	3	3	9	3	3	3	3	9	9	3
4	3	3	3	9	4	9	3	3	3	4	9	3	3	9

[Si mi l a r i t y T a b l e]

[C o m p l e m e n t a r y]

[C o l l e c t i v e]

[Compositions of 8]

1/	2/	1#	1/	3/	2#
0 2 1	1 0 2	2 7 6	0 2 1	1 2 0	2 9 4
2 1 0	2 1 0	9 5 1	2 1 0	0 1 2	7 5 3
1 0 2	0 2 1	4 3 8	1 0 2	2 0 1	6 1 8
2/	1/	3#	2/	4/	4#
1 0 2	0 2 1	4 3 8	1 0 2	2 0 1	6 1 8
2 1 0	2 1 0	9 5 1	2 1 0	0 1 2	7 5 3
0 2 1	1 0 2	2 7 6	0 2 1	1 2 0	2 9 4
3/	1/	5#	3/	4/	6#
1 2 0	0 2 1	4 9 2	1 2 0	2 0 1	6 7 2
0 1 2	2 1 0	3 5 7	0 1 2	0 1 2	1 5 9
2 0 1	1 0 2	8 1 6	2 0 1	1 2 0	8 3 4
4/	2/	7#	4/	3/	8#
2 0 1	1 0 2	8 1 6	2 0 1	1 2 0	8 3 4
0 1 2	2 1 0	3 5 7	0 1 2	0 1 2	1 5 9
1 2 0	0 2 1	4 9 2	1 2 0	2 0 1	6 7 2

[Count = 8]

These 8 compositions are no longer wrong ones against the definition (3) and they are considered to be 'primitive solutions'.

Each of them is such an imitation of any other solution as made by simple reflection or simple rotation. We cannot recognize each as an independent solution.

The only one correct solution must be listed out as a result, what can pass under the next list-forming inequality conditions complex.

```

    if(fc==9){/* prans(); */
        if((nm[1]<nm[9])&&(nm[1]<nm[3])&&(nm[1]<nm[7])){
            if(nm[2]>nm[4]){prans();}}
        }
    }
    /**/
    if(cnt3==1){pr2ans(); cnt3=0;}

```

[Standard Solution of SC MS33]

1/	3/	1#	0/	0/	0#
0 2 1	1 2 0	2 9 4	0 0 0	0 0 0	0 0 0
2 1 0	0 1 2	7 5 3	0 0 0	0 0 0	0 0 0
1 0 2	2 0 1	6 1 8	0 0 0	0 0 0	0 0 0

[Count = 1] OK!

We have got it! But I am afraid you might feel it is too hard to follow this big experiment just for the smallest magic square 3x3. Of course, you don't have to do the same thing again by yourself, I suppose. You may well only read this and understand that our New Euler's Method using PNS of Base 3 is really effective here.

Every step demonstrates quite reasonable, especially the result 2x1x2x2/8=1 solution is just the same with the one we got before in our former study.

We now know the Self-Complementary Magic Square 3x3 also has the beautiful structure, what we call 'Euler Square' of order 3.

2. How about Magic Cube of Order 3?

Let's build Magic Cubes of order 3 by this New Euler's Method using the Positional Number System of Base 3.

Let me show you an example solution of our object below with the decomposed layer figures of Base 3.

**** Example Solution of S-C magic Cube 3x3x3 ****

[#] Solution	*9/	*3/	*1/																																																																																																																											
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Between those figures above the following equations are active:

$$\begin{aligned}
 2 &= 0x9 + 0x3 + 1x1 + 1; & 15 &= 1x9 + 1x3 + 2x1 + 1; & 25 &= 2x9 + 2x3 + 0x1 + 1; \\
 24 &= 2x9 + 1x3 + 2x1 + 1; & 7 &= 0x9 + 2x3 + 0x1 + 1; & 11 &= 1x9 + 0x3 + 1x1 + 1; \\
 &\dots & & & & \\
 V_n &= Anx9 + Bnx3 + Cnx1 + 1 = (Anx3 + Bn)x3 + Cn + 1 \quad (n=1, 2, 3, 4, \dots, 26, 27)
 \end{aligned}$$

First of all let's design and compose each layer unit for our object.
I prepared the following basic form and basic conditions:

**** Basic Positions and Basic Conditions ****

<table border="1" style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>1</td><td>-----10</td><td>-----19</td></tr> <tr><td> \</td><td>:</td><td> \</td></tr> <tr><td> 2</td><td>:</td><td> 11</td><td> 20</td></tr> <tr><td>4</td><td> \</td><td>13</td><td>22</td><td> \</td></tr> <tr><td> </td><td>3</td><td>-----12</td><td>-----21</td></tr> <tr><td> 5</td><td> </td><td>:</td><td>14</td><td>:</td><td> 23</td><td> </td></tr> <tr><td>7</td><td>---</td><td>16</td><td>-----25</td><td> </td></tr> <tr><td> \</td><td>6</td><td>15</td><td> \</td><td>24</td></tr> <tr><td>8</td><td> </td><td>17</td><td>:</td><td>26</td><td> </td></tr> <tr><td> \</td><td>:</td><td> \</td><td></td><td></td></tr> <tr><td>9</td><td>-----18</td><td>-----27</td></tr> </table>	1	-----10	-----19	\	:	\	2	:	11	20	4	\	13	22	\		3	-----12	-----21	5		:	14	:	23		7	---	16	-----25		\	6	15	\	24	8		17	:	26		\	:	\			9	-----18	-----27	$n_1+n_2+n_3=C \quad \dots \text{df01}$ $n_4+n_5+n_6=C \quad \dots \text{df02}$ $n_7+n_8+n_9=C \quad \dots \text{df03}$ $n_{10}+n_{11}+n_{12}=C \quad \dots \text{df04}$ $n_{13}+n_{14}+n_{15}=C \quad \dots \text{df05}$ $n_{16}+n_{17}+n_{18}=C \quad \dots \text{df06}$ $n_{19}+n_{20}+n_{21}=C \quad \dots \text{df07}$ $n_{22}+n_{23}+n_{24}=C \quad \dots \text{df08}$ $n_{25}+n_{26}+n_{27}=C \quad \dots \text{df09}$ $n_1+n_4+n_7=C \quad \dots \text{df11}$ $n_2+n_5+n_8=C \quad \dots \text{df12}$ $n_3+n_6+n_9=C \quad \dots \text{df13}$ $n_{10}+n_{13}+n_{16}=C \quad \dots \text{df14}$ $n_{11}+n_{14}+n_{17}=C \quad \dots \text{df15}$ $n_{12}+n_{15}+n_{18}=C \quad \dots \text{df16}$ $n_{19}+n_{22}+n_{25}=C \quad \dots \text{df17}$ $n_{20}+n_{23}+n_{26}=C \quad \dots \text{df18}$ $n_{21}+n_{24}+n_{27}=C \quad \dots \text{df19}$ $n_1+n_{10}+n_{19}=C \quad \dots \text{df21}$ $n_2+n_{11}+n_{20}=C \quad \dots \text{df22}$ $n_3+n_{12}+n_{21}=C \quad \dots \text{df23}$ $n_4+n_{13}+n_{22}=C \quad \dots \text{df24}$ $n_5+n_{14}+n_{23}=C \quad \dots \text{df25}$ $n_6+n_{15}+n_{24}=C \quad \dots \text{df26}$ $n_7+n_{16}+n_{25}=C \quad \dots \text{df27}$ $n_8+n_{17}+n_{26}=C \quad \dots \text{df28}$ $n_9+n_{18}+n_{27}=C \quad \dots \text{df29}$
1	-----10	-----19																																																	
\	:	\																																																	
2	:	11	20																																																
4	\	13	22	\																																															
	3	-----12	-----21																																																
5		:	14	:	23																																														
7	---	16	-----25																																																
\	6	15	\	24																																															
8		17	:	26																																															
\	:	\																																																	
9	-----18	-----27																																																	

**** Self-Complementary Conditions ****

$$\begin{aligned}
 n_1+n_{27}=CC; & \quad n_2+n_{26}=CC; & \quad n_3+n_{25}=CC; & \quad n_4+n_{24}=CC; \\
 n_5+n_{23}=CC; & \quad n_6+n_{22}=CC; & \quad n_7+n_{21}=CC; & \quad n_8+n_{20}=CC;
 \end{aligned}$$

n9+n19=CC; n10+n18=CC; n11+n17=CC; n12+n16=CC;
n13+n15=CC; n14+n14=CC; n15+n13=CC;

In each layer the value of magic constants must be C=3 and CC=2.
I do not define anything here about the 4 primary triagonals:

n1+n14+n27 ...pt1; n3+n14+n25 ...pt2;
n7+n14+n21 ...pt3; n9+n14+n19 ...pt4;

But every sum of these 4 triagonals must be equal to C, because n14=CC/2=1; and
(n1+n27)+n14 = CC+CC/2 = 2+1=3 = C; (n3+n25)+n14 = CC+CC/2 = C; ...
according to the Self-Complementary Conditions above.

But we do not define any number patterns about the 4 triagonals.

It may be {1,1,1}, or may be {0,1,2}.

In each layer we define every line in all three dimensions must be made of {0,1,2},
and set each flag to watch precisely how often '0', '1' or '2' is used there.

The next program is dictated for our experiment as follows.

If you want to have the whole list of it, please click at the file end to get it.

```

/**/
/** Core Part of ' EulerMC3.c' **/
/**/
#include <stdio.h>
/**/
/* Variables */
short int cnt;
short CC;
short u1, u2, u3, u4;
short tlu[9][28];
short mtc[9][9];
short nm[31], flg[28];
/* Flags */
short d0[10][3], d1[10][3], d2[10][3];
/**/
/* Main Program */
/**/
main(){
short m, n;
printf("\n** Self-Complementary Magic Cubes of Order 3 made by **\n");
printf("*** 'New Euler's Method' with Posional NS of Base 3 ***\n");
for(n=0; n<31; n++){nm[n]=0;}
for(m=0; m<10; m++){
for(n=0; n<3; n++){
d0[m][n]=0; d1[m][n]=0; d2[m][n]=0;
}}
CC=2; cnt=0;
nm[14]=1;
d0[5][1]=1; d1[5][1]=1; d2[5][1]=1;
printf("\n [Layer Units]\n");
deep01(); /* Make the Layer Units */
prlunit(); /* Print the Layer Units */
cnt=0;
printf("\n [Composition of Standard MC333]\n");
cmbcmp(); /* Combi be, Compose and Print */
printf("\n [Count of Solutions = %d]\n", cnt);
printf(" OK! \n");
return 0;
}

```

```

/**/
/* Make the Layer Units */
/* Set n1 & n27 */
void deep01(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if((d0[1][a]==0)&&(d1[1][a]==0)&&(d2[1][a]==0)){
if((d0[9][b]==0)&&(d1[9][b]==0)&&(d2[9][b]==0)){
nm[1]=a; nm[27]=b;
d0[1][a]=1; d1[1][a]=1; d2[1][a]=1;
d0[9][b]=1; d1[9][b]=1; d2[9][b]=1;
deep02();
d0[1][a]=0; d1[1][a]=0; d2[1][a]=0;
d0[9][b]=0; d1[9][b]=0; d2[9][b]=0;
}}
}
}
/* Set n2 & n26 */
void deep02(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if((d0[1][a]==0)&&(d1[2][a]==0)&&(d2[2][a]==0)){
if((d0[9][b]==0)&&(d1[8][b]==0)&&(d2[8][b]==0)){
nm[2]=a; nm[26]=b;
d0[1][a]=1; d1[2][a]=1; d2[2][a]=1;
d0[9][b]=1; d1[8][b]=1; d2[8][b]=1;
deep03();
d0[1][a]=0; d1[2][a]=0; d2[2][a]=0;
d0[9][b]=0; d1[8][b]=0; d2[8][b]=0;
}}
}
}
/* Set n3 & n25 */
void deep03(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if((d0[1][a]==0)&&(d1[3][a]==0)&&(d2[3][a]==0)){
if((d0[9][b]==0)&&(d1[7][b]==0)&&(d2[7][b]==0)){
nm[3]=a; nm[25]=b;
d0[1][a]=1; d1[3][a]=1; d2[3][a]=1;
d0[9][b]=1; d1[7][b]=1; d2[7][b]=1;
deep04();
d0[1][a]=0; d1[3][a]=0; d2[3][a]=0;
d0[9][b]=0; d1[7][b]=0; d2[7][b]=0;
}}
}
}
/* Set n4 & n24 */
void deep04(){
short a,b;
for(a=0;a<3;a++){b=CC-a;
if((d0[2][a]==0)&&(d1[1][a]==0)&&(d2[4][a]==0)){
if((d0[8][b]==0)&&(d1[9][b]==0)&&(d2[6][b]==0)){
nm[4]=a; nm[24]=b;
d0[2][a]=1; d1[1][a]=1; d2[4][a]=1;
d0[8][b]=1; d1[9][b]=1; d2[6][b]=1;
deep05();
d0[2][a]=0; d1[1][a]=0; d2[4][a]=0;
d0[8][b]=0; d1[9][b]=0; d2[6][b]=0;
}
}
}
}

```

```

    }}
}
}
/* Set n5 & n23 */
void deep05(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((d0[2][a]==0)&&(d1[2][a]==0)&&(d2[5][a]==0)){
if((d0[8][b]==0)&&(d1[8][b]==0)&&(d2[5][b]==0)){
nm[5]=a; nm[23]=b;
d0[2][a]=1; d1[2][a]=1; d2[5][a]=1;
d0[8][b]=1; d1[8][b]=1; d2[5][b]=1;
deep06();
d0[2][a]=0; d1[2][a]=0; d2[5][a]=0;
d0[8][b]=0; d1[8][b]=0; d2[5][b]=0;
}}}
}
}
/* Set n6 & n22 */
void deep06(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((d0[2][a]==0)&&(d1[3][a]==0)&&(d2[6][a]==0)){
if((d0[8][b]==0)&&(d1[7][b]==0)&&(d2[4][b]==0)){
nm[6]=a; nm[22]=b;
d0[2][a]=1; d1[3][a]=1; d2[6][a]=1;
d0[8][b]=1; d1[7][b]=1; d2[4][b]=1;
deep07();
d0[2][a]=0; d1[3][a]=0; d2[6][a]=0;
d0[8][b]=0; d1[7][b]=0; d2[4][b]=0;
}}}
}
}
/**/
. . . . .
/**/
/* Set n13 & n15 */
void deep13(){
short a, b;
for(a=0; a<3; a++){b=CC-a;
if((d0[5][a]==0)&&(d1[4][a]==0)&&(d2[4][a]==0)){
if((d0[5][b]==0)&&(d1[6][b]==0)&&(d2[6][b]==0)){
nm[13]=a; nm[15]=b;
d0[5][a]=1; d1[4][a]=1; d2[4][a]=1;
d0[5][b]=1; d1[6][b]=1; d2[6][b]=1;
recordans();
d0[5][a]=0; d1[4][a]=0; d2[4][a]=0;
d0[5][b]=0; d1[6][b]=0; d2[6][b]=0;
}}}
}
}
/**/
/* Record the Layer Units */
void recordans(){
short n;
tlu[cnt][0]=cnt+1;
for(n=1; n<28; n++){tlu[cnt][n]=nm[n]; }
cnt++;
}
}

```

/**/

The next list shows the result of my recent calculation:

** Self-Complementary Magic Cubes of Order 3 made by **
** 'New Euler's Method' with Positional NS of Base 3 **

[Layer Units]

1/ 0 2 1 1 0 2 2 1 0 1 0 2 2 1 0 0 2 1 1 0 2	2/ 0 1 2 1 2 0 2 0 1 0 1 2 1 2 0 2 0 1 0 1 2	3/ 0 1 2 2 0 1 1 2 0 0 1 2 2 0 1 1 2 0 0 1 2	4/ 1 0 2 0 2 1 0 2 1 2 1 0 2 1 0 1 0 2 0 2 1
5/ 1 2 0 2 0 1 2 0 1 0 1 2 0 1 2 1 2 0 2 0 1	6/ 2 1 0 0 2 1 1 0 2 2 1 0 0 2 1 1 0 2 2 1 0	7/ 2 1 0 1 0 2 0 2 1 2 1 0 1 0 2 0 2 1 2 1 0	8/ 2 0 1 1 2 0 0 1 2 1 2 0 2 0 1 0 1 2 1 2 0

[Count of Layer Units = 8]

[Reference Table for the Best Combinations]

SC	1	2	3	4	5	6	7	8
1	27	9	9	9	9	9	9	27
2	9	27	9	9	9	9	27	9
3	9	9	27	9	9	27	9	9
4	9	9	9	27	27	9	9	9
5	9	9	9	27	27	9	9	9
6	9	9	27	9	9	27	9	9
7	9	27	9	9	9	9	27	9
8	27	9	9	9	9	9	9	27

[Composition of Standard MC333]

1/ 0 2 1 1 0 2 2 1 0 1 0 2 2 1 0 0 2 1 2 1 0 0 2 1 1 0 2	2/ 0 1 2 1 2 0 2 0 1 2 0 1 0 1 2 1 2 0 1 2 0 2 0 1 0 1 2	3/ 0 1 2 2 0 1 1 2 0 0 1 2 2 0 1 1 2 0 2 0 1 1 2 0 0 1 2	1# 1-----23-----18 15 7 20 26-----12-----4 17 3 22 19 14 9 6 25 11 24-----16-----2 8 21 13 10-----5-----27
---	---	---	--

1/	2/	5/	2#
0 2 1	0 1 2	1 2 0	2-----24-----16
1 0 2	1 2 0	2 0 1	15 7 20
2 1 0	2 0 1	0 1 2	25-----11-----+----- 6
1 0 2	2 0 1	2 0 1	18 1 23
2 1 0	0 1 2	0 1 2	19 14 9
0 2 1	1 2 0	1 2 0	5 27 10
2 1 0	1 2 0	0 1 2	22-----+-----17----- 3
0 2 1	2 0 1	1 2 0	8 21 13
1 0 2	0 1 2	2 0 1	12----- 4-----26
1/	5/	2/	3#
0 2 1	1 2 0	0 1 2	4-----26-----12
1 0 2	2 0 1	1 2 0	17 3 22
2 1 0	0 1 2	2 0 1	21-----13-----+----- 8
1 0 2	2 0 1	2 0 1	18 1 23
2 1 0	0 1 2	0 1 2	19 14 9
0 2 1	1 2 0	1 2 0	5 27 10
2 1 0	0 1 2	1 2 0	20-----+-----15----- 7
0 2 1	1 2 0	2 0 1	6 25 11
1 0 2	2 0 1	0 1 2	16----- 2-----24
1/	5/	6/	4#
0 2 1	1 2 0	2 1 0	6-----26-----10
1 0 2	2 0 1	0 2 1	16 3 23
2 1 0	0 1 2	1 0 2	20-----13-----+----- 9
1 0 2	2 0 1	1 0 2	17 1 24
2 1 0	0 1 2	2 1 0	21 14 7
0 2 1	1 2 0	0 2 1	4 27 11
2 1 0	0 1 2	0 2 1	19-----+-----15----- 8
0 2 1	1 2 0	1 0 2	5 25 12
1 0 2	2 0 1	2 1 0	18----- 2-----22

[Count of Solutions = 4] OK!

All the result solutions are just the same with the ones we got in our former study. Our New Euler's Method could certainly demonstrate here how powerful it is.

3. How to Compose Pan-Diagonal MS55 by this Method of PNS Base 5

Let's examine our New Euler's Method of PNS Base 5 applying to the case of Pan-Diagonal Magic Squares 5x5 and also Simultaneous type.

We once made them by our 'Knight's Compositions', but we have not yet applied our New Euler's Method to those objects.

Let's compose each layer unit by our new method at first, and then combine, compose and test our answers.

I would like to modify a little about the way how to compose layer units.

Since we have to make only two layers high and low, we may modify each layer into more compact and effective one, especially about high layer.

Why not apply the list-forming inequality conditions to the earlier stage of making each layer? We can get the smaller set of high layers, can't we?

I prepared the following basic form and basic conditions for our layer units:

*** Basic Diagram for Magic Square 5x5 ***
 ** with Basic Conditions and Pandiagonals **

```

23 24 25 21 22 23 24 25 21 22 23 24
 3  4  5 | 1 | 2 | 3 | 4 | 5 | 1  2  3  4
 8  9 10 | 6 | 7 | 8 | 9 |10 | 6  7  8  9
13 14 15 |11 |12 |13 |14 |15 |11 12 13 14
18 19 20 |16 |17 |18 |19 |20 |16 17 18 19
23 24 25 |21 |22 |23 |24 |25 |21 22 23 24
 3  4  5 | 1 | 2 | 3 | 4 | 5 | 1  2  3  4

```

```

n1+n2+n3+n4+n5=C      ... rw1;    n1+n6+n11+n16+n21=C    ... cl 1;
n6+n7+n8+n9+n10=C     ... rw2;    n2+n7+n12+n17+n22=C    ... cl 2;
n11+n12+n13+n14+n15=C ... rw3;    n3+n8+n13+n18+n23=C    ... cl 3;
n16+n17+n18+n19+n20=C ... rw4;    n4+n9+n14+n19+n24=C    ... cl 4;
n21+n22+n23+n24+n25=C ... rw5;    n5+n10+n15+n20+n25=C   ... cl 5;

```

* Pandiagonals *

```

n1+n7+n13+n19+n25=C    ... pd1;    n1+n10+n14+n18+n22=C    ... pb1;
n2+n8+n14+n20+n21=C    ... pd2;    n2+n6+n15+n19+n23=C    ... pb2;
n3+n9+n15+n16+n22=C    ... pd3;    n3+n7+n11+n20+n24=C    ... pb3;
n4+n10+n11+n17+n23=C    ... pd4;    n4+n8+n12+n16+n25=C    ... pb4;
n5+n6+n12+n18+n24=C    ... pd5;    n5+n9+n13+n17+n21=C    ... pb5;

```

Let me show you the new program I dictated for our object recently.

If you want to have the whole list of it, please click at the file end to get that.

```

/**/
/** Core Part of 'CES55PD.c' **/
/**/
#include <stdio.h>
/**/
short int cnt, cnt2;
short u1, u2;
short nm[26], uflg[26];
short st[3601][28];
char thlu[31][26], tllu[241][26];
char mtc[31][241];
/**/
short rw1[5], cl1[5], pd1[5], pb1[5];
short rw2[5], cl2[5], pd2[5], pb2[5];
short rw3[5], cl3[5], pd3[5], pb3[5];
short rw4[5], cl4[5], pd4[5], pb4[5];
short rw5[5], cl5[5], pd5[5], pb5[5];
/**/
/* Main Program */
main(){
short n;
printf("\n** Pan-diagonal Magic Squares of Order 5: **\n");
printf("** Made by 'New Euler's Method' with /D5i **\n");
for(n=0; n<26; n++){nm[n]=0;}
for(n=0; n<5; n++){
rw1[n]=0; cl1[n]=0; pd1[n]=0; pb1[n]=0;

```

```

    rw2[n]=0; cl 2[n]=0; pd2[n]=0; pb2[n]=0;
    rw3[n]=0; cl 3[n]=0; pd3[n]=0; pb3[n]=0;
    rw4[n]=0; cl 4[n]=0; pd4[n]=0; pb4[n]=0;
    rw5[n]=0; cl 5[n]=0; pd5[n]=0; pb5[n]=0;
}
cnt=0; cnt2=0;
stp01();      /* Make the Layer Units */
printf("\n [Layer Units]\n");
prlunit();    /* Print the Layer Units */
cnt=0;
printf("\n [Compositions: Used Units// List Number#]\n");
cmbcmp();     /* Combine and Compose */
rtsol(cnt);   /* Sort the Solutions */
prsol(1,cnt); /* Print the Solution List */
printf(" [Count = %d]\n",cnt);
printf("  OK!\n");
return 0;
}
/* Make the Layer Units */
/* Set n1 */
void stp01(){
short a;
for(a=0;a<5;a++){
    if((rw1[a]==0)&&(cl 1[a]==0)&&(pd1[a]==0)&&(pb1[a]==0)){
        nm[1]=a;
        rw1[a]=1; cl 1[a]=1; pd1[a]=1; pb1[a]=1;
        stp02();
        rw1[a]=0; cl 1[a]=0; pd1[a]=0; pb1[a]=0;
    }
}
}
/* Set n2 */
void stp02(){
short a;
for(a=0;a<5;a++){
    if((rw1[a]==0)&&(cl 2[a]==0)&&(pd2[a]==0)&&(pb2[a]==0)){
        nm[2]=a;
        rw1[a]=1; cl 2[a]=1; pd2[a]=1; pb2[a]=1;
        stp03();
        rw1[a]=0; cl 2[a]=0; pd2[a]=0; pb2[a]=0;
    }
}
}
/* Set n3 */
void stp03(){
short a;
for(a=0;a<5;a++){
    if((rw1[a]==0)&&(cl 3[a]==0)&&(pd3[a]==0)&&(pb3[a]==0)){
        nm[3]=a;
        rw1[a]=1; cl 3[a]=1; pd3[a]=1; pb3[a]=1;
        stp04();
        rw1[a]=0; cl 3[a]=0; pd3[a]=0; pb3[a]=0;
    }
}
}
/* Set n4 */
void stp04(){
short a;
for(a=0;a<5;a++){
    if((rw1[a]==0)&&(cl 4[a]==0)&&(pd4[a]==0)&&(pb4[a]==0)){
        nm[4]=a;
        rw1[a]=1; cl 4[a]=1; pd4[a]=1; pb4[a]=1;
    }
}
}

```

```

        stp05();
        rw1[a]=0; cl 4[a]=0; pd4[a]=0; pb4[a]=0;
    }}
}
/* Set n5 */
void stp05(){
    short a;
    for(a=0; a<5; a++){
        if((rw1[a]==0)&&(cl 5[a]==0)&&(pd5[a]==0)&&(pb5[a]==0)){
            nm[5]=a;
            rw1[a]=1; cl 5[a]=1; pd5[a]=1; pb5[a]=1;
            stp06();
            rw1[a]=0; cl 5[a]=0; pd5[a]=0; pb5[a]=0;
        }
    }
}
/* Level 2: */
/* Set n6 */
void stp06(){
    short a;
    for(a=0; a<5; a++){
        if((rw2[a]==0)&&(cl 1[a]==0)&&(pd5[a]==0)&&(pb2[a]==0)){
            nm[6]=a;
            rw2[a]=1; cl 1[a]=1; pd5[a]=1; pb2[a]=1;
            stp07();
            rw2[a]=0; cl 1[a]=0; pd5[a]=0; pb2[a]=0;
        }
    }
}
/* Set n7 */
void stp07(){
    short a;
    for(a=0; a<5; a++){
        if((rw2[a]==0)&&(cl 2[a]==0)&&(pd1[a]==0)&&(pb3[a]==0)){
            nm[7]=a;
            rw2[a]=1; cl 2[a]=1; pd1[a]=1; pb3[a]=1;
            stp08();
            rw2[a]=0; cl 2[a]=0; pd1[a]=0; pb3[a]=0;
        }
    }
}
/**/
. . . . .
/**/
/* Set n25 */
void stp25(){
    short a;
    for(a=0; a<5; a++){
        if((rw5[a]==0)&&(cl 5[a]==0)&&(pd1[a]==0)&&(pb4[a]==0)){
            nm[25]=a;
            rw5[a]=1; cl 5[a]=1; pd1[a]=1; pb4[a]=1;
            ansrecord();
            rw5[a]=0; cl 5[a]=0; pd1[a]=0; pb4[a]=0;
        }
    }
}
/**/
/* Record the Answers */
void ansrecord(){
    short n;
    tll u[cnt][0]=cnt+1;
    for(n=1; n<26; n++){tll u[cnt][n]=nm[n]; }
    if((nm[1]<nm[25])&&(nm[1]<=nm[5])&&(nm[1]<=nm[21])&&(nm[2]>=nm[6])){

```

```

    thl u[cnt2][0]=cnt2+1;
    for(n=1; n<26; n++){ thl u[cnt2][n]=nm[n]; }
    cnt2++; }
cnt++;
}
/**/
. . . . .

```

I could get the set of high layer units smaller into 1/8 of original size.
The next list shows the total result of my recent calculation:

** Pan-diagonal Magic Squares of Order 5: **
** Made by 'New Euler's Method' with /D5i **

[Layer Units]

[for High]

1/H	2/H	3/H	4/H	5/H	6/H
0 2 1 3 4	0 2 1 4 3	0 2 3 1 4	0 2 4 1 3	0 3 1 2 4	0 3 1 2 4
1 3 4 0 2	1 4 3 0 2	1 4 0 2 3	1 3 0 2 4	1 2 4 0 3	2 4 0 3 1
4 0 2 1 3	3 0 2 1 4	2 3 1 4 0	2 4 1 3 0	4 0 3 1 2	3 1 2 4 0
2 1 3 4 0	2 1 4 3 0	4 0 2 3 1	3 0 2 4 1	3 1 2 4 0	4 0 3 1 2
3 4 0 2 1	4 3 0 2 1	3 1 4 0 2	4 1 3 0 2	2 4 0 3 1	1 2 4 0 3
7/H	8/H	9/H	10/H	11/H	12/H
0 3 1 4 2	0 3 2 1 4	0 3 2 1 4	0 3 2 4 1	0 3 4 1 2	0 3 4 2 1
1 4 2 0 3	1 4 0 3 2	2 1 4 0 3	2 4 1 0 3	1 2 0 3 4	2 1 0 3 4
2 0 3 1 4	3 2 1 4 0	4 0 3 2 1	1 0 3 2 4	3 4 1 2 0	3 4 2 1 0
3 1 4 2 0	4 0 3 2 1	3 2 1 4 0	3 2 4 1 0	2 0 3 4 1	1 0 3 4 2
4 2 0 3 1	2 1 4 0 3	1 4 0 3 2	4 1 0 3 2	4 1 2 0 3	4 2 1 0 3
13/H	14/H	15/H	16/H	17/H	18/H
0 4 1 2 3	0 4 1 2 3	0 4 1 3 2	0 4 1 3 2	0 4 2 1 3	0 4 2 1 3
1 2 3 0 4	2 3 0 4 1	1 3 2 0 4	3 2 0 4 1	1 3 0 4 2	2 1 3 0 4
3 0 4 1 2	4 1 2 3 0	2 0 4 1 3	4 1 3 2 0	4 2 1 3 0	3 0 4 2 1
4 1 2 3 0	3 0 4 1 2	4 1 3 2 0	2 0 4 1 3	3 0 4 2 1	4 2 1 3 0
2 3 0 4 1	1 2 3 0 4	3 2 0 4 1	1 3 2 0 4	2 1 3 0 4	1 3 0 4 2
19/H	20/H	21/H	22/H	23/H	24/H
0 4 2 3 1	0 4 2 3 1	0 4 3 1 2	0 4 3 1 2	0 4 3 2 1	0 4 3 2 1
2 3 1 0 4	3 1 0 4 2	1 2 0 4 3	3 1 2 0 4	2 1 0 4 3	3 2 1 0 4
1 0 4 2 3	4 2 3 1 0	4 3 1 2 0	2 0 4 3 1	4 3 2 1 0	1 0 4 3 2
4 2 3 1 0	1 0 4 2 3	2 0 4 3 1	4 3 1 2 0	1 0 4 3 2	4 3 2 1 0
3 1 0 4 2	2 3 1 0 4	3 1 2 0 4	1 2 0 4 3	3 2 1 0 4	2 1 0 4 3
25/H	26/H	27/H	28/H	29/H	30/H
1 2 3 0 4	1 2 4 0 3	1 3 2 0 4	1 3 4 0 2	1 4 2 0 3	1 4 3 0 2
0 4 1 2 3	0 3 1 2 4	0 4 1 3 2	0 2 1 3 4	0 3 1 4 2	0 2 1 4 3
2 3 0 4 1	2 4 0 3 1	3 2 0 4 1	3 4 0 2 1	4 2 0 3 1	4 3 0 2 1
4 1 2 3 0	3 1 2 4 0	4 1 3 2 0	2 1 3 4 0	3 1 4 2 0	2 1 4 3 0
3 0 4 1 2	4 0 3 1 2	2 0 4 1 3	4 0 2 1 3	2 0 3 1 4	3 0 2 1 4

[for Low]

1/L	2/L	3/L	4/L	5/L	6/L
0 1 2 3 4	0 1 2 3 4	0 1 2 4 3	0 1 2 4 3	0 1 3 2 4	0 1 3 2 4
2 3 4 0 1	3 4 0 1 2	2 4 3 0 1	4 3 0 1 2	2 4 0 1 3	3 2 4 0 1
4 0 1 2 3	1 2 3 4 0	3 0 1 2 4	1 2 4 3 0	1 3 2 4 0	4 0 1 3 2
1 2 3 4 0	4 0 1 2 3	1 2 4 3 0	3 0 1 2 4	4 0 1 3 2	1 3 2 4 0
3 4 0 1 2	2 3 4 0 1	4 3 0 1 2	2 4 3 0 1	3 2 4 0 1	2 4 0 1 3
41/L	42/L	43/L	44/L	45/L	46/L
0 4 2 1 3	0 4 2 1 3	0 4 2 3 1	0 4 2 3 1	0 4 3 1 2	0 4 3 1 2
1 3 0 4 2	2 1 3 0 4	2 3 1 0 4	3 1 0 4 2	1 2 0 4 3	3 1 2 0 4
4 2 1 3 0	3 0 4 2 1	1 0 4 2 3	4 2 3 1 0	4 3 1 2 0	2 0 4 3 1
3 0 4 2 1	4 2 1 3 0	4 2 3 1 0	1 0 4 2 3	2 0 4 3 1	4 3 1 2 0
2 1 3 0 4	1 3 0 4 2	3 1 0 4 2	2 3 1 0 4	3 1 2 0 4	1 2 0 4 3

81/L	82/L	83/L	84/L	85/L	86/L
1 3 4 0 2	1 3 4 0 2	1 3 4 2 0	1 3 4 2 0	1 4 0 2 3	1 4 0 2 3
0 2 1 3 4	4 0 2 1 3	2 0 1 3 4	4 2 0 1 3	0 2 3 1 4	2 3 1 4 0
3 4 0 2 1	2 1 3 4 0	3 4 2 0 1	0 1 3 4 2	3 1 4 0 2	4 0 2 3 1
2 1 3 4 0	3 4 0 2 1	0 1 3 4 2	3 4 2 0 1	4 0 2 3 1	3 1 4 0 2
4 0 2 1 3	0 2 1 3 4	4 2 0 1 3	2 0 1 3 4	2 3 1 4 0	0 2 3 1 4
121/L	122/L	123/L	124/L	125/L	126/L
2 3 0 1 4	2 3 0 1 4	2 3 0 4 1	2 3 0 4 1	2 3 1 0 4	2 3 1 0 4
0 1 4 2 3	1 4 2 3 0	0 4 1 2 3	4 1 2 3 0	0 4 2 3 1	1 0 4 2 3
4 2 3 0 1	3 0 1 4 2	1 2 3 0 4	3 0 4 1 2	3 1 0 4 2	4 2 3 1 0
3 0 1 4 2	4 2 3 0 1	3 0 4 1 2	1 2 3 0 4	4 2 3 1 0	3 1 0 4 2
1 4 2 3 0	0 1 4 2 3	4 1 2 3 0	0 4 1 2 3	1 0 4 2 3	0 4 2 3 1
161/L	162/L	163/L	164/L	165/L	166/L
3 1 2 0 4	3 1 2 0 4	3 1 2 4 0	3 1 2 4 0	3 1 4 0 2	3 1 4 0 2
0 4 3 1 2	2 0 4 3 1	2 4 0 3 1	4 0 3 1 2	0 2 3 1 4	4 0 2 3 1
1 2 0 4 3	4 3 1 2 0	0 3 1 2 4	1 2 4 0 3	1 4 0 2 3	2 3 1 4 0
4 3 1 2 0	1 2 0 4 3	1 2 4 0 3	0 3 1 2 4	2 3 1 4 0	1 4 0 2 3
2 0 4 3 1	0 4 3 1 2	4 0 3 1 2	2 4 0 3 1	4 0 2 3 1	0 2 3 1 4
201/L	202/L	203/L	204/L	205/L	206/L
4 0 3 1 2	4 0 3 1 2	4 0 3 2 1	4 0 3 2 1	4 1 0 2 3	4 1 0 2 3
1 2 4 0 3	3 1 2 4 0	2 1 4 0 3	3 2 1 4 0	0 2 3 4 1	2 3 4 1 0
0 3 1 2 4	2 4 0 3 1	0 3 2 1 4	1 4 0 3 2	3 4 1 0 2	1 0 2 3 4
2 4 0 3 1	0 3 1 2 4	1 4 0 3 2	0 3 2 1 4	1 0 2 3 4	3 4 1 0 2
3 1 2 4 0	1 2 4 0 3	3 2 1 4 0	2 1 4 0 3	2 3 4 1 0	0 2 3 4 1

[Count of Layer Units = 30H/240L]

[Reference Table for the Best Combination]

SC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	15	5	5	5	5	10	5	5	5	5	5	10	25	5	15	5	5	15	10	5	5	10	5	15
2	5	5	15	5	5	5	10	5	5	10	5	5	15	5	25	5	5	10	15	5	5	15	5	10
3	5	10	5	5	15	5	5	10	5	5	5	5	5	15	5	10	25	5	5	15	15	5	10	5
4	5	5	5	10	5	5	5	5	15	5	10	5	5	10	5	15	15	5	5	10	25	5	15	5
5	10	5	5	5	5	15	10	5	5	10	5	5	15	5	10	5	5	10	5	5	5	5	5	5
6	5	10	5	5	15	5	5	10	10	5	5	5	5	15	5	10	10	5	5	5	5	5	5	5
7	5	5	10	5	5	10	15	5	5	5	5	10	10	5	15	5	5	5	10	5	5	5	5	5
8	5	15	5	10	10	5	5	5	5	5	10	5	5	10	5	5	15	5	5	5	10	5	5	5
9	15	5	10	5	5	10	5	5	5	5	5	10	10	5	5	5	5	15	5	5	5	10	5	5
10	10	5	15	5	5	5	10	5	5	10	5	5	5	5	10	5	5	5	15	5	5	5	5	10
11	5	10	5	5	5	5	5	10	10	5	15	5	5	5	5	5	10	5	5	5	15	5	10	5
12	5	5	5	10	10	5	5	5	15	5	10	5	5	5	5	5	5	5	5	10	10	5	15	5
13	5	5	10	5	5	10	5	5	5	15	5	5	10	5	15	5	5	5	10	5	5	10	5	5
14	5	5	5	10	10	5	5	5	15	5	5	5	5	10	5	15	5	5	5	10	10	5	5	5
15	10	5	5	5	5	5	10	5	5	5	5	15	15	5	10	5	5	10	5	5	5	5	5	10
16	5	10	5	5	5	5	5	10	5	5	15	5	5	15	5	10	10	5	5	5	5	5	10	5
17	5	10	5	15	5	5	5	5	10	5	5	5	5	5	5	10	10	5	5	5	15	5	10	5
18	10	5	15	5	5	5	5	5	5	10	5	5	5	5	10	5	5	10	5	5	5	15	5	10
19	15	5	10	5	5	5	5	5	5	5	5	10	10	5	5	5	5	5	10	5	5	10	5	15
20	5	15	5	10	5	5	5	5	5	5	10	5	5	10	5	5	5	5	5	10	10	5	15	5
21	5	5	5	5	10	5	5	15	5	5	10	5	5	10	5	5	15	5	5	10	10	5	5	5
22	5	5	5	5	5	10	15	5	5	5	5	10	10	5	5	5	5	15	10	5	5	10	5	5
23	5	5	5	5	15	5	5	10	10	5	5	5	5	5	5	10	10	5	5	15	5	5	10	5
24	5	5	5	5	5	15	10	5	5	10	5	5	5	5	10	5	5	10	15	5	5	5	5	10
25	5	5	5	5	10	5	5	5	0	5	0	5	5	10	5	15	15	5	5	10	5	5	5	5
26	5	0	5	5	0	5	5	5	10	5	5	5	5	5	5	10	5	5	5	15	15	5	10	5
27	5	10	5	15	5	5	5	10	5	5	5	5	5	5	5	5	5	5	5	5	5	0	5	0
28	5	5	5	10	5	5	5	15	5	5	10	5	5	0	5	5	0	5	5	5	5	5	5	5
29	5	5	5	10	0	5	5	5	5	5	0	5	5	0	5	5	0	5	5	10	5	5	5	5
30	5	0	5	5	5	5	5	10	0	5	5	5	5	5	5	10	5	5	5	5	0	5	0	5

#2	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
1	15	5	10	5	5	10	15	5	5	15	5	25	10	5	5	5	5	5	10	5	5	5	5	15
2	10	5	15	5	5	15	10	5	5	25	5	15	5	5	10	5	5	10	5	5	5	15	5	5
3	5	10	5	15	15	5	5	25	10	5	15	5	5	5	5	10	5	5	15	5	5	10	5	5
4	5	15	5	25	10	5	5	15	15	5	10	5	5	10	5	15	5	5	5	5	10	5	5	5
5	5	5	5	5	5	5	10	5	5	10	5	15	5	5	10	5	5	10	15	5	5	5	5	10
6	5	5	5	5	5	5	5	10	10	5	15	5	5	5	5	10	10	5	5	15	5	5	10	5
7	5	5	5	5	5	10	5	5	5	15	5	10	10	5	5	5	5	15	10	5	5	10	5	5
8	5	5	5	10	5	5	5	15	5	5	10	5	5	10	5	5	5	5	5	10	10	5	15	5
9	5	5	10	5	5	5	15	5	5	5	5	10	10	5	5	5	5	5	10	5	5	10	5	15
10	10	5	5	5	5	15	5	5	5	10	5	5	5	5	10	5	5	10	5	5	5	15	5	10
11	5	10	5	15	5	5	5	10	5	5	5	5	5	15	5	10	10	5	5	5	5	5	10	5
12	5	15	5	10	10	5	5	5	5	5	5	5	5	10	5	15	5	5	5	10	10	5	5	5
13	5	5	10	5	5	10	5	5	5	15	5	10	5	5	15	5	5	5	10	5	5	10	5	5
14	5	5	5	10	10	5	5	5	15	5	10	5	5	5	5	15	5	5	5	10	10	5	5	5
15	10	5	5	5	5	5	10	5	5	10	5	15	15	5	5	5	5	10	5	5	5	5	5	10
16	5	10	5	5	5	5	5	10	10	5	15	5	5	15	5	5	10	5	5	5	5	5	10	5
17	5	10	5	15	5	5	5	10	10	5	5	5	5	5	5	10	5	5	5	5	15	5	10	5
18	10	5	15	5	5	5	10	5	5	10	5	5	5	5	10	5	5	5	5	5	5	15	5	10
19	15	5	10	5	5	10	5	5	5	5	5	10	10	5	5	5	5	5	5	5	5	10	5	15
20	5	15	5	10	10	5	5	5	5	5	10	5	5	10	5	5	5	5	5	5	5	10	5	15
21	5	5	5	10	10	5	5	15	5	5	10	5	5	10	5	5	15	5	5	10	5	5	5	5
22	5	5	10	5	5	10	15	5	5	5	5	10	10	5	5	5	5	15	10	5	5	5	5	5
23	5	10	5	5	15	5	5	10	10	5	5	5	5	5	5	10	10	5	5	15	5	5	5	5
24	10	5	5	5	5	15	10	5	5	10	5	5	5	5	10	5	5	10	15	5	5	5	5	5
25	5	5	5	5	10	5	5	15	15	5	10	5	5	5	0	5	5	5	10	5	5	5	5	5
26	5	10	5	15	15	5	5	5	10	5	5	5	5	5	5	10	5	5	5	5	0	5	5	0
27	5	0	5	0	5	5	5	5	5	5	5	5	5	5	5	10	5	5	5	15	5	10	5	5
28	5	5	5	5	5	5	5	0	5	5	0	5	5	10	5	5	15	5	5	5	10	5	5	5
29	5	5	5	5	10	5	5	0	5	5	0	5	5	0	5	5	5	5	5	0	10	5	5	5
30	5	0	5	0	5	5	5	5	10	5	5	5	5	5	5	0	10	5	5	5	5	5	0	5

[Compositions: Used Units// List Number#(Sorted)]

1/H	2/L	1#	1/H	4/L	2#
0 2 1 3 4	0 1 2 3 4	1 12 8 19 25	0 2 1 3 4	0 1 2 4 3	1 12 8 20 24
1 3 4 0 2	3 4 0 1 2	9 20 21 2 13	1 3 4 0 2	4 3 0 1 2	10 19 21 2 13
4 0 2 1 3	1 2 3 4 0	22 3 14 10 16	4 0 2 1 3	1 2 4 3 0	22 3 15 9 16
2 1 3 4 0	4 0 1 2 3	15 6 17 23 4	2 1 3 4 0	3 0 1 2 4	14 6 17 23 5
3 4 0 2 1	2 3 4 0 1	18 24 5 11 7	3 4 0 2 1	2 4 3 0 1	18 25 4 11 7
1/H	5/L	5#	1/H	8/L	6#
0 2 1 3 4	0 1 3 2 4	1 12 9 18 25	0 2 1 3 4	0 1 3 4 2	1 12 9 20 23
1 3 4 0 2	2 4 0 1 3	8 20 21 2 14	1 3 4 0 2	4 2 0 1 3	10 18 21 2 14
4 0 2 1 3	1 3 2 4 0	22 4 13 10 16	4 0 2 1 3	1 3 4 2 0	22 4 15 8 16
2 1 3 4 0	4 0 1 3 2	15 6 17 24 3	2 1 3 4 0	2 0 1 3 4	13 6 17 24 5
3 4 0 2 1	3 2 4 0 1	19 23 5 11 7	3 4 0 2 1	3 4 2 0 1	19 25 3 11 7
1/H	9/L	9#	1/H	11/L	10#
0 2 1 3 4	0 1 4 2 3	1 12 10 18 24	0 2 1 3 4	0 1 4 3 2	1 12 10 19 23
1 3 4 0 2	2 3 0 1 4	8 19 21 2 15	1 3 4 0 2	3 2 0 1 4	9 18 21 2 15
4 0 2 1 3	1 4 2 3 0	22 5 13 9 16	4 0 2 1 3	1 4 3 2 0	22 5 14 8 16
2 1 3 4 0	3 0 1 4 2	14 6 17 25 3	2 1 3 4 0	2 0 1 4 3	13 6 17 25 4
3 4 0 2 1	4 2 3 0 1	20 23 4 11 7	3 4 0 2 1	4 3 2 0 1	20 24 3 11 7
3/H	1/L	13#	3/H	3/L	14#
0 2 3 1 4	0 1 2 3 4	1 12 18 9 25	0 2 3 1 4	0 1 2 4 3	1 12 18 10 24
1 4 0 2 3	2 3 4 0 1	8 24 5 11 17	1 4 0 2 3	2 4 3 0 1	8 25 4 11 17
2 3 1 4 0	4 0 1 2 3	15 16 7 23 4	2 3 1 4 0	3 0 1 2 4	14 16 7 23 5
4 0 2 3 1	1 2 3 4 0	22 3 14 20 6	4 0 2 3 1	1 2 4 3 0	22 3 15 19 6
3 1 4 0 2	3 4 0 1 2	19 10 21 2 13	3 1 4 0 2	4 3 0 1 2	20 9 21 2 13

3/H	6/L	15#	3/H	7/L	16#
0 2 3 1 4	0 1 3 2 4	1 12 19 8 25	0 2 3 1 4	0 1 3 4 2	1 12 19 10 23
1 4 0 2 3	3 2 4 0 1	9 23 5 11 17	1 4 0 2 3	3 4 2 0 1	9 25 3 11 17
2 3 1 4 0	4 0 1 3 2	15 16 7 24 3	2 3 1 4 0	2 0 1 3 4	13 16 7 24 5
4 0 2 3 1	1 3 2 4 0	22 4 13 20 6	4 0 2 3 1	1 3 4 2 0	22 4 15 18 6
3 1 4 0 2	2 4 0 1 3	18 10 21 2 14	3 1 4 0 2	4 2 0 1 3	20 8 21 2 14
3/H	10/L	17#	3/H	12/L	18#
0 2 3 1 4	0 1 4 2 3	1 12 20 8 24	0 2 3 1 4	0 1 4 3 2	1 12 20 9 23
1 4 0 2 3	4 2 3 0 1	10 23 4 11 17	1 4 0 2 3	4 3 2 0 1	10 24 3 11 17
2 3 1 4 0	3 0 1 4 2	14 16 7 25 3	2 3 1 4 0	2 0 1 4 3	13 16 7 25 4
4 0 2 3 1	1 4 2 3 0	22 5 13 19 6	4 0 2 3 1	1 4 3 2 0	22 5 14 18 6
3 1 4 0 2	2 3 0 1 4	18 9 21 2 15	3 1 4 0 2	3 2 0 1 4	19 8 21 2 15
4/H	1/L	19#	4/H	3/L	20#
0 2 4 1 3	0 1 2 3 4	1 12 23 9 20	0 2 4 1 3	0 1 2 4 3	1 12 23 10 19
1 3 0 2 4	2 3 4 0 1	8 19 5 11 22	1 3 0 2 4	2 4 3 0 1	8 20 4 11 22
2 4 1 3 0	4 0 1 2 3	15 21 7 18 4	2 4 1 3 0	3 0 1 2 4	14 21 7 18 5
3 0 2 4 1	1 2 3 4 0	17 3 14 25 6	3 0 2 4 1	1 2 4 3 0	17 3 15 24 6
4 1 3 0 2	3 4 0 1 2	24 10 16 2 13	4 1 3 0 2	4 3 0 1 2	25 9 16 2 13
4/H	6/L	21#	4/H	7/L	22#
0 2 4 1 3	0 1 3 2 4	1 12 24 8 20	0 2 4 1 3	0 1 3 4 2	1 12 24 10 18
1 3 0 2 4	3 2 4 0 1	9 18 5 11 22	1 3 0 2 4	3 4 2 0 1	9 20 3 11 22
2 4 1 3 0	4 0 1 3 2	15 21 7 19 3	2 4 1 3 0	2 0 1 3 4	13 21 7 19 5
3 0 2 4 1	1 3 2 4 0	17 4 13 25 6	3 0 2 4 1	1 3 4 2 0	17 4 15 23 6
4 1 3 0 2	2 4 0 1 3	23 10 16 2 14	4 1 3 0 2	4 2 0 1 3	25 8 16 2 14
4/H	10/L	23#	4/H	12/L	24#
0 2 4 1 3	0 1 4 2 3	1 12 25 8 19	0 2 4 1 3	0 1 4 3 2	1 12 25 9 18
1 3 0 2 4	4 2 3 0 1	10 18 4 11 22	1 3 0 2 4	4 3 2 0 1	10 19 3 11 22
2 4 1 3 0	3 0 1 4 2	14 21 7 20 3	2 4 1 3 0	2 0 1 4 3	13 21 7 20 4
3 0 2 4 1	1 4 2 3 0	17 5 13 24 6	3 0 2 4 1	1 4 3 2 0	17 5 14 23 6
4 1 3 0 2	2 3 0 1 4	23 9 16 2 15	4 1 3 0 2	3 2 0 1 4	24 8 16 2 15
1/H	14/L	25#	1/H	16/L	26#
0 2 1 3 4	0 2 1 3 4	1 13 7 19 25	0 2 1 3 4	0 2 1 4 3	1 13 7 20 24
1 3 4 0 2	3 4 0 2 1	9 20 21 3 12	1 3 4 0 2	4 3 0 2 1	10 19 21 3 12
4 0 2 1 3	2 1 3 4 0	23 2 14 10 16	4 0 2 1 3	2 1 4 3 0	23 2 15 9 16
2 1 3 4 0	4 0 2 1 3	15 6 18 22 4	2 1 3 4 0	3 0 2 1 4	14 6 18 22 5
3 4 0 2 1	1 3 4 0 2	17 24 5 11 8	3 4 0 2 1	1 4 3 0 2	17 25 4 11 8
1/H	26/L	49#	1/H	28/L	50#
0 2 1 3 4	0 3 1 2 4	1 14 7 18 25	0 2 1 3 4	0 3 1 4 2	1 14 7 20 23
1 3 4 0 2	2 4 0 3 1	8 20 21 4 12	1 3 4 0 2	4 2 0 3 1	10 18 21 4 12
4 0 2 1 3	3 1 2 4 0	24 2 13 10 16	4 0 2 1 3	3 1 4 2 0	24 2 15 8 16
2 1 3 4 0	4 0 3 1 2	15 6 19 22 3	2 1 3 4 0	2 0 3 1 4	13 6 19 22 5
3 4 0 2 1	1 2 4 0 3	17 23 5 11 9	3 4 0 2 1	1 4 2 0 3	17 25 3 11 9
1/H	38/L	73#	1/H	40/L	74#
0 2 1 3 4	0 4 1 2 3	1 15 7 18 24	0 2 1 3 4	0 4 1 3 2	1 15 7 19 23
1 3 4 0 2	2 3 0 4 1	8 19 21 5 12	1 3 4 0 2	3 2 0 4 1	9 18 21 5 12
4 0 2 1 3	4 1 2 3 0	25 2 13 9 16	4 0 2 1 3	4 1 3 2 0	25 2 14 8 16
2 1 3 4 0	3 0 4 1 2	14 6 20 22 3	2 1 3 4 0	2 0 4 1 3	13 6 20 22 4
3 4 0 2 1	1 2 3 0 4	17 23 4 11 10	3 4 0 2 1	1 3 2 0 4	17 24 3 11 10
5/H	2/L	97#	6/H	1/L	98#
0 3 1 2 4	0 1 2 3 4	1 17 8 14 25	0 3 1 2 4	0 1 2 3 4	1 17 8 14 25
1 2 4 0 3	3 4 0 1 2	9 15 21 2 18	2 4 0 3 1	2 3 4 0 1	13 24 5 16 7
4 0 3 1 2	1 2 3 4 0	22 3 19 10 11	3 1 2 4 0	4 0 1 2 3	20 6 12 23 4
3 1 2 4 0	4 0 1 2 3	20 6 12 23 4	4 0 3 1 2	1 2 3 4 0	22 3 19 10 11
2 4 0 3 1	2 3 4 0 1	13 24 5 16 7	1 2 4 0 3	3 4 0 1 2	9 15 21 2 18

5/H	14/L	145#	6/H	13/L	146#
0 3 1 2 4	0 2 1 3 4	1 18 7 14 25	0 3 1 2 4	0 2 1 3 4	1 18 7 14 25
1 2 4 0 3	3 4 0 2 1	9 15 21 3 17	2 4 0 3 1	1 3 4 0 2	12 24 5 16 8
4 0 3 1 2	2 1 3 4 0	23 2 19 10 11	3 1 2 4 0	4 0 2 1 3	20 6 13 22 4
3 1 2 4 0	4 0 2 1 3	20 6 13 22 4	4 0 3 1 2	2 1 3 4 0	23 2 19 10 11
2 4 0 3 1	1 3 4 0 2	12 24 5 16 8	1 2 4 0 3	3 4 0 2 1	9 15 21 3 17
5/H	26/L	193#	6/H	25/L	194#
0 3 1 2 4	0 3 1 2 4	1 19 7 13 25	0 3 1 2 4	0 3 1 2 4	1 19 7 13 25
1 2 4 0 3	2 4 0 3 1	8 15 21 4 17	2 4 0 3 1	1 2 4 0 3	12 23 5 16 9
4 0 3 1 2	3 1 2 4 0	24 2 18 10 11	3 1 2 4 0	4 0 3 1 2	20 6 14 22 3
3 1 2 4 0	4 0 3 1 2	20 6 14 22 3	4 0 3 1 2	3 1 2 4 0	24 2 18 10 11
2 4 0 3 1	1 2 4 0 3	12 23 5 16 9	1 2 4 0 3	2 4 0 3 1	8 15 21 4 17
5/H	38/L	241#	6/H	37/L	242#
0 3 1 2 4	0 4 1 2 3	1 20 7 13 24	0 3 1 2 4	0 4 1 2 3	1 20 7 13 24
1 2 4 0 3	2 3 0 4 1	8 14 21 5 17	2 4 0 3 1	1 2 3 0 4	12 23 4 16 10
4 0 3 1 2	4 1 2 3 0	25 2 18 9 11	3 1 2 4 0	3 0 4 1 2	19 6 15 22 3
3 1 2 4 0	3 0 4 1 2	19 6 15 22 3	4 0 3 1 2	4 1 2 3 0	25 2 18 9 11
2 4 0 3 1	1 2 3 0 4	12 23 4 16 10	1 2 4 0 3	2 3 0 4 1	8 14 21 5 17
13/H	2/L	289#	14/H	1/L	290#
0 4 1 2 3	0 1 2 3 4	1 22 8 14 20	0 4 1 2 3	0 1 2 3 4	1 22 8 14 20
1 2 3 0 4	3 4 0 1 2	9 15 16 2 23	2 3 0 4 1	2 3 4 0 1	13 19 5 21 7
3 0 4 1 2	1 2 3 4 0	17 3 24 10 11	4 1 2 3 0	4 0 1 2 3	25 6 12 18 4
4 1 2 3 0	4 0 1 2 3	25 6 12 18 4	3 0 4 1 2	1 2 3 4 0	17 3 24 10 11
2 3 0 4 1	2 3 4 0 1	13 19 5 21 7	1 2 3 0 4	3 4 0 1 2	9 15 16 2 23
13/H	14/L	361#	14/H	13/L	362#
0 4 1 2 3	0 2 1 3 4	1 23 7 14 20	0 4 1 2 3	0 2 1 3 4	1 23 7 14 20
1 2 3 0 4	3 4 0 2 1	9 15 16 3 22	2 3 0 4 1	1 3 4 0 2	12 19 5 21 8
3 0 4 1 2	2 1 3 4 0	18 2 24 10 11	4 1 2 3 0	4 0 2 1 3	25 6 13 17 4
4 1 2 3 0	4 0 2 1 3	25 6 13 17 4	3 0 4 1 2	2 1 3 4 0	18 2 24 10 11
2 3 0 4 1	1 3 4 0 2	12 19 5 21 8	1 2 3 0 4	3 4 0 2 1	9 15 16 3 22
13/H	26/L	433#	14/H	25/L	434#
0 4 1 2 3	0 3 1 2 4	1 24 7 13 20	0 4 1 2 3	0 3 1 2 4	1 24 7 13 20
1 2 3 0 4	2 4 0 3 1	8 15 16 4 22	2 3 0 4 1	1 2 4 0 3	12 18 5 21 9
3 0 4 1 2	3 1 2 4 0	19 2 23 10 11	4 1 2 3 0	4 0 3 1 2	25 6 14 17 3
4 1 2 3 0	4 0 3 1 2	25 6 14 17 3	3 0 4 1 2	3 1 2 4 0	19 2 23 10 11
2 3 0 4 1	1 2 4 0 3	12 18 5 21 9	1 2 3 0 4	2 4 0 3 1	8 15 16 4 22
13/H	38/L	505#	14/H	37/L	506#
0 4 1 2 3	0 4 1 2 3	1 25 7 13 19	0 4 1 2 3	0 4 1 2 3	1 25 7 13 19
1 2 3 0 4	2 3 0 4 1	8 14 16 5 22	2 3 0 4 1	1 2 3 0 4	12 18 4 21 10
3 0 4 1 2	4 1 2 3 0	20 2 23 9 11	4 1 2 3 0	3 0 4 1 2	24 6 15 17 3
4 1 2 3 0	3 0 4 1 2	24 6 15 17 3	3 0 4 1 2	4 1 2 3 0	20 2 23 9 11
2 3 0 4 1	1 2 3 0 4	12 18 4 21 10	1 2 3 0 4	2 3 0 4 1	8 14 16 5 22
1/H	50/L	577#	1/H	52/L	578#
0 2 1 3 4	1 0 2 3 4	2 11 8 19 25	0 2 1 3 4	1 0 2 4 3	2 11 8 20 24
1 3 4 0 2	3 4 1 0 2	9 20 22 1 13	1 3 4 0 2	4 3 1 0 2	10 19 22 1 13
4 0 2 1 3	0 2 3 4 1	21 3 14 10 17	4 0 2 1 3	0 2 4 3 1	21 3 15 9 17
2 1 3 4 0	4 1 0 2 3	15 7 16 23 4	2 1 3 4 0	3 1 0 2 4	14 7 16 23 5
3 4 0 2 1	2 3 4 1 0	18 24 5 12 6	3 4 0 2 1	2 4 3 1 0	18 25 4 12 6
1/H	98/L	1153#	1/H	100/L	1154#
0 2 1 3 4	2 0 1 3 4	3 11 7 19 25	0 2 1 3 4	2 0 1 4 3	3 11 7 20 24
1 3 4 0 2	3 4 2 0 1	9 20 23 1 12	1 3 4 0 2	4 3 2 0 1	10 19 23 1 12
4 0 2 1 3	0 1 3 4 2	21 2 14 10 18	4 0 2 1 3	0 1 4 3 2	21 2 15 9 18
2 1 3 4 0	4 2 0 1 3	15 8 16 22 4	2 1 3 4 0	3 2 0 1 4	14 8 16 22 5
3 4 0 2 1	1 3 4 2 0	17 24 5 13 6	3 4 0 2 1	1 4 3 2 0	17 25 4 13 6

1/H	146/L	1729#	1/H	148/L	1730#
0 2 1 3 4	3 0 1 2 4	4 11 7 18 25	0 2 1 3 4	3 0 1 4 2	4 11 7 20 23
1 3 4 0 2	2 4 3 0 1	8 20 24 1 12	1 3 4 0 2	4 2 3 0 1	10 18 24 1 12
4 0 2 1 3	0 1 2 4 3	21 2 13 10 19	4 0 2 1 3	0 1 4 2 3	21 2 15 8 19
2 1 3 4 0	4 3 0 1 2	15 9 16 22 3	2 1 3 4 0	2 3 0 1 4	13 9 16 22 5
3 4 0 2 1	1 2 4 3 0	17 23 5 14 6	3 4 0 2 1	1 4 2 3 0	17 25 3 14 6
1/H	194/L	2305#	1/H	196/L	2306#
0 2 1 3 4	4 0 1 2 3	5 11 7 18 24	0 2 1 3 4	4 0 1 3 2	5 11 7 19 23
1 3 4 0 2	2 3 4 0 1	8 19 25 1 12	1 3 4 0 2	3 2 4 0 1	9 18 25 1 12
4 0 2 1 3	0 1 2 3 4	21 2 13 9 20	4 0 2 1 3	0 1 3 2 4	21 2 14 8 20
2 1 3 4 0	3 4 0 1 2	14 10 16 22 3	2 1 3 4 0	2 4 0 1 3	13 10 16 22 4
3 4 0 2 1	1 2 3 4 0	17 23 4 15 6	3 4 0 2 1	1 3 2 4 0	17 24 3 15 6
25/H	1/L	2881#	25/H	3/L	2882#
1 2 3 0 4	0 1 2 3 4	6 12 18 4 25	1 2 3 0 4	0 1 2 4 3	6 12 18 5 24
0 4 1 2 3	2 3 4 0 1	3 24 10 11 17	0 4 1 2 3	2 4 3 0 1	3 25 9 11 17
2 3 0 4 1	4 0 1 2 3	15 16 2 23 9	2 3 0 4 1	3 0 1 2 4	14 16 2 23 10
4 1 2 3 0	1 2 3 4 0	22 8 14 20 1	4 1 2 3 0	1 2 4 3 0	22 8 15 19 1
3 0 4 1 2	3 4 0 1 2	19 5 21 7 13	3 0 4 1 2	4 3 0 1 2	20 4 21 7 13
25/H	49/L	3025#	25/H	51/L	3026#
1 2 3 0 4	1 0 2 3 4	7 11 18 4 25	1 2 3 0 4	1 0 2 4 3	7 11 18 5 24
0 4 1 2 3	2 3 4 1 0	3 24 10 12 16	0 4 1 2 3	2 4 3 1 0	3 25 9 12 16
2 3 0 4 1	4 1 0 2 3	15 17 1 23 9	2 3 0 4 1	3 1 0 2 4	14 17 1 23 10
4 1 2 3 0	0 2 3 4 1	21 8 14 20 2	4 1 2 3 0	0 2 4 3 1	21 8 15 19 2
3 0 4 1 2	3 4 1 0 2	19 5 22 6 13	3 0 4 1 2	4 3 1 0 2	20 4 22 6 13
25/H	97/L	3169#	25/H	99/L	3170#
1 2 3 0 4	2 0 1 3 4	8 11 17 4 25	1 2 3 0 4	2 0 1 4 3	8 11 17 5 24
0 4 1 2 3	1 3 4 2 0	2 24 10 13 16	0 4 1 2 3	1 4 3 2 0	2 25 9 13 16
2 3 0 4 1	4 2 0 1 3	15 18 1 22 9	2 3 0 4 1	3 2 0 1 4	14 18 1 22 10
4 1 2 3 0	0 1 3 4 2	21 7 14 20 3	4 1 2 3 0	0 1 4 3 2	21 7 15 19 3
3 0 4 1 2	3 4 2 0 1	19 5 23 6 12	3 0 4 1 2	4 3 2 0 1	20 4 23 6 12
25/H	145/L	3313#	25/H	147/L	3314#
1 2 3 0 4	3 0 1 2 4	9 11 17 3 25	1 2 3 0 4	3 0 1 4 2	9 11 17 5 23
0 4 1 2 3	1 2 4 3 0	2 23 10 14 16	0 4 1 2 3	1 4 2 3 0	2 25 8 14 16
2 3 0 4 1	4 3 0 1 2	15 19 1 22 8	2 3 0 4 1	2 3 0 1 4	13 19 1 22 10
4 1 2 3 0	0 1 2 4 3	21 7 13 20 4	4 1 2 3 0	0 1 4 2 3	21 7 15 18 4
3 0 4 1 2	2 4 3 0 1	18 5 24 6 12	3 0 4 1 2	4 2 3 0 1	20 3 24 6 12
25/H	193/L	3457#	25/H	195/L	3458#
1 2 3 0 4	4 0 1 2 3	10 11 17 3 24	1 2 3 0 4	4 0 1 3 2	10 11 17 4 23
0 4 1 2 3	1 2 3 4 0	2 23 9 15 16	0 4 1 2 3	1 3 2 4 0	2 24 8 15 16
2 3 0 4 1	3 4 0 1 2	14 20 1 22 8	2 3 0 4 1	2 4 0 1 3	13 20 1 22 9
4 1 2 3 0	0 1 2 3 4	21 7 13 19 5	4 1 2 3 0	0 1 3 2 4	21 7 14 18 5
3 0 4 1 2	2 3 4 0 1	18 4 25 6 12	3 0 4 1 2	3 2 4 0 1	19 3 25 6 12

[Count = 3600] OK!

I must mention now that I used the sort program before listing out the solutions to rearrange them, and made the list smarter so that you could read it easier.

3600 solutions composed here are just the same with the ones we have got before. It makes me happy, because it means the verification of our "Knight's Movement".

How fast this program could run! Down-sizing of high layer unit set really makes our program run faster, because it surely saves a lot of memories in our computer.

I must report here about those 16 Simultaneous MS55 made by the same method.

*** Simultaneous Magic Squares of Order 5: ***
 ** Both Self-Complementary and Pan-diagonal **
 *** Made by 'New Euler's Method' with /D5i ***

[Layer Units]

1/	2/	3/	4/	5/	6/	7/	8/
02413	02431	04123	04321	12304	12340	13024	13420
41302	43102	23041	21043	30412	34012	24130	20134
30241	10243	41230	43210	41230	01234	30241	34201
24130	24310	30412	10432	23041	23401	41302	01342
13024	31024	12304	32104	04123	40123	02413	42013
9/	10/	11/	12/	13/	14/	15/	16/
31024	31420	32104	32140	40123	40321	42013	42031
24310	20314	10432	14032	23401	21403	01342	03142
10243	14203	43210	03214	01234	03214	34201	14203
43102	03142	21043	21403	34012	14032	20134	20314
02431	42031	04321	40321	12340	32140	13420	31420

[Count of Layer Units = 16]

** Reference Tables for the Best Combination **

SC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	25	15	5	5	5	5	5	5	5	5	5	5	5	5	15	25
2	15	25	5	5	5	5	5	5	5	5	5	5	5	5	25	15
3	5	5	25	15	5	5	5	5	5	5	5	5	15	25	5	5
4	5	5	15	25	5	5	5	5	5	5	5	5	25	15	5	5
5	5	5	5	5	25	15	5	5	5	5	15	25	5	5	5	5
6	5	5	5	5	15	25	5	5	5	5	25	15	5	5	5	5
7	5	5	5	5	5	5	25	15	15	25	5	5	5	5	5	5
8	5	5	5	5	5	15	25	25	15	5	5	5	5	5	5	5
9	5	5	5	5	5	5	15	25	25	15	5	5	5	5	5	5
10	5	5	5	5	5	5	25	15	15	25	5	5	5	5	5	5
11	5	5	5	5	15	25	5	5	5	5	25	15	5	5	5	5
12	5	5	5	5	25	15	5	5	5	5	15	25	5	5	5	5
13	5	5	15	25	5	5	5	5	5	5	5	5	25	15	5	5
14	5	5	25	15	5	5	5	5	5	5	5	5	15	25	5	5
15	15	25	5	5	5	5	5	5	5	5	5	5	5	5	25	15
16	25	15	5	5	5	5	5	5	5	5	5	5	5	5	15	25

[Compositions: Used Units// List Number#(Sorted)]

2/H	3/L	1#	2/H	4/L	2#
0 2 4 3 1	0 4 1 2 3	1 15 22 18 9	0 2 4 3 1	0 4 3 2 1	1 15 24 18 7
4 3 1 0 2	2 3 0 4 1	23 19 6 5 12	4 3 1 0 2	2 1 0 4 3	23 17 6 5 14
1 0 2 4 3	4 1 2 3 0	10 2 13 24 16	1 0 2 4 3	4 3 2 1 0	10 4 13 22 16
2 4 3 1 0	3 0 4 1 2	14 21 20 7 3	2 4 3 1 0	1 0 4 3 2	12 21 20 9 3
3 1 0 2 4	1 2 3 0 4	17 8 4 11 25	3 1 0 2 4	3 2 1 0 4	19 8 2 11 25
4/H	1/L	3#	4/H	2/L	4#
0 4 3 2 1	0 2 4 1 3	1 23 20 12 9	0 4 3 2 1	0 2 4 3 1	1 23 20 14 7
2 1 0 4 3	4 1 3 0 2	15 7 4 21 18	2 1 0 4 3	4 3 1 0 2	15 9 2 21 18
4 3 2 1 0	3 0 2 4 1	24 16 13 10 2	4 3 2 1 0	1 0 2 4 3	22 16 13 10 4
1 0 4 3 2	2 4 1 3 0	8 5 22 19 11	1 0 4 3 2	2 4 3 1 0	8 5 24 17 11
3 2 1 0 4	1 3 0 2 4	17 14 6 3 25	3 2 1 0 4	3 1 0 2 4	19 12 6 3 25
2/H	7/L	5#	2/H	8/L	6#
0 2 4 3 1	1 3 0 2 4	2 14 21 18 10	0 2 4 3 1	1 3 4 2 0	2 14 25 18 6
4 3 1 0 2	2 4 1 3 0	23 20 7 4 11	4 3 1 0 2	2 0 1 3 4	23 16 7 4 15
1 0 2 4 3	3 0 2 4 1	9 1 13 25 17	1 0 2 4 3	3 4 2 0 1	9 5 13 21 17
2 4 3 1 0	4 1 3 0 2	15 22 19 6 3	2 4 3 1 0	0 1 3 4 2	11 22 19 10 3
3 1 0 2 4	0 2 4 1 3	16 8 5 12 24	3 1 0 2 4	4 2 0 1 3	20 8 1 12 24

4/H	5/L	7#	4/H	6/L	8#
0 4 3 2 1	1 2 3 0 4	2 23 19 11 10	0 4 3 2 1	1 2 3 4 0	2 23 19 15 6
2 1 0 4 3	3 0 4 1 2	14 6 5 22 18	2 1 0 4 3	3 4 0 1 2	14 10 1 22 18
4 3 2 1 0	4 1 2 3 0	25 17 13 9 1	4 3 2 1 0	0 1 2 3 4	21 17 13 9 5
1 0 4 3 2	2 3 0 4 1	8 4 21 20 12	1 0 4 3 2	2 3 4 0 1	8 4 25 16 12
3 2 1 0 4	0 4 1 2 3	16 15 7 3 24	3 2 1 0 4	4 0 1 2 3	20 11 7 3 24
2/H	9/L	9#	2/H	10/L	10#
0 2 4 3 1	3 1 0 2 4	4 12 21 18 10	0 2 4 3 1	3 1 4 2 0	4 12 25 18 6
4 3 1 0 2	2 4 3 1 0	23 20 9 2 11	4 3 1 0 2	2 0 3 1 4	23 16 9 2 15
1 0 2 4 3	1 0 2 4 3	7 1 13 25 19	1 0 2 4 3	1 4 2 0 3	7 5 13 21 19
2 4 3 1 0	4 3 1 0 2	15 24 17 6 3	2 4 3 1 0	0 3 1 4 2	11 24 17 10 3
3 1 0 2 4	0 2 4 3 1	16 8 5 14 22	3 1 0 2 4	4 2 0 3 1	20 8 1 14 22
4/H	11/L	11#	4/H	12/L	12#
0 4 3 2 1	3 2 1 0 4	4 23 17 11 10	0 4 3 2 1	3 2 1 4 0	4 23 17 15 6
2 1 0 4 3	1 0 4 3 2	12 6 5 24 18	2 1 0 4 3	1 4 0 3 2	12 10 1 24 18
4 3 2 1 0	4 3 2 1 0	25 19 13 7 1	4 3 2 1 0	0 3 2 1 4	21 19 13 7 5
1 0 4 3 2	2 1 0 4 3	8 2 21 20 14	1 0 4 3 2	2 1 4 0 3	8 2 25 16 14
3 2 1 0 4	0 4 3 2 1	16 15 9 3 22	3 2 1 0 4	4 0 3 2 1	20 11 9 3 22
2/H	13/L	13#	2/H	14/L	14#
0 2 4 3 1	4 0 1 2 3	5 11 22 18 9	0 2 4 3 1	4 0 3 2 1	5 11 24 18 7
4 3 1 0 2	2 3 4 0 1	23 19 10 1 12	4 3 1 0 2	2 1 4 0 3	23 17 10 1 14
1 0 2 4 3	0 1 2 3 4	6 2 13 24 20	1 0 2 4 3	0 3 2 1 4	6 4 13 22 20
2 4 3 1 0	3 4 0 1 2	14 25 16 7 3	2 4 3 1 0	1 4 0 3 2	12 25 16 9 3
3 1 0 2 4	1 2 3 4 0	17 8 4 15 21	3 1 0 2 4	3 2 1 4 0	19 8 2 15 21
4/H	15/L	15#	4/H	16/L	16#
0 4 3 2 1	4 2 0 1 3	5 23 16 12 9	0 4 3 2 1	4 2 0 3 1	5 23 16 14 7
2 1 0 4 3	0 1 3 4 2	11 7 4 25 18	2 1 0 4 3	0 3 1 4 2	11 9 2 25 18
4 3 2 1 0	3 4 2 0 1	24 20 13 6 2	4 3 2 1 0	1 4 2 0 3	22 20 13 6 4
1 0 4 3 2	2 0 1 3 4	8 1 22 19 15	1 0 4 3 2	2 0 3 1 4	8 1 24 17 15
3 2 1 0 4	1 3 4 2 0	17 14 10 3 21	3 2 1 0 4	3 1 4 2 0	19 12 10 3 21

[Count = 16] OK!

4. How about Simultaneous Magic Squares 7x7: Both S-C & P-D Type?

Can we make Simultaneous MS77 both Self-Complementary and Pan-Diagonal by this new method of Base 7? I prepared the following basic form and basic conditions for our object. First of all let's compose two sets of layer units high and low decomposed by PNS of Base 7.

*** Basic Diagram for Simultaneous MS77 ***
 ** with Basic Conditions and Pan-Diagonals **

3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5
10 11 12 13 14	8 9 10 11 12 13 14	8 9 10 11 12
17 18 19 20 21	15 16 17 18 19 20 21	15 16 17 18 19
24 25 26 27 28	22 23 24 25 26 27 28	22 23 24 25 26
31 32 33 34 35	29 30 31 32 33 34 35	29 30 31 32 33
38 39 40 41 42	36 37 38 39 40 41 42	36 37 38 39 40
45 46 47 48 49	43 44 45 46 47 48 49	43 44 45 46 47

**** Basic Conditions: ****

$n1+n2+n3+n4+n5+n6+n7=C$... rw1;	$n1+n8+n15+n22+n29+n36+n43=C$... cl 1;
$n8+n9+n10+n11+n12+n13+n14=C$... rw2;	$n2+n9+n16+n23+n30+n37+n44=C$... cl 2;
$n15+n16+n17+n18+n19+n20+n21=C$... rw3;	$n3+n10+n17+n24+n31+n38+n45=C$... cl 3;
$n22+n23+n24+n25+n26+n27+n28=C$... rw4;	$n4+n11+n18+n25+n32+n39+n46=C$... cl 4;
$n29+n30+n31+n32+n33+n34+n35=C$... rw5;	$n5+n12+n19+n26+n33+n40+n47=C$... cl 5;
$n36+n37+n38+n39+n40+n41+n42=C$... rw6;	$n6+n13+n20+n27+n34+n41+n48=C$... cl 6;
$n43+n44+n45+n46+n47+n48+n49=C$... rw7;	$n7+n14+n21+n28+n35+n42+n49=C$... cl 7;

**** Pan-diagonal Conditions: ****

$n1+n9+n17+n25+n33+n41+n49=C$... pd1;	$n1+n14+n20+n26+n32+n38+n44=C$... pb1;
$n2+n10+n18+n26+n34+n42+n43=C$... pd2;	$n2+n8+n21+n27+n33+n39+n45=C$... pb2;
$n3+n11+n19+n27+n35+n36+n44=C$... pd3;	$n3+n9+n15+n28+n34+n40+n46=C$... pb3;
$n4+n12+n20+n28+n29+n37+n45=C$... pd4;	$n4+n10+n16+n22+n35+n41+n47=C$... pb4;
$n5+n13+n21+n22+n30+n38+n46=C$... pd5;	$n5+n11+n17+n23+n29+n42+n48=C$... pb5;
$n6+n14+n15+n23+n31+n39+n47=C$... pd6;	$n6+n12+n18+n24+n30+n36+n49=C$... pb6;
$n7+n8+n16+n24+n32+n40+n48=C$... pd7;	$n7+n13+n19+n25+n31+n37+n43=C$... pb7;

**** Self-complementary Conditions: ****

$n1+n49=n2+n48=n3+n47=n4+n46=n5+n45=n6+n44=n7+n43=n8+n42=n9+n41$
 $=n10+n40=n11+n39=n12+n38=n13+n37=n14+n36=n15+n35=n16+n34=n17+n33$
 $=n18+n32=n19+n31=n20+n30=n21+n29=n22+n28=n23+n27=n24+n26=C=50$

The next list shows part of my computer program dictated recently for our object. If you want to have the whole list of it, please click at the file end to get that.

```

/**/
/** Core Part of 'CES7Sml.c' **/
/**/
#include <stdio.h>
/**/
short int cnt, bcnt, cnt2, cnt3;
short CC;
short u1, u2;
short nm[50], flg[50];
short anm[4][53];
short tn[3457][53];
char thlu[25][50], tllu[193][50];
char mtc[25][193];
/**/
short rw1[7], cl 1[7], pd1[7], pb1[7];
short rw2[7], cl 2[7], pd2[7], pb2[7];
short rw3[7], cl 3[7], pd3[7], pb3[7];
short rw4[7], cl 4[7], pd4[7], pb4[7];
short rw5[7], cl 5[7], pd5[7], pb5[7];
short rw6[7], cl 6[7], pd6[7], pb6[7];
short rw7[7], cl 7[7], pd7[7], pb7[7];
/**/
/* Main Program */
int main(){
short n;
printf("\n*** 'Complete Euler Squares' of Order 7 for Simul taneous Magi c Squares: ***\n");
printf("*** Both Self-Complementary and Pan-diagonal built by 'New Euler's Method' ***\n");
for(n=0; n<50; n++){nm[n]=0;}
for(n=0; n<7; n++){
rw1[n]=0; cl 1[n]=0; pd1[n]=0; pb1[n]=0;
rw2[n]=0; cl 2[n]=0; pd2[n]=0; pb2[n]=0;
rw3[n]=0; cl 3[n]=0; pd3[n]=0; pb3[n]=0;
rw4[n]=0; cl 4[n]=0; pd4[n]=0; pb4[n]=0;
rw5[n]=0; cl 5[n]=0; pd5[n]=0; pb5[n]=0;
rw6[n]=0; cl 6[n]=0; pd6[n]=0; pb6[n]=0;
rw7[n]=0; cl 7[n]=0; pd7[n]=0; pb7[n]=0;
}
}

```

```

}
CC=6; cnt=0; bcnt=0; cnt3=0;
nm[25]=3; rw4[3]=1; cl 4[3]=1; pd1[3]=1; pb7[3]=1;
stp01(); /* Make the Layer Units */
rw4[3]=0; cl 4[3]=0; pd1[3]=0; pb7[3]=0;
printf("\n [Layer Units]\n");
prlunit(); /* Print the Layer Units */
cnt=0;
printf("\n [Compositions of Simultaneous MS77]\n");
cmbcmp(); /* Combine and Compose */
sol sort(cnt); /* Sort the Solutions */
sol print(1, cnt); /* Print the Solution List */
if(cnt3==1){pr1sol();} /* Print The Rest One */
printf(" [Count = %d]\n", cnt);
printf(" OK!\n");
return 0;
}
/* Make the Layer Units */
/* Set n1 & n49 */
void stp01(){
short a, b;
for(a=0; a<7; a++){b=CC-a;
if((rw1[a]==0)&&(cl 1[a]==0)&&(pd1[a]==0)&&(pb1[a]==0)){
if((rw7[b]==0)&&(cl 7[b]==0)&&(pd1[b]==0)&&(pb6[b]==0)){
nm[1]=a; nm[49]=b;
rw1[a]=1; cl 1[a]=1; pd1[a]=1; pb1[a]=1;
rw7[b]=1; cl 7[b]=1; pd1[b]=1; pb6[b]=1;
stp02();
rw1[a]=0; cl 1[a]=0; pd1[a]=0; pb1[a]=0;
rw7[b]=0; cl 7[b]=0; pd1[b]=0; pb6[b]=0;
}}
}
}
/* Set n2 & n48 */
void stp02(){
short a, b;
for(a=0; a<7; a++){b=CC-a;
if((rw1[a]==0)&&(cl 2[a]==0)&&(pd2[a]==0)&&(pb2[a]==0)){
if((rw7[b]==0)&&(cl 6[b]==0)&&(pd7[b]==0)&&(pb5[b]==0)){
nm[2]=a; nm[48]=b;
rw1[a]=1; cl 2[a]=1; pd2[a]=1; pb2[a]=1;
rw7[b]=1; cl 6[b]=1; pd7[b]=1; pb5[b]=1;
stp03();
rw1[a]=0; cl 2[a]=0; pd2[a]=0; pb2[a]=0;
rw7[b]=0; cl 6[b]=0; pd7[b]=0; pb5[b]=0;
}}
}
}
/* Set n3 & n47 */
void stp03(){
short a, b;
for(a=0; a<7; a++){b=CC-a;
if((rw1[a]==0)&&(cl 3[a]==0)&&(pd3[a]==0)&&(pb3[a]==0)){
if((rw7[b]==0)&&(cl 5[b]==0)&&(pd6[b]==0)&&(pb4[b]==0)){
nm[3]=a; nm[47]=b;
rw1[a]=1; cl 3[a]=1; pd3[a]=1; pb3[a]=1;
rw7[b]=1; cl 5[b]=1; pd6[b]=1; pb4[b]=1;
stp04();
rw1[a]=0; cl 3[a]=0; pd3[a]=0; pb3[a]=0;
rw7[b]=0; cl 5[b]=0; pd6[b]=0; pb4[b]=0;
}}
}
}
/* Set n4 & n46 */
void stp04(){
short a, b;
for(a=0; a<7; a++){b=CC-a;

```

```

    if((rw1[a]==0)&&(cl4[a]==0)&&(pd4[a]==0)&&(pb4[a]==0)){
        if((rw7[b]==0)&&(cl4[b]==0)&&(pd5[b]==0)&&(pb3[b]==0)){
            nm[4]=a; nm[46]=b;
            rw1[a]=1; cl4[a]=1; pd4[a]=1; pb4[a]=1;
            rw7[b]=1; cl4[b]=1; pd5[b]=1; pb3[b]=1;
            stp05();
            rw1[a]=0; cl4[a]=0; pd4[a]=0; pb4[a]=0;
            rw7[b]=0; cl4[b]=0; pd5[b]=0; pb3[b]=0;
        }}
    }
    /* Set n5 & n45 */
    void stp05(){
        short a, b;
        for(a=0; a<7; a++){b=CC-a;
            if((rw1[a]==0)&&(cl5[a]==0)&&(pd5[a]==0)&&(pb5[a]==0)){
                if((rw7[b]==0)&&(cl3[b]==0)&&(pd4[b]==0)&&(pb2[b]==0)){
                    nm[5]=a; nm[45]=b;
                    rw1[a]=1; cl5[a]=1; pd5[a]=1; pb5[a]=1;
                    rw7[b]=1; cl3[b]=1; pd4[b]=1; pb2[b]=1;
                    stp06();
                    rw1[a]=0; cl5[a]=0; pd5[a]=0; pb5[a]=0;
                    rw7[b]=0; cl3[b]=0; pd4[b]=0; pb2[b]=0;
                }}
        }}
    }
    /* Set n6 & n44 */
    void stp06(){
        short a, b;
        for(a=0; a<7; a++){b=CC-a;
            if((rw1[a]==0)&&(cl6[a]==0)&&(pd6[a]==0)&&(pb6[a]==0)){
                if((rw7[b]==0)&&(cl2[b]==0)&&(pd3[b]==0)&&(pb1[b]==0)){
                    nm[6]=a; nm[44]=b;
                    rw1[a]=1; cl6[a]=1; pd6[a]=1; pb6[a]=1;
                    rw7[b]=1; cl2[b]=1; pd3[b]=1; pb1[b]=1;
                    stp07();
                    rw1[a]=0; cl6[a]=0; pd6[a]=0; pb6[a]=0;
                    rw7[b]=0; cl2[b]=0; pd3[b]=0; pb1[b]=0;
                }}
        }}
    }
    /* Set n7 & n43 */
    void stp07(){
        short a, b;
        for(a=0; a<7; a++){b=CC-a;
            if((rw1[a]==0)&&(cl7[a]==0)&&(pd7[a]==0)&&(pb7[a]==0)){
                if((rw7[b]==0)&&(cl1[b]==0)&&(pd2[b]==0)&&(pb7[b]==0)){
                    nm[7]=a; nm[43]=b;
                    rw1[a]=1; cl7[a]=1; pd7[a]=1; pb7[a]=1;
                    rw7[b]=1; cl1[b]=1; pd2[b]=1; pb7[b]=1;
                    stp08();
                    rw1[a]=0; cl7[a]=0; pd7[a]=0; pb7[a]=0;
                    rw7[b]=0; cl1[b]=0; pd2[b]=0; pb7[b]=0;
                }}
        }}
    }
    /* Set n8 & n42 */
    void stp08(){
        short a, b;
        for(a=0; a<7; a++){b=CC-a;
            if((rw2[a]==0)&&(cl1[a]==0)&&(pd7[a]==0)&&(pb2[a]==0)){
                if((rw6[b]==0)&&(cl7[b]==0)&&(pd2[b]==0)&&(pb5[b]==0)){
                    nm[8]=a; nm[42]=b;
                    rw2[a]=1; cl1[a]=1; pd7[a]=1; pb2[a]=1;
                    rw6[b]=1; cl7[b]=1; pd2[b]=1; pb5[b]=1;
                    stp09();
                    rw2[a]=0; cl1[a]=0; pd7[a]=0; pb2[a]=0;
                    rw6[b]=0; cl7[b]=0; pd2[b]=0; pb5[b]=0;
                }}
        }}
    }

```

```

    }}}
}
/**/
.....
/**/
/* Set n24 & n26 */
void stp24(){
short a, b;
for(a=0; a<7; a++){b=CC-a;
if((rw4[a]==0)&&(cl3[a]==0)&&(pd7[a]==0)&&(pb6[a]==0)){
if((rw4[b]==0)&&(cl5[b]==0)&&(pd2[b]==0)&&(pb1[b]==0)){
nm[24]=a; nm[26]=b;
rw4[a]=1; cl3[a]=1; pd7[a]=1; pb6[a]=1;
rw4[b]=1; cl5[b]=1; pd2[b]=1; pb1[b]=1;
lurecord();
rw4[a]=0; cl3[a]=0; pd7[a]=0; pb6[a]=0;
rw4[b]=0; cl5[b]=0; pd2[b]=0; pb1[b]=0;
}}}
}
/**/
/* Record the Layer Units */
void lurecord(){
short n;
cnt++;
thlu[cnt-1][0]=cnt;
for(n=1; n<50; n++){thlu[cnt-1][n]=nm[n];}
if((nm[1]<nm[49])&&(nm[1]<=nm[7])&&(nm[7]<=nm[43])){
cnt2++;
thlu[cnt2-1][0]=cnt2;
for(n=1; n<50; n++){thlu[cnt2-1][n]=nm[n];}
}
}
/**/
.....

```

We have got 30 layer units for high, and 240 ones for low. Let's combine them, compose solutions and test them.

*** 'Complete Euler Squares' of Order 7 for Simultaneous Magic Squares: ***
 ** Both Self-Complementary and Pan-diagonal built by 'New Euler's Method' **

[Layer Units] [for High]

1/	2/	3/	4/	5/	6/	7/	8/
013564	015643	023465	024653	036415	036451	036524	036542
356420	643201	346510	653102	152036	512036	241036	421036
642013	201564	651023	102465	364152	364512	365241	365421
201356	564320	102346	465310	520364	120364	410365	210365
135642	320156	234651	310246	641520	645120	652410	654210
564201	156432	465102	246531	203641	203645	103652	103654
420135	432015	510234	531024	415203	451203	524103	542103
9/	10/	11/	12/	13/	14/	15/	16/
042653	043265	051643	053164	064135	064531	065234	065432
653104	326510	643205	316420	520641	120645	410652	210654
104265	651043	205164	642053	135206	531206	234106	432106
265310	104326	164320	205316	641352	645312	652341	654321
310426	432651	320516	531642	206413	206453	106523	106543
426531	265104	516432	164205	352064	312064	341065	321065
531042	510432	432051	420531	413520	453120	523410	543210
17/	18/	19/	20/	21/	22/	23/	24/
103654	106543	135406	135460	154036	154630	160543	163054
365421	543210	062135	602135	621540	021546	543216	305421
542103	210654	354062	354602	036215	630215	216054	542163
210365	654321	621354	021354	540362	546302	054321	216305
036542	321065	540621	546021	215403	215463	321605	630542
654210	065432	213540	213546	362154	302154	605432	054216
421036	432106	406213	460213	403621	463021	432160	421630

[for Low]

1/	2/	3/	4/	5/	6/	7/	8/
013562	013564	015623	015643	023461	023465	024613	024653
356240	356420	623401	643201	346150	346510	613502	653102
624013	642013	401562	201564	615023	651023	502461	102465
401356	201356	562340	564320	502346	102346	461350	465310
135624	135642	340156	320156	234615	234651	350246	310246
562401	564201	156234	156432	461502	465102	246135	246531
240135	420135	234015	432015	150234	510234	135024	531024
49/	50/	51/	52/	53/	54/	55/	56/
142503	142563	143250	143256	150234	150432	152036	152630
503614	563014	325061	325601	461502	261504	641520	041526
614250	014256	506143	560143	234615	432615	036415	630415
250361	256301	614325	014325	502346	504326	520364	526304
361425	301425	432506	432560	615023	615043	415203	415263
425036	425630	250614	256014	346150	326150	364152	304152
036142	630142	061432	601432	023461	043261	203641	263041
97/	98/	99/	100/	101/	102/	103/	104/
403621	403625	406213	406253	413520	413526	415203	415263
362154	362514	213540	253140	352064	352604	203641	263041
215403	251403	540621	140625	206413	260413	641520	041526
540362	140362	621354	625314	641352	041352	520364	526304
036215	036251	354062	314062	135206	135260	364152	304152
621540	625140	062135	062531	520641	526041	152036	152630
154036	514036	135406	531406	064135	604135	036415	630415
145/	146/	147/	148/	149/	150/	151/	152/
531024	531042	531206	531260	531406	531460	531624	531642
246531	426531	064531	604531	062531	602531	240531	420531
310246	310426	312064	312604	314062	314602	316240	316420
465310	265310	645312	045312	625314	025314	405316	205316
102465	104265	120645	126045	140625	146025	162405	164205
653102	653104	453120	453126	253140	253146	053162	053164
024653	042653	206453	260453	406253	460253	624053	642053

[Count of Layer Units = 24H/192L]

[Reference Table for the Best Combination = 3840]

#1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1H	35	49	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7	35	35	
2H	7	7	35	49	7	7	21	21	7	7	7	7	7	7	7	7	7	21	21	7	7	35	35	7	7
3H	21	21	7	7	35	49	7	7	7	7	7	7	7	7	7	7	7	35	35	7	7	21	21	7	7
4H	7	7	21	21	7	7	35	49	7	7	7	7	7	7	7	7	35	35	7	7	21	21	7	7	7
5H	7	7	7	7	7	7	7	7	21	21	35	35	49	35	21	21	7	7	7	7	7	7	7	7	7
6H	7	7	7	7	7	7	7	7	21	21	35	35	35	49	21	21	7	7	7	7	7	7	7	7	7
7H	7	7	7	7	7	7	7	7	35	35	21	21	21	21	49	35	7	7	7	7	7	7	7	7	7
8H	7	7	7	7	7	7	7	7	35	35	21	21	21	21	35	49	7	7	7	7	7	7	7	7	7
9H	7	7	21	21	7	7	35	35	7	7	7	7	7	7	7	7	35	49	7	7	21	21	7	7	7
10H	21	21	7	7	35	35	7	7	7	7	7	7	7	7	7	7	7	35	49	7	7	21	21	7	7
11H	7	7	35	35	7	7	21	21	7	7	7	7	7	7	7	7	21	21	7	7	35	49	7	7	7
12H	35	35	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7	35	49	7	7
13H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
14H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
15H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
16H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
17H	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7
18H	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7
19H	7	7	7	7	7	7	7	7	7	7	21	21	21	21	7	7	7	7	7	7	7	7	7	7	7
20H	7	7	7	7	7	7	7	7	7	21	21	21	21	21	7	7	7	7	7	7	7	7	7	7	7
21H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
22H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
23H	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7
24H	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7
#2	97	98	99	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1H	35	35	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7	49	35	
2H	7	7	35	35	7	7	21	21	7	7	7	7	7	7	7	7	21	21	7	7	49	35	7	7	
3H	21	21	7	7	35	35	7	7	7	7	7	7	7	7	7	7	7	49	35	7	7	21	21	7	

4H	7	7	21	21	7	7	35	35	7	7	7	7	7	7	7	7	7	49	35	7	7	21	21	7	7
5H	7	7	7	7	7	7	7	21	21	35	49	35	35	21	21	7	7	7	7	7	7	7	7	7	7
6H	7	7	7	7	7	7	7	21	21	49	35	35	35	21	21	7	7	7	7	7	7	7	7	7	7
7H	7	7	7	7	7	7	7	35	49	21	21	21	21	35	35	7	7	7	7	7	7	7	7	7	7
8H	7	7	7	7	7	7	7	49	35	21	21	21	21	35	35	7	7	7	7	7	7	7	7	7	7
9H	7	7	21	21	7	7	49	35	7	7	7	7	7	7	7	7	7	35	35	7	7	21	21	7	7
10H	21	21	7	7	49	35	7	7	7	7	7	7	7	7	7	7	7	7	35	35	7	7	21	21	
11H	7	7	49	35	7	7	21	21	7	7	7	7	7	7	7	7	7	21	21	7	7	35	35	7	7
12H	49	35	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7	35	35	
13H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
14H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
15H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
16H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
17H	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	
18H	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7
19H	7	7	7	7	7	7	7	7	7	21	21	21	21	7	7	7	7	7	7	7	7	7	7	7	7
20H	7	7	7	7	7	7	7	7	7	21	21	21	21	7	7	7	7	7	7	7	7	7	7	7	7
21H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
22H	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
23H	7	7	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	7	7
24H	21	21	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	21	21	

.

[Composi ti ons of Si mul taneous MS77]

1/H	3/L	1#	1/H	4/L	2#
0135642	0156234	1 9 27 42 45 32 19	0135642	0156432	1 9 27 42 47 32 17
3564201	6234015	28 38 46 33 15 2 13	3564201	6432015	28 40 46 31 15 2 13
6420135	4015623	47 29 16 6 14 24 39	6420135	2015643	45 29 16 6 14 26 39
2013564	5623401	20 7 10 25 40 43 30	2013564	5643201	20 7 12 25 38 43 30
1356420	3401562	11 26 36 44 34 21 3	1356420	3201564	11 24 36 44 34 21 5
5642013	1562340	37 48 35 17 4 12 22	5642013	1564320	37 48 35 19 4 10 22
4201356	2340156	31 18 5 8 23 41 49	4201356	4320156	33 18 3 8 23 41 49
2/H	1/L	3#	1/H	7/L	5#
0156432	0135624	1 9 39 48 35 24 19	0135642	0246135	1 10 26 42 44 32 20
6432015	3562401	46 34 28 17 5 8 37	3564201	6135024	28 37 46 34 15 3 12
2015643	6240135	21 3 12 36 44 32 27	6420135	5024613	48 29 17 5 14 23 39
5643201	4013562	40 43 30 25 20 7 10	2013564	4613502	19 7 9 25 41 43 31
3201564	1356240	23 18 6 14 38 47 29	1356420	3502461	11 27 36 45 33 21 2
1564320	5624013	13 42 45 33 22 16 4	5642013	2461350	38 47 35 16 4 13 22
4320156	2401356	31 26 15 2 11 41 49	4201356	1350246	30 18 6 8 24 40 49
1/H	8/L	6#	2/H	5/L	7#
0135642	0246531	1 10 26 42 48 32 16	0156432	0234615	1 10 39 47 35 23 20
3564201	6531024	28 41 46 30 15 3 12	6432015	3461502	46 33 28 16 6 8 38
6420135	1024653	44 29 17 5 14 27 39	2015643	6150234	21 2 13 36 45 32 26
2013564	4653102	19 7 13 25 37 43 31	5643201	5023461	41 43 31 25 19 7 9
1356420	3102465	11 23 36 45 33 21 6	3201564	2346150	24 18 5 14 37 48 29
5642013	2465310	38 47 35 20 4 9 22	1564320	4615023	12 42 44 34 22 17 4
4201356	5310246	34 18 2 8 24 40 49	4320156	1502346	30 27 15 3 11 40 49
1/H	9/L	9#	2/H	9/L	17#
0135642	0361245	1 11 28 37 45 33 20	0156432	0361245	1 11 42 44 31 26 20
3564201	2450361	24 40 48 29 18 7 9	6432015	2450361	45 33 27 15 4 14 37
6420135	3612450	46 35 16 3 12 27 36	2015643	3612450	18 7 9 38 47 34 22
2013564	4503612	19 6 8 25 42 44 31	5643201	4503612	40 48 29 25 21 2 10
1356420	6124503	14 23 38 47 34 15 4	3201564	6124503	28 16 3 12 41 43 32
5642013	5036124	41 43 32 21 2 10 26	1564320	5036124	13 36 46 35 23 17 5
4201356	1245036	30 17 5 13 22 39 49	4320156	1245036	30 24 19 6 8 39 49

1/H	17/L					25#	2/H	19/L					27#				
0135642	0426135	1	12	24	42	44	32	20	0156432	0432615	1	12	39	45	35	23	20
3564201	6135042	28	37	46	34	15	5	10	6432015	3261504	46	31	28	16	6	8	40
6420135	5042613	48	29	19	3	14	23	39	2015643	6150432	21	2	13	36	47	32	24
2013564	2613504	17	7	9	25	41	43	33	5643201	5043261	41	43	33	25	17	7	9
1356420	3504261	11	27	36	47	31	21	2	3201564	4326150	26	18	3	14	37	48	29
5642013	4261350	40	45	35	16	4	13	22	1564320	2615043	10	42	44	34	22	19	4
4201356	1350426	30	18	6	8	26	38	49	4320156	1504326	30	27	15	5	11	38	49
1/H	21/L					29#	1/H	25/L					33#				
0135642	0516234	1	13	23	42	45	32	19	0135642	0612345	1	14	23	38	46	33	20
3564201	6234051	28	38	46	33	15	6	9	3564201	4506123	26	41	43	35	16	3	11
6420135	4051623	47	29	20	2	14	24	39	6420135	2345061	45	32	19	6	8	28	37
2013564	1623405	16	7	10	25	40	43	34	2013564	6123450	21	2	10	25	40	48	29
1356420	3405162	11	26	36	48	30	21	3	1356420	5061234	13	22	42	44	31	18	5
5642013	5162340	41	44	35	17	4	12	22	5642013	3450612	39	47	34	15	7	9	24
4201356	2340516	31	18	5	8	27	37	49	4201356	1234506	30	17	4	12	27	36	49
3/H	3/L					49#	3/H	7/L					53#				
0234651	0156234	1	16	27	35	45	39	12	0234651	0246135	1	17	26	35	44	39	13
3465102	6234015	28	31	46	40	8	2	20	3465102	6135024	28	30	46	41	8	3	19
6510234	4015623	47	36	9	6	21	24	32	6510234	5024613	48	36	10	5	21	23	32
1023465	5623401	13	7	17	25	33	43	37	1023465	4613502	12	7	16	25	34	43	38
2346510	3401562	18	26	29	44	41	14	3	2346510	3502461	18	27	29	45	40	14	2
4651023	1562340	30	48	42	10	4	19	22	4651023	2461350	31	47	42	9	4	20	22
5102346	2340156	38	11	5	15	23	34	49	5102346	1350246	37	11	6	15	24	33	49
3/H	9/L					57#	3/H	17/L					73#				
0234651	0361245	1	18	28	30	45	40	13	0234651	0426135	1	19	24	35	44	39	13
3465102	2450361	24	33	48	36	11	7	16	3465102	6135042	28	30	46	41	8	5	17
6510234	3612450	46	42	9	3	19	27	29	6510234	5042613	48	36	12	3	21	23	32
1023465	4503612	12	6	15	25	35	44	38	1023465	2613504	10	7	16	25	34	43	40
2346510	6124503	21	23	31	47	41	8	4	2346510	3504261	18	27	29	47	38	14	2
4651023	5036124	34	43	39	14	2	17	26	4651023	4261350	33	45	42	9	4	20	22
5102346	1245036	37	10	5	20	22	32	49	5102346	1350426	37	11	6	15	26	31	49
3/H	21/L					77#	3/H	25/L					81#				
0234651	0516234	1	20	23	35	45	39	12	0234651	0612345	1	21	23	31	46	40	13
3465102	6234051	28	31	46	40	8	6	16	3465102	4506123	26	34	43	42	9	3	18
6510234	4051623	47	36	13	2	21	24	32	6510234	2345061	45	39	12	6	15	28	30
1023465	1623405	9	7	17	25	33	43	41	1023465	6123450	14	2	17	25	33	48	36
2346510	3405162	18	26	29	48	37	14	3	2346510	5061234	20	22	35	44	38	11	5
4651023	5162340	34	44	42	10	4	19	22	4651023	3450612	32	47	41	8	7	16	24
5102346	2340516	38	11	5	15	27	30	49	5102346	1234506	37	10	4	19	27	29	49
5/H	1/L					97#	5/H	5/L					113#				
0364152	0135624	1	23	46	34	14	38	19	0364152	0234615	1	24	46	33	14	37	20
1520364	3562401	11	41	21	3	26	43	30	1520364	3461502	11	40	21	2	27	43	31
3641520	6240135	28	45	33	8	37	18	6	3641520	6150234	28	44	34	8	38	18	5
5203641	4013562	40	15	2	25	48	35	10	5203641	5023461	41	15	3	25	47	35	9
6415203	1356240	44	32	13	42	17	5	22	6415203	2346150	45	32	12	42	16	6	22
2036415	5624013	20	7	24	47	29	9	39	2036415	4615023	19	7	23	48	29	10	39
4152036	2401356	31	12	36	16	4	27	49	4152036	1502346	30	13	36	17	4	26	49
5/H	17/L					129#	5/H	21/L					145#				
0364152	0426135	1	26	45	35	9	39	20	0364152	0516234	1	27	44	35	10	39	19
1520364	6135042	14	37	18	6	22	47	31	1520364	6234051	14	38	18	5	22	48	30
3641520	5042613	27	43	33	10	42	16	4	3641520	4051623	26	43	34	9	42	17	4
5203641	2613504	38	21	2	25	48	29	12	5203641	1623405	37	21	3	25	47	29	13
6415203	3504261	46	34	8	40	17	7	23	6415203	3405162	46	33	8	41	16	7	24
2036415	4261350	19	3	28	44	32	13	36	2036415	5162340	20	2	28	45	32	12	36
4152036	1350426	30	11	41	15	5	24	49	4152036	2340516	31	11	40	15	6	23	49

5/H	25/L					161#	9/H	1/L							193#		
0364152	0612345	1	28	44	31	11	40	20	0426531	0135624	1	30	18	48	42	24	12
1520364	4506123	12	41	15	7	23	45	32	6531042	3562401	46	41	28	10	5	29	16
3641520	2345061	24	46	33	13	36	21	2	1042653	6240135	14	3	33	15	44	39	27
5203641	6123450	42	16	3	25	47	34	8	2653104	4013562	19	43	37	25	13	7	31
6415203	5061234	48	29	14	37	17	4	26	3104265	1356240	23	11	6	35	17	47	36
2036415	3450612	18	5	27	43	35	9	38	4265310	5624013	34	21	45	40	22	9	4
4152036	1234506	30	10	39	19	6	22	49	5310426	2401356	38	26	8	2	32	20	49
9/H	5/L					197#	9/H	9/L							201#		
0426531	0234615	1	31	18	47	42	23	13	0426531	0361245	1	32	21	44	38	26	13
6531042	3461502	46	40	28	9	6	29	17	6531042	2450361	45	40	27	8	4	35	16
1042653	6150234	14	2	34	15	45	39	26	1042653	3612450	11	7	30	17	47	41	22
2653104	5023461	20	43	38	25	12	7	30	2653104	4503612	19	48	36	25	14	2	31
3104265	2346150	24	11	5	35	16	48	36	3104265	6124503	28	9	3	33	20	43	39
4265310	4615023	33	21	44	41	22	10	4	4265310	5036124	34	15	46	42	23	10	5
5310426	1502346	37	27	8	3	32	19	49	5310426	1245036	37	24	12	6	29	18	49
9/H	19/L					217#	9/H	23/L							221#		
0426531	0432615	1	33	18	45	42	23	13	0426531	0531624	1	34	18	44	42	24	12
6531042	3261504	46	38	28	9	6	29	19	6531042	3162405	46	37	28	10	5	29	20
1042653	6150432	14	2	34	15	47	39	24	1042653	6240531	14	3	33	15	48	39	23
2653104	5043261	20	43	40	25	10	7	30	2653104	4053162	19	43	41	25	9	7	31
3104265	4326150	26	11	3	35	16	48	36	3104265	5316240	27	11	2	35	17	47	36
4265310	2615043	31	21	44	41	22	12	4	4265310	1624053	30	21	45	40	22	13	4
5310426	1504326	37	27	8	5	32	17	49	5310426	2405316	38	26	8	6	32	16	49
9/H	25/L					225#	11/H	1/L							241#		
0426531	0612345	1	35	16	45	39	26	13	0516432	0135624	1	37	11	48	35	24	19
6531042	4506123	47	41	22	14	2	31	18	6432051	3562401	46	34	28	17	5	36	9
1042653	2345061	10	4	33	20	43	42	23	2051643	6240135	21	3	40	8	44	32	27
2653104	6123450	21	44	38	25	12	6	29	1643205	4013562	12	43	30	25	20	7	38
3104265	5061234	27	8	7	30	17	46	40	3205164	1356240	23	18	6	42	10	47	29
4265310	3450612	32	19	48	36	28	9	3	5164320	5624013	41	14	45	33	22	16	4
5310426	1234506	37	24	11	5	34	15	49	4320516	2401356	31	26	15	2	39	13	49
11/H	5/L					245#	11/H	9/L							249#		
0516432	0234615	1	38	11	47	35	23	20	0516432	0361245	1	39	14	44	31	26	20
6432051	3461502	46	33	28	16	6	36	10	6432051	2450361	45	33	27	15	4	42	9
2051643	6150234	21	2	41	8	45	32	26	2051643	3612450	18	7	37	10	47	34	22
1643205	5023461	13	43	31	25	19	7	37	1643205	4503612	12	48	29	25	21	2	38
3205164	2346150	24	18	5	42	9	48	29	3205164	6124503	28	16	3	40	13	43	32
5164320	4615023	40	14	44	34	22	17	4	5164320	5036124	41	8	46	35	23	17	5
4320516	1502346	30	27	15	3	39	12	49	4320516	1245036	30	24	19	6	36	11	49
11/H	19/L					265#	11/H	23/L							269#		
0516432	0432615	1	40	11	45	35	23	20	0516432	0531624	1	41	11	44	35	24	19
6432051	3261504	46	31	28	16	6	36	12	6432051	3162405	46	30	28	17	5	36	13
2051643	6150432	21	2	41	8	47	32	24	2051643	6240531	21	3	40	8	48	32	23
1643205	5043261	13	43	33	25	17	7	37	1643205	4053162	12	43	34	25	16	7	38
3205164	4326150	26	18	3	42	9	48	29	3205164	5316240	27	18	2	42	10	47	29
5164320	2615043	38	14	44	34	22	19	4	5164320	1624053	37	14	45	33	22	20	4
4320516	1504326	30	27	15	5	39	10	49	4320516	2405316	31	26	15	6	39	9	49
11/H	25/L					273#	13/H	1/L							289#		
0516432	0612345	1	42	9	45	32	26	20	0641352	0135624	1	44	32	13	28	38	19
6432051	4506123	47	34	22	21	2	38	11	5206413	3562401	39	20	7	45	33	8	23
2051643	2345061	17	4	40	13	43	35	23	1352064	6240135	14	24	40	15	2	46	34
1643205	6123450	14	44	31	25	19	6	36	6413520	4013562	47	29	9	25	41	21	3
3205164	5061234	27	15	7	37	10	46	33	2064135	1356240	16	4	48	35	10	26	36
5164320	3450612	39	12	48	29	28	16	3	3520641	5624013	27	42	17	5	43	30	11
4320516	1234506	30	24	18	5	41	8	49	4135206	2401356	31	12	22	37	18	6	49

13/H	5/L		305#	13/H	9/L		321#
0641352	0234615	1 45 32 12 28 37 20		0641352	0361245	1 46 35 9 24 40 20	
5206413	3461502	39 19 7 44 34 8 24		5206413	2450361	38 19 6 43 32 14 23	
1352064	6150234	14 23 41 15 3 46 33		1352064	3612450	11 28 37 17 5 48 29	
6413520	5023461	48 29 10 25 40 21 2		6413520	4503612	47 34 8 25 42 16 3	
2064135	2346150	17 4 47 35 9 27 36		2064135	6124503	21 2 45 33 13 22 39	
3520641	4615023	26 42 16 6 43 31 11		3520641	5036124	27 36 18 7 44 31 12	
4135206	1502346	30 13 22 38 18 5 49		4135206	1245036	30 10 26 41 15 4 49	

13/H	17/L		353#	13/H	21/L		369#
0641352	0426135	1 47 31 14 23 39 20		0641352	0516234	1 48 30 14 24 39 19	
5206413	6135042	42 16 4 48 29 12 24		5206413	6234051	42 17 4 47 29 13 23	
1352064	5042613	13 22 40 17 7 44 32		1352064	4051623	12 22 41 16 7 45 32	
6413520	2613504	45 35 9 25 41 15 5		6413520	1623405	44 35 10 25 40 15 6	
2064135	3504261	18 6 43 33 10 28 37		2064135	3405162	18 5 43 34 9 28 38	
3520641	4261350	26 38 21 2 46 34 8		3520641	5162340	27 37 21 3 46 33 8	
4135206	1350426	30 11 27 36 19 3 49		4135206	2340516	31 11 26 36 20 2 49	

1/H	35/L		385#	1/H	39/L		389#
0135642	1065234	2 8 28 41 45 32 19		0135642	1245036	2 10 26 41 43 32 21	
3564201	5234106	27 38 46 33 16 1 14		3564201	5036124	27 36 46 35 16 3 12	
6420135	4106523	47 30 15 7 13 24 39		6420135	6124503	49 30 17 5 13 22 39	
2013564	6523410	21 6 10 25 40 44 29		2013564	4503612	19 6 8 25 42 44 31	
1356420	3410652	11 26 37 43 35 20 3		1356420	3612450	11 28 37 45 33 20 1	
5642013	0652341	36 49 34 17 4 12 23		5642013	2450361	38 47 34 15 4 14 23	
4201356	2341065	31 18 5 9 22 42 48		4201356	0361245	29 18 7 9 24 40 48	

1/H	41/L		393#	1/H	49/L		409#
0135642	1350246	2 11 27 36 45 33 21		0135642	1425036	2 12 24 41 43 32 21	
3564201	2461350	24 40 49 30 18 6 8		3564201	5036142	27 36 46 35 16 5 10	
6420135	3502461	46 34 15 3 12 28 37		6420135	6142503	49 30 19 3 13 22 39	
2013564	4613502	19 7 9 25 41 43 31		2013564	2503614	17 6 8 25 42 44 33	
1356420	5024613	13 22 38 47 35 16 4		1356420	3614250	11 28 37 47 31 20 1	
5642013	6135024	42 44 32 20 1 10 26		5642013	4250361	40 45 34 15 4 14 23	
4201356	0246135	29 17 5 14 23 39 48		4201356	0361425	29 18 7 9 26 38 48	

1/H	53/L		413#	1/H	61/L		429#
0135642	1502346	2 13 22 38 46 33 21		0135642	1605234	2 14 22 41 45 32 19	
3564201	4615023	26 42 44 34 15 3 11		3564201	5234160	27 38 46 33 16 7 8	
6420135	2346150	45 32 19 7 9 27 36		6420135	4160523	47 30 21 1 13 24 39	
2013564	5023461	20 1 10 25 40 49 30		2013564	0523416	15 6 10 25 40 44 35	
1356420	6150234	14 23 41 43 31 18 5		1356420	3416052	11 26 37 49 29 20 3	
5642013	3461502	39 47 35 16 6 8 24		5642013	6052341	42 43 34 17 4 12 23	
4201356	0234615	29 17 4 12 28 37 48		4201356	2341605	31 18 5 9 28 36 48	

13/H	49/L		737#	13/H	61/L		753#
0641352	1425036	2 47 31 13 22 39 21		0641352	1605234	2 49 29 13 24 39 19	
5206413	5036142	41 15 4 49 30 12 24		5206413	5234160	41 17 4 47 30 14 22	
1352064	6142503	14 23 40 17 6 43 32		1352064	4160523	12 23 42 15 6 45 32	
6413520	2503614	45 34 8 25 42 16 5		6413520	0523416	43 34 10 25 40 16 7	
2064135	3614250	18 7 44 33 10 27 36		2064135	3416052	18 5 44 35 8 27 38	
3520641	4250361	26 38 20 1 46 35 9		3520641	6052341	28 36 20 3 46 33 9	
4135206	0361425	29 11 28 37 19 3 48		4135206	2341605	31 11 26 37 21 1 48	

1/H	67/L		769#	1/H	99/L		1153#
0135642	2064135	3 8 28 40 44 32 20		0135642	4062135	5 8 28 38 44 32 20	
3564201	4135206	26 37 46 34 17 1 14		3564201	2135406	24 37 46 34 19 1 14	
6420135	5206413	48 31 15 7 12 23 39		6420135	5406213	48 33 15 7 10 23 39	
2013564	6413520	21 5 9 25 41 45 29		2013564	6213540	21 3 9 25 41 47 29	
1356420	3520641	11 27 38 43 35 19 2		1356420	3540621	11 27 40 43 35 17 2	
5642013	0641352	36 49 33 16 4 13 24		5642013	0621354	36 49 31 16 4 13 26	
4201356	1352064	30 18 6 10 22 42 47		4201356	1354062	30 18 6 12 22 42 45	

1/H	131/L							1537#	1/H	161/L							1921#
0135642	5061234	6	8	28	37	45	32	19	0135642	6012345	7	8	23	38	46	33	20
3564201	1234506	23	38	46	33	20	1	14	3564201	4560123	26	41	49	29	16	3	11
6420135	4506123	47	34	15	7	9	24	39	6420135	2345601	45	32	19	6	14	22	37
2013564	6123450	21	2	10	25	40	48	29	2013564	0123456	15	2	10	25	40	48	35
1356420	3450612	11	26	41	43	35	16	3	1356420	5601234	13	28	36	44	31	18	5
5642013	0612345	36	49	30	17	4	12	27	5642013	3456012	39	47	34	21	1	9	24
4201356	2345061	31	18	5	13	22	42	44	4201356	1234560	30	17	4	12	27	42	43
17/H	3/L							2305#	17/H	35/L							2497#
1036542	0156234	8	2	27	49	38	32	19	1036542	1065234	9	1	28	48	38	32	19
3654210	6234015	28	45	39	33	15	9	6	3654210	5234106	27	45	39	33	16	8	7
5421036	4015623	40	29	16	13	7	24	46	5421036	4106523	40	30	15	14	6	24	46
2103654	5623401	20	14	3	25	47	36	30	2103654	6523410	21	13	3	25	47	37	29
0365421	3401562	4	26	43	37	34	21	10	0365421	3410652	4	26	44	36	35	20	10
6542103	1562340	44	41	35	17	11	5	22	6542103	0652341	43	42	34	17	11	5	23
4210365	2340156	31	18	12	1	23	48	42	4210365	2341065	31	18	12	2	22	49	41
17/H	67/L							2689#	17/H	99/L							2881#
1036542	2064135	10	1	28	47	37	32	20	1036542	4062135	12	1	28	45	37	32	20
3654210	4135206	26	44	39	34	17	8	7	3654210	2135406	24	44	39	34	19	8	7
5421036	5206413	41	31	15	14	5	23	46	5421036	5406213	41	33	15	14	3	23	46
2103654	6413520	21	12	2	25	48	38	29	2103654	6213540	21	10	2	25	48	40	29
0365421	3520641	4	27	45	36	35	19	9	0365421	3540621	4	27	47	36	35	17	9
6542103	0641352	43	42	33	16	11	6	24	6542103	0621354	43	42	31	16	11	6	26
4210365	1352064	30	18	13	3	22	49	40	4210365	1354062	30	18	13	5	22	49	38
17/H	131/L							3073#	17/H	161/L							3265#
1036542	5061234	13	1	28	44	38	32	19	1036542	6012345	14	1	23	45	39	33	20
3654210	1234506	23	45	39	33	20	8	7	3654210	4560123	26	48	42	29	16	10	4
5421036	4506123	40	34	15	14	2	24	46	5421036	2345601	38	32	19	13	7	22	44
2103654	6123450	21	9	3	25	47	41	29	2103654	0123456	15	9	3	25	47	41	35
0365421	3450612	4	26	48	36	35	16	10	0365421	5601234	6	28	43	37	31	18	12
6542103	0612345	43	42	30	17	11	5	27	6542103	3456012	46	40	34	21	8	2	24
4210365	2345061	31	18	12	6	22	49	37	4210365	1234560	30	17	11	5	27	49	36

[Count = 3456] OK!

These 3456 solutions are just the same ones we have got before. We have made it! This means we could verify our "Knight's Compositions" was correct for order 7.

I must report that I could reconstruct 38102400 solutions of Pandiagonal MS77 by this method of Base 7, though I would like to skip my exact report here. If you want to have my report, please click at the file end to get the outline of that.

5. Why are 'Complete Euler Squares' 7x7 rare?

In the end I would like you to think about the reason why 'Complete Euler Squares' are found rare in the case of any odd order. There are many "Non-Euler" type of MS77, say, far more than 'Complete Euler Squares'. Why?

Please think about the next questions:

- (1) If $n_1+n_2+n_3+n_4+n_5+n_6+n_7=21$, then how many correct number patterns are possible to it? Every variable must take any one of {0,1,2,3,4,5,6}. But it can take any one number twice or more often and can neglect any other to take.
- (2) If $n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8=4$, then how many correct number patterns are possible to it? Every variable must take either one of {0,1}.

You see these questions are often asked when you compose any layer unit by the Positional Number System of Base 7 or 2.

**** Possible Number Patterns to the Equation of Layer Units of Order N ****

[Order 3: Base 3]

If $[n1+n2+n3==3]$ then $[{n1,n2,n3}=\{0, 1, 2\}, \{1, 1, 1\}]$

[Order 4: Base 2]

If $[n1+n2+n3+n4==2]$ then $[{n1,n2,n3,n4}=\{0, 0, 1, 1\}]$

[Order 5: Base 5]

If $[n1+n2+n3+n4+n5==10]$ then $[{n1,n2,n3,n4,n5}=\{0, 0, 2, 4, 4\}, \{0, 0, 3, 3, 4\}, \{0, 1, 1, 4, 4\}, \{0, 1, 2, 3, 4\}, \{0, 1, 3, 3, 3\}, \{0, 2, 2, 2, 4\}, \{0, 2, 2, 3, 3\}, \{1, 1, 1, 3, 4\}, \{1, 1, 2, 2, 4\}, \{1, 1, 2, 3, 3\}, \{1, 2, 2, 2, 3\}, \{2, 2, 2, 2, 2\}]$

[Order 7: Base 7]

If $[n1+n2+n3+n4+n5+n6+n7==21]$ then $[{n1,n2,n3,n4,n5,n6,n7}=\{0, 0, 0, 3, 6, 6, 6\}, \{0, 0, 0, 4, 5, 6, 6\}, \{0, 0, 0, 5, 5, 5, 6\}, \{0, 0, 1, 2, 6, 6, 6\}, \{0, 0, 1, 3, 5, 6, 6\}, \{0, 0, 1, 4, 4, 6, 6\}, \{0, 0, 1, 4, 5, 5, 6\}, \{0, 0, 1, 5, 5, 5, 5\}, \{0, 0, 2, 2, 5, 6, 6\}, \{0, 0, 2, 3, 4, 6, 6\}, \{0, 0, 2, 3, 5, 5, 6\}, \{0, 0, 2, 4, 4, 5, 6\}, \{0, 0, 2, 4, 5, 5, 5\}, \{0, 0, 3, 3, 3, 6, 6\}, \{0, 0, 3, 3, 4, 5, 6\}, \{0, 0, 3, 3, 5, 5, 5\}, \{0, 0, 3, 4, 4, 4, 6\}, \{0, 0, 3, 4, 4, 5, 5\}, \{0, 0, 4, 4, 4, 4, 5\}, \{0, 1, 1, 1, 6, 6, 6\}, \{0, 1, 1, 2, 5, 6, 6\}, \{0, 1, 1, 3, 4, 6, 6\}, \{0, 1, 1, 3, 5, 5, 6\}, \{0, 1, 1, 4, 4, 5, 6\}, \{0, 1, 1, 4, 5, 5, 5\}, \{0, 1, 2, 2, 4, 6, 6\}, \{0, 1, 2, 2, 5, 5, 6\}, \{0, 1, 2, 3, 3, 6, 6\}, \{0, 1, 2, 3, 4, 5, 6\}, \{0, 1, 2, 3, 5, 5, 5\}, \{0, 1, 2, 4, 4, 4, 6\}, \{0, 1, 2, 4, 4, 5, 5\}, \{0, 1, 3, 3, 3, 5, 6\}, \{0, 1, 3, 3, 4, 4, 6\}, \{0, 1, 3, 3, 4, 5, 5\}, \{0, 1, 3, 4, 4, 4, 5\}, \{0, 1, 4, 4, 4, 4, 4\}, \{0, 2, 2, 2, 3, 6, 6\}, \{0, 2, 2, 2, 4, 5, 6\}, \{0, 2, 2, 2, 5, 5, 5\}, \{0, 2, 2, 3, 3, 5, 6\}, \{0, 2, 2, 3, 4, 4, 6\}, \{0, 2, 2, 3, 4, 5, 5\}, \{0, 2, 2, 4, 4, 4, 5\}, \{0, 2, 3, 3, 3, 4, 6\}, \{0, 2, 3, 3, 3, 5, 5\}, \{0, 2, 3, 3, 4, 4, 5\}, \{0, 2, 3, 4, 4, 4, 4\}, \{0, 3, 3, 3, 3, 3, 6\}, \{0, 3, 3, 3, 3, 4, 5\}, \{0, 3, 3, 3, 4, 4, 4\}, \{1, 1, 1, 1, 5, 6, 6\}, \{1, 1, 1, 2, 4, 6, 6\}, \{1, 1, 1, 2, 5, 5, 6\}, \{1, 1, 1, 3, 3, 6, 6\}, \{1, 1, 1, 3, 4, 5, 6\}, \{1, 1, 1, 3, 5, 5, 5\}, \{1, 1, 1, 4, 4, 4, 6\}, \{1, 1, 1, 4, 4, 5, 5\}, \{1, 1, 2, 2, 3, 6, 6\}, \{1, 1, 2, 2, 4, 5, 6\}, \{1, 1, 2, 2, 5, 5, 5\}, \{1, 1, 2, 3, 3, 5, 6\}, \{1, 1, 2, 3, 4, 4, 6\}, \{1, 1, 2, 3, 4, 5, 5\}, \{1, 1, 2, 4, 4, 4, 5\}, \{1, 1, 3, 3, 3, 4, 6\}, \{1, 1, 3, 3, 3, 5, 5\}, \{1, 1, 3, 3, 4, 4, 5\}, \{1, 1, 3, 4, 4, 4, 4\}, \{1, 2, 2, 2, 2, 6, 6\}, \{1, 2, 2, 2, 3, 5, 6\}, \{1, 2, 2, 2, 4, 4, 6\}, \{1, 2, 2, 2, 4, 5, 5\}, \{1, 2, 2, 3, 3, 4, 6\}, \{1, 2, 2, 3, 3, 5, 5\}, \{1, 2, 2, 3, 4, 4, 5\}, \{1, 2, 2, 4, 4, 4, 4\}, \{1, 2, 3, 3, 3, 3, 6\}, \{1, 2, 3, 3, 3, 4, 5\}, \{1, 2, 3, 3, 4, 4, 4\}, \{1, 3, 3, 3, 3, 3, 5\}, \{1, 3, 3, 3, 3, 4, 4\}, \{2, 2, 2, 2, 2, 5, 6\}, \{2, 2, 2, 2, 3, 4, 6\}, \{2, 2, 2, 2, 3, 5, 5\}, \{2, 2, 2, 2, 4, 4, 5\}, \{2, 2, 2, 3, 3, 3, 6\}, \{2, 2, 2, 3, 3, 4, 5\}, \{2, 2, 2, 3, 4, 4, 4\}, \{2, 2, 3, 3, 3, 3, 5\}, \{2, 2, 3, 3, 3, 4, 4\}, \{2, 3, 3, 3, 3, 3, 4\}, \{3, 3, 3, 3, 3, 3, 3\}]$

[Order 8: Base 2]

If $[n1+n2+n3+n4+n5+n6+n7+n8==4]$ then $[{n1,n2,n3,n4,n5,n6,n7,n8}=\{0, 0, 0, 0, 1, 1, 1, 1\}]$

[Order 9: Base 3]

If $[n1+n2+n3+n4+n5+n6+n7+n8+n9==9]$ then $[{n1,n2,n3,n4,n5,n6,n7,n8,n9}=\{0, 0, 0, 0, 1, 2, 2, 2, 2\}, \{0, 0, 0, 1, 1, 1, 2, 2, 2\}, \{0, 0, 1, 1, 1, 1, 1, 2, 2\}, \{0, 1, 1, 1, 1, 1, 1, 1, 2\}, \{1, 1, 1, 1, 1, 1, 1, 1, 1\}]$

The only one number pattern can make the 'Euler Square' in each case.
I hope you could see how rare it is, especially in the case of order 7.

(October 16, 2003; Revised on September 15, 2005;
New Calculation by Kanji Setsuda on MacOSX and Xcode 1.5)

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