Part 1 : "Basic Studies of Magic Squares and Cubes"
Chapter 4 : "Advanced Algebraic Study of Magic Cubes"

Section 1 : Pan-Magic Cubes of Order 4: Kanji Setsuda

#1. Basic Form, Basic Conditions and Extended Space

Let's study about the pan-diagonal magic cube of order 4. It should be built in the basic form shown below. You can make it up of the natural numbers 1 to 64, using each strictly once, and place them in the 4x4x4 regular array. The basic form shows their positions and let's call the variables "n1, n2, n3, n4, ..., n63 and n64."

We must assume all four entries on every row, every column and every pillar in the three directions add up to the magic constant K(=130), and even all four entries on 64 pan-diagonals (I should say "pan-triagonals") must add up to the same sum.

[Figure 1: Basic Form of Magic Cube 4x4x4]

```
 1--------17--------33--------49
 | 2       |18       |34       |50
 5   3     21  19    37  35    53  51
 | 6   4----22--20----38--36----54--52
 9   7 |   25  23 |  41  39 |  57  55 |
 |10   8   |26  24   |42  40   |58  56
13--11-|--29--27-|--45--43-|--61  59 |
  14  12     30  28    46  44    62  60
    15 |       31 |      47 |      63 |
      16--------32--------48--------64
```

[Basic Conditions:]

```
  n1+ n2+ n3+ n4=K;  n1+ n5+ n9+n13=K;  n1+n17+n33+n49=K;
  n5+ n6+ n7+ n8=K;  n2+ n6+n10+n14=K;  n2+n18+n34+n50=K;
  n9+n10+n11+n12=K;  n3+ n7+n11+n15=K;  n3+n19+n35+n51=K;
 n13+n14+n15+n16=K;  n4+ n8+n12+n16=K;  n4+n20+n36+n52=K;
 n17+n18+n19+n20=K; n17+n21+n25+n29=K;  n5+n21+n37+n53=K;
 n21+n22+n23+n24=K; n18+n22+n26+n30=K;  n6+n22+n38+n54=K;
 .....              .....               .....
 n57+n58+n59+n60=K; n51+n55+n59+n63=K; n15+n31+n47+n63=K;
 n61+n62+n63+n64=K; n52+n56+n60+n64=K; n16+n32+n48+n64=K;
```

[Pan-Triagonals:]

```
 n1+n22+n43+n64=K;  n1+n24+n43+n62=K;  n1+n30+n43+n56=K;  n1+n32+n43+n54=K;
 n2+n23+n44+n61=K;  n2+n21+n44+n63=K;  n2+n31+n44+n53=K;  n2+n29+n44+n55=K;
 n3+n24+n41+n62=K;  n3+n22+n41+n64=K;  n3+n32+n41+n54=K;  n3+n30+n41+n56=K;
 n4+n21+n42+n63=K;  n4+n23+n42+n61=K;  n4+n29+n42+n55=K;  n4+n31+n42+n53=K;
 n5+n26+n47+n52=K;  n5+n28+n47+n50=K;  n5+n18+n47+n60=K;  n5+n20+n47+n58=K;
 n6+n27+n48+n49=K;  n6+n25+n48+n51=K;  n6+n19+n48+n57=K;  n6+n17+n48+n59=K;
 n7+n28+n45+n50=K;  n7+n26+n45+n52=K;  n7+n20+n45+n58=K;  n7+n18+n45+n60=K;
 n8+n25+n46+n51=K;  n8+n27+n46+n49=K;  n8+n17+n46+n59=K;  n8+n19+n46+n57=K;
 n9+n30+n35+n56=K;  n9+n32+n35+n54=K;  n9+n22+n35+n64=K;  n9+n24+n35+n62=K;
n10+n31+n36+n53=K; n10+n29+n36+n55=K; n10+n23+n36+n61=K; n10+n21+n36+n63=K;
n11+n32+n33+n54=K; n11+n30+n33+n56=K; n11+n24+n33+n62=K; n11+n22+n33+n64=K;
n12+n29+n34+n55=K; n12+n31+n34+n53=K; n12+n21+n34+n63=K; n12+n23+n34+n61=K;
n13+n18+n39+n60=K; n13+n20+n39+n58=K; n13+n26+n39+n52=K; n13+n28+n39+n50=K;
n14+n19+n40+n57=K; n14+n17+n40+n59=K; n14+n27+n40+n49=K; n14+n25+n40+n51=K;
n15+n20+n37+n58=K; n15+n18+n37+n60=K; n15+n28+n37+n50=K; n15+n26+n37+n52=K;
n16+n17+n38+n59=K; n16+n19+n38+n57=K; n16+n25+n38+n51=K; n16+n27+n38+n49=K
```

We must assume such simultaneous equations written as above, so many as 112 equations in all (112=48+64).

The next diagram is drawn for the 'Extended Space' of pan-magic cube of order 4.

I invented it so that you could easily see any pan-triagonal in a 'straight' line and find some other cubes from your different viewpoints.

[Fig.2: Extended Space for Pan-Triagonals of Magic Cube 4*4*4]

```
11
  12                                                              12
15   9                                                               9
  16  10          32              48              64              16  10
 3  13  11          29              45              61              13  11
   4  14  12      20  30          36  46          52  62           4  14  12
 7   1--15---9------17--31----------33--47----------49--63-----------1  15   9
   8 | 2  16  10  24  18  32      40  34  48      56  50  64       8 | 2  16  10
11   5   3  13      21  19  29      37  35  45      53  51  61       5   3  13  11
  12 | 6   4  14  28  22--20--30--44--38--36--46--60--54  52  62  12 | 6   4  14
15   9   7   1------25--23--17------41--39--33------57--55--49-------9---7---1  15
  16 |10   8 | 2  32  26  24--18--48--42--40--34--64--58--56  50  16 |10   8 | 2
 3  13  11   5      29| 27 |21      45| 43 |37      61| 59 |53      13  11   5   3
   4 |14  12 | 6  20  30--28--22--36--46--44--38--52--62  60  54   4 |14  12 | 6
 7   1--15---9------17--31-|25------33--47-|41------49--63-|57-------1  15   9   7
   8   2  16 |10  24  18  32--26------34--48--42--56--50--64  58   8   2  16 |10
11   5   3  13      21  19  29          35  45      53  51  61       5   3  13  11
  12   6   4 |14      22  20  30          36  46      54  52  62  12   6   4 |14
     9   7   1----------23--17--------------33----------55--49-------9---7---1  15
      10   8   2          24  18              34          56  50      10   8   2
        11   5              21                              53          11   5   3
          12   6              22                              54          12   6
             9                                                               9   7
              10                                                              10
                                                                                11
```

#2. Let's Calculate and Find some Interesting Properties

Let's pick up some equations of pan-triagonals and compare and calculate them.

Compare those equations and take out complementary pairs as many as possible.

```
n1+n22+n43+n64=K;  n1+n24+n43+n62=K;  n1+n30+n43+n56=K;  n1+n32+n43+n54=K;
 (n1+n43)=C1; n22+n64=n24+n62=n30+n56=n32+n54=K-C1=D1; C1+D1=K       ...(130)
n2+n23+n44+n61=K;  n2+n21+n44+n63=K;  n2+n31+n44+n53=K;  n2+n29+n44+n55=K;
 (n2+n44)=C2; n23+n61=n21+n63=n31+n53=n29+n55=K-C2=D2; C2+D2=K       ...(131)
n3+n24+n41+n62=K;  n3+n22+n41+n64=K;  n3+n32+n41+n54=K;  n3+n30+n41+n56=K;
 (n3+n41)=C3; n24+n62=n22+n64=n32+n54=n30+n56=K-C3=D3; C3+D3=K       ...(132)
n4+n21+n42+n63=K;  n4+n23+n42+n61=K;  n4+n29+n42+n55=K;  n4+n31+n42+n53=K;
 (n4+n42)=C4; n21+n63=n23+n61=n29+n55=n31+n53=K-C4=D4; C4+D4=K       ...(133)
 Compare (130)~(133), and you will find D3=D1; D4=D2; C3=C1; and C4=C2;

n5+n26+n47+n52=K;  n5+n28+n47+n50=K;  n5+n18+n47+n60=K;  n5+n20+n47+n58=K;
 (n5+n47)=C5; n26+n52=n28+n50=n18+n60=n20+n58=K-C5=D5; C5+D5=K       ...(134)
n6+n27+n48+n49=K;  n6+n25+n48+n51=K;  n6+n19+n48+n57=K;  n6+n17+n48+n59=K;
 (n6+n48)=C6; n27+n49=n25+n51=n19+n57=n17+n59=K-C6=D6; C6+D6=K       ...(135)
n7+n28+n45+n50=K;  n7+n26+n45+n52=K;  n7+n20+n45+n58=K;  n7+n18+n45+n60=K;
 (n7+n45)=C7; n28+n50=n26+n52=n20+n58=n18+n60=K-C7=D7; C7+D7=K       ...(136)
n8+n25+n46+n51=K;  n8+n27+n46+n49=K;  n8+n17+n46+n59=K;  n8+n19+n46+n57=K;
 (n8+n46)=C8; n25+n51=n27+n49=n17+n59=n19+n57=K-C8=D8; C8+D9=K       ...(137)
 Compare (134)~(137), and you will find D7=D5; D8=D6; C7=C5; and C8=C6;

 (n9+n35)=C9;  n30+n56=n32+n54=n22+n64=n24+n62=K-C9=D9;   C9+D9=K    ...(138)
(n10+n36)=C10; n31+n53=n29+n55=n23+n61=n21+n63=K-C10=D10; C10+D10=K ...(139)
(n11+n33)=C11; n32+n54=n30+n56=n24+n62=n22+n64=K-C11=D11; C11+D11=K ...(140)
```

```
(n12+n34)=C12; n29+n55=n31+n53=n21+n63=n23+n61=K-C12=D12; C12+D12=K ...(141)
 Compare (138)~(141) with (130)~(133), you will find
   D9=D1; D10=D2; D11=D1; D12=D2; C9=C1; C10=C2; C11=C1; and C12=C2

(n13+n39)=C13; n18+n60=n20+n58=n26+n52=n28+n50=K-C13=D13; C13+D13=K ...(142)
(n14+n40)=C14; n19+n57=n17+n59=n27+n49=n25+n51=K-C14=D14; C14+D14=K ...(143)
(n15+n37)=C15; n20+n58=n18+n60=n28+n50=n26+n52=K-C15=D15; C15+D15=K ...(144)
(n16+n38)=C16; n17+n59=n19+n57=n25+n51=n27+n49=K-C16=D16; C16+D16=K ...(145)
 Compare (142)~(145) with (134)~(137), you will find
   D13=D5; D14=D6; D15=D5; D16=D6; C13=C5; C14=C6; C15=C5; and C16=C6

n6+n27+n48+n49=K;  n8+n27+n46+n49=K; n14+n27+n40+n49=K; n16+n27+n38+n49=K;
 (n27+n49)=D6; n6+n48=n8+n46=n14+n40=n16+n38=K-D6=C6
     D6           C6  =  C8  =   C14 =   C16

n1+n22+n43+n64=K;  n3+n22+n41+n64=K;  n9+n22+n35+n64=K; n11+n22+n33+n64=K;
 (n22+n64)=D1; n1+n43=n3+n41=n9+n35=n11+n33=K-D1=C1
                  C1  =  C3  =  C9  =   C11

 Make 4 groups according to its values:
   C1+D1 Group[C1+D1, C3+D3, C9+D9, C11+D11]
   C2+D2 Group[C2+D2, C4+D4, C10+D10, C12+D12]
   C5+D5 Group[C5+D5, C7+D7, C13+D13, C15+D15]
   C6+D6 Group[C6+D6, C8+D8, C14+D14, C16+D16]

C1+C2+C3+C4
 =(n1+n43)+(n2+n44)+(n3+n41)+(n4+n42)=(n1+n2+n3+n4)+(n41+n42+n43+n44)=2*K
C3=C1; C4=C2; --> C1+C2+C3+C4=C1+C2+C1+C2=2*(C1+C2)=2*K; C1+C2=K;
But C1+D1=K; C2+D2=K; Consequently C2=D1; C1=D2

C5+C6+C7+C8
 =(n5+n47)+(n6+n48)+(n7+n45)+(n8+n46)=(n5+n6+n7+n8)+(n45+n46+n47+n48)=2*K
C7=C5; C8=C6; --> C5+C6+C7+C8=C5+C6+C5+C6=2*(C5+C6)=2*K; C5+C6=K;
But C5+D5=K; C6+D6=K; As a consequence C6=D5; C5=D6

C1+C5+C9+c13
 =(n1+n43)+(n5+n47)+(n9+n35)+(n13+n39)=(n1+n5+n9+n13)+(n35+n39+n43+n47)=2*K
C9=C1; C13=C5; --> C1+C5+C9+c13=C1+C5+C1+C5=2*(C1+C5)=2*K; C1+C5=K;
But C1+D1=K; C5+D5=K; Therefore C1=D5; C5=D1

C2+C6+C10+C14
 =(n2+n44)+(n6+n48)+(n10+n36)+(n14+n40)=(n2+n6+n10+n14)+(n36+n40+n44+n48)=2*K
C10=C2; C14=C6; --> C2+C6+C10+C14=C2+C6+C2+C6=2*(C2+C6)=2*K; C2+C6=K;
But C2+D2=K; C6+D6=K; As a result C2=D6; C6=D2

 Remake the groups according to their values:
   CC Group [C1,C3,C6,C8,C9,C11,C14,C16,D2,D4,D5,D7,D10,D12,D13,D15]
   DC Group [C2,C4,C5,C7,C10,C12,C13,C15,D1,D3,D6,D8,D9,D11,D14,D16]
 That means:
 n1+n43=n3+n41=n6+n48=n8+n46=n9+n35=n11+n33=n14+n40=n16+n38
   =n18+n60=n20+n58=n21+n63=n23+n61=n26+n52=n28+n50=n29+n55=n31+n53=CC   ...(146)
 n2+n44=n4+n42=n5+n47=n7+n45=n10+n36=n12+n34=n13+n39=n15+n37
   =n17+n59=n19+n57=n22+n64=n24+n62=n25+n51=n27+n49=n30+n56=n32+n54=DD   ...(147)
 (CC+DD=K)
```

All complementary pairs on pan-triagonals are classified into two groups according to their values. One group has 16 pairs of the same sum CC and the other has 16 pairs of DD. Let's call CC, DD simply as 'C, D' from now on.

All pan-triagonals are made of complementary pairs of C and of D.
Take some of the primary triagonals for instance:

```
  n1+n22+n43+n64=(n1+n43)+(n22+n64)=C+D=K
  n4+n23+n42+n61=(n4+n42)+(n23+n61)=D+C=K
 n13+n26+n39+n52=(n13+n39)+(n26+n52)=D+C=K
 n16+n27+n38+n49=(n16+n38)+(n27+n49)=C+D=K
  . . . . . . . . .
If you put the value 1 to n1 and 32 to n43,
  then n1+n43=1+32=33=C; n22+n64=D=K-C=130-33=97.
If you put n1=1 and n43=64,
  then C=n1+n43=1+64=65; and n22+n64=D=K-C=130-65=65; That means C=D.
```

What values could any pair of {C, D} take, then? Of course, C+D=K. Are there so many pairs of {C, D} as 16?

#3. How many Pairs of C and D could we find?

For instance, suppose n1=1 and n43=16; and you will know C=n1+n43=1+16=17; D=K-C=130-17=113

How many pairs could we make then?

```
  C1(1,16),D1(64,49); C2(2,15),D2(63,50); C3(3,14),D3(62,51);
  C4(4,13),D4(61,52); C5(5,12),D5(60,53); C6(6,11),D6(59,54);
  C7(7,10),D7(58,55); C8(8,9),D8(57,56); C9(9,8),D9(56,57)? ...
```

But C9=C8, D9=D8. This means the end of making new pairs.
We could only make 8 pairs of C and also 8 pairs of D.
With so few pairs we cannot make any pan-triagonal magic cube of order 4 at all.

The next list below is the result of making pairs according to the values of C and D.
I found we could only make less than 16 pairs when C<33.

** Find Possible Complementary Pairs for Magic Cube 4x4x4 **

```
CC=32:  C1(1,31),C2(2,30),C3(3,29),C4(4,28),C5(5,27),C6(6,26),
        C7(7,25),C8(8,24),C9(9,23),C10(10,22),C11(11,21),C12(12,20),
        C13(13,19),C14(14,18),C15(15,17),
DD=98:  D1(64,34),D2(63,35),D3(62,36),D4(61,37),D5(60,38),D6(59,39),
        D7(58,40),D8(57,41),D9(56,42),D10(55,43),D11(54,44),D12(53,45),
        D13(52,46),D14(51,47),D15(50,48),

CC=33:  C1(1,32),C2(2,31),C3(3,30),C4(4,29),C5(5,28),C6(6,27),
        C7(7,26),C8(8,25),C9(9,24),C10(10,23),C11(11,22),C12(12,21),
        C13(13,20),C14(14,19),C15(15,18),C16(16,17),
DD=97:  D1(64,33),D2(63,34),D3(62,35),D4(61,36),D5(60,37),D6(59,38),
        D7(58,39),D8(57,40),D9(56,41),D10(55,42),D11(54,43),D12(53,44),
        D13(52,45),D14(51,46),D15(50,47),D16(49,48)    ......[OK!]

CC=34:  C1(1,33),C2(3,31),C3(4,30),C4(5,29),C5(6,28),C6(7,27),
        C7(8,26),C8(9,25),C9(10,24),C10(11,23),C11(12,22),C12(13,21),
        C13(14,20),C14(15,19),C15(16,18),
DD=96:  D1(64,32),D2(62,34),D3(61,35),D4(60,36),D5(59,37),D6(58,38),
        D7(57,39),D8(56,40),D9(55,41),D10(54,42),D11(53,43),D12(52,44),
        D13(51,45),D14(50,46),D15(49,47),

   . . . . .

CC=48:  C1(1,47),C2(2,46),C3(3,45),C4(4,44),C5(5,43),C6(6,42),
        C7(7,41),C8(8,40),C9(9,39),C10(10,38),C11(11,37),C12(12,36),
        C13(13,35),C14(14,34),C15(15,33),
```

```
DD=82:  D1(64,18),D2(63,19),D3(62,20),D4(61,21),D5(60,22),D6(59,23),
        D7(58,24),D8(57,25),D9(56,26),D10(55,27),D11(54,28),D12(53,29),
        D13(52,30),D14(51,31),D15(50,32),

CC=49:  C1(1,48),C2(2,47),C3(3,46),C4(4,45),C5(5,44),C6(6,43),
        C7(7,42),C8(8,41),C9(9,40),C10(10,39),C11(11,38),C12(12,37),
        C13(13,36),C14(14,35),C15(15,34),C16(16,33),
DD=81:  D1(64,17),D2(63,18),D3(62,19),D4(61,20),D5(60,21),D6(59,22),
        D7(58,23),D8(57,24),D9(56,25),D10(55,26),D11(54,27),D12(53,28),
        D13(52,29),D14(51,30),D15(50,31),D16(49,32)    ......[OK!]

CC=50:  C1(1,49),C2(2,48),C3(3,47),C4(4,46),C5(5,45),C6(6,44),
        C7(7,43),C8(8,42),C9(9,41),C10(10,40),C11(11,39),C12(12,38),
        C13(13,37),C14(14,36),C15(15,35),
DD=80:  D1(64,16),D2(63,17),D3(62,18),D4(61,19),D5(60,20),D6(59,21),
        D7(58,22),D8(57,23),D9(56,24),D10(55,25),D11(54,26),D12(53,27),
        D13(52,28),D14(51,29),D15(50,30),


CC=56:  C1(1,55),C2(2,54),C3(3,53),C4(4,52),C5(5,51),C6(6,50),
        C7(7,49),C8(8,48),C9(9,47),C10(19,37),C11(20,36),C12(21,35),
        C13(22,34),C14(23,33),
DD=74:  D1(64,10),D2(63,11),D3(62,12),D4(61,13),D5(60,14),D6(59,15),
        D7(58,16),D8(57,17),D9(56,18),D10(46,28),D11(45,29),D12(44,30),
        D13(43,31),D14(42,32),

CC=57:  C1(1,56),C2(2,55),C3(3,54),C4(4,53),C5(5,52),C6(6,51),
        C7(7,50),C8(8,49),C9(17,40),C10(18,39),C11(19,38),C12(20,37),
        C13(21,36),C14(22,35),C15(23,34),C16(24,33),
DD=73:  D1(64,9),D2(63,10),D3(62,11),D4(61,12),D5(60,13),D6(59,14),
        D7(58,15),D8(57,16),D9(48,25),D10(47,26),D11(46,27),D12(45,28),
        D13(44,29),D14(43,30),D15(42,31),D16(41,32)    ......[OK!]

CC=58:  C1(1,57),C2(2,56),C3(3,55),C4(4,54),C5(5,53),C6(6,52),
        C7(7,51),C8(15,43),C9(16,42),C10(17,41),C11(18,40),C12(19,39),
        C13(20,38),C14(21,37),
DD=72:  D1(64,8),D2(63,9),D3(62,10),D4(61,11),D5(60,12),D6(59,13),
        D7(58,14),D8(50,22),D9(49,23),D10(48,24),D11(47,25),D12(46,26),
        D13(45,27),D14(44,28),

CC=59:  C1(1,58),C2(2,57),C3(3,56),C4(4,55),C5(5,54),C6(6,53),
        C7(13,46),C8(14,45),C9(15,44),C10(16,43),C11(17,42),C12(18,41),
        C13(25,34),C14(26,33),C15(29,30),
DD=71:  D1(64,7),D2(63,8),D3(62,9),D4(61,10),D5(60,11),D6(59,12),
        D7(52,19),D8(51,20),D9(50,21),D10(49,22),D11(48,23),D12(47,24),
        D13(40,31),D14(39,32),D15(36,35),

CC=60:  C1(1,59),C2(2,58),C3(3,57),C4(4,56),C5(5,55),C6(11,49),
        C7(12,48),C8(13,47),C9(14,46),C10(15,45),C11(21,39),C12(22,38),
        C13(23,37),C14(24,36),C15(25,35),
DD=70:  D1(64,6),D2(63,7),D3(62,8),D4(61,9),D5(60,10),D6(54,16),
        D7(53,17),D8(52,18),D9(51,19),D10(50,20),D11(44,26),D12(43,27),
        D13(42,28),D14(41,29),D15(40,30),

CC=61:  C1(1,60),C2(2,59),C3(3,58),C4(4,57),C5(9,52),C6(10,51),
        C7(11,50),C8(12,49),C9(17,44),C10(18,43),C11(19,42),C12(20,41),
        C13(25,36),C14(26,35),C15(27,34),C16(28,33),
DD=69:  D1(64,5),D2(63,6),D3(62,7),D4(61,8),D5(56,13),D6(55,14),
        D7(54,15),D8(53,16),D9(48,21),D10(47,22),D11(46,23),D12(45,24),
        D13(40,29),D14(39,30),D15(38,31),D16(37,32)    ......[OK!]
```

```
CC=62:  C1(1,61),C2(2,60),C3(3,59),C4(7,55),C5(8,54),C6(9,53),
        C7(13,49),C8(14,48),C9(15,47),C10(19,43),C11(20,42),C12(21,41),
        C13(25,37),C14(26,36),C15(27,35),
DD=68:  D1(64,4),D2(63,5),D3(62,6),D4(58,10),D5(57,11),D6(56,12),
        D7(52,16),D8(51,17),D9(50,18),D10(46,22),D11(45,23),D12(44,24),
        D13(40,28),D14(39,29),D15(38,30),

CC=63:  C1(1,62),C2(2,61),C3(5,58),C4(6,57),C5(9,54),C6(10,53),
        C7(13,50),C8(14,49),C9(17,46),C10(18,45),C11(21,42),C12(22,41),
        C13(25,38),C14(26,37),C15(29,34),C16(30,33),
DD=67:  D1(64,3),D2(63,4),D3(60,7),D4(59,8),D5(56,11),D6(55,12),
        D7(52,15),D8(51,16),D9(48,19),D10(47,20),D11(44,23),D12(43,24),
        D13(40,27),D14(39,28),D15(36,31),D16(35,32)    ......[OK!]

CC=64:  C1(1,63),C2(3,61),C3(5,59),C4(7,57),C5(9,55),C6(11,53),
        C7(13,51),C8(15,49),C9(17,47),C10(19,45),C11(21,43),C12(23,41),
        C13(25,39),C14(27,37),C15(29,35),C16(31,33),
DD=66:  D1(64,2),D2(62,4),D3(60,6),D4(58,8),D5(56,10),D6(54,12),
        D7(52,14),D8(50,16),D9(48,18),D10(46,20),D11(44,22),D12(42,24),
        D13(40,26),D14(38,28),D15(36,30),D16(34,32)    ......[OK!]

CC=DD=65: 1(1,64),2(2,63),3(3,62),4(4,61),5(5,60),6(6,59),7(7,58),8(8,57),
        9(9,56),10(10,55),11(11,54),12(12,53),13(13,52),14(14,51),
        15(15,50),16(16,49),17(17,48),18(18,47),19(19,46),20(20,45),
        21(21,44),22(22,43),23(23,42),24(24,41),25(25,40),26(26,39),
        27(27,38),28(28,37),29(29,36),30(30,35),31(31,34),32(32,33)  ....[OK!]
```

I found that the next seven cases could only satisfy our request:

(C,D)=(33,97), (49,81), (57,73), (61,69), (63,67), (64,66) and (65,65)

The last case (C=D=65) needs not be classified into two groups of C and D, because we can find no difference between them. We must accept it has 32 pairs of 65 in all, and all pairs should be placed anywhere only on pan-triagonals but no one elsewhere.

I would like to call this type "Complete pan-magic" from now on.

Notice every case has the pair C1(1,N) on the list head.

Each of (1,32), (1,48), (1,56), (1,60), (1,62), (1,63) and (1,64) is supposed to be the index pair of the group of which it must be a representative member.

If you put the value 1 to n1, then you must put any of {32, 48, 56, 60, 62, 63 or 64} to n43, and you must not put anything else there.

However surprising it might look, it is certainly one of the destinies of our pan-magic cube of order 4.

#4. How many Solutions could we find?

Take the first Basic Form and the 112 equations of our definition stage, and assume the 'standard form' with n1=1 and put some inequality conditions to it:

n2>n5>n17; n2>n4; n5>n13; and n17>n49

It is because you can exchange the 2nd and 4th planes with each other in all three directions, and you can identify the patterns of which the 2nd-4th planes were exchanged with the original ones.

We want to count only the 'fundamental solutions' and prevent us from double counting of any simple reflections or rotations.

(n1, n43)=(1, 32): 469,921 solutions are found.
(n1, n43)=(1, 48): 377,803 ...
(n1, n43)=(1, 56): 355,173 ...

```
(n1, n43)=(1, 60): 355,173 ...
(n1, n43)=(1, 62): 377,803 ...
(n1, n43)=(1, 63): 469,921 ...
(n1, n43)=(1, 64): ???
```

In the last case (C=D=65) I could not yet count up through, because it takes too much time to calculate and count up. But the counts seem to exceed far over the sum of all the former six cases.

[Figure 3: **List of Pan-Magic Cubes 4*4*4** (Part)]

```
** Pan-Triagonal Type: n43=32; CC=33; DD=97 **
  1/                                            2/
   1        60        10        59               1        60        13        56
    64         5        52         9              64         5        52         9
  62   2    13  53    39  27    16  48           62   2    19  47    42  27     7  54
     3  63    55  12    29  41    43  14            3  63    46  18    23  38    58  11
   6  47    49  22    31  36    44  25             6  53    43  16    31  36    50  25
    56  18    19  40    34  26    21  46            59  12    22  49    34  29    15  40
  61  23     8  38    50  32    11  37           61  20     8  41    44  32    17  37
     7  45    51  24    15  33    57  28             4  45    57  24    21  33    48  28
      58        17        35        20                55        26        35        14
         4        54        30        42                10        39        30        51
2715/                                         2716/
   1        60        13        56               1        60        13        56
    64        18        39         9              64         7        50         9
  63   3     8  45    49  28    10  54           63   3    19  45    38  28    10  54
     2  62    44   7    29  50    55  11            2  62    44  18    29  39    55  11
   5  59    43  21    30  36    52  14             5  48    43  21    30  36    52  25
    47   6    22  57    35  16    26  51            58  17    22  46    35  27    15  40
  61  20    19  41    38  32    12  37           61  20     8  41    49  32    12  37
    17  58    46  24    27  33    40  15             6  47    57  24    16  33    51  26
      48        23        34        25                59        23        34        14
         4        42        31        53                 4        42        31        53
5295/                                         5296/
   1        62        10        57               1        62        10        57
    64         3        55         8              64         3        55         8
  63   4     7  54    44  27    16  45           63   4     7  54    44  27    16  45
     2  61    58  11    21  38    49  20            2  61    56  11    21  38    51  20
   6  50    52  19    29  37    43  24             6  50    52  19    29  37    43  24
    59  15    13  46    36  28    22  41            59  15    13  48    36  28    22  39
  60  23     9  40    47  32    14  35           60  23     9  40    47  32    14  35
     5  42    56  25    18  33    51  30             5  42    58  25    18  33    49  30
      53        17        34        26                53        17        34        26
        12        48        31        39                12        46        31        41
```

....

```
** Pan-Triagonal Type: n43=48; CC=49; DD=81 **
  1/                                            2/
   1        59        38        32               1        59        38        32
    64        10        23        33              64         7        26        33
  62   2     9  54    24  43    35  31           62   2    12  54    21  43    35  31
     3  63    52   7    45  26    30  34            3  63    52  10    45  23    30  34
   6  60    50  13    47  20    27  37             6  57    50  13    47  20    27  40
    55   5    15  56    18  41    42  28            58   8    15  53    18  44    39  25
  61  11    12  49    21  48    36  22           61  11     9  49    24  48    36  22
     8  58    53  16    44  17    25  39             5  55    56  16    41  17    28  42
      57        14        19        40                60        14        19        37
         4        51        46        29                 4        51        46        29
```

```
1357/                                     1358/
   1        60        13        56           1        60        13        56
    64        34        23         9          64         7        50         9
  63   3     8  29    49  44    10  54      63   3    35  29    22  44    10  54
     2  62    28   7    45  50    55  11       2  62    28  34    45  23    55  11
   5  59    27  37    46  20    52  14       5  32    27  37    46  20    52  41
    31   6    38  57    19  16    42  51      58  33    38  30    19  43    15  24
  61  36    35  25    22  48    12  21      61  36     8  25    49  48    12  21
    33  58    30  40    43  17    24  15       6  31    57  40    16  17    51  42
      32        39        18        41          59        39        18        14
         4        26        47        53           4        26        47        53

2819/                                     2820/
   1        62        11        56           1        62        11        56
    64         3        54         9          64         3        54         9
  63   4     7  30    50  43    10  53      63   4    10  53    24  43    33  30
     2  61    58  35    15  22    55  12       2  61    55  12    41  22    32  35
   6  57    28  36    45  21    51  16       6  31    51  36    45  21    28  42
    59   8    37  29    20  44    14  49      59  34    14  29    20  44    37  23
  60  38    33  25    24  48    13  19      60  38     7  25    50  48    13  19
     5  27    32  40    41  17    52  46       5  27    58  40    15  17    52  46
      31        39        18        42          57        16        18        39
        34        26        47        23           8        49        47        26
.........
```

** Pan-Triagonal Type: n43=56; CC=57; DD=73 **

```
  1/                                        2/
   1        60        21        48           1        60        22        47
    64         5        44        17          64         5        43        18
  62   2    35  31    26  51     7  46      62   2     7  31    28  51    33  46
     3  63    30  34    39  14    58  19       3  63    58  34    37  14    32  19
   6  45    27  24    55  12    42  49       6  48    27  49    55  12    42  21
    59  20    38  41    10  53    23  16      59  17    38  16    10  53    23  44
  61  36     8  25    28  56    33  13      61  35    36  26    25  56     8  13
     4  29    57  40    37   9    32  52       4  30    29  39    40   9    57  52
      47        50        11        22          45        24        11        50
        18        15        54        43          20        41        54        15

977/                                      978/
   1        60        21        48           1        60        21        48
    64        34        15        17          64         7        42        17
  63   3     8  29    41  52    18  46      63   3    35  29    14  52    18  46
     2  62    28   7    53  42    47  19       2  62    28  34    53  15    47  19
   5  59    27  37    54  12    44  22       5  32    27  37    54  12    44  49
    31   6    38  57    11  24    50  43      58  33    38  30    11  51    23  16
  61  36    35  25    14  56    20  13      61  36     8  25    41  56    20  13
    33  58    30  40    51   9    16  23       6  31    57  40    24   9    43  50
      32        39        10        49          59        39        10        22
         4        26        55        45           4        26        55        45

2318/                                     2319/
   1        62        19        48           1        62        19        48
    64         3        46        17          64        35        14        17
  63   4     7  30    42  51    18  45      63   4     7  30    26  51    34  45
     2  61    58  35    23  14    47  20      18  61    42   3    55  46    15  20
   6  57    28  36    53  13    43  24       6  41    28  52    53  13    43  24
    59   8    37  29    12  52    22  41      27   8    37  29    12  36    54  57
  60  38    33  25    16  56    21  11      60  38    33  25    32  56     5  11
     5  27    32  40    49   9    44  54      21  59    16  40    49   9    44  22
      31        39        10        50          47        23        10        50
        34        26        55        15           2        58        39        31
.........
```

** Pan-Triagonal Type: n43=60; CC=61; DD=69 **

1/
```
 1         61         20         48
  64          4         45         17
62    2    12   31    37   50    19   47
   3   63    53   34    28   15    46   18
11   55    22   36    59   13    38   26
  54   10    43   29     6   52    27   39
56   41    35   21    14   60    25    8
   9   24    30   44    51    5    40   57
    32         42          7         49
       33         23         58         16
```

2/
```
 1         61         20         48
  64          4         45         17
62    2    19   47    14   50    35   31
   3   63    46   18    51   15    30   34
11   32    38   36    59   13    22   49
  54   33    27   29     6   52    43   16
56   41    12   21    37   60    25    8
   9   24    53   44    28    5    40   57
    55         26          7         42
       10         39         58         23
```

1128/
```
 1         61         20         48
  64          4         45         17
63    3    12   30    37   51    18   46
   2   62    53   35    28   14    47   19
10   54    23   36    58   13    39   27
  55   11    42   29     7   52    26   38
56   41    34   21    15   60    25    8
   9   24    31   44    50    5    40   57
    32         43          6         49
       33         22         59         16
```

1129/
```
 1         61         20         48
  64          4         45         17
63    3    18   46    15   51    34   30
   2   62    47   19    50   14    31   35
10   32    39   36    58   13    23   49
  55   33    26   29     7   52    42   16
56   41    12   21    37   60    25    8
   9   24    53   44    28    5    40   57
    54         27          6         43
       11         38         59         22
```

2618/
```
 1         62         19         48
  64          3         46         17
63    4    11   30    38   51    18   45
   2   61    54   35    27   14    47   20
10   53    24   36    57   13    39   28
  55   12    41   29     8   52    26   37
56   42    33   21    16   60    25    7
   9   23    32   44    49    5    40   58
    31         43          6         50
       34         22         59         15
```

2619/
```
 1         62         19         48
  64         35         14         17
63    4    11   30    22   51    34   45
  18   61    38    3    59   46    15   20
10   37    24   52    57   13    39   28
  23   12    41   29     8   36    58   53
56   42    33   21    32   60     9    7
  25   55    16   44    49    5    40   26
    47         27          6         50
        2         54         43         31
```

....

** Pan-Triagonal Type: n43=62; CC=63; DD=67 **

1/
```
 1         56         25         48
  64          9         40         17
60    2    37   31    20   53    13   44
   5   63    28   34    45   12    52   21
10   43    23   30    61    8    36   49
  55   22    42   35     4   57    29   16
59   38    14   19    24   62    33   11
   6   27    51   46    41    3    32   54
    47         50          7         26
       18         15         58         39
```

2/
```
 1         56         26         47
  64          9         39         18
60    2    13   31    24   53    33   44
   5   63    52   34    41   12    32   21
10   48    23   49    61    8    36   25
  55   17    42   16     4   57    29   40
59   37    38   20    19   62    14   11
   6   28    27   45    46    3    51   54
    43         30          7         50
       22         35         58         15
```

1854/
```
 1         59         22         48
  64          6         43         17
63    5    14   28    35   53    18   44
   2   60    51   37    30   12    47   21
10   52    23   38    58   11    39   29
  55   13    42   27     7   54    26   36
56   41    34   19    15   62    25    8
   9   24    31   46    50    3    40   57
    32         45          4         49
       33         20         61         16
```

1855/
```
 1         59         22         48
  64         17         24         25
63    5     6   52    27   53    34   20
   9   60    55    2    50   31    16   37
10   44    47   30    58   11    15   45
  36   14    26   39     7   42    61   35
56   41    18   19    23   62    33    8
  21   43    32   38    49    3    28   46
    40         29          4         57
       13         51         54         12
```

```
3210/                                    3211/
   1         60         18         51       1         60         18         51
     64         5         47         14       64         5         47         14
  63    6   13   44   24   53   30   27    63    6   13   44   24   53   30   27
      2   59   52   21   41   12   35   38      2   59   48   21   41   12   39   38
  10   36   40   37   57   11   23   46    10   36   40   37   57   11   23   46
    55   29   25   28    8   54   42   19     55   29   25   32    8   54   42   15
  56   45   17   16   31   62   26    7    56   45   17   16   31   62   26    7
      9   20   48   49   34    3   39   58      9   20   52   49   34    3   35   58
       43         33          4         50        43         33          4         50
          22         32         61         15        22         28         61         19
.........
```

** Pan-Triagonal Type: n43=63; CC=64; DD=66 **

```
 1/                                         2/
   1         56         19         54       1         56         25         48
     64          9         40         17      64          9         40         17
  60    3   25   42   14   53   31   32    60    3   37   30   20   53   13   44
      5   62   46   23   57   18   22   27      5   62   28   35   45   12   52   21
  11   30   34   43   61    8   24   49    11   42   22   31   61    8   36   49
    48   35   37   16    4   51   41   28     54   23   43   34    4   57   29   16
  58   45   15   12   36   63   21   10    58   39   15   18   24   63   33   10
    13   26   38   47   29    2   50   55      7   26   50   47   41    2   32   55
       52         33          6         39        46         51          6         27
           7         44         59         20        19         14         59         38

2715/                                    2716/
   1         56         25         48       1         56         25         48
     64         35         14         17      64         13         36         17
  62    5   15   26   34   55   19   44    62    5   37   26   12   55   19   44
      3   60   24   13   57   36   46   21      3   60   24   35   57   14   46   21
   9   54   22   41   59    8   40   27     9   32   22   41   59    8   40   49
    30   11   43   50    6   31   51   38     52   33   43   28    6   53   29   16
  58   39   37   18   12   63   23   10    58   39   15   18   34   63   23   10
    33   52   28   47   53    2   16   29     11   30   50   47   31    2   38   51
       32         45          4         49        54         45          4         27
           7         20         61         42         7         20         61         42

5295/                                    5296/
   1         60         19         50       1         60         19         50
     64          5         46         15      64          5         46         15
  62    7   13   44   24   53   31   26    62    7   13   44   24   53   31   26
      3   58   52   21   41   12   34   39      3   58   48   21   41   12   38   39
  11   36   40   37   57   10   22   47    11   36   40   37   57   10   22   47
    54   29   25   28    8   55   43   18     54   29   25   32    8   55   43   14
  56   45   17   16   30   63   27    6    56   45   17   16   30   63   27    6
      9   20   48   49   35    2   38   59      9   20   52   49   35    2   34   59
       42         33          4         51        42         33          4         51
          23         32         61         14        23         28         61         18
........
```

** Complete Pan-Magic Type: n43=64; CC=DD=65 **

```
 1/                                         2/
   1         58         14         57       1         58         14         57
     63         20         21         26      63         11         38         18
  60    4   16   33   37   55   17   38    60    4    9   40   33   55   28   31
    12   62   23   19   52   40   43    9     12   62   30   21   39   23   49   24
  10   47   27   41   61    6   32   36    10   43   34   45   61    6   25   36
    25   11   56   50    3   35   46   34     42   15   41   46    3   52   44   17
  59   51   29    8   18   64   24    7    59   51   29    8   22   64   20    7
    30   44   31   39   54    2   15   45     13   27   48   47   50    2   19   54
       28         48          5         49        32         37          5         56
          13         22         53         42        26         16         53         35
```

```
374/                                   375/
  1       59       13       57          1       59       13       57
   63       21       26       20         63       10       30       27
 60   4   22  31   38  54   10  41      60   4   21  43   37  54   12  29
    9  62    23  19   50  37   48  12      15  62   49  18   41  33   25  17
 11  47   24  36   61   7   34  40      11  46   36  26   61   7   22  51
   28  14   53  49    3  35   46  32      32   9   48  34    3  45   47  42
 58  52   25   8   18  64   29   6      58  52   14   8   19  64   39   6
   30  39   33  45   51   2   16  44      20  35   23  38   56   2   31  55
     27       55        5       43         28       53        5       44
       15       17       56       42         24       40       50       16

722/                                   723/
  1       59       14       56          1       59       14       56
   63       29       28       10         63       21       33       13
 60   4   13  27   32  53   25  46      60   4   23  19   29  53   18  54
   21  62   20  15   41  35   48  18      15  62   27  31   45  30   43   7
 12  42   19  54   61   8   38  26      12  39   11  55   61   8   46  28
   30   7   47  43    3  49   50  31      35  16   58  25    3  48   34  41
 57  51   39   9   23  64   11   6      57  51   37   9   26  64   10   6
   16  37   34  55   58   2   22  36      17  32   24  52   49   2   40  44
     33       40        5       52         36       47        5       42
       24       17       44       45         20       22       50       38

........
```

In autumn 2005 I tried again to count up the last type: Complete pan-magic cubes of order 4 through, but only all of 'Complete Euler type' by my newest method.

I could successfully have known the total count of standard solutions with n1=1: 133856760. How big the count was! But I was more surprised with the fact I could count them all through during my life. I really had no idea about that.

But, I could not yet count up the total count of all Complete pan-magic cubes including 'Non-Euler' type. I can not yet imagine how big it might be, either.

(Originally Written in English on September 10, 2001;
Revised on February 27, 2007 with MacOSX(10.4.8) by Kanji Setsuda)

Section 2: Self-Complementary Magic Cubes of Order 4

#1. Basic Form and Basic Conditions

Let's study about the self-complementary type of magic cube 4x4x4.

I prepared the next basic form and basic conditions for our discussion.

```
[Basic Conditions:]
  n1+ n2+ n3+ n4=K;   n1+ n5+ n9+n13=K;   n1+n17+n33+n49=K;
  n5+ n6+ n7+ n8=K;   n2+ n6+n10+n14=K;   n2+n18+n34+n50=K;
  n9+n10+n11+n12=K;   n3+ n7+n11+n15=K;   n3+n19+n35+n51=K;
 n13+n14+n15+n16=K;   n4+ n8+n12+n16=K;   n4+n20+n36+n52=K;
 n17+n18+n19+n20=K;  n17+n21+n25+n29=K;   n5+n21+n37+n53=K;
 n21+n22+n23+n24=K;  n18+n22+n26+n30=K;   n6+n22+n38+n54=K;
 .....               .....                .....
 n57+n58+n59+n60=K;  n51+n55+n59+n63=K;  n15+n31+n47+n63=K;
 n61+n62+n63+n64=K;  n52+n56+n60+n64=K;  n16+n32+n48+n64=K;

[Four Primary Triagonals:]
  n1+n22+n43+n64=K;   n4+n23+n42+n61=K;
```

n13+n26+n39+n52=K; n16+n27+n38+n49=K;

[Figure 1: Basic Form of Magic Cube 4x4x4]

```
 1--------17--------33--------49
 | 2       |18       |34       |50
 5   3    21   19    37   35    53   51
 | 6   4----22--20----38--36----54--52
 9   7 |  25   23 |  41   39 |  57   55 |
 |10   8  |26   24  |42   40  |58   56
13--11-|--29--27-|--45--43-|--61   59 |
   14   12    30   28    46   44    62   60
     15 |       31 |       47 |       63 |
      16--------32--------48--------64
```

[Complementary Pairs:]

n1+n64=n2+n63=n3+n62=n4+n61=n5+n60=n6+n59=n7+n58=n8+n57
=n9+n56=n10+n55=n11+n54=n12+n53=n13+n52=n14+n51=n15+n50=n16+n49
=n17+n48=n18+n47=n19+n46=n20+n45=n21+n44=n22+n43=n23+n42=n24+n41
=n25+n40=n26+n39=n27+n38=n28+n37=n29+n36=n30+n35=n31+n34=n32+n33=C=65

All complementary pairs of 65 are placed symmetrically with respect to the geometric center of our cube.

The next form is invented to express complementary relations explicitly so that you could easily find where any complementary pair exists.

New Notation for Complementary Pairs:

n64=C-n1=n1; n63=C-n2=n2; n62=C-n3=n3; n61=C-n4=n4;
n60=C-n5=n5; n59=C-n6=n6; n58=C-n7=n7; n57=C-n8=n8;
....
n36=C-n29=n29; n35=C-n30=n30; n34=C-n31=n31; n33=C-32=n32;

[Figure 2: Self-Complementary Magic Cube 4x4x4]

```
 1--------17--------32--------16
 | 2       |18       |31       |15
 5   3    21   19    28   30    12   14
 | 6   4----22--20----27--29----11--13
 9   7 |  25   23 |  24   26 |   8   10 |
 |10   8  |26   24  |23   25  |  7    9
13--11-|--29--27-|--20--22-|---4    6 |
   14   12    30   28    19   21     3    5
     15 |       31 |       18 |        2 |
      16--------32--------17---------1
```

n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7=n8+n8
=n9+n9=n10+n10=....=n29+n29=n30+n30=n31+n31=n32+n32=C=65

Look at the diagram above, and see if all complementary pairs of 65 are located symmetrically with respect to the geometric center of the cube.

#2. How many Solutions could we find?

Take those basic forms and equations of the definition stage, and assume the 'Standard Form' with n1=1 on the top left back and add some inequalities such as:

n2>n5>n17; n2>n3; n5>n9; n17>n33

It is because you can exchange the 2nd and 3rd planes with each other in all three directions, and you can identify the patterns of which the 2nd-3rd planes were exchanged or not.

I want to count only the 'fundamental solutions' and prevent us from double counting of any simple reflections or rotations.

The next list shows some extract examples of the Self-complementary type and the Complete pan-magic type corresponding to each other.

They look quite similar, don't you think?

Yes, there is surely the 'one-to-one correspondence' between those two types.

Therefore we could expect that there are as many Self-complementary solutions as this Complete pan-magic type, however big the total solution counts of those two types might be, although I have not yet count them up to the last.

```
** Magic Cube 4*4*4: Self-Complementary vs. Complete Pan-Magic type **

  1/S                                       1/C
   1        58        57        14           1        58        14        57
    63        20        26        21          63        20        21        26
  60  62    16  19    17   9    37  40      60   4    16  33    37  55    17  38
    12   4    23  33    43  38    52  55      12  62    23  19    52  40    43   9
  59  11    29  50    24  34    18  35      10  47    27  41    61   6    32  36
    30  47    31  41    15  36    54   6      25  11    56  50     3  35    46  34
  10  13    27  22    32  42    61  53      59  51    29   8    18  64    24   7
    25  28    56  48    46  49     3   5      30  44    31  39    54   2    15  45
      44        39        45         2          28        48         5        49
        51         8         7        64          13        22        53        42
374/S                                     374/C
   1        59        57        13           1        59        13        57
    63        21        20        26          63        21        26        20
  60  62    22  19    10  12    38  37      60   4    22  31    38  54    10  41
     9   4    23  31    48  41    50  54       9  62    23  19    50  37    48  12
  58  14    25  49    29  32    18  35      11  47    24  36    61   7    34  40
    30  47    33  36    16  40    51   7      28  14    53  49     3  35    46  32
  11  15    24  17    34  42    61  56      58  52    25   8    18  64    29   6
    28  27    53  55    46  43     3   5      30  39    33  45    51   2    16  44
      39        45        44         2          27        55         5        43
        52         8         6        64          15        17        56        42
722/S                                     722/C
   1        59        56        14           1        59        14        56
    63        29        10        28          63        29        28        10
  60  62    13  15    25  18    32  35      60   4    13  27    32  53    25  46
    21   4    20  27    48  46    41  53      21  62    20  15    41  35    48  18
  57   7    39  43    11  31    23  49      12  42    19  54    61   8    38  26
    16  42    34  54    22  26    58   8      30   7    47  43     3  49    50  31
  12  24    19  17    38  45    61  44      57  51    39   9    23  64    11   6
    30  33    47  40    50  52     3   5      16  37    34  55    58   2    22  36
      37        55        36         2          33        40         5        52
        51         9         6        64          24        17        44        45
1796/S                                    1796/C
   1        59        58        12           1        59        12        58
    63        25        18        24          63        25        24        18
  60  62    11  16    22  10    37  42      60   4    11  30    37  52    22  44
    17   4    36  30    31  44    46  52      17  62    36  16    46  42    31  10
  56   8    39  33    15  51    20  38      13  45    21  50    61   9    35  26
    27  45    14  50    32  26    57   9      23   8    55  33     3  38    49  51
  13  19    21  34    35  29    61  48      56  53    39   7    20  64    15   6
    23  28    55  43    49  54     3   5      27  41    14  47    57   2    32  40
      41        47        40         2          28        43         5        54
        53         7         6        64          19        34        48        29
```

2356/S

```
  1        59        58        12
   63        8        24        35
 60  62    21  16    11  20    38  32
    9   4    48  47    31  28    42  51
 55  15    13  39    43  36    19  40
   25  46    29  22    26  52    50  10
 14  23    37  34    18  17    61  56
   33  27    45  54    49  44     3   5
     30        41        57         2
       53         7         6        64
```

2356/C

```
  1        59        12        58
   63        8        35        24
 60   4    21  47    38  51    11  28
    9  62    48  16    42  32    31  20
 14  46    37  22    61  10    18  52
   33  15    45  39     3  40    49  36
 55  53    13   7    19  64    43   6
   25  30    29  41    50   2    26  57
     27        54         5        44
       23        34        56        17
```

3570/S

```
  1        59        58        12
   63       17        24        26
 60  62    14   8    20  18    36  42
   10   4    28  46    40  30    52  50
 54  16    22  56    33  27    21  31
   34  44    38  32     9  43    49  11
 15  13    35  25    19  37    61  55
   23  29    47  45    57  51     3   5
     39        41        48         2
       53         7         6        64
```

3570/C

```
  1        59        12        58
   63       17        26        24
 60   4    14  46    36  50    20  30
   10  62    28   8    52  42    40  18
 15  44    35  32    61  11    19  43
   23  16    47  56     3  31    57  27
 54  53    22   7    21  64    33   6
   34  39    38  41    49   2     9  48
     29        45         5        51
       13        25        55        37
```

4411/S

```
  1        59        56        14
   63       17        10        40
 60  62    22   8    13  33    35  27
   18   4    44  46    39  31    29  49
 53   7    15  41    42  28    20  54
   11  45    37  23    24  50    58  12
 16  36    34  26    19  21    61  47
   38  30    32  52    57  43     3   5
     25        55        48         2
       51         9         6        64
```

4411/C

```
  1        59        14        56
   63       17        40        10
 60   4    22  46    35  49    13  31
   18  62    44   8    29  27    39  33
 16  45    34  23    61  12    19  50
   38   7    32  41     3  54    57  28
 53  51    15   9    20  64    42   6
   11  25    37  55    58   2    24  48
     30        52         5        43
       36        26        47        21
```

6868/S

```
  1        59        58        12
   63       29        18        20
 60  62    23   8    22  10    25  50
   28   4    35  34    16  44    51  48
 52   9    27  26    19  54    32  41
   24  33    11  46    39  38    56  13
 17  14    21  49    31  30    61  37
   15  40    55  43    57  42     3   5
     45        47        36         2
       53         7         6        64
```

6868/C

```
  1        59        12        58
   63       29        20        18
 60   4    23  34    25  48    22  44
   28  62    35   8    51  50    16  10
 17  33    21  46    61  13    31  38
   15   9    55  26     3  41    57  54
 52  53    27   7    32  64    19   6
   24  45    11  47    56   2    39  36
     40        43         5        42
       14        49        37        30
```

....

#3. Type Conversion

The Self-complementary type could be transformed into the Complete pan-magic type whose complementary pairs of 65 are placed only on its pantriagonals.

Let me show you the way how to transform the one type into the other by the next step-diagrams below, instead of showing the proof of 'one-to-one correspondence'.

It is the key point of this method that you should exchange the 3-rd plane with the 4-th one each other equally in all three directions at the same time.

You can also convert the Complete pan-magic type back to the Self-complementary one, only when all complementary pairs of 65 are located on its pan-triagonals.

You cannot always make all types of Pan-magic cubes into the Self-complementary ones. The only type you can convert is limited to the one with n1=1 and n43=64.

```
 [Figure 7: Magic Cube 4x4x4: Type Conversions]
     Self-Complementary Type                        Complete Type
Start(1)      Original                   |              Original
   1--------17--------32--------16       |   1--------17--------11--------27
   | 2       |18       |31       |15     |   | 2       |18       |12       |28
   5   3    21  19    28  30    12  14   |   5   3    21  19    15   9    31  25
   | 6   4----22--20----27--29----11--13 |   | 6   4----22--20----16--10----32--26
   9   7 |  25  23 |  24  26 |   8  10 | |   9   7 |  25  23 |   3  13 |  19  29 |
   |10   8   |26  24   |23  25   | 7   9 |   |10   8   |26  24   | 4  14   |20  30
  13--11-|--29--27-|--20--22-|---4   6 | |  13--11-|--29--27-|---7---1-|--23  17 |
    14  12    30  28    19  21     3   5 |    14  12    30  28     8   2    24  18
      15 |      31 |      18 |       2 | |      15 |      31 |       5 |      21 |
        16--------32--------17---------1 |        16--------32---------6--------22
                                         V
Step(2)            [Left-Right(3-4)]     |                   [Left-Right(3-4)]
   1--------17--------16--------32       |   1--------17--------27--------11
   | 2       |18       |15       |31     |   | 2       |18       |28       |12
   5   3    21  19    12  14    28  30   |   5   3    21  19    31  25    15   9
   | 6   4----22--20----11--13----27--29 |   | 6   4----22--20----32--26----16--10
   9   7 |  25  23 |   8  10 |  24  26 | |   9   7 |  25  23 |  19  29 |   3  13 |
   |10   8   |26  24   | 7   9   |23  25 |   |10   8   |26  24   |20  30   | 4  14
  13--11-|--29--27-|---4---6-|--20  22 | |  13--11-|--29--27-|--23--17-|---7   1 |
    14  12    30  28     3   5    19  21 |    14  12    30  28    24  18     8   2
      15 |      31 |       2 |      18 | |      15 |      31 |      21 |       5 |
        16--------32---------1--------17 |        16--------32--------22---------6
                                         V
Step(3)    [Up-Down(3-4)]                |           [Up-Down(3-4)]
   1--------17--------16--------32       |   1--------17--------27--------11
   | 2       |18       |15       |31     |   | 2       |18       |28       |12
   5   3    21  19    12  14    28  30   |   5   3    21  19    31  25    15   9
   | 6   4----22--20----11--13----27--29 |   | 6   4----22--20----32--26----16--10
  13   7 |  29  23 |   4  10 |  20  26 | |  13   7 |  29  23 |  23  29 |   7  13 |
   |14   8   |30  24   | 3   9   |19  25 |   |14   8   |30  24   |24  30   | 8  14
   9--15-|--25--31-|---8---2-|--24  18 | |   9--15-|--25--31-|--19--21-|---3   5 |
    10  16    26  32     7   1    23  17 |    10  16    26  32    20  22     4   6
      11 |      27 |       6 |      22 | |      11 |      27 |      17 |       1 |
        12--------28---------5--------21 |        12--------28--------18---------2
                                         V
Step(4)    [Back-Front(3-4)]             |           [Back-Front(3-4)]
   1--------17--------16--------32       |   1--------17--------27--------11
   | 2       |18       |15       |31     |   | 2       |18       |28       |12
   5   4    21  20    12  13    28  29   |   5   4    21  20    31  26    15  10
   | 6   3----22--19----11--14----27--30 |   | 6   3----22--19----32--25----16---9
  13   8 |  29  24 |   4   9 |  20  25 | |  13   8 |  29  24 |  23  30 |   7  14 |
   |14   7   |30  23   | 3  10   |19  26 |   |14   7   |30  23   |24  29   | 8  13
   9--16-|--25--32-|---8---1-|--24  17 | |   9--16-|--25--32-|--19--22-|---3   6 |
    10  15    26  31     7   2    23  18 |    10  15    26  31    20  21     4   5
      12 |      28 |       5 |      21 | |      12 |      28 |      18 |       2 |
        11--------27---------6--------22 |        11--------27--------17---------1
      [Goal: Complete Type]              |    [Goal: Self-Complementary Type]
                                         |
   Check all pantriagonals:              |  Check all complementary pairs:
    n1+n22+n1+n22=K; n1+n23+n1+n23=K;    |  n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6
    n1+n26+n1+n26=K; n1+n27+n1+n27=K;    |  =n7+n7=n8+n8=n9+n9=n10+n10=n11+n11
    n2+n24+n2+n24=K; n2+n21+n2+n21=K;    |  =n12+n12=n13+n13=n14+n14=n15+n15
    n2+n28+n2+n28=K; n2+n25+n2+n25=K;    |  =n16+n16=n17+n17=.......
    n3+n24+n3+n24=K; n3+n21+n3+n21=K;    |  ..=n30+n30=n31+n31=n32+n32=C
    n3+n28+n3+n28=K; n3+n25+n3+n25=K;    |
    n4+n23+n4+n23=K; n4+n22+n4+n22=K;    |  All pairs are located symmetrically
```

```
  ..........                             | with respect to the geometric center.
 n16+n27+n16+n27=K; n16+n22+n16+n22=K;   |
```

I would like to revive the name 'Complete pan-magic' back for this special type that has all complementary pairs of 65 only on its pan-triagonals and can be transformed into the Self-complementary type by such simple rules as shown above. This request arose from our Japanese tradition. We have loved them best of all for a long time.

The next list below shows some extract examples of 'Complete' pan-magic cubes and their converted patterns back to the Self-complementary type. Compare this list with the last one above, and you will find the beautiful reversibility.
This beautiful reversibility indicates the basis of the 'one-to-one correspondence' between those two types, I think.

```
** Magic Cube 4*4*4: 'Complete Pan-magic' vs. 'Self-Complementary' **
 1/C                                       1/S
  1         58         14         57         1         58         57         14
    63         20         21         26        63         20         26         21
 60   4    16  33    37  55    17  38      60  62    16  19    17   9    37  40
   12  62    23  19    52  40    43   9      12   4    23  33    43  38    52  55
 10  47    27  41    61   6    32  36      59  11    29  50    24  34    18  35
   25  11    56  50     3  35    46  34      30  47    31  41    15  36    54   6
 59  51    29   8    18  64    24   7      10  13    27  22    32  42    61  53
   30  44    31  39    54   2    15  45      25  28    56  48    46  49     3   5
      28         48          5         49        44         39         45          2
         13         22         53         42        51          8          7         64

374/C                                     374/S
  1         59         13         57         1         59         57         13
    63         21         26         20        63         21         20         26
 60   4    22  31    38  54    10  41      60  62    22  19    10  12    38  37
    9  62    23  19    50  37    48  12       9   4    23  31    48  41    50  54
 11  47    24  36    61   7    34  40      58  14    25  49    29  32    18  35
   28  14    53  49     3  35    46  32      30  47    33  36    16  40    51   7
 58  52    25   8    18  64    29   6      11  15    24  17    34  42    61  56
   30  39    33  45    51   2    16  44      28  27    53  55    46  43     3   5
      27         55          5         43        39         45         44          2
         15         17         56         42        52          8          6         64

722/C                                     722/S
  1         59         14         56         1         59         56         14
    63         29         28         10        63         29         10         28
 60   4    13  27    32  53    25  46      60  62    13  15    25  18    32  35
   21  62    20  15    41  35    48  18      21   4    20  27    48  46    41  53
 12  42    19  54    61   8    38  26      57   7    39  43    11  31    23  49
   30   7    47  43     3  49    50  31      16  42    34  54    22  26    58   8
 57  51    39   9    23  64    11   6      12  24    19  17    38  45    61  44
   16  37    34  55    58   2    22  36      30  33    47  40    50  52     3   5
      33         40          5         52        37         55         36          2
         24         17         44         45        51          9          6         64

1796/C                                    1796/S
  1         59         12         58         1         59         58         12
    63         25         24         18        63         25         18         24
 60   4    11  30    37  52    22  44      60  62    11  16    22  10    37  42
   17  62    36  16    46  42    31  10      17   4    36  30    31  44    46  52
 13  45    21  50    61   9    35  26      56   8    39  33    15  51    20  38
   23   8    55  33     3  38    49  51      27  45    14  50    32  26    57   9
 56  53    39   7    20  64    15   6      13  19    21  34    35  29    61  48
   27  41    14  47    57   2    32  40      23  28    55  43    49  54     3   5
      28         43          5         54        41         47         40          2
         19         34         48         29        53          7          6         64
```

2356/C

```
 1        59        12        58
  63         8        35        24
 60   4   21  47    38  51    11  28
    9  62    48  16    42  32    31  20
 14  46   37  22    61  10    18  52
   33  15    45  39     3  40    49  36
 55  53   13   7    19  64    43   6
   25  30    29  41    50   2    26  57
      27        54         5        44
         23        34        56        17
```

2356/S

```
 1        59        58        12
  63         8        24        35
 60  62   21  16    11  20    38  32
    9   4    48  47    31  28    42  51
 55  15   13  39    43  36    19  40
   25  46    29  22    26  52    50  10
 14  23   37  34    18  17    61  56
   33  27    45  54    49  44     3   5
      30        41        57         2
         53         7         6        64
```

3570/C

```
 1        59        12        58
  63        17        26        24
 60   4   14  46    36  50    20  30
   10  62    28   8    52  42    40  18
 15  44   35  32    61  11    19  43
   23  16    47  56     3  31    57  27
 54  53   22   7    21  64    33   6
   34  39    38  41    49   2     9  48
      29        45         5        51
         13        25        55        37
```

3570/S

```
 1        59        58        12
  63        17        24        26
 60  62   14   8    20  18    36  42
   10   4    28  46    40  30    52  50
 54  16   22  56    33  27    21  31
   34  44    38  32     9  43    49  11
 15  13   35  25    19  37    61  55
   23  29    47  45    57  51     3   5
      39        41        48         2
         53         7         6        64
```

4411/C

```
 1        59        14        56
  63        17        40        10
 60   4   22  46    35  49    13  31
   18  62    44   8    29  27    39  33
 16  45   34  23    61  12    19  50
   38   7    32  41     3  54    57  28
 53  51   15   9    20  64    42   6
   11  25    37  55    58   2    24  48
      30        52         5        43
         36        26        47        21
```

4411/S

```
 1        59        56        14
  63        17        10        40
 60  62   22   8    13  33    35  27
   18   4    44  46    39  31    29  49
 53   7   15  41    42  28    20  54
   11  45    37  23    24  50    58  12
 16  36   34  26    19  21    61  47
   38  30    32  52    57  43     3   5
      25        55        48         2
         51         9         6        64
```

6868/C

```
 1        59        12        58
  63        29        20        18
 60   4   23  34    25  48    22  44
   28  62    35   8    51  50    16  10
 17  33   21  46    61  13    31  38
   15   9    55  26     3  41    57  54
 52  53   27   7    32  64    19   6
   24  45    11  47    56   2    39  36
      40        43         5        42
         14        49        37        30
```

6868/S

```
 1        59        58        12
  63        29        18        20
 60  62   23   8    22  10    25  50
   28   4    35  34    16  44    51  48
 52   9   27  26    19  54    32  41
   24  33    11  46    39  38    56  13
 17  14   21  49    31  30    61  37
   15  40    55  43    57  42     3   5
      45        47        36         2
         53         7         6        64
```

7336/C

```
 1        59        12        58
  63        21        30        16
 60   4    8  40    42  47    20  39
    9  62    32  10    43  41    46  17
 18  50   26  38    61  14    25  28
   24  11    48  52     3  31    55  36
 51  53   37   7    15  64    27   6
   34  35    29  49    54   2    13  44
      23        45         5        57
         22        19        56        33
```

7336/S

```
 1        59        58        12
  63        21        16        30
 60  62    8  10    20  17    42  41
    9   4    32  40    46  39    43  47
 51  11   37  52    27  36    15  31
   34  50    29  38    13  28    54  14
 18  22   26  19    25  33    61  56
   24  23    48  45    55  57     3   5
      35        49        44         2
         53         7         6        64
```

....

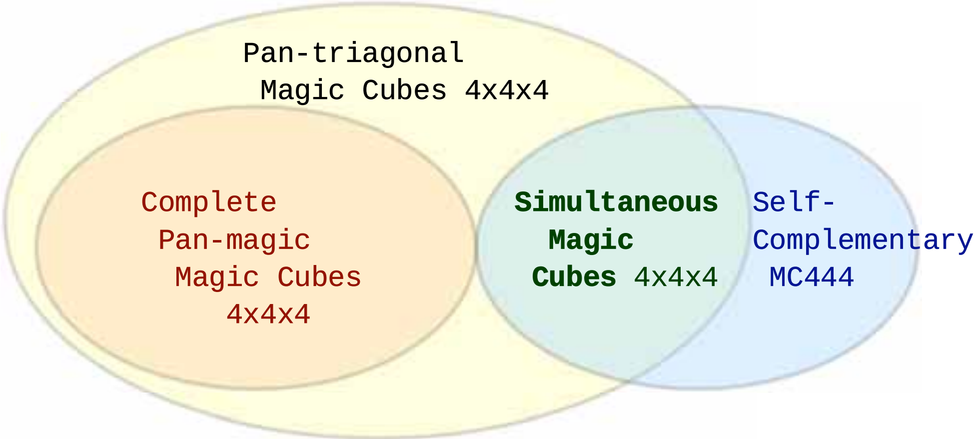
(Originally Written in English on September 10, 2001;
Retyped on February 27, 2007 with MacOSX(10.4.8) by Kanji Setsuda)

Section 3: 'Simultaneous' Type of Magic Cubes of Order 4: Both Self-complementary and Pan-triagonal

#1. How about Simultaneous MC444: both Self-complementary and Pan-triagonal?

I heard some European researcher discovered and reported of the 'Simultaneous' magic cubes of order 4: both Self-complementary and Pan-triagonal.

Set Diagrams for **'Simultaneous'** Type



I put some basic conditions and basic form at the first definition stage as follows:

```
* Basic Conditions: C=130 *
 n1+n2+n3+n4=C       ...d1;   |  n1+n17+n33+n49=C  ...d17;   |  n1+n5+n9+n13=C    ...d33;
 n5+n6+n7+n8=C       ...d2;   |  n2+n18+n34+n50=C  ...d18;   |  n2+n6+n10+n14=C   ...d34;
 n9+n10+n11+n12=C    ...d3;   |  n3+n19+n35+n51=C  ...d19;   |  n3+n7+n11+n15=C   ...d35;
 n13+n14+n15+n16=C   ...d4;   |  n4+n20+n36+n52=C  ...d20;   |  n4+n8+n12+n16=C   ...d36;
 n17+n18+n19+n20=C   ...d5;   |  n5+n21+n37+n53=C  ...d21;   |  n17+n21+n25+n29=C ...d37;
 n21+n22+n23+n24=C   ...d6;   |  n6+n22+n38+n54=C  ...d22;   |  n18+n22+n26+n30=C ...d38;
 n25+n26+n27+n28=C   ...d7;   |  n7+n23+n39+n55=C  ...d23;   |  n19+n23+n27+n31=C ...d39;
 n29+n30+n31+n32=C   ...d8;   |  n8+n24+n40+n56=C  ...d24;   |  n20+n24+n28+n32=C ...d40;
 n33+n34+n35+n36=C   ...d9;   |  n9+n25+n41+n57=C  ...d25;   |  n33+n37+n41+n45=C ...d41;
 n37+n38+n39+n40=C   ...d10;  |  n10+n26+n42+n58=C ...d26;   |  n34+n38+n42+n46=C ...d42;
 n41+n42+n43+n44=C   ...d11;  |  n11+n27+n43+n59=C ...d27;   |  n35+n39+n43+n47=C ...d43;
 n45+n46+n47+n48=C   ...d12;  |  n12+n28+n44+n60=C ...d28;   |  n36+n40+n44+n48=C ...d44;
 n49+n50+n51+n52=C   ...d13;  |  n13+n29+n45+n61=C ...d29;   |  n49+n53+n57+n61=C ...d45;
 n53+n54+n55+n56=C   ...d14;  |  n14+n30+n46+n62=C ...d30;   |  n50+n54+n58+n62=C ...d46;
 n57+n58+n59+n60=C   ...d15;  |  n15+n31+n47+n63=C ...d31;   |  n51+n55+n59+n63=C ...d47;
 n61+n62+n63+n64=C   ...d16;  |  n16+n32+n48+n64=C ...d32;   |  n52+n56+n60+n64=C ...d48;
```

```
16                               64
  13                               61
 4  14                           52  62
   1--15-----17---------33---------49  63
 8 | 2  16      18          34     56 |50  64
   5   3  13 21  19      37  35      53  51  61
12 | 6   4-----22--20-----38--36-60--54--52
   9   7 | 1 25  23      41  39      57  55 |49
16 |10   8      26  24      42  40 64 |58  56
  13--11-|-5-29--27-----45--43-----61  59 |53
 4  14  12      30  28      46  44 52  62  60
   1  15 | 9     31          47      49  63 |57
     2  16---------32---------48-----50--64
       3  13                           51  61
         4                               52
           1                               49
```

```
* Self-Complementary Conditions: *

n1+n64=n2+n63=n2+n62=n2+n61=
n5+n60=n5+n59=n5+n58=n5+n57=
n9+n56=n10+n55=n11+n54=n12+n53=
n13+n52=n14+n51=n15+n50=n16+n49=
n17+n48=n18+n47=n19+n46=n20+n45=
n21+n44=n22+n43=n23+n42=n24+n41=
n25+n40=n26+n39=n27+n38=n28+n37=
n29+n36=n30+n35=n31+n34=n32+n33=
n33+n32=n34+n31=n35+n30=n36+n29=
n37+n28=n38+n27=n39+n26=n40+n25=
 . . . . .
 . . . . . =65 ....(cmp1)
```

```
* Pan-Triagonal Conditions: C=130 *
 n1+n22+n43+n64=C ... pd01;   | n1+n24+n43+n62=C ... pd02;
 n1+n30+n43+n56=C ... pd03;   | n1+n32+n43+n54=C ... pd04;
 n2+n23+n44+n61=C ... pd05;   | n2+n21+n44+n63=C ... pd06;
 n2+n31+n44+n53=C ... pd07;   | n2+n29+n44+n55=C ... pd08;
 n3+n24+n41+n62=C ... pd09;   | n3+n22+n41+n64=C ... pd10;
 n3+n32+n41+n54=C ... pd11;   | n3+n30+n41+n56=C ... pd12;
 n4+n21+n42+n63=C ... pd13;   | n4+n23+n42+n61=C ... pd14;
 n4+n29+n42+n55=C ... pd15;   | n4+n31+n42+n53=C ... pd16;
 n5+n26+n47+n52=C ... pd17;   | n5+n28+n47+n50=C ... pd18;
 n5+n18+n47+n60=C ... pd19;   | n5+n20+n47+n58=C ... pd20;
 n6+n27+n48+n49=C ... pd21;   | n6+n25+n48+n51=C ... pd22;
 n6+n19+n48+n57=C ... pd23;   | n6+n17+n48+n59=C ... pd24;
 n7+n28+n45+n50=C ... pd25;   | n7+n26+n45+n52=C ... pd26;
 n7+n20+n45+n58=C ... pd27;   | n7+n18+n45+n60=C ... pd28;
 n8+n25+n46+n51=C ... pd29;   | n8+n27+n46+n49=C ... pd30;
 n8+n17+n46+n59=C ... pd31;   | n8+n19+n46+n57=C ... pd32;
 n9+n30+n35+n56=C ... pd33;   | n9+n32+n35+n54=C ... pd34;
 n9+n22+n35+n64=C ... pd35;   | n9+n24+n35+n62=C ... pd36;
 n10+n31+n36+n53=C .. pd37;   | n10+n29+n36+n55=C .. pd38;
 n10+n23+n36+n61=C .. pd39;   | n10+n21+n36+n63=C .. pd40;
 n11+n32+n33+n54=C .. pd41;   | n11+n30+n33+n56=C .. pd42;
 n11+n24+n33+n62=C .. pd43;   | n11+n22+n33+n64=C .. pd44;
 n12+n29+n34+n55=C .. pd45;   | n12+n31+n34+n53=C .. pd46;
 n12+n21+n34+n63=C .. pd47;   | n12+n23+n34+n61=C .. pd48;
 n13+n18+n39+n60=C .. pd49;   | n13+n20+n39+n58=C .. pd50;
 n13+n26+n39+n52=C .. pd51;   | n13+n28+n39+n50=C .. pd52;
 n14+n19+n40+n57=C .. pd53;   | n14+n17+n40+n59=C .. pd54;
 n14+n27+n40+n49=C .. pd55;   | n14+n25+n40+n51=C .. pd56;
 n15+n20+n37+n58=C .. pd57;   | n15+n18+n37+n60=C .. pd58;
 n15+n28+n37+n50=C .. pd59;   | n15+n26+n37+n52=C .. pd60;
 n16+n17+n38+n59=C .. pd61;   | n16+n19+n38+n57=C .. pd62;
 n16+n25+n38+n51=C .. pd63;   | n16+n27+n38+n49=C .. pd64;
```

I was extremely surprised at the news and wondered if it was true. For I knew that we could not compose any 'Simultaneous' type of MC444: both Self-complementary and **Complete** pan-magic. But I thought we might be able to make any simultaneous type, if they should be of both Self-complementary and **Pan-triagonal** type.

Let's follow him and try to examine and count up all the possible solutions of them.

2. How many Simultaneous MC444 are there in all?

But, it has always been one of the hardest jobs for us to compose them in such an ordinary way as usual, without using any new method of composition. It took too much time and energy to go on doing our job, and we often gave it up on our way.

I had to design and write the program carefully, using all our knowledge and skills. I tried as hard as possible. I have now got it! I am glad to report here of our result.

```
** Simultaneous Magic Cubes 4*4*4: Self-Complementary & Pan-Triagonal **
**  Solutions with n1=1 & n64=64 & n43=32   **

 1/                                        49/
  1        50        16        63          1        42        24        63
    60        23        41         6         61        19        44         6
 55    8    28   43    38   21     9   58   51    8    29   47    38   17    12   58
   18   61    33   14    31   52    48    3    10   60    33   22    31   45    56    3
 12   46    39   29    25   35    54   20   16   54    39   28    25   35    50   13
   45   11    30   40    36   26    19   53    52   15    30   40    37   26    11   49
 62   17    13   34    51   32     4   47   62    9    20   34    43   32     5   55
     7   56    44   27    22   37    57   10     7   53    48   27    18   36    57   14
      59        24        42         5          59        21        46         4
         2        49        15        64            2        41        23        64
```

```
55/                                       109/
   1        42        24        63           1        43        24        62
    62        20        43         5          63        20        42         5
  52   8    30  47    37  17    11  58      52   8    31  46    37  17    10  59
    10  59    33  21    31  46    56   4      11  58    33  21    30  47    56   4
  16  53    39  27    25  36    50  14      16  53    38  26    25  36    51  15
    51  15    29  40    38  26    12  49      50  14    29  40    39  27    12  49
  61   9    19  34    44  32     6  55      61   9    18  35    44  32     7  54
     7  54    48  28    18  35    57  13       6  55    48  28    19  34    57  13
      60        22        45         3          60        23        45         2
         2        41        23        64           3        41        22        64
175/                                      181/
   1        36        30        63           1        38        28        63
    59        19        43         9          61        19        40        10
  51  13    27  49    41  15    11  53      51  12    29  47    42  17     8  54
     4  57    33  26    31  42    62   5       6  56    33  26    31  45    60   3
  18  58    44  25    20  37    48  10      16  58    43  24    21  35    50  13
    55  17    28  45    40  21     7  47      52  15    30  44    41  22     7  49
  60   3    23  34    39  32     8  61      62   5    20  34    39  32     9  59
    12  54    50  24    16  38    52  14      11  57    48  23    18  36    53  14
      56        22        46         6          55        25        46         4
         2        35        29        64           2        37        27        64
199/                                      259/
   1        38        28        63           1        39        28        62
    62        20        39         9          63        20        38         9
  52  12    30  47    41  17     7  54      52  12    31  46    41  17     6  55
     6  55    33  25    31  46    60   4       7  54    33  25    30  47    60   4
  16  57    43  23    21  36    50  14      16  57    42  22    21  36    51  15
    51  15    29  44    42  22     8  49      50  14    29  44    43  23     8  49
  61   5    19  34    40  32    10  59      61   5    18  35    40  32    11  58
    11  58    48  24    18  35    53  13      10  59    48  24    19  34    53  13
      56        26        45         3          56        27        45         2
         2        37        27        64           3        37        26        64
337/                                      421/
   1        39        30        60           1        38        28        63
    63        22        36         9          61        11        40        18
  54  14    31  44    41  17     4  55      43  20    29  55    50   9     8  46
     7  52    33  25    28  47    62   6       6  48    33  26    31  53    60   3
  16  57    42  20    19  38    53  15      24  58    51  16    13  35    42  21
    50  12    27  46    45  23     8  49      44  23    30  52    49  14     7  41
  59   3    18  37    40  32    13  58      62   5    12  34    39  32    17  59
    10  61    48  24    21  34    51  11      19  57    56  15    10  36    45  22
      56        29        43         2          47        25        54         4
         5        35        26        64           2        37        27        64
457/                                      523/
   1        38        28        63           1        39        28        62
    62        12        39        17          63        12        38        17
  44  20    30  55    49   9     7  46      44  20    31  54    49   9     6  47
     6  47    33  25    31  54    60   4       7  46    33  25    30  55    60   4
  24  57    51  15    13  36    42  22      24  57    50  14    13  36    43  23
    43  23    29  52    50  14     8  41      42  22    29  52    51  15     8  41
  61   5    11  34    40  32    18  59      61   5    10  35    40  32    19  58
    19  58    56  16    10  35    45  21      18  59    56  16    11  34    45  21
      48        26        53         3          48        27        53         2
         2        37        27        64           3        37        26        64
601/                                      607/
   1        36        31        62           1        39        30        60
    59        19        41        11          63        14        36        17
  51  25    27  58    43   5     9  42      46  22    31  52    49   9     4  47
     4  45    33  17    30  53    63  15       7  44    33  25    28  55    62   6
  28  49    55  13     8  47    39  21      24  57    50  12    11  38    45  23
    44  26    18  57    52  10    16  37      42  20    27  54    53  15     8  41
  50   2    12  35    48  32    20  61      59   3    10  37    40  32    21  58
    23  56    60  22     7  38    40  14      18  61    56  16    13  34    43  19
      54        24        46         6          48        29        51         2
         3        34        29        64           5        35        26        64
[Count = 804]
```

** Solutions with n1=1 & n64=64 & n43=48 **

1/
```
 1        50        16        63
  60        39        25         6
55   8    44  27    22  37     9  58
  34  61    17  14    47  52    32   3
12  30    23  45    41  19    54  36
  29  11    46  24    20  42    35  53
62  33    13  18    51  48     4  31
   7  56    28  43    38  21    57  10
    59        40        26         5
       2        49        15        64
```

49/
```
 1        26        40        63
  61        35        28         6
51   8    45  31    22  33    12  58
  10  60    17  38    47  29    56   3
16  54    23  44    41  19    50  13
  52  15    46  24    21  42    11  49
62   9    36  18    27  48     5  55
   7  53    32  43    34  20    57  14
    59        37        30         4
       2        25        39        64
```

55/
```
 1        26        40        63
  62        36        27         5
52   8    46  31    21  33    11  58
  10  59    17  37    47  30    56   4
16  53    23  43    41  20    50  14
  51  15    45  24    22  42    12  49
61   9    35  18    28  48     6  55
   7  54    32  44    34  19    57  13
    60        38        29         3
       2        25        39        64
```

109/
```
 1        27        40        62
  63        36        26         5
52   8    47  30    21  33    10  59
  11  58    17  37    46  31    56   4
16  53    22  42    41  20    51  15
  50  14    45  24    23  43    12  49
61   9    34  19    28  48     7  54
   6  55    32  44    35  18    57  13
    60        39        29         2
       3        25        38        64
```

175/
```
 1        22        44        63
  61        35        24        10
51  12    45  31    26  33     8  54
   6  56    17  42    47  29    60   3
16  58    27  40    37  19    50  13
  52  15    46  28    25  38     7  49
62   5    36  18    23  48     9  59
  11  57    32  39    34  20    53  14
    55        41        30         4
       2        21        43        64
```

193/
```
 1        22        44        63
  62        36        23         9
52  12    46  31    25  33     7  54
   6  55    17  41    47  30    60   4
16  57    27  39    37  20    50  14
  51  15    45  28    26  38     8  49
61   5    35  18    24  48    10  59
  11  58    32  40    34  19    53  13
    56        42        29         3
       2        21        43        64
```

253/
```
 1        23        44        62
  63        36        22         9
52  12    47  30    25  33     6  55
   7  54    17  41    46  31    60   4
16  57    26  38    37  20    51  15
  50  14    45  28    27  39     8  49
61   5    34  19    24  48    11  58
  10  59    32  40    35  18    53  13
    56        43        29         2
       3        21        42        64
```

325/
```
 1        23        46        60
  63        38        20         9
54  14    47  28    25  33     4  55
   7  52    17  41    44  31    62   6
16  57    26  36    35  22    53  15
  50  12    43  30    29  39     8  49
59   3    34  21    24  48    13  58
  10  61    32  40    37  18    51  11
    56        45        27         2
       5        19        42        64
```

409/
```
 1        22        44        63
  61        11        24        34
27  36    45  55    50   9     8  30
   6  32    17  42    47  53    60   3
40  58    51  16    13  19    26  37
  28  39    46  52    49  14     7  25
62   5    12  18    23  48    33  59
  35  57    56  15    10  20    29  38
    31        41        54         4
       2        21        43        64
```

445/
```
 1        22        44        63
  62        12        23        33
28  36    46  55    49   9     7  30
   6  31    17  41    47  54    60   4
40  57    51  15    13  20    26  38
  27  39    45  52    50  14     8  25
61   5    11  18    24  48    34  59
  35  58    56  16    10  19    29  37
    32        42        53         3
       2        21        43        64
```

511/
```
 1        23        44        62
  63        12        22        33
28  36    47  54    49   9     6  31
   7  30    17  41    46  55    60   4
40  57    50  14    13  20    27  39
  26  38    45  52    51  15     8  25
61   5    10  19    24  48    35  58
  34  59    56  16    11  18    29  37
    32        43        53         2
       3        21        42        64
```

589/
```
 1        23        46        60
  63        14        20        33
30  38    47  52    49   9     4  31
   7  28    17  41    44  55    62   6
40  57    50  12    11  22    29  39
  26  36    43  54    53  15     8  25
59   3    10  21    24  48    37  58
  34  61    56  16    13  18    27  35
    32        45        51         2
       5        19        42        64
```

[Count = 780]

** Solutions with n1=1 & n64=64 & n43=56 **

1/
```
 1          42          24          63
  60          39          25           6
47   8      52  27      14  37      17  58
  34  61       9  22      55  44      32   3
20  30      15  53      49  11      46  36
  29  19      54  16      12  50      35  45
62  33      21  10      43  56       4  31
   7  48      28  51      38  13      57  18
    59          40          26           5
        2          41          23          64
```

49/
```
 1          26          40          63
  61          35          28           6
43   8      53  31      14  33      20  58
  18  60       9  38      55  29      48   3
24  46      15  52      49  11      42  21
  44  23      54  16      13  50      19  41
62  17      36  10      27  56       5  47
   7  45      32  51      34  12      57  22
    59          37          30           4
        2          25          39          64
```

55/
```
 1          26          40          63
  62          36          27           5
44   8      54  31      13  33      19  58
  18  59       9  37      55  30      48   4
24  45      15  51      49  12      42  22
  43  23      53  16      14  50      20  41
61  17      35  10      28  56       6  47
   7  46      32  52      34  11      57  21
    60          38          29           3
        2          25          39          64
```

109/
```
 1          27          40          62
  63          36          26           5
44   8      55  30      13  33      18  59
  19  58       9  37      54  31      48   4
24  45      14  50      49  12      43  23
  42  22      53  16      15  51      20  41
61  17      34  11      28  56       7  46
   6  47      32  52      35  10      57  21
    60          39          29           2
        3          25          38          64
```

175/
```
 1          14          52          63
  61          35          16          18
43  20      53  31      26  33       8  46
   6  48       9  50      55  29      60   3
24  58      27  40      37  11      42  21
  44  23      54  28      25  38       7  41
62   5      36  10      15  56      17  59
  19  57      32  39      34  12      45  22
    47          49          30           4
        2          13          51          64
```

193/
```
 1          14          52          63
  62          36          15          17
44  20      54  31      25  33       7  46
   6  47       9  49      55  30      60   4
24  57      27  39      37  12      42  22
  43  23      53  28      26  38       8  41
61   5      35  10      16  56      18  59
  19  58      32  40      34  11      45  21
    48          50          29           3
        2          13          51          64
```

253/
```
 1          15          52          62
  63          36          14          17
44  20      55  30      25  33       6  47
   7  46       9  49      54  31      60   4
24  57      26  38      37  12      43  23
  42  22      53  28      27  39       8  41
61   5      34  11      16  56      19  58
  18  59      32  40      35  10      45  21
    48          51          29           2
        3          13          50          64
```

325/
```
 1          15          54          60
  63          38          12          17
46  22      55  28      25  33       4  47
   7  44       9  49      52  31      62   6
24  57      26  36      35  14      45  23
  42  20      51  30      29  39       8  41
59   3      34  13      16  56      21  58
  18  61      32  40      37  10      43  19
    48          53          27           2
        5          11          50          64
```

409/
```
 1          14          52          63
  61          19          16          34
27  36      53  47      42  17       8  30
   6  32       9  50      55  45      60   3
40  58      43  24      21  11      26  37
  28  39      54  44      41  22       7  25
62   5      20  10      15  56      33  59
  35  57      48  23      18  12      29  38
    31          49          46           4
        2          13          51          64
```

445/
```
 1          14          52          63
  62          20          15          33
28  36      54  47      41  17       7  30
   6  31       9  49      55  46      60   4
40  57      43  23      21  12      26  38
  27  39      53  44      42  22       8  25
61   5      19  10      16  56      34  59
  35  58      48  24      18  11      29  37
    32          50          45           3
        2          13          51          64
```

511/
```
 1          15          52          62
  63          20          14          33
28  36      55  46      41  17       6  31
   7  30       9  49      54  47      60   4
40  57      42  22      21  12      27  39
  26  38      53  44      43  23       8  25
61   5      18  11      16  56      35  58
  34  59      48  24      19  10      29  37
    32          51          45           2
        3          13          50          64
```

589/
```
 1          15          54          60
  63          22          12          33
30  38      55  44      41  17       4  31
   7  28       9  49      52  47      62   6
40  57      42  20      19  14      29  39
  26  36      51  46      45  23       8  25
59   3      18  13      16  56      37  58
  34  61      48  24      21  10      27  35
    32          53          43           2
        5          11          50          64
```

[Count = 780]

** Solutions with n1=1 & n64=64 & n43=60 **

1/
```
 1        38        28        63
  56        43        21        10
47  12    52  23    14  41    17  54
  34  61     5  26    59  40    32   3
20  30    15  57    49   7    46  36
  29  19    58  16     8  50    35  45
62  33    25   6    39  60     4  31
  11  48    24  51    42  13    53  18
    55        44        22         9
       2        37        27        64
```

49/
```
 1        22        44        63
  61        35        24        10
39  12    57  31    14  33    20  54
  18  56     5  42    59  29    48   3
28  46    15  52    49   7    38  25
  40  27    58  16    13  50    19  37
62  17    36   6    23  60     9  47
  11  45    32  51    34   8    53  26
    55        41        30         4
       2        21        43        64
```

55/
```
 1        22        44        63
  62        36        23         9
40  12    58  31    13  33    19  54
  18  55     5  41    59  30    48   4
28  45    15  51    49   8    38  26
  39  27    57  16    14  50    20  37
61  17    35   6    24  60    10  47
  11  46    32  52    34   7    53  25
    56        42        29         3
       2        21        43        64
```

109/
```
 1        23        44        62
  63        36        22         9
40  12    59  30    13  33    18  55
  19  54     5  41    58  31    48   4
28  45    14  50    49   8    39  27
  38  26    57  16    15  51    20  37
61  17    34   7    24  60    11  46
  10  47    32  52    35   6    53  25
    56        43        29         2
       3        21        42        64
```

175/
```
 1        14        52        63
  61        35        16        18
39  20    57  31    22  33    12  46
  10  48     5  50    59  29    56   3
28  54    23  44    41   7    38  25
  40  27    58  24    21  42    11  37
62   9    36   6    15  60    17  55
  19  53    32  43    34   8    45  26
    47        49        30         4
       2        13        51        64
```

193/
```
 1        14        52        63
  62        36        15        17
40  20    58  31    21  33    11  46
  10  47     5  49    59  30    56   4
28  53    23  43    41   8    38  26
  39  27    57  24    22  42    12  37
61   9    35   6    16  60    18  55
  19  54    32  44    34   7    45  25
    48        50        29         3
       2        13        51        64
```

253/
```
 1        15        52        62
  63        36        14        17
40  20    59  30    21  33    10  47
  11  46     5  49    58  31    56   4
28  53    22  42    41   8    39  27
  38  26    57  24    23  43    12  37
61   9    34   7    16  60    19  54
  18  55    32  44    35   6    45  25
    48        51        29         2
       3        13        50        64
```

325/
```
 1        15        58        56
  63        42         8        17
46  26    59  24    21  33     4  47
  11  40     5  49    52  31    62  10
28  53    22  36    35  14    45  27
  38  20    51  30    29  43    12  37
55   3    34  13    16  60    25  54
  18  61    32  44    41   6    39  19
    48        57        23         2
       9         7        50        64
```

409/
```
 1        14        52        63
  61        19        16        34
23  36    57  47    38  17    12  30
  10  32     5  50    59  45    56   3
44  54    39  28    25   7    22  41
  24  43    58  40    37  26    11  21
62   9    20   6    15  60    33  55
  35  53    48  27    18   8    29  42
    31        49        46         4
       2        13        51        64
```

445/
```
 1        14        52        63
  62        20        15        33
24  36    58  47    37  17    11  30
  10  31     5  49    59  46    56   4
44  53    39  27    25   8    22  42
  23  43    57  40    38  26    12  21
61   9    19   6    16  60    34  55
  35  54    48  28    18   7    29  41
    32        50        45         3
       2        13        51        64
```

511/
```
 1        15        52        62
  63        20        14        33
24  36    59  46    37  17    10  31
  11  30     5  49    58  47    56   4
44  53    38  26    25   8    23  43
  22  42    57  40    39  27    12  21
61   9    18   7    16  60    35  54
  34  55    48  28    19   6    29  41
    32        51        45         2
       3        13        50        64
```

589/
```
 1        15        58        56
  63        26         8        33
30  42    59  40    37  17     4  31
  11  24     5  49    52  47    62  10
44  53    38  20    19  14    29  43
  22  36    51  46    45  27    12  21
55   3    18  13    16  60    41  54
  34  61    48  28    25   6    23  35
    32        57        39         2
       9         7        50        64
```

[Count = 780]

** Solutions with n1=1 & n64=64 & n43=62 **

```
1/
 1         36         30         63
   56         45         19         10
 47  14     54  23     12  41     17  52
   34  59      3  26     61  40     32   5
 22  28     15  57     49   7     44  38
   27  21     58  16      8  50     37  43
 60  33     25   4     39  62      6  31
   13  48     24  53     42  11     51  18
      55         46         20          9
         2         35         29         64

49/
 1         20         46         63
   59         37         24         10
 39  14     57  31     12  33     22  52
   18  56      3  42     61  27     48   5
 30  44     15  54     49   7     36  25
   40  29     58  16     11  50     21  35
 60  17     38   4     23  62      9  47
   13  43     32  53     34   8     51  26
      55         41         28          6
         2         19         45         64

55/
 1         20         46         63
   60         38         23          9
 40  14     58  31     11  33     21  52
   18  55      3  41     61  28     48   6
 30  43     15  53     49   8     36  26
   39  29     57  16     12  50     22  35
 59  17     37   4     24  62     10  47
   13  44     32  54     34   7     51  25
      56         42         27          5
         2         19         45         64

109/
 1         23         46         60
   63         38         20          9
 40  14     61  28     11  33     18  55
   21  52      3  41     58  31     48   6
 30  43     12  50     49   8     39  29
   36  26     57  16     15  53     22  35
 59  17     34   7     24  62     13  44
   10  47     32  54     37   4     51  25
      56         45         27          2
         5         19         42         64

175/
 1         12         54         63
   59         37         16         18
 39  22     57  31     20  33     14  44
   10  48      3  50     61  27     56   5
 30  52     23  46     41   7     36  25
   40  29     58  24     19  42     13  35
 60   9     38   4     15  62     17  55
   21  51     32  45     34   8     43  26
      47         49         28          6
         2         11         53         64

193/
 1         12         54         63
   60         38         15         17
 40  22     58  31     19  33     13  44
   10  47      3  49     61  28     56   6
 30  51     23  45     41   8     36  26
   39  29     57  24     20  42     14  35
 59   9     37   4     16  62     18  55
   21  52     32  46     34   7     43  25
      48         50         27          5
         2         11         53         64

253/
 1         15         54         60
   63         38         12         17
 40  22     61  28     19  33     10  47
   13  44      3  49     58  31     56   6
 30  51     20  42     41   8     39  29
   36  26     57  24     23  45     14  35
 59   9     34   7     16  62     21  52
   18  55     32  46     37   4     43  25
      48         53         27          2
         5         11         50         64

325/
 1         15         58         56
   63         42          8         17
 44  26     61  24     19  33      6  47
   13  40      3  49     54  31     60  10
 30  51     20  38     37  12     43  29
   36  22     53  28     27  45     14  35
 55   5     34  11     16  62     25  52
   18  59     32  46     41   4     39  21
      48         57         23          2
         9          7         50         64

409/
 1         12         54         63
   59         21         16         34
 23  38     57  47     36  17     14  28
   10  32      3  50     61  43     56   5
 46  52     39  30     25   7     20  41
   24  45     58  40     35  26     13  19
 60   9     22   4     15  62     33  55
   37  51     48  29     18   8     27  42
      31         49         44          6
         2         11         53         64

445/
 1         12         54         63
   60         22         15         33
 24  38     58  47     35  17     13  28
   10  31      3  49     61  44     56   6
 46  51     39  29     25   8     20  42
   23  45     57  40     36  26     14  19
 59   9     21   4     16  62     34  55
   37  52     48  30     18   7     27  41
      32         50         43          5
         2         11         53         64

511/
 1         15         54         60
   63         22         12         33
 24  38     61  44     35  17     10  31
   13  28      3  49     58  47     56   6
 46  51     36  26     25   8     23  45
   20  42     57  40     39  29     14  19
 59   9     18   7     16  62     37  52
   34  55     48  30     21   4     27  41
      32         53         43          2
         5         11         50         64

589/
 1         15         58         56
   63         26          8         33
 28  42     61  40     35  17      6  31
   13  24      3  49     54  47     60  10
 46  51     36  22     21  12     27  45
   20  38     53  44     43  29     14  19
 55   5     18  11     16  62     41  52
   34  59     48  30     25   4     23  37
      32         57         39          2
         9          7         50         64
```

[Count = 780]

** Solutions with n1=1 & n64=64 & n43=63 **

1/
```
 1        36        31        62
  56        45        18        11
46  15    55  22    12  41    17  52
  35  58     2  27    61  40    32   5
23  28    14  57    49   6    44  39
  26  21    59  16     8  51    37  42
60  33    25   4    38  63     7  30
  13  48    24  53    43  10    50  19
    54        47        20         9
       3        34        29        64
```

49/
```
 1        20        47        62
  58        37        24        11
38  15    57  30    12  33    23  52
  19  56     2  43    61  26    48   5
31  44    14  55    49   6    36  25
  40  29    59  16    10  51    21  34
60  17    39   4    22  63     9  46
  13  42    32  53    35   8    50  27
    54        41        28         7
       3        18        45        64
```

55/
```
 1        20        47        62
  60        39        22         9
40  15    59  30    10  33    21  52
  19  54     2  41    61  28    48   7
31  42    14  53    49   8    36  27
  38  29    57  16    12  51    23  34
58  17    37   4    24  63    11  46
  13  44    32  55    35   6    50  25
    56        43        26         5
       3        18        45        64
```

109/
```
 1        22        47        60
  62        39        20         9
40  15    61  28    10  33    19  54
  21  52     2  41    59  30    48   7
31  42    12  51    49   8    38  29
  36  27    57  16    14  53    23  34
58  17    35   6    24  63    13  44
  11  46    32  55    37   4    50  25
    56        45        26         3
       5        18        43        64
```

175/
```
 1         8        59        62
  54        37        22        17
38  25    53  34    18  29    21  42
   7  50     2  51    61  20    60   9
35  52    24  49    39  10    32  19
  46  33    55  26    16  41    13  30
56   5    45   4    14  63    15  58
  23  44    36  47    31  12    40  27
    48        43        28        11
       3         6        57        64
```

181/
```
 1        12        55        62
  58        37        16        19
38  23    57  30    20  33    15  44
  11  48     2  51    61  26    56   5
31  52    22  47    41   6    36  25
  40  29    59  24    18  43    13  34
60   9    39   4    14  63    17  54
  21  50    32  45    35   8    42  27
    46        49        28         7
       3        10        53        64
```

199/
```
 1        12        55        62
  60        39        14        17
40  23    59  30    18  33    13  44
  11  46     2  49    61  28    56   7
31  50    22  45    41   8    36  27
  38  29    57  24    20  43    15  34
58   9    37   4    16  63    19  54
  21  52    32  47    35   6    42  25
    48        51        26         5
       3        10        53        64
```

259/
```
 1        14        55        60
  62        39        12        17
40  23    61  28    18  33    11  46
  13  44     2  49    59  30    56   7
31  50    20  43    41   8    38  29
  36  27    57  24    22  45    15  34
58   9    35   6    16  63    21  52
  19  54    32  47    37   4    42  25
    48        53        26         3
       5        10        51        64
```

337/
```
 1        14        59        56
  62        43         8        17
44  27    61  24    18  33     7  46
  13  40     2  49    55  30    60  11
31  50    20  39    37  12    42  29
  36  23    53  28    26  45    15  34
54   5    35  10    16  63    25  52
  19  58    32  47    41   4    38  21
    48        57        22         3
       9         6        51        64
```

421/
```
 1        12        55        62
  58        21        16        35
22  39    57  46    36  17    15  28
  11  32     2  51    61  42    56   5
47  52    38  31    25   6    20  41
  24  45    59  40    34  27    13  18
60   9    23   4    14  63    33  54
  37  50    48  29    19   8    26  43
    30        49        44         7
       3        10        53        64
```

457/
```
 1        12        55        62
  60        23        14        33
24  39    59  46    34  17    13  28
  11  30     2  49    61  44    56   7
47  50    38  29    25   8    20  43
  22  45    57  40    36  27    15  18
58   9    21   4    16  63    35  54
  37  52    48  31    19   6    26  41
    32        51        42         5
       3        10        53        64
```

523/
```
 1        14        55        60
  62        23        12        33
24  39    61  44    34  17    11  30
  13  28     2  49    59  46    56   7
47  50    36  27    25   8    22  45
  20  43    57  40    38  29    15  18
58   9    19   6    16  63    37  52
  35  54    48  31    21   4    26  41
    32        53        42         3
       5        10        51        64
```

```
601/                                   607/
   1        8       61       60           1       14       59       56
    54       37       18       21          62       27        8       33
  38  49   53  52   22   9   17  20      28  43   61  40   34  17    7  30
     7  26    2  33   59  42   62  29      13  24    2  49   55  46   60  11
  55  34   46  25   15  30   14  41      47  50   36  23   21  12   26  45
    24  51   35  50   40  19   31  10      20  39   53  44   42  29   15  18
  36   3   23   6   32  63   39  58      54   5   19  10   16  63   41  52
    45  48   56  43   13  12   16  27      35  58   48  31   25   4   22  37
      44       47       28       11          32       57       38        3
         5        4        57       64           9        6       51       64
[Count = 804]
```

```
 [Counts according to the Value of n43]
   1: ...,   2: ...,   3: ...,   4: ...,   5: ...,   6: ...,   7: ...,   8: ...,
   9: ...,  10: ...,  11: ...,  12: ...,  13: ...,  14: ...,  15: ...,  16: ...,
  17: ...,  18: ...,  19: ...,  20: ...,  21: ...,  22: ...,  23: ...,  24: ...,
  25: ...,  26: ...,  27: ...,  28: ...,  29: ...,  30: ...,  31: ...,  32: 804,
  33: ...,  34: ...,  35: ...,  36: ...,  37: ...,  38: ...,  39: ...,  40: ...,
  41: ...,  42: ...,  43: ...,  44: ...,  45: ...,  46: ...,  47: ...,  48: 780,
  49: ...,  50: ...,  51: ...,  52: ...,  53: ...,  54: ...,  55: ...,  56: 780,
  57: ...,  58: ...,  59: ...,  60: 780,  61: ...,  62: 780,  63: 804,  64: ...
 [Total Count = 4728]  OK!
```

There are 4680 'Complete Euler type' by binary number system among all 4728 Simultaneous MC444, and you may guess there are 48 'Non-Euler type' of them.

I would like you to read my newest article about them in Part 4, Chapter 4.

(Originally written on Sep. 22, 2003; Feb. 22, 2005 by Kanji Setsuda;
Revised on Feb. 28, 2007 by Kanji Setsuda on MacOSX and Xcode 2.2)

E-Mail Address <jag12001@nifty.com>