Part 3: New Algebraic Studies of Magic Squares: Kanji Setsuda

Chapter 4: Algebraic Study of Magic Squares of Order 6

Section 1: Self-Complementary Magic Square of Order 6

#1. Let's start our discussion about a self-complementary magic type with the next diagram and simultaneous equations below. They are the basic form and conditions assumed at the first definition stage. Any one of serial Natural Numbers {1, 2, 3, 4, 5, ..., 34, 35, 36} must be used strictly once and be placed in the 6 x 6 regular array.

The equation (15) below means all 18 complementary pairs of 37 must be located symmetrically with respect to the geometric center(C) of the square. They should surely make this square into a self-complementary type.

[Figure 1: **Basic Form and Conditions**]

```
  .--.--.--.--.--.--.    n1+ n2+ n3+ n4+ n5+ n6=C ....(1)
  | 1| 2| 3| 4| 5| 6|    n7+ n8+ n9+n10+n11+n12=C ....(2)
  |--+--+--+--+--+--|   n13+n14+n15+n16+n17+n18=C ....(3)
  | 7| 8| 9|10|11|12|   n19+n20+n21+n22+n23+n24=C ....(4)
  |--+--+--+--+--+--|   n25+n26+n27+n28+n29+n30=C ....(5)
  |13|14|15|16|17|18|   n31+n32+n33+n34+n35+n36=C ....(6)
  |--+--+--C--+--+--|    n1+ n7+n13+n19+n25+n31=C ....(7)
  |19|20|21|22|23|24|    n2+ n8+n14+n20+n26+n32=C ....(8)
  |--+--+--+--+--+--|    n3+ n9+n15+n21+n27+n33=C ....(9)
  |25|26|27|28|29|30|    n4+n10+n16+n22+n28+n34=C ...(10)
  |--+--+--+--+--+--|    n5+n11+n17+n23+n29+n35=C ...(11)
  |31|32|33|34|35|36|    n6+n12+n18+n24+n30+n36=C ...(12)
  '--'--'--'--'--'--'    n1+ n8+n15+n22+n29+n36=C ...(13)
                         n6+n11+n16+n21+n26+n31=C ...(14)
  n1+n36=n2+n35=n3+n34                       (C=111)
 =n4+n33=n5+n32=n6+n31
 =n7+n30=n8+n29=n9+n28=n10+n27=n11+n26=n12+n25=n13+n24
 =n14+n23=n15+n22=n16+n21=n17+n20=n18+n19=CS=37  ...(15)
```

The next figure is rewritten with the new notation of complementary pairs.

```
n36=CS-n1=n1; n35=CS-n2=n2; n34=CS-n3=n3; ... ; n20=CS-n17=n17; n19=CS-n18=n18
```

[Figure 2: Basic Form and Conditions again **with New Notation**]

```
   Block P   |   Block Q
   .--.--.--+--.--.--.     n1+ n2+ n3+ n4+ n5+ n6=C ....(1)
   | 1| 2| 3| 4| 5| 6|     n7+ n8+ n9+n10+n11+n12=C ....(2)
   |--+--+--+--+--+--|    n13+n14+n15+n16+n17+n18=C ....(3)
   | 7| 8| 9|10|11|12|    n18+n17+n16+n15+n14+n13=C ....(4)
   |--+--+--+--+--+--|    n12+n11+n10+ n9+ n8+ n7=C ....(5)
   |13|14|15|16|17|18|     n6+ n5+ n4+ n3+ n2+ n1=C ....(6)
 --+--+--+--+--+--+--+--   n1+ n7+n13+n18+n12+ n6=C ....(7)
   |18|17|16|15|14|13|     n2+ n8+n14+n17+n11+ n5=C ....(8)
   |--+--+--+--+--+--|     n3+ n9+n15+n16+n10+ n4=C ....(9)
   |12|11|10| 9| 8| 7|     n4+n10+n16+n15+ n9+ n3=C ...(10)
   |--+--+--+--+--+--|     n5+n11+n17+n14+ n8+ n2=C ...(11)
   | 6| 5| 4| 3| 2| 1|     n6+n12+n18+n13+ n7+ n1=C ...(12)
   '--'--'--+--'--'--'     n1+ n8+n15+n15+ n8+ n1=C ...(13)
   Block R   |   Block S   n6+n11+n16+n16+n11+ n6=C ...(14)
                                                 (C=111)
  n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7
  =n8+n8=n9+n9=n10+n10=n11+n11=n12+n12=n13+n13
  =n14+n14=n15+n15=n16+n16=n17+n17=n18+n18=CS=37   ...(15)
```

It is very useful for you to calculate such a sum of each primary diagonal as:

n1+n8+n15+n22+n29+n36=n1+n8+n15+n15+n8+n1
=(n1+n1)+(n8+n8)+(n15+n15)=CS+CS+CS=3x37=111=C
n6+n11+n16+n21+n26+n31=(n6+n6)+(n11+n11)+(n16+n16)=3xCS=C

#2. It is known that we can never make a self-complementary type of magic square of order 6. No answers could really be found.

Are there any ways of proving that could demonstrate its impossibility?

Let me introduce you such a proof example by 'reductio ad absurdum' as follows.

Let me put the name to each block of the figure above:

P=n1+n2+n3+n7+n8+n9+n13+n14+n15;
Q=n4+n5+n6+n10+n11+n12+n16+n17+n18;
R=n19+n20+n21+n25+n26+n27+n31+n32+n33
=n18+n17+n16+n12+n11+n10+n6+n5+n4;
S=n22+n23+n24+n28+n29+n30+n34+n35+n36
=n15+n14+n13+n9+n8+n7+n3+n2+n1;

P+Q=3C [Because n1+n2+n3+n4+n5+n6=C; n7+n8+n9+n10+n11+n12=C; and n13+n14+n15+n16+n17+n18=C]
P+S=3C [Because n1+n2+n3+n3+n2+n1=3CS=C; n7+n8+n9+n9+n8+n7=3CS=C; and n13+n14+n15+n15+n14+n13=3CS=C]
Q+S=3C [Because n4+n10+n16+n22+n28+n34=C; n5+n11+n17+n23+n29+35=C; and n6+n12+n18+n24+n30+n36=C]

Therefore Q = S = 3C/2 = 333/2

This quotient is not an integer at all.

But the block-sums Q and S must be integers, because the sum of integers must be an integer.

These conclusions are contradictory to each other. They should demonstrate that any self-complementary magic square of order 6 is impossible for us to build.

[This proof was worked out by Prof. M. Suzuki and K. Setsuda.]

Section 2: Pan-Diagonal Magic Square of Order 6

[Figure 3: **Extended Space for PMS66**]

```
 34 35 36 31 32 33 34 35 36 31 32 33 34    n1+ n2+ n3+ n4+ n5+ n6=C ....(1)
          .--.--.--.--.--.--.              n7+ n8+ n9+n10+n11+n12=C ....(2)
  4  5  6| 1| 2| 3| 4| 5| 6| 1  2  3  4   n13+n14+n15+n16+n17+n18=C ....(3)
          |--+--+--+--+--+--|             n19+n20+n21+n22+n23+n24=C ....(4)
 10 11 12| 7| 8| 9|10|11|12| 7  8  9 10   n25+n26+n27+n28+n29+n30=C ....(5)
          |--+--+--+--+--+--|             n31+n32+n33+n34+n35+n36=C ....(6)
 16 17 18|13|14|15|16|17|18|13 14 15 16    n1+ n7+n13+n19+n25+n31=C ....(7)
          |--+--+--+--+--+--|              n2+ n8+n14+n20+n26+n32=C ....(8)
 22 23 24|19|20|21|22|23|24|19 20 21 22    n3+ n9+n15+n21+n27+n33=C ....(9)
          |--+--+--+--+--+--|              n4+n10+n16+n22+n28+n34=C ...(10)
 28 29 30|25|26|27|28|29|30|25 26 27 28    n5+n11+n17+n23+n29+n35=C ...(11)
          |--+--+--+--+--+--|              n6+n12+n18+n24+n30+n36=C ...(12)
 34 35 36|31|32|33|34|35|36|31 32 33 34   Pan-Diagonals:
          '--'--'--'--'--'--'              n1+ n8+n15+n22+n29+n36=C ...(13)
  4  5  6  1  2  3  4  5  6  1  2  3  4    n1+n12+n17+n22+n27+n32=C ...(14)
           [Classical Style]               n2+ n9+n16+n23+n30+n31=C ...(15)
                                           n2+ n7+n18+n23+n28+n33=C ...(16)
```

```
n3+n10+n17+n24+n25+n32=C ...(17); n3+ n8+n13+n24+n29+n34=C ...(18)
n4+n11+n18+n19+n26+n33=C ...(19); n4+ n9+n14+n19+n30+n35=C ...(20)
n5+n12+n13+n20+n27+n34=C ...(21); n5+n10+n15+n20+n25+n36=C ...(22)
n6+ n7+n14+n21+n28+n35=C ...(23); n6+n11+n16+n21+n26+n31=C ...(24)
```

```
n1+n22=n2+n23=n3+n24=n4+n19=n5+n20=n6+n21=n7+n28=n8+n29=n9+n30=n10+n25=
  n11+n26=n12+n27=n13+n34=n14+n35=n15+n36=n16+n31=n17+n32=n18+n33=CS=37...(25)
```

[Figure 4: **Basic Concept of the 'Complete' Type**]

$n1+\underline{n1}=n2+\underline{n2}=n3+\underline{n3}=n4+\underline{n4}=n5+\underline{n5}=n6+\underline{n6}=n7+\underline{n7}=n8+\underline{n8}=n9+\underline{n9}=n10+\underline{n10}=$
$n11+\underline{n11}=n12+\underline{n12}=n13+\underline{n13}=n14+\underline{n14}=n15+\underline{n15}=n16+\underline{n16}=n17+\underline{n17}=n18+\underline{n18}=CS=37$ ...(25)

```
                      |
13 14 15 16 17 18|13 14 15 16 17 18 13
              .--.--.--+--.--.--.
 4  5  6| 1| 2| 3| 4| 5| 6| 1  2  3  4
        |--P--+--+--+--Q--|
10 11 12| 7| 8| 9|10|11|12| 7  8  9 10
        |--+--+--+--+--+--|
16 17 18|13|14|15|16|17|18|13 14 15 16
      --+--+--+--+--+--+--+--
 1  2  3| 4| 5| 6| 1| 2| 3| 4  5  6  1
        |--+--+--+--+--+--|
 7  8  9|10|11|12| 7| 8| 9|10 11 12  7
        |--R--+--+--+--S--|
13 14 15|16|17|18|13|14|15|16 17 18 13
        '--'--'--+--'--'--'
 4  5  6  1  2  3| 4  5  6  1  2  3  4
                 |
```

n1+ n2+ n3+ n4+ n5+ n6=C ....(1)
n7+ n8+ n9+n10+n11+n12=C ....(2)
n13+n14+n15+n16+n17+n18=C ....(3)
$\underline{n4}+\underline{n5}+\underline{n6}+\underline{n1}+\underline{n2}+\underline{n3}=C$ ....(4)
$\underline{n10}+\underline{n11}+\underline{n12}+\underline{n7}+\underline{n8}+\underline{n9}=C$ ....(5)
$\underline{n16}+\underline{n17}+\underline{n18}+\underline{n13}+\underline{n14}+\underline{n15}=C$ ....(6)
$n1+n7+n13+\underline{n4}+\underline{n10}+\underline{n16}=C$ ....(7)
$n2+n8+n14+\underline{n5}+\underline{n11}+\underline{n17}=C$ ....(8)
$n3+n9+n15+\underline{n6}+\underline{n12}+\underline{n18}=C$ ....(9)
$n4+n10+n16+\underline{n1}+\underline{n7}+\underline{n13}=C$ ...(10)
$n5+n11+n17+\underline{n2}+\underline{n8}+\underline{n14}=C$ ...(11)
$n6+n12+n18+\underline{n3}+\underline{n9}+\underline{n15}=C$ ...(12)

Pan-Diagonals:
$n1+n8+n15+\underline{n1}+\underline{n8}+\underline{n15}=C$ ...(13)
$n1+n12+n17+\underline{n1}+\underline{n12}+\underline{n17}=C$ ...(14)
$n2+n9+n16+\underline{n2}+\underline{n9}+\underline{n16}=C$ ...(15)
$n2+n7+n18+\underline{n2}+\underline{n7}+\underline{n18}=C$ ...(16)

$n3+n10+n17+\underline{n3}+\underline{n10}+\underline{n17}=C$ ...(17); $n3+n8+n13+\underline{n3}+\underline{n8}+\underline{n13}=C$ ...(18)
$n4+n11+n18+\underline{n4}+\underline{n11}+\underline{n18}=C$ ...(19); $n4+n9+n14+\underline{n4}+\underline{n9}+\underline{n14}=C$ ...(20)
$n5+n12+n13+\underline{n5}+\underline{n12}+\underline{n13}=C$ ...(21); $n5+n10+n15+\underline{n5}+\underline{n10}+\underline{n15}=C$ ...(22)
$n6+n7+n14+\underline{n6}+\underline{n7}+\underline{n14}=C$ ...(23); $n6+n11+n16+\underline{n6}+\underline{n11}+\underline{n16}=C$ ...(24)

#1. It is also known that we cannot make any pan-diagonal magic squares of order 6. No answers could really be found. Are there any proofs to show its impossibility?
See the next diagram [Figure 3] and simultaneous equations above. Let's take them for the first definitions of basic form and the basic conditions.

At first I like to start our discussion about 'Complete' magic squares of order 6. It is a special type of pan-diagonal magic squares. All complementary pairs of 37 exist only on its pan-diagonals and no one elsewhere. See the equation (25).
Imagine what form the complete type could take if it did exist.
The next figure [Figure 4] above shows its concept of basic form and simultaneous equations assumed.

Let me introduce you such a proof example by 'reductio ad absurdum' as follows.

Let me put the name to each block of the Figure 4 above:

P=n1+n2+n3+n7+n8+n9+n13+n14+n15;
Q=n4+n5+n6+n10+n11+n12+n16+n17+n18;
R=n19+n20+n21+n25+n26+n27+n31+n32+n33
$=\underline{n4}+\underline{n5}+\underline{n6}+\underline{n10}+\underline{n11}+\underline{n12}+\underline{n16}+\underline{n17}+\underline{n18}$;
S=n22+n23+n24+n28+n29+n30+n34+n35+n36
$=\underline{n1}+\underline{n2}+\underline{n3}+\underline{n7}+\underline{n8}+\underline{n9}+\underline{n13}+\underline{n14}+\underline{n15}$;

P+Q=3C [Because n1+n2+n3+n4+n5+n6=C; n7+n8+n9+n10+n11+n12=C; and n13+n14+n15+n16+n17+n18=C]

P+S=3C [Because n1+n2+n3+n1+n2+n3=3CS=C; n7+n8+n9+n7+n8+n9=3CS=C; and n13+n14+n15+n13+n14+n15=3CS=C]

Q+S=3C [Because n4+n10+n16+n22+n28+n34=C; n5+n11+n17+n23+n29+35=C; and n6+n12+n18+n24+n30+n36=C]

Therefore Q = S = 3C/2 = 333/2

This quotient is not an integer at all.

But the block-sums Q and S must be integers, because the sum of integers must be an integer.

These conclusions are contradictory to each other. They should demonstrate that we could not make any 'complete' magic squares of order 6 at all.

[This proof was worked out by Prof. M. Suzuki and K. Setsuda.]

#2. How about the other pan-diagonal type? It doesn't always have to have all complementary pairs of 37 on its pandiagonals, though the sum of every pandiagonal must be equal to the magic constant 111. Could we find any solutions of this type of pan-diagonal magic squares?

No answers could really be found. Then how do we prove that?

I don't know what to do with it yet. But I think the Euler's method seems to help us. We will probably be able to find it by the figures of binary notation, especially by binary decomposition, I hope.

#3. If you want to know more about the new notation, Positional Writing System of Numbers, Binary Numbers, and especially about "Euler Squares", would you please read my first article in Chapter 4?

(1) Watch the next list below carefully and please find out the beautiful law in it.

**** Pan-Magic Square of Order 4 Decomposed by the Base 2 ****

```
 1/Classic                   /Math(Ext)
 .--.--.--.--.              .--.--.--.--.
 | 1| 8|13|12|   12 11|  0|  7|12|11|  0   7
 |--+--+--+--|              |--+--+--+--|
 |14|11| 2| 7|    1  6|13|10|  1|  6|13  10
 |--+--+--+--|              |--+--+--+--|
 | 4| 5|16| 9|   15  8|  3|  4|15|  8|  3   4
 |--+--+--+--|              |--+--+--+--|
 |15|10| 3| 6|    2  5|14|  9|  2|  5|14   9
 '--'--'--'--'              '--'--'--'--'
  /N2i                       /D2i
  0000 0111 1100 1011        0011 0110 0101 0101
  1101 1010 0001 0110        1100 1001 0101 1010
  0011 0100 1111 1000        0011 0110 1010 1010
  1110 1001 0010 0101        1100 1001 1010 0101
                              *8/  *4/  *2/  *1/
* Check Line-Sums and See if they are equal to 30:
  0+7+12+11=(0+0+1+1)*8+(0+1+1+0)*4+(0+1+0+1)*2+(0+1+0+1)= 30 ... rw1
  13+10+1+6=(1+1+0+0)*8+(1+0+0+1)*4+(0+1+0+1)*2+(1+0+1+0)= 30 ... rw2
  3+4+15+8= (0+0+1+1)*8+(0+1+1+0)*4+(1+0+1+0)*2+(1+0+1+0)= 30 ... rw3
  14+9+2+5= (1+1+0+0)*8+(1+0+0+1)*4+(1+0+1+0)*2+(0+1+0+1)= 30 ... rw4
  0+13+3+14=(0+1+0+1)*8+(0+1+0+1)*4+(0+0+1+1)*2+(0+1+1+0)= 30 ... cl1
  7+10+4+9= (0+1+0+1)*8+(1+0+1+0)*4+(1+1+0+0)*2+(1+0+0+1)= 30 ... cl2
  12+1+15+2=(1+0+1+0)*8+(1+0+1+0)*4+(0+0+1+1)*2+(0+1+1+0)= 30 ... cl3
  11+6+8+5= (1+0+1+0)*8+(0+1+0+1)*4+(1+1+0+0)*2+(1+0+0+1)= 30 ... cl4
```

```
... This is certainly a Basic type of MS44!
 0+10+15+5=(0+1+1+0)*8+(0+0+1+1)*4+(0+1+1+0)*2+(0+0+1+1)= 30 ... pd1
 7+1+8+14= (0+0+1+1)*8+(1+0+0+1)*4+(1+0+0+1)*2+(1+1+0+0)= 30 ... pd2
 12+6+3+9= (1+0+0+1)*8+(1+1+0+0)*4+(0+1+1+0)*2+(0+0+1+1)= 30 ... pd3
 11+13+4+2=(1+1+0+0)*8+(0+1+1+0)*4+(1+0+0+1)*2+(1+1+0+0)= 30 ... pd4
 0+6+15+9= (0+0+1+1)*8+(0+1+1+0)*4+(0+1+1+0)*2+(0+0+1+1)= 30 ... pb1
 7+13+8+2= (0+1+1+0)*8+(1+1+0+0)*4+(1+0+0+1)*2+(1+1+0+0)= 30 ... pb2
 12+10+3+5=(1+1+0+0)*8+(1+0+0+1)*4+(0+1+1+0)*2+(0+0+1+1)= 30 ... pb3
 11+1+4+14=(1+0+0+1)*8+(0+0+1+1)*4+(1+0+0+1)*2+(1+1+0+0)= 30 ... pb4
... This is certainly a Pan-diagonal type!
 0+5=5   ... cp1;  7+2=9    ... cp2; 12+9=21 ... cp3;  11+14=25 ... cp4;
 13+8=21 ... cp5;  10+15=25 ... cp6; 1+4=5   ... cp7;  6+3=9    ... cp8;
... This is not a Self-complementary type!

2/Classic     /Math.--.--.--.--.        /N2i                /D2i
  1  8 11 14   10 13| 0| 7|10|13| 0  7   0000 0111 1010 1101  0011 0101 0110 0101
 12 13  2  7    1  6|11|12| 1| 6|11 12   1011 1100 0001 0110  1100 0101 1001 1010
  6  3 16  9   15  8| 5| 2|15| 8| 5  2   0101 0010 1111 1000  0011 1010 0110 1010
 15 10  5  4    4  3|14| 9| 4| 3|14  9   1110 1001 0100 0011  1100 1010 1001 0101
                    '--'--'--'--'                              *8/  *4/  *2/  *1/
3/Classic     /Math.--.--.--.--.        /N2i                /D2i
  1 12  7 14    6 13| 0|11| 6|13| 0 11   0000 1011 0110 1101  0101 0011 0110 0101
  8 13  2 11    1 10| 7|12| 1|10| 7 12   0111 1100 0001 1010  0101 1100 1001 1010
 10  3 16  5   15  4| 9| 2|15| 4| 9  2   1001 0010 1111 0100  1010 0011 0110 1010
 15  6  9  4    8  3|14| 5| 8| 3|14  5   1110 0101 1000 0011  1010 1100 1001 0101
                    '--'--'--'--'                              *8/  *4/  *2/  *1/
```

I would just like you to know that any pandiagonal magic square has the constant combination of same number patterns in each line in each decomposed layer such as {0,0,1,1}, {0,1,0,1}, {0,1,1,0}, ..., {1,0,1,0}, {1,1,0,0}.

It makes each line add up to the same sum, so called 'magic constant' by the next form of equation:

(0+0+1+1)*8+(0+0+1+1)*4+(0+0+1+1)*2+(0+0+1+1)=30

It is supposed to be the basic condition for every pandiagonal magic square 4x4.

(2) In the case of order 8, those which have the same property in each line in each decomposed layer as the ones of order 4 are all the pandiagonal magic squares:

```
** Pan-Magic Square of Order 8 Decomposed by the Base 2 **
 1/Classic                            /Math(Ext)
.--.--.--.--.--.--.--.--.              .--.--.--.--.--.--.--.--.
| 1|63| 4|62| 8|58| 5|59|   7 57  4 58| 0|62| 3|61| 7|57| 4|58| 0 62  3 61
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|56|10|53|11|49|15|52|14|  48 14 51 13|55| 9|52|10|48|14|51|13|55  9 52 10
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|25|39|28|38|32|34|29|35|  31 33 28 34|24|38|27|37|31|33|28|34|24 38 27 37
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|48|18|45|19|41|23|44|22|  40 22 43 21|47|17|44|18|40|22|43|21|47 17 44 18
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|57| 7|60| 6|64| 2|61| 3|  63  1 60  2|56| 6|59| 5|63| 1|60| 2|56  6 59  5
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|16|50|13|51| 9|55|12|54|   8 54 11 53|15|49|12|50| 8|54|11|53|15 49 12 50
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|33|31|36|30|40|26|37|27|  39 25 36 26|32|30|35|29|39|25|36|26|32 30 35 29
|--+--+--+--+--+--+--+--|             |--+--+--+--+--+--+--+--|
|24|42|21|43|17|47|20|46|  16 46 19 45|23|41|20|42|16|46|19|45|23 41 20 42
'--'--'--'--'--'--'--'--'              '--'--'--'--'--'--'--'--'
```

```
/N2i
000000 111110 000011 111101 000111 111001 000100 111010
110111 001001 110100 001010 110000 001110 110011 001101
011000 100110 011011 100101 011111 100001 011100 100010
101111 010001 101100 010010 101000 010110 101011 010101
111000 000110 111011 000101 111111 000001 111100 000010
001111 110001 001100 110010 001000 110110 001011 110101
100000 011110 100011 011101 100111 011001 100100 011010
010111 101001 010100 101010 010000 101110 010011 101101
 /D2i*32/     *16/      *8/      *4/      *2/      *1/
 01010101 01010101 01010101 01011010 01101001 00111100
 10101010 10101010 01010101 10100101 10010110 11000011
 01010101 10101010 10101010 01011010 01101001 00111100
 10101010 01010101 10101010 10100101 10010110 11000011
 10101010 10101010 10101010 01011010 01101001 00111100
 01010101 01010101 10101010 10100101 10010110 11000011
 10101010 01010101 01010101 01011010 01101001 00111100
 01010101 10101010 01010101 10100101 10010110 11000011
```

* Check Line-Sums and See if they are equal to 252:

```
0+62+3+61+7+57+4+58=     (0+1+0+1+0+1+0+1)*32+(0+1+0+1+0+1+0+1)*16+(0+1+0+1+0+1+0+1)*8+
  (0+1+0+1+1+0+1+0)*4+(0+1+1+0+1+0+0+1)*2+(0+0+1+1+1+1+0+0)= 252 ... rw1
55+9+52+10+48+14+51+13=  (1+0+1+0+1+0+1+0)*32+(1+0+1+0+1+0+1+0)*16+(0+1+0+1+0+1+0+1)*8+
  (1+0+1+0+0+1+0+1)*4+(1+0+0+1+0+1+1+0)*2+(1+1+0+0+0+0+1+1)= 252 ... rw2
24+38+27+37+31+33+28+34= (0+1+0+1+0+1+0+1)*32+(1+0+1+0+1+0+1+0)*16+(1+0+1+0+1+0+1+0)*8+
  (0+1+0+1+1+0+1+0)*4+(0+1+1+0+1+0+0+1)*2+(0+0+1+1+1+1+0+0)= 252 ... rw3
47+17+44+18+40+22+43+21= (1+0+1+0+1+0+1+0)*32+(0+1+0+1+0+1+0+1)*16+(1+0+1+0+1+0+1+0)*8+
  (1+0+1+0+0+1+0+1)*4+(1+0+0+1+0+1+1+0)*2+(1+1+0+0+0+0+1+1)= 252 ... rw4

0+55+24+47+56+15+32+23=  (0+1+0+1+1+0+1+0)*32+(0+1+1+0+1+0+0+1)*16+(0+0+1+1+1+1+0+0)*8+
  (0+1+0+1+0+1+0+1)*4+(0+1+0+1+0+1+0+1)*2+(0+1+0+1+0+1+0+1)= 252 ... cl1
62+9+38+17+6+49+30+41=   (1+0+1+0+0+1+0+1)*32+(1+0+0+1+0+1+1+0)*16+(1+1+0+0+0+0+1+1)*8+
  (1+0+1+0+1+0+1+0)*4+(1+0+1+0+1+0+1+0)*2+(0+1+0+1+0+1+0+1)= 252 ... cl2
3+52+27+44+59+12+35+20=  (0+1+0+1+1+0+1+0)*32+(0+1+1+0+1+0+0+1)*16+(0+0+1+1+1+1+0+0)*8+
  (0+1+0+1+0+1+0+1)*4+(1+0+1+0+1+0+1+0)*2+(1+0+1+0+1+0+1+0)= 252 ... cl3
61+10+37+18+5+50+29+42=  (1+0+1+0+0+1+0+1)*32+(1+0+0+1+0+1+1+0)*16+(1+1+0+0+0+0+1+1)*8+
  (1+0+1+0+1+0+1+0)*4+(0+1+0+1+0+1+0+1)*2+(1+0+1+0+1+0+1+0)= 252 ... cl4
```

... This is certainly a Basic type of MS88!

```
0+9+27+18+63+54+36+45=   (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
  (0+0+0+0+1+1+1+1)*4+(0+0+1+1+1+1+0+0)*2+(0+1+1+0+1+0+0+1)= 252 ... pd1
62+52+37+40+1+11+26+23=  (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
  (1+1+1+0+0+0+0+1)*4+(1+0+0+0+0+1+1+1)*2+(0+0+1+0+1+1+0+1)= 252 ... pd2
3+10+31+22+60+53+32+41=  (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
  (0+0+1+1+1+1+0+0)*4+(1+1+1+1+0+0+0+0)*2+(1+0+1+0+0+1+0+1)= 252 ... pd3
61+48+33+43+2+15+30+20=  (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
  (1+0+0+0+0+1+1+1)*4+(0+0+0+1+1+1+1+0)*2+(1+0+1+1+0+1+0+0)= 252 ... pd4

7+10+27+17+56+53+36+46=  (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
  (1+0+0+0+0+1+1+1)*4+(1+1+1+0+0+0+0+1)*2+(1+0+1+1+0+1+0+0)= 252 ... pb5
57+48+37+44+6+15+26+19=  (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
  (0+0+1+1+1+1+0+0)*4+(0+0+0+0+1+1+1+1)*2+(1+0+1+0+0+1+0+1)= 252 ... pb6
4+14+31+18+59+49+32+45=  (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
  (1+1+1+0+0+0+0+1)*4+(0+1+1+1+1+0+0+0)*2+(0+0+1+0+1+1+0+1)= 252 ... pb7
58+51+33+40+5+12+30+23=  (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
  (0+0+0+0+1+1+1+1)*4+(1+1+0+0+0+0+1+1)*2+(0+1+1+0+1+0+0+1)= 252 ... pb8
```

... This is certainly a Pan-diagonal type!

```
0+45=45   ... cp1;  62+19=81 ... cp2;  3+46=49   ... cp3;  61+16=77 ... cp4;
7+42=49   ... cp5;  57+20=77 ... cp6;  4+41=45   ... cp7;  58+23=81 ... cp8;
55+26=81 ... cp9;  9+36=45   ... cp10; 52+25=77 ... cp11; 10+39=49 ... cp12;
48+29=77 ... cp13; 14+35=49 ... cp14; 51+30=81 ... cp15; 13+32=45 ... cp16;
24+53=77 ... cp17; 38+11=49 ... cp18; 27+54=81 ... cp19; 37+8=45   ... cp20;
31+50=81 ... cp21; 33+12=45 ... cp22; 28+49=77 ... cp23; 34+15=49 ... cp24;
```

```
47+2=49   ... cp25; 17+60=77 ... cp26; 44+1=45  ... cp27; 18+63=81 ... cp28;
40+5=45   ... cp29; 22+59=81 ... cp30; 43+6=49  ... cp31; 21+56=77 ... cp32;
... This is not a Self-complementary type!
```

* New Euler's Condition for order 8:

(0+0+0+0+1+1+1+1)*2^5+(0+0+0+0+1+1+1+1)*2^4+(0+0+0+0+1+1+1+1)*2^3+
(0+0+0+0+1+1+1+1)*2^2+(0+0+0+0+1+1+1+1)*2^1+(0+0+0+0+1+1+1+1)*2^0 = 252

This beautiful structure is well known as 'Greco-Latin Square' or 'Euler Square'.

I would like to call "**Complete** Euler Squares" for those ones whose pandiagonals all obey the same condition at the same time along with all rows and columns.

#4. How about the case of order 6?

Could we make such a kind of 'Euler Squares' of order 6 just as order 4 and 8?

If we could, then we would know there certainly exist any pandiagonal magic squares of order 6. If we could not, then we would finally know there are no such things as Pandiagonal MS66.

At first let me decompose each of our Basic Form by the base 6, 3 and 2 as follows:

See the lists below. It doesn't seem that decompositions by the base 3 and 2 could possibly make any "Complete Euler Squares", but it seems that the decomposition by the base 6 might make any 'CES'.

```
** Basic Forms of Order 6 Decomposed by the Base 6 **
 0/Classic                         /Math(Ext)
 .--.--.--.--.--.--.               .--.--.--.--.--.--.
 | 1| 2| 3| 4| 5| 6|    3  4  5| 0| 1| 2| 3| 4| 5| 0  1  2
 |--+--+--+--+--+--|               |--+--+--+--+--+--|
 | 7| 8| 9|10|11|12|    9 10 11| 6| 7| 8| 9|10|11| 6  7  8
 |--+--+--+--+--+--|               |--+--+--+--+--+--|
 |13|14|15|16|17|18|   15 16 17|12|13|14|15|16|17|12 13 14
 |--+--+--+--+--+--|               |--+--+--+--+--+--|
 |19|20|21|22|23|24|   21 22 23|18|19|20|21|22|23|18 19 20
 |--+--+--+--+--+--|               |--+--+--+--+--+--|
 |25|26|27|28|29|30|   27 28 29|24|25|26|27|28|29|24 25 26
 |--+--+--+--+--+--|               |--+--+--+--+--+--|
 |31|32|33|34|35|36|   33 34 35|30|31|32|33|34|35|30 31 32
 '--'--'--'--'--'--'               '--'--'--'--'--'--'

  /N6i                /D6i    *6/         *1/
  00 01 02 03 04 05   0 0 0 0 0 0  0 1 2 3 4 5
  10 11 12 13 14 15   1 1 1 1 1 1  0 1 2 3 4 5
  20 21 22 23 24 25   2 2 2 2 2 2  0 1 2 3 4 5
  30 31 32 33 34 35   3 3 3 3 3 3  0 1 2 3 4 5
  40 41 42 43 44 45   4 4 4 4 4 4  0 1 2 3 4 5
  50 51 52 53 54 55   5 5 5 5 5 5  0 1 2 3 4 5

** Decomposed by the Base 3 **
  /N3i                            /D3i
  0000 0001 0002 0010 0011 0012   000000 000000 000111 012012
  0020 0021 0022 0100 0101 0102   000000 000111 222000 012012
  0110 0111 0112 0120 0121 0122   000000 111111 111222 012012
  0200 0201 0202 0210 0211 0212   000000 222222 000111 012012
  0220 0221 0222 1000 1001 1002   000111 222000 222000 012012
  1010 1011 1012 1020 1021 1022   111111 000000 111222 012012
                                     *27/     *9/     *3/     *1/

** Decomposed by the Base 2 **
```

```
/N2i
000000 000001 000010 000011 000100 000101
000110 000111 001000 001001 001010 001011
001100 001101 001110 001111 010000 010001
010010 010011 010100 010101 010110 010111
011000 011001 011010 011011 011100 011101
011110 011111 100000 100001 100010 100011
```

```
/D2i
000000 000000 000000 000011 001100 010101
000000 000000 001111 110000 110011 010101
000000 000011 111100 111100 001100 010101
000000 111111 000000 001111 110011 010101
000000 111111 111111 000011 001100 010101
001111 110000 110000 110000 110011 010101
  *32/    *16/   *8/    *4/    *2/    *1/
```

Do you notice the left-most layer of the highest position $*3^3/$ is not yet filled full with the same counts of {0, 1 and 2}. There are only 9 counts of 1, which stand for the values exceeding 26. It is because the positional layer $*3^3/$ is made for the values from 27 to 80. But our square of order 6 could only give the largest value 35 (=6^2 -1).

The left-most layer of the highest position $*2^5/$ is not yet filled full with 18 counts of {0 and 1}. There are only 4 counts of 1, which stand for the numbers exceeding 31. It is because the positional layer $*2^5/$ is intended to express the values from 32 to 63. But our 6x6 square could only give the largest value 35.

Because you cannot fill the layer of the highest position full with equal counts of every value, you cannot make the beautiful structure 'Euler Square' of order 6 at all.

The square could fill the highest layer full with its binary values, only when it is of order 4, 8, 16, ... It is because 4x4 = 2^4; 8x8 = 2^6; 16x16 = 2^8; ...

But 2^5 < 6x6 < 2^6; 2^6 < 10x10 < 2^7; 2^7 < 14x14 < 2^8; ...

As a consequence the pan-diagonal magic squares of order 6, 10, 14, ... prove to be impossible for us to construct by any ordinary method.

I believe Legendary Leonhard Euler(1707-1783) must probably have known this fact. They say he already knew the impossibility of those squares about 250 years ago.

I actually tried to compose "Complete Euler Squares" of order 6 by our "New Euler's Method" the other day, but I could not even compose any 'Latin Squares' decomposed by the base 6. It proves to be impossible for us to compose any CES of order 6.

That seems to mean there are no such things as Pan-diagonal MS66 at all.

Section 3: Standard Magic Squares of Order 6

What type of magic squares can we make for order 6, then? Can we really make the most basic type of standard magic squares of order 6? Let's try to make that here.

#1. The following diagram and list of equations demonstrate our assumption for the first definitions. We want to make the most basic type of standard magic squares of order 6, whose rows and columns add up to the magic constant, and two primary diagonals also add up to the same sum. We add no other conditions than those.

We use each of the serial natural numbers {1, 2, 3, 4, 5, ..., 34, 35, 36} strictly once and never use any number twice or more often.

** **Basic Diagram** for Magic Square 6x6 and **Basic Equations**: **

```
       .-----------------.
  5  6| 1| 2| 3| 4| 5| 6| 1  2
      |--+--+--+--+--+--|
 11 12| 7| 8| 9|10|11|12| 7  8
      |--+--+--+--+--+--|
 17 18|13|14|15|16|17|18|13 14
      |--+--+--+--+--+--|
 23 24|19|20|21|22|23|24|19 20
      |--+--+--+--+--+--|
 29 30|25|26|27|28|29|30|25 26
      |--+--+--+--+--+--|
 35 36|31|32|33|34|35|36|31 32
       '-----------------'
```

```
n1+n2+n3+n4+n5+n6=C         ....(rw1)
n7+n8+n9+n10+n11+n12=C      ....(rw2)
n13+n14+n15+n16+n17+n18=C   ....(rw3)
n19+n20+n21+n22+n23+n24=C   ....(rw4)
n25+n26+n27+n28+n29+n30=C   ....(rw5)
n31+n32+n33+n34+n35+n36=C   ....(rw6)
n1+n7+n13+n19+n25+n31=C     ....(cl1)
n2+n8+n14+n20+n26+n32=C     ....(cl2)
n3+n9+n15+n21+n27+n33=C     ....(cl3)
n4+n10+n16+n22+n28+n34=C    ....(cl4)
n5+n11+n17+n23+n29+n35=C    ....(cl5)
n6+n12+n18+n24+n30+n36=C    ....(cl6)
n1+n8+n15+n22+n29+n36=C     ....(pd1)
n6+n11+n16+n21+n26+n31=C    ....(pd2)
```

#2. The next list of solutions shows the result of my recent calculation, only a part of innumerable solutions. I don't yet know how many solutions exist in all. I am afraid it will take too long a time for you to count them through.

** **Standard Magic Squares** of Order 6: Standard Solutions **

```
               1/                39/                75/               131/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 14 25  7 13 21  31 16 25  7 20 12  31 16 26  7 18 13  31 16 25 10 17 12
  6 20 26 16 24 19   6 18 27 14 22 24   6 20 24 15 25 21   6 23 27 15 21 19
 30 11 18 12 17 23  30 15 17 10 13 26  30 14 19 11 10 27  30 11 20  7 14 29
  8 27 29 15 22 10   8 23 29 19 21 11   8 22 29 17 23 12   8 22 26 18 24 13
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
             284/               374/               545/               745/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 16 26 10 17 11  31 18 24 12 16 10  31 20 23 10 14 13  31 14 29  7 20 10
  6 21 27 15 23 19   6 20 27 14 25 19   6 19 25 15 29 17   6 24 25 15 22 19
 30 13 20  7 12 29  30 11 21  7 13 29  30 12 24  7 11 27  30 13 18 12 11 27
  8 22 25 18 24 14   8 23 26 17 22 15   8 21 26 18 22 16   8 21 26 16 23 17
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
             941/              1167/              1459/              1654/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 12 24 14 19 11  31 17 22 10 15 16  31 18 21 10 16 15  31 16 25  7 20 12
  6 25 29 13 21 17   6 25 26 20 23 11   6 25 26 19 24 11   6 24 26 19 23 13
 30 15 22  7 10 27  30 12 21  7 14 27  30 12 22  7 13 27  30 15 18 10 11 27
  8 20 23 16 26 18   8 18 29 13 24 19   8 17 29 14 23 20   8 17 29 14 22 21
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
            2036/              2275/              2573/              2784/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 14 26 10 19 11  31 14 27 10 17 12  31 15 26 12 16 11  31 15 22 13 16 14
  6 25 29 17 21 13   6 21 29 22 20 13   6 27 29 17 19 13   6 27 29 19 20 10
 30 15 20  7 12 27  30 18 16  7 15 25  30 10 21  7 18 25  30 12 21  7 17 24
  8 18 23 16 24 22   8 19 26 11 24 23   8 20 22 14 23 24   8 18 26 11 23 25
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
            3103/              3303/              3599/              3850/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 19 22 11 21  7  31 15 25  7 21 12  31 18 27  7 16 12  31 12 22 10 15 21
  6 24 27 16 23 15   6 24 29 19 22 11   6 24 26 19 25 11   6 23 29 18 16 19
 30 12 20 10 14 25  30 17 18 10 13 23  30 13 22 10 15 21  27 13 17  8 20 26
  8 17 29 13 18 26   8 16 26 14 20 27   8 17 23 14 20 29  11 24 30 14 25  7
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
```

```
            3875/              3928/              4098/              4247/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 12 22 10 16 20  31 16 29  8 13 14  31 17 20 10 15 18  31  8 23 16 14 19
  6 24 30 18 14 19   6 21 25 18 22 19   6 25 29 14 24 13   6 24 29 17 20 15
 27 13 17  7 21 26  27 12 20  7 15 30  27  8 23  7 16 30  27 18 21  7 12 26
 11 23 29 15 25  8  11 23 24 17 26 10  11 22 26 19 21 12  11 22 25 10 30 13
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
            4335/              4605/              4722/              4963/
  1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2   1 34  9 33 32  2
 31 15 26 13 16 10  31 16 25  8 24  7  31 18 29  7 12 14  31 18 20 14 21  7
  6 25 29 12 20 19   6 23 26 13 22 21   6 21 26 20 25 13   6 26 25 13 22 19
 27  8 22  7 17 30  27 14 18 12 10 30  27 10 19  8 17 30  27 12 24  8 10 30
 11 24 21 18 23 14  11 19 29 17 20 15  11 23 24 15 22 16  11 16 29 15 23 17
 35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36  35  5  4 28  3 36
 . . . . . .
```

#3. I tried to pick up an example of those solutions above and examined it carefully by decomposing of the base 6, 3 and 2. The following lists show the exam results.

**** Standard Magic Square of Order 6 Examined ****

```
 1/Classic                     /Math(Ext)
 .--.--.--.--.--.--.            .--.--.--.--.--.--.
 | 1|34| 9|33|32| 2|  32 31  1| 0|33| 8|32|31| 1| 0 33  8
 |--+--+--+--+--+--|            |--+--+--+--+--+--|
 |31|16|20|12|14|18|  11 13 17|30|15|19|11|13|17|30 15 19
 |--+--+--+--+--+--|            |--+--+--+--+--+--|
 | 7|21|27|13|24|19|  12 23 18| 6|20|26|12|23|18| 6 20 26
 |--+--+--+--+--+--|            |--+--+--+--+--+--|
 |26|10|22| 8|15|30|   7 14 29|25| 9|21| 7|14|29|25  9 21
 |--+--+--+--+--+--|            |--+--+--+--+--+--|
 |11|25|29|17|23| 6|  16 22  5|10|24|28|16|22| 5|10 24 28
 |--+--+--+--+--+--|            |--+--+--+--+--+--|
 |35| 5| 4|28| 3|36|  27  2 35|34| 4| 3|27| 2|35|34  4  3
 '--'--'--'--'--'--'            '--'--'--'--'--'--'

  /N6i                /D6i    *6/         *1/
  00 53 12 52 51 01   0 5 1 5 5 0   0 3 2 2 1 1
  50 23 31 15 21 25   5 2 3 1 2 2   0 3 1 5 1 5
  10 32 42 20 35 30   1 3 4 2 3 3   0 2 2 0 5 0
  41 13 33 11 22 45   4 1 3 1 2 4   1 3 3 1 2 5
  14 40 44 24 34 05   1 4 4 2 3 0   4 0 4 4 4 5
  54 04 03 43 02 55   5 0 0 4 0 5   4 4 3 3 2 5
```

```
* Check Line-Sums and See if they are equal to 105:
  0+33+8+32+31+1=     (0+5+1+5+5+0)*6+(0+3+2+2+1+1)=  105 ... rw1
  30+15+19+11+13+17=(5+2+3+1+2+2)*6+(0+3+1+5+1+5)=  105 ... rw2
  6+20+26+12+23+18=  (1+3+4+2+3+3)*6+(0+2+2+0+5+0)=  105 ... rw3
  25+9+21+7+14+29=   (4+1+3+1+2+4)*6+(1+3+3+1+2+5)=  105 ... rw4
  10+24+28+16+22+5=  (1+4+4+2+3+0)*6+(4+0+4+4+4+5)=  105 ... rw5
  34+4+3+27+2+35=    (5+0+0+4+0+5)*6+(4+4+3+3+2+5)=  105 ... rw6
  0+30+6+25+10+34=   (0+5+1+4+1+5)*6+(0+0+0+1+4+4)=  105 ... cl1
  33+15+20+9+24+4=   (5+2+3+1+4+0)*6+(3+3+2+3+0+4)=  105 ... cl2
  8+19+26+21+28+3=   (1+3+4+3+4+0)*6+(2+1+2+3+4+3)=  105 ... cl3
  32+11+12+7+16+27=  (5+1+2+1+2+4)*6+(2+5+0+1+4+3)=  105 ... cl4
  31+13+23+14+22+2=  (5+2+3+2+3+0)*6+(1+1+5+2+4+2)=  105 ... cl5
  1+17+18+29+5+35=   (0+2+3+4+0+5)*6+(1+5+0+5+5+5)=  105 ... cl6
  0+15+26+7+22+35=   (0+2+4+1+3+5)*6+(0+3+2+1+4+5)=  105 ... pd1
  1+13+12+21+24+34=  (0+2+2+3+4+5)*6+(1+1+0+3+0+4)=  105 ... pb6
 ... This is certainly a basic type of Standard MS66!
 ... But, this is not a Pan-diagonal one!
  0+35=35  ... cp1; 33+2=35  ... cp2; 8+27=35  ... cp3; 32+3=35  ... cp4;
  31+4=35  ... cp5; 1+34=35  ... cp6; 30+5=35  ... cp7; 15+22=37 ... cp8;
```

```
19+16=35 ... cp9; 11+28=39 ... cp10;13+24=37 ... cp11;17+10=27 ... cp12;
6+29=35  ... cp13;20+14=34 ... cp14;26+7=33  ... cp15;12+21=33 ... cp16;
23+9=32  ... cp17;18+25=43 ... cp18;
```

... This is not a Self-complementary type!

** The Same Solution Decomposed by **the Base 3** **

```
/N3i                               /D3i
0000 1020 0022 1012 1011 0001      010110 000000 022110 002211
1010 0120 0201 0102 0111 0122      100000 012111 120012 001212
0020 0202 0222 0110 0212 0200      000000 022122 202110 022020
0221 0100 0210 0021 0112 1002      000001 212010 201210 100122
0101 0220 1001 0121 0211 0012      001000 120120 020211 101112
1021 0011 0010 1000 0002 1022      100101 000000 211002 110022
```

** The Same Solution Decomposed by **the Base 2** **

```
/N2i
000000 100001 001000 100000 011111 000001
011110 001111 010011 001011 001101 010001
000110 010100 011010 001100 010111 010010
011001 001001 010101 000111 001110 011101
001010 011000 011100 010000 010110 000101
100010 000100 000011 011011 000010 100011
/D2i
010100 000010 001010 000010 000010 010011
000000 101001 110110 110010 111100 011111
000000 011011 001100 110110 101011 000010
000000 101001 110011 001111 000110 111101
000000 011110 111000 001011 100010 000001
100001 000100 000100 010000 101111 001101
```

As you see, this is certainly not an example of Euler Squares by any means. Any line in any layer of the decompositions does not always obey the Euler's Condition. It is not always expressed in the same forms to the next equations:

```
(0+1+2+3+4+5)x6+(0+1+2+3+4+5)=105                                .../D6i
(0+0+1+1+2+2)x27+(0+0+1+1+2+2)x9+(0+0+1+1+2+2)x3+(0+0+1+1+2+2)=105 .../D3i
(0+0+0+1+1+1)x32+(0+0+0+1+1+1)x16+(0+0+0+1+1+1)x8+
  (0+0+0+1+1+1)x4+(0+0+0+1+1+1)x2+(0+0+0+1+1+1) = 105            .../D2i
```

#4. The next list shows the same solutions with /D6i diagrams.

It seems that all the innumerable solutions of standard magic squares of order 6 may be "Non-Euler" type of Magic Squares, doesn't it? How surprising it is!

** Standard Magic Squares of Order 6: Standard Solutions with /D6i **

```
                1/  /D6i                               3/  /D6i
 1 34  9 33 32  2   051550 032211    1 34  9 33 32  2   051550 032211
31 14 25  7 13 21   524123 010002   31 11 25  7 18 19   514123 040050
 6 20 26 16 24 19   034233 511350    6 20 27 13 24 21   034233 512052
30 11 18 12 17 23   412123 545544   30 15 17 14 12 23   422213 524154
 8 27 29 15 22 10   144231 124233    8 26 29 16 22 10   144231 114333
35  5  4 28  3 36   500405 443325   35  5  4 28  3 36   500405 443325
                8/  /D6i                               9/  /D6i
 1 34  9 33 32  2   051550 032211    1 34  9 33 32  2   051550 032211
31 12 27  7 19 15   514132 052002   31 11 25  7 18 19   514123 040050
 6 20 24 14 21 26   033234 515121    6 27 24 14 17 23   043223 525144
30 17 18 13 11 22   422213 545043   30 12 20 13 15 21   413223 551022
 8 23 29 16 25 10   134241 144303    8 22 29 16 26 10   134241 134313
35  5  4 28  3 36   500405 443325   35  5  4 28  3 36   500405 443325
```

```
                  13/  /D6i                                       16/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 12 24 13 16 15  513222 055032  31 14 25 11 17 13  524122 010440
 6 18 29 11 20 27  024134 554412   6 19 27 12 18 29  034124 502554
30 17 22  7 14 21  423123 543012  30 16 22  7 15 21  423123 533022
 8 25 23 19 26 10  143341 104013   8 23 24 20 26 10  133341 145113
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
                  17/  /D6i                                       19/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 11 26  7 17 19  514123 041040  31 14 18 12 13 23  522123 015504
 6 21 25 12 23 24  034133 520545   6 25 27 15 17 21  044223 502242
30 13 18 16 14 20  422223 505311  30 11 24  7 20 19  413133 545010
 8 27 29 15 22 10  144231 124233   8 22 29 16 26 10  134241 134313
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
                  20/  /D6i                                       21/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 16 22 13 14 15  523222 033012  31 11 24 12 14 19  513123 045510
 6 20 27 12 17 29  034124 512544   6 17 27 15 20 26  024234 542211
30 11 23  7 21 19  413133 544020  30 21 22  7 13 18  433122 523005
 8 25 26 18 24 10  144231 101553   8 23 25 16 29 10  134241 140343
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
                  25/  /D6i                                       27/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 14 23  7 11 25  523114 014040  31 11 22 14 12 21  513213 043152
 6 18 26 16 24 21  024233 551352   6 20 29 13 18 25  034224 514050
30 13 20 12 19 17  423132 501504  30 15 23  7 19 17  423132 524004
 8 27 29 15 22 10  144231 124233   8 26 24 16 27 10  143241 115323
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
                  32/  /D6i                                       33/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 16 23  7 15 19  523123 034020  31 12 19  7 15 27  513124 050022
 6 24 20 12 22 27  033134 551532   6 22 26 14 23 20  034233 531141
30 11 26 13 14 17  414222 541014  30 17 24 11 13 16  423122 545403
 8 21 29 18 25 10  134241 124503   8 21 29 18 25 10  134241 124503
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
                  34/  /D6i                                       38/  /D6i
 1 34  9 33 32  2  051550 032211   1 34  9 33 32  2  051550 032211
31 11 22  7 19 21  513133 043002  31 12 19  7 15 27  513124 050022
 6 25 24 12 17 27  043124 505542   6 22 26 14 20 23  034233 531114
30 16 23 13 14 15  423222 534012  30 17 24 11 16 13  423122 545430
 8 20 29 18 26 10  134241 114513   8 21 29 18 25 10  134241 124503
35  5  4 28  3 36  500405 443325  35  5  4 28  3 36  500405 443325
. . . . . .
```

I cannot help wondering whether there might be any Euler Squares among them or not. The solution count would surely be so big, almost as uncountable, that there might probably exist any 'Euler Type' among those solutions, don't you think?

I made up my mind to examine it at once. Let's make a new experiment of trying to compose Euler Squares of order 6 by our "New Euler's Method".

I gave up my idea of composing any "Complete Euler Squares", for I know it is really impossible. I just want to compose any basic type of Euler Squares, only the two primary diagonals of which obey the Euler's Condition beside their rows and columns. And I use the decompositions by the base 6 this time.

#5. The next list demonstrates the core part of my recent program.

```c
/** Composing 'Euler Squares' of Order 6 **/
/**   by "New Euler's Method" with /D6i  **/
/** 'EulerMS6.c' built by Kanji Setsuda  **/
/** on Nov. 5, 2005; Apr.11, 2006 **/
/** Working on MacOSX & Xcode 2.1 **/
/**/
#include <stdio.h>
/**/
long int cnt, bcnt;
long cntr[4], cfc[37];
short hcnt, lcnt, cnt2, cnt3;
short u1,u2;
short nm[37], uflg[37];
short anm[3][39];
char thlu[7681][37], tllu[92161][37];
char mtc[7681][92161];
/**/
short rw[7][7], cl[7][7];
short pd[7], pb[7];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void), stp05(void);
void stp06(void), stp07(void), stp08(void), stp09(void), stp10(void);
void stp11(void), stp12(void), stp13(void), stp14(void), stp15(void);
void stp16(void), stp17(void), stp18(void), stp19(void), stp20(void);
void stp21(void), stp22(void), stp23(void), stp24(void), stp25(void);
void stp26(void), stp27(void), stp28(void), stp29(void), stp30(void);
void stp31(void), stp32(void), stp33(void), stp34(void), stp35(void);
void stp36(void), ansrecord(void);
void prlunit(void);
void cmbcmp(void);
void prans(void), pr3ans(void), pr2ans(void), pr1ans(void);
void cmissn(void);
/**/
int main(){
 short m,n;
  printf("\n** Composing 'Euler Squares' of Order 6 **\n");
  printf("***  by 'New Euler's Method' with /D6i ***\n");
  for(m=0;m<7;m++){
    for(n=0;n<7;n++){
      rw[m][n]=0; cl[m][n]=0;
  }}
  for(n=0;n<7;n++){pd[n]=0; pb[n]=0;}
  for(n=0;n<37;n++){nm[n]=0;}
  hcnt=0; cnt=0; bcnt=0; cnt3=0;
  stp01();           /* Make the Latin Squares */
  printf("\n [List of Latin Squares]\n");
  prlunit();         /* Print the Latin Squares */
  lcnt=cnt; cnt=0; cnt3=0;
  printf("\n [List of Composed Solutions of 'Euler' MS66]\n");
//printf("\n [Examples of Wrong Compositions of 'Euler' MS66]\n");
  cmbcmp();          /* Combine, Compose and Print */
  if(cnt3==1){pr1ans();}  /* Print the Rest One */
  printf(" [Count = %d]\n",cnt);
  cmissn();
  printf(" [OK!]\n");
 return 0;
}
/* Make the Latin Squares */
/* Line #1 & #6 */
```

```
/* Set n1 */
void stp01(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[1][i]==0)&&(pd[i]==0)){nm[1]=i;
      rw[1][i]=1; cl[1][i]=1; pd[i]=1;
      stp02();
      rw[1][i]=0; cl[1][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n36 */
void stp02(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[6][i]==0)&&(cl[6][i]==0)&&(pd[i]==0)){nm[36]=i;
      rw[6][i]=1; cl[6][i]=1; pd[i]=1;
      stp03();
      rw[6][i]=0; cl[6][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n2 */
void stp03(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[2][i]==0)){nm[2]=i;
      rw[1][i]=1; cl[2][i]=1;
      stp04();
      rw[1][i]=0; cl[2][i]=0;}
  }
}
/**/
/* Set n35 */
void stp04(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[6][i]==0)&&(cl[5][i]==0)){nm[35]=i;
      rw[6][i]=1; cl[5][i]=1;
      stp05();
      rw[6][i]=0; cl[5][i]=0;}
  }
}
/**/
/* Set n3 */
void stp05(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[3][i]==0)){nm[3]=i;
      rw[1][i]=1; cl[3][i]=1;
      stp06();
      rw[1][i]=0; cl[3][i]=0;}
  }
}
/**/
/* Set n34 */
void stp06(){
 short i;
  for(i=5;i>=0;i--){
```

```
    if((rw[6][i]==0)&&(cl[4][i]==0)){nm[34]=i;
      rw[6][i]=1; cl[4][i]=1;
      stp07();
      rw[6][i]=0; cl[4][i]=0;}
  }
}
/**/
/* Set n4 */
void stp07(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[4][i]==0)){nm[4]=i;
      rw[1][i]=1; cl[4][i]=1;
      stp08();
      rw[1][i]=0; cl[4][i]=0;}
  }
}
/**/
/* Set n33 */
void stp08(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[6][i]==0)&&(cl[3][i]==0)){nm[33]=i;
      rw[6][i]=1; cl[3][i]=1;
      stp09();
      rw[6][i]=0; cl[3][i]=0;}
  }
}
/**/
/* Set n5 */
void stp09(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[5][i]==0)){nm[5]=i;
      rw[1][i]=1; cl[5][i]=1;
      stp10();
      rw[1][i]=0; cl[5][i]=0;}
  }
}
/**/
/* Set n32 */
void stp10(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[6][i]==0)&&(cl[2][i]==0)){nm[32]=i;
      rw[6][i]=1; cl[2][i]=1;
      stp11();
      rw[6][i]=0; cl[2][i]=0;}
  }
}
/**/
/* Set n6 */
void stp11(){
 short i;
  for(i=0;i<6;i++){
    if((rw[1][i]==0)&&(cl[6][i]==0)&&(pb[i]==0)){nm[6]=i;
      rw[1][i]=1; cl[6][i]=1; pb[i]=1;
      stp12();
      rw[1][i]=0; cl[6][i]=0; pb[i]=0;}
```

```
  }
}
/**/
/* Set n31 */
void stp12(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[6][i]==0)&&(cl[1][i]==0)&&(pb[i]==0)){nm[31]=i;
      rw[6][i]=1; cl[1][i]=1; pb[i]=1;
      stp13();
      rw[6][i]=0; cl[1][i]=0; pb[i]=0;}
  }
}
/**/
/* Line #2 & #5 */
/* Set n7 */
void stp13(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[1][i]==0)){nm[7]=i;
      rw[2][i]=1; cl[1][i]=1;
      stp14();
      rw[2][i]=0; cl[1][i]=0;}
  }
}
/* Set n30 */
void stp14(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[6][i]==0)){nm[30]=i;
      rw[5][i]=1; cl[6][i]=1;
      stp15();
      rw[5][i]=0; cl[6][i]=0;}
  }
}
/**/
/* Set n8 */
void stp15(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[2][i]==0)&&(pd[i]==0)){nm[8]=i;
      rw[2][i]=1; cl[2][i]=1; pd[i]=1;
      stp16();
      rw[2][i]=0; cl[2][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n29 */
void stp16(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[5][i]==0)&&(pd[i]==0)){nm[29]=i;
      rw[5][i]=1; cl[5][i]=1; pd[i]=1;
      stp17();
      rw[5][i]=0; cl[5][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n9 */
```

```
void stp17(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[3][i]==0)){nm[9]=i;
      rw[2][i]=1; cl[3][i]=1;
      stp18();
      rw[2][i]=0; cl[3][i]=0;}
  }
}
/**/
/* Set n28 */
void stp18(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[4][i]==0)){nm[28]=i;
      rw[5][i]=1; cl[4][i]=1;
      stp19();
      rw[5][i]=0; cl[4][i]=0;}
  }
}
/**/
/* Set n10 */
void stp19(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[4][i]==0)){nm[10]=i;
      rw[2][i]=1; cl[4][i]=1;
      stp20();
      rw[2][i]=0; cl[4][i]=0;}
  }
}
/**/
/* Set n27 */
void stp20(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[3][i]==0)){nm[27]=i;
      rw[5][i]=1; cl[3][i]=1;
      stp21();
      rw[5][i]=0; cl[3][i]=0;}
  }
}
/**/
/* Set n11 */
void stp21(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[5][i]==0)&&(pb[i]==0)){nm[11]=i;
      rw[2][i]=1; cl[5][i]=1; pb[i]=1;
      stp22();
      rw[2][i]=0; cl[5][i]=0; pb[i]=0;}
  }
}
/**/
/* Set n26 */
void stp22(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[2][i]==0)&&(pb[i]==0)){nm[26]=i;
```

```
      rw[5][i]=1; cl[2][i]=1; pb[i]=1;
      stp23();
      rw[5][i]=0; cl[2][i]=0; pb[i]=0;}
  }
}
/**/
/* Set n12 */
void stp23(){
 short i;
  for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[6][i]==0)){nm[12]=i;
      rw[2][i]=1; cl[6][i]=1;
      stp24();
      rw[2][i]=0; cl[6][i]=0;}
  }
}
/**/
/* Set n25 */
void stp24(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[5][i]==0)&&(cl[1][i]==0)){nm[25]=i;
      rw[5][i]=1; cl[1][i]=1;
      stp25();
      rw[5][i]=0; cl[1][i]=0;}
  }
}
/**/
/* Line #3 & #4 */
/* Set n13 */
void stp25(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[1][i]==0)){nm[13]=i;
      rw[3][i]=1; cl[1][i]=1;
      stp26();
      rw[3][i]=0; cl[1][i]=0;}
  }
}
/**/
/* Set n24 */
void stp26(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[6][i]==0)){nm[24]=i;
      rw[4][i]=1; cl[6][i]=1;
      stp27();
      rw[4][i]=0; cl[6][i]=0;}
  }
}
/**/
/* Set n14 */
void stp27(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[2][i]==0)){nm[14]=i;
      rw[3][i]=1; cl[2][i]=1;
      stp28();
      rw[3][i]=0; cl[2][i]=0;}
```

```
  }
}
/**/
/* Set n23 */
void stp28(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[5][i]==0)){nm[23]=i;
      rw[4][i]=1; cl[5][i]=1;
      stp29();
      rw[4][i]=0; cl[5][i]=0;}
  }
}
/**/
/* Set n15 */
void stp29(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[3][i]==0)&&(pd[i]==0)){nm[15]=i;
      rw[3][i]=1; cl[3][i]=1; pd[i]=1;
      stp30();
      rw[3][i]=0; cl[3][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n22 */
void stp30(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[4][i]==0)&&(pd[i]==0)){nm[22]=i;
      rw[4][i]=1; cl[4][i]=1; pd[i]=1;
      stp31();
      rw[4][i]=0; cl[4][i]=0; pd[i]=0;}
  }
}
/**/
/* Set n16 */
void stp31(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[4][i]==0)&&(pb[i]==0)){nm[16]=i;
      rw[3][i]=1; cl[4][i]=1; pb[i]=1;
      stp32();
      rw[3][i]=0; cl[4][i]=0; pb[i]=0;}
  }
}
/**/
/* Set n21 */
void stp32(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[3][i]==0)&&(pb[i]==0)){nm[21]=i;
      rw[4][i]=1; cl[3][i]=1; pb[i]=1;
      stp33();
      rw[4][i]=0; cl[3][i]=0; pb[i]=0;}
  }
}
/**/
/* Set n17 */
```

```
void stp33(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[5][i]==0)){nm[17]=i;
      rw[3][i]=1; cl[5][i]=1;
      stp34();
      rw[3][i]=0; cl[5][i]=0;}
  }
}
/**/
/* Set n20 */
void stp34(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[2][i]==0)){nm[20]=i;
      rw[4][i]=1; cl[2][i]=1;
      stp35();
      rw[4][i]=0; cl[2][i]=0;}
  }
}
/**/
/* Set n18 */
void stp35(){
 short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[6][i]==0)){nm[18]=i;
      rw[3][i]=1; cl[6][i]=1;
      stp36();
      rw[3][i]=0; cl[6][i]=0;}
  }
}
/**/
/* Set n19 */
void stp36(){
 short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[1][i]==0)){nm[19]=i;
      rw[4][i]=1; cl[1][i]=1;
      ansrecord();
      rw[4][i]=0; cl[1][i]=0;}
  }
}
/**/
/* Record the Answers */
void ansrecord(){
 short n;
  tllu[cnt][0]=cnt+1;
  for(n=1;n<37;n++){tllu[cnt][n]=nm[n];}
  if((nm[1]==0)&&(nm[6]<nm[31])){
    thlu[hcnt][0]=hcnt+1;
    for(n=1;n<37;n++){thlu[hcnt][n]=nm[n];}
    hcnt++;
  }
  cnt++;
}
/**/
/* Classify and Print the Latin Squares */
void prlunit(){
 int t,m,n;
```

```
 short l,l6;
  printf("  [for High]");
  for(t=0;t<hcnt;t=t+768){
    printf("\n%7d/%7d/%7d/%7d/%7d/%7d/%7d/%7d/\n",
      t+1,t+2,t+3,t+4,t+5,t+6,t+7,t+8);
    for(l=0;l<6;l++){l6=l*6;
      for(m=t;m<(t+8);m++){
        printf("  ");
        for(n=1;n<7;n++){printf("%d",thlu[m][l6+n]);}
      }
      printf("\n");
    }
  }
  printf("\n  [for Low]");
  for(t=0;t<cnt;t=t+9216){
    printf("\n%7d/%7d/%7d/%7d/%7d/%7d/%7d/%7d/\n",
      t+1,t+2,t+3,t+4,t+5,t+6,t+7,t+8);
    for(l=0;l<6;l++){l6=l*6;
      for(m=t;m<(t+8);m++){
        printf("  ");
        for(n=1;n<7;n++){printf("%d",tllu[m][l6+n]);}
      }
      printf("\n");
    }
  }
  printf("\n [Counts of Latin Squares = %dH/%dL]\n",hcnt,cnt);
  for(m=0;m<hcnt;m++){
    for(n=0;n<cnt;n++){
      t=0;
      for(l=1;l<37;l++){if(thlu[m][l]==tllu[n][l]){t++;}}
      mtc[m][n]=t;
      if(mtc[m][n]==6){bcnt++;}
    }
  }
  printf("\n [Reference Table for the Best Combinations = %d]\n",bcnt);
  printf("\n T1|",t);
  for(n=0;n<20;n++){printf("%3d",n+1);}
  printf("\n---+-------------------------------------------------------------");
  for(m=0;m<20;m++){
    printf("\n%3d|",m+1);
    for(n=0;n<20;n++){printf("%3d",mtc[m][n]);}
  }
  printf("\n\n T2|",t);
  for(n=20;n<40;n++){printf("%3d",n+1);}
  printf("\n---+-------------------------------------------------------------");
  for(m=20;m<40;m++){
    printf("\n%3d|",m+1);
    for(n=20;n<40;n++){printf("%3d",mtc[m][n]);}
  }
  printf("\n");
}
/**/
/* Combine, Compose and Print */
void cmbcmp(){
 short n,d,fc;
  for(u1=0;u1<hcnt;u1++){cnt2=0;
    for(u2=0;u2<lcnt;u2++){
      if(mtc[u1][u2]==6){
        for(n=1;n<37;n++){uflg[n]=0;}
```

```
        for(n=1;n<37;n++){
          d=thlu[u1][n]*6+tllu[u2][n]+1;
          nm[n]=d; uflg[d]++;
        }
        fc=0;
        for(n=1;n<37;n++){if(uflg[n]==0){fc++;}}
        cfc[fc]++;
        if(fc==0){  //if(fc<=6){
          if((nm[1]==1)&&(nm[6]<nm[31])){prans();}
        }
      }
    }
  }
}
/**/
/* Print the Solution List */
void prans(){
 short n;
  cnt++; cnt2++;
  if(cnt2==1){
    cntr[cnt3]=cnt; cntr[cnt3+1]=0;
    for(n=1;n<37;n++){anm[cnt3][n]=nm[n]; anm[cnt3+1][n]=0;}
    anm[cnt3][37]=u1+1; anm[cnt3][38]=u2+1;
    cnt3++;
    if(cnt3==2){pr2ans(); cnt3=0;}
  }
}
/**/
/* Print 2 Answers */
void pr2ans(){
 short lu1,lu2,l,l6,m,n;
  printf("%6d/%6d/%18d#%8d/%6d/%18d#\n",
    anm[0][37],anm[0][38],cntr[0],anm[1][37],anm[1][38],cntr[1]);
  for(l=0;l<6;l++){l6=l*6;
    for(m=0;m<2;m++){
      lu1=anm[m][37]-1; lu2=anm[m][38]-1;
      printf(" ");
      for(n=1;n<7;n++){printf("%d",thlu[lu1][l6+n]);}
      printf(" ");
      for(n=1;n<7;n++){printf("%d",tllu[lu2][l6+n]);}
      printf(" ");
      for(n=1;n<7;n++){printf("%3d",anm[m][l6+n]);}
      if(m<1){printf("  ");}
    }
    printf("\n");
  }
}
/**/
/* Print the Rest One */
void pr1ans(){
  short lu1,lu2,l,l6,n;
    lu1=anm[0][37]; lu2=anm[0][38];
    printf("%6d/%6d/  /N6i%31d#\n",lu1,lu2,cntr[0]);
    for(l=0;l<6;l++){l6=l*6;
      printf(" ");
      for(n=1;n<7;n++){printf("%d",thlu[lu1-1][l6+n]);}
      printf(" ");
      for(n=1;n<7;n++){printf("%d",tllu[lu2-1][l6+n]);}
      printf(" ");
```

```
      for(n=1;n<7;n++){printf(" %d%d",thlu[lu1-1][l6+n],tllu[lu2-1][l6+n]);}
      printf(" ");
      for(n=1;n<7;n++){printf("%3d",anm[0][l6+n]);}
      printf("\n");
    }
}
/**/
void cmissn(){
 short n;
  printf("\n [Count of Missing Numbers]");
  for(n=0;n<36;n++){
    if(n%6==0){printf("\n");}
    printf("%2d:%7d,",36-n,cfc[36-n]);
  }
  printf("\n [No Missing: %d]\n",cfc[0]);
}
/**/
```

I was extremely surprised when I knew I had to compose a lot of Latin Squares. Let me show you the next report of my newest result.

```
** Composing 'Euler Squares' of Order 6 **
***  by 'New Euler's Method' with /D6i ***

 [List of Latin Squares]
  [for High]
     1/      2/      3/      4/      5/      6/      7/      8/
  012453  012453  012453  012453  012453  012453  012453  012453
  340521  340521  340512  340512  540321  540321  540312  540312
  253140  253104  153240  153204  253140  253104  153240  153204
  135204  135240  235104  235140  135204  135240  235104  235140
  504312  504312  504321  504321  304512  304512  304521  304521
  421035  421035  421035  421035  421035  421035  421035  421035

   769/    770/    771/    772/    773/    774/    775/    776/
  041352  041352  041352  041352  041352  041352  041352  041352
  124503  324501  532014  532014  124503  324501  530214  530214
  253140  253140  324501  124503  253140  253140  324501  124503
  305421  105423  253140  253140  305421  105423  253140  253140
  532014  532014  105423  305421  530214  530214  105423  305421
  410235  410235  410235  410235  412035  412035  412035  412035

  1537/   1538/   1539/   1540/   1541/   1542/   1543/   1544/
  012543  012543  012543  012543  012543  012543  012543  012543
  350421  350421  350412  350412  450321  450321  450312  450312
  243150  243105  143250  143205  243150  243105  143250  143205
  134205  134250  234105  234150  134205  134250  234105  234150
  405312  405312  405321  405321  305412  305412  305421  305421
  521034  521034  521034  521034  521034  521034  521034  521034

  2305/   2306/   2307/   2308/   2309/   2310/   2311/   2312/
  041532  041532  041532  041532  041532  041532  041532  041532
  123405  452013  452013  523401  123405  450213  450213  523401
  235140  523401  123405  235140  235140  523401  123405  235140
  504321  235140  235140  104325  504321  235140  235140  104325
  452013  104325  504321  452013  450213  104325  504321  450213
  310254  310254  310254  310254  312054  312054  312054  312054
```

3073/	3074/	3075/	3076/	3077/	3078/	3079/	3080/
012534	012534	012534	012534	012534	012534	012534	012534
350421	350421	350412	350412	450321	450321	450312	450312
234150	234105	134250	134205	234150	234105	134250	134205
143205	143250	243105	243150	143205	143250	243105	243150
405312	405312	405321	405321	305412	305412	305421	305421
521043	521043	521043	521043	521043	521043	521043	521043

3841/	3842/	3843/	3844/	3845/	3846/	3847/	3848/
031542	031542	031542	031542	031524	031524	031524	031524
345201	345201	315204	315204	325401	325401	315402	315402
152430	102435	452130	402135	154230	104235	254130	204135
403125	453120	103425	153420	203145	253140	103245	153240
210354	210354	240351	240351	410352	410352	420351	420351
524013	524013	524013	524013	542013	542013	542013	542013

4609/	4610/	4611/	4612/	4613/	4614/	4615/	4616/
012534	012534	012534	012534	012534	012534	012534	012534
154023	354021	435210	435210	150423	350421	435210	435210
243105	243105	354021	154023	324051	124053	241305	241305
320451	120453	243105	243105	241305	241305	350421	150423
435210	435210	120453	320451	435210	435210	124053	324051
501342	501342	501342	501342	503142	503142	503142	503142

5377/	5378/	5379/	5380/	5381/	5382/	5383/	5384/
032451	032451	032451	032451	032541	032541	032541	032541
415230	415230	410235	410235	145230	145230	350124	450123
153024	253014	153024	253014	203415	203415	421053	321054
204513	104523	204513	104523	450123	350124	203415	203415
320145	320145	325140	325140	321054	421053	145230	145230
541302	541302	541302	541302	514302	514302	514302	514302

6145/	6146/	6147/	6148/	6149/	6150/	6151/	6152/
012543	012543	012543	012543	012543	012543	012543	012543
153024	153024	234105	534102	150324	150324	234105	534102
534102	234105	345210	345210	534102	234105	345210	345210
345210	345210	501432	201435	345210	345210	501432	201435
201435	501432	153024	153024	201435	501432	150324	150324
420351	420351	420351	420351	423051	423051	423051	423051

6913/	6914/	6915/	6916/	6917/	6918/	6919/	6920/
031524	031524	031524	031524	031542	031542	031542	031542
250413	350412	425130	425130	324150	324150	320154	320154
314052	214053	103245	103245	145023	245013	145023	245013
103245	103245	350412	250413	203415	103425	203415	103425
425130	425130	214053	314052	510234	510234	514230	514230
542301	542301	542301	542301	452301	452301	452301	452301

[for Low]

1/	2/	3/	4/	5/	6/	7/	8/
012453	012453	012453	012453	012534	012534	012534	012534
231504	431502	543021	543021	124053	324051	435210	435210
354210	354210	431502	231504	543102	543102	324051	124053
405132	205134	354210	354210	350421	150423	543102	543102
543021	543021	205134	405132	435210	435210	150423	350421
120345	120345	120345	120345	201345	201345	201345	201345

9217/	9218/	9219/	9220/	9221/	9222/	9223/	9224/
012345	012345	012345	012345	012543	012543	012543	012543
135024	435021	543210	543210	451320	451320	450321	450321
254103	254103	435021	135024	234015	324015	234015	324015
420531	120534	254103	254103	325104	235104	325104	235104
543210	543210	120534	420531	540231	540231	541230	541230
301452	301452	301452	301452	103452	103452	103452	103452

18433/	18434/	18435/	18436/	18437/	18438/	18439/	18440/
102435	102435	102435	102435	102543	102543	102543	102543
025143	325140	534201	534201	230415	453120	453120	530412
453012	453012	325140	025143	345201	530412	230415	345201
341520	041523	453012	453012	514032	345201	345201	214035
534201	534201	041523	341520	453120	214035	514032	453120
210354	210354	210354	210354	021354	021354	021354	021354

27649/	27650/	27651/	27652/	27653/	27654/	27655/	27656/
102534	102534	102534	102534	102543	102543	102543	102543
054123	054123	243015	543012	053214	053214	043215	043215
543012	243015	435201	435201	524031	514032	425031	415032
435201	435201	510342	210345	415302	425301	514302	524301
210345	510342	054123	054123	340125	340125	350124	350124
321450	321450	321450	321450	231450	231450	231450	231450

36865/	36866/	36867/	36868/	36869/	36870/	36871/	36872/
201345	201345	201345	201345	201534	201534	201534	201534
015234	415230	543102	543102	140325	354210	354210	540321
354021	354021	415230	015234	435102	540321	140325	435102
432510	032514	354021	354021	523041	435102	435102	123045
543102	543102	032514	432510	354210	123045	523041	354210
120453	120453	120453	120453	012453	012453	012453	012453

46081/	46082/	46083/	46084/	46085/	46086/	46087/	46088/
301254	301254	301254	301254	301254	301254	301254	301254
423510	423510	423501	423501	523410	523410	523401	523401
154032	154023	054132	054123	154032	154023	054132	054123
045123	045132	145023	145032	045123	045132	145023	145032
532401	532401	532410	532410	432501	432501	432510	432510
210345	210345	210345	210345	210345	210345	210345	210345

55297/	55298/	55299/	55300/	55301/	55302/	55303/	55304/
301245	301245	301245	301245	301542	301542	301542	301542
025314	425310	542103	542103	453210	453210	450213	450213
154032	154032	425310	025314	124305	214305	124305	214305
413520	013524	154032	154032	215034	125034	215034	125034
542103	542103	013524	413520	540123	540123	543120	543120
230451	230451	230451	230451	032451	032451	032451	032451

64513/	64514/	64515/	64516/	64517/	64518/	64519/	64520/
401235	401235	401235	401235	401235	401235	401235	401235
324510	324510	324501	324501	524310	524310	524301	524301
135042	135024	035142	035124	135042	135024	035142	035124
053124	053142	153024	153042	053124	053142	153024	153042
542301	542301	542310	542310	342501	342501	342510	342510
210453	210453	210453	210453	210453	210453	210453	210453

73729/	73730/	73731/	73732/	73733/	73734/	73735/	73736/
401523	401523	401523	401523	401532	401532	401532	401532
054312	054312	132045	532041	054213	054213	123045	523041
532041	132045	325104	325104	523041	123045	235104	235104
325104	325104	540231	140235	235104	235104	540321	140325
140235	540231	054312	054312	140325	540321	054213	054213
213450	213450	213450	213450	312450	312450	312450	312450

82945/	82946/	82947/	82948/	82949/	82950/	82951/	82952/
501234	501234	501234	501234	501234	501234	501234	501234
042513	042513	042315	042315	142503	142503	142305	142305
253041	453021	253041	453021	253140	453120	253140	453120
435120	235140	435120	235140	435021	235041	435021	235041
124305	124305	124503	124503	024315	024315	024513	024513
310452	310452	310452	310452	310452	310452	310452	310452

[Counts of Latin Squares = 7680H/92160L]

[Reference Table for the Best Combinations = 112008960]

T1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	13	14	13	15	8	11	6	6	13	11	11	9	11	9	9	7	8	10	5	6
2	12	12	15	18	9	12	5	5	11	13	9	11	9	11	7	9	7	9	6	7
3	17	20	10	10	6	7	9	11	12	10	12	10	10	8	10	8	7	9	7	8
4	16	18	12	13	7	8	8	10	10	12	10	12	8	10	8	10	6	8	8	9
5	11	12	14	16	8	10	6	7	12	10	10	8	12	10	10	8	8	9	5	7
6	10	10	16	19	9	11	5	6	10	12	8	10	10	12	8	10	7	8	6	8
7	15	18	11	11	6	6	9	12	11	9	11	9	11	9	11	9	7	8	7	9
8	14	16	13	14	7	7	8	11	9	11	9	11	9	11	9	11	6	7	8	10
9	4	6	11	12	18	16	7	8	6	7	5	6	7	8	6	7	11	10	9	9
10	6	4	12	13	16	18	8	9	8	8	7	7	9	9	8	8	11	12	8	10
11	5	6	4	6	9	10	18	16	8	7	8	7	7	6	7	6	11	11	11	10
12	6	7	6	4	10	11	16	18	9	9	9	9	8	8	8	8	10	12	11	12
13	10	12	15	16	6	8	6	7	18	16	14	12	16	14	12	10	8	11	5	5
14	9	10	17	19	7	9	5	6	16	18	12	14	14	16	10	12	7	10	6	6
15	12	14	14	15	5	7	8	9	14	12	18	16	12	10	16	14	6	7	8	10
16	11	12	16	18	6	8	7	8	12	14	16	18	10	12	14	16	5	6	9	11
17	8	10	18	19	7	8	5	7	16	14	12	10	18	16	14	12	9	11	4	5
18	7	8	20	22	8	9	4	6	14	16	10	12	16	18	12	14	8	10	5	6
19	10	12	17	18	6	7	7	9	12	10	16	14	14	12	18	16	7	7	7	10
20	9	10	19	21	7	8	6	8	10	12	14	16	12	14	16	18	6	6	8	11

T2	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
21	10	9	8	7	9	8	7	6	5	7	9	10	5	6	6	5	13	11	13	11
22	10	9	8	7	9	8	7	6	6	6	11	10	4	5	7	6	11	13	11	13
23	6	7	8	9	7	8	9	10	5	6	7	9	6	5	7	8	11	9	15	13
24	6	7	8	9	7	8	9	10	7	6	8	8	7	6	6	7	9	11	13	15
25	6	7	6	7	7	8	7	8	5	7	9	10	5	5	12	12	22	18	8	8
26	8	9	4	5	9	10	5	6	7	5	10	11	5	5	12	12	18	22	8	8
27	8	7	8	7	7	6	7	6	9	10	5	7	10	10	5	5	10	10	22	18
28	6	5	10	9	5	4	9	8	10	11	7	5	10	10	5	5	10	10	18	22
29	9	8	7	6	10	9	8	7	6	8	9	10	6	7	6	5	11	9	11	9
30	9	8	7	6	10	9	8	7	7	7	11	10	5	6	7	6	9	11	9	11
31	7	8	9	10	6	7	8	9	7	8	8	10	8	7	8	9	11	9	13	11
32	7	8	9	10	6	7	8	9	9	8	9	9	9	8	7	8	9	11	11	13
33	10	13	7	10	10	13	7	10	10	8	8	7	9	7	12	11	24	22	9	10
34	7	5	9	7	7	5	9	7	6	5	10	8	8	7	9	7	9	10	24	22
35	7	5	11	9	8	6	12	10	7	6	8	10	9	8	7	9	10	11	22	24
36	13	16	8	11	12	15	7	10	8	10	9	8	7	9	13	12	22	24	10	11
37	16	12	14	10	15	11	13	9	12	10	11	10	14	12	15	14	10	12	9	10
38	12	16	10	14	11	15	9	13	12	10	13	12	12	11	18	16	13	15	7	8
39	14	10	14	10	13	9	13	9	12	10	12	11	14	12	16	15	9	11	9	10
40	10	14	10	14	9	13	9	13	12	10	14	13	12	11	19	17	12	14	7	8

[List of Composed Solutions of 'Euler' MS66]
[Count = 0]

I was really surprised at the big counts of Latin Squares. All of them are of Euler type, as you see. I must confess I was delighted by these wonderful results at first.

But I was shocked at the next moment when I read the report from my computer that it could not really make any compositions at all. It checked all the cases of 7680 x 92160 combinations, but it could find no correct answers at all.

I wondered what was wrong with it. I tried first to find any bugs in my program, tested and checked everything many times. But finally I came to know everything tells me that it is impossible for us to compose any 'Greco-Latin' Squares of order 6, even when we make only the basic type of standard magic squares 6x6.

#6. The next list is printed out so that we could know what the problem is.

```
[Count of Missing Numbers]
  36:       0, 35:       0, 34:       0, 33:       0, 32:       0, 31:       0,
  30: 585568, 29:       0, 28: 628176, 27:       0, 26: 725728, 25:       0,
  24: 232656, 23:       0, 22:       0, 21: 120954, 20: 112112, 19:       0,
  18:1062664, 17:1028688, 16:1424014, 15:1912124, 14:5295216, 13:1384918,
  12:5973378, 11:1640468, 10:7582804,  9: 988896,  8:       0,  7:       0,
   6:1236860,  5:       0,  4:       0,  3:       0,  2:       0,  1:       0,
 [No Missing: 0]
```

```
 [Examples of Wrong Compositions of 'Euler' MS66]
     1/     51/                  1#        2/     82/                 199#
 012453 013524    1  8 16 30 33 23    012453 014532    1  8 17 30 34 21
 340521 425310   23 27  6 34 14  7    340521 342150   22 29  3 32 18  7
 253140 541203   18 35 20  9 25  4    253104 521403   18 33 20 11  1 28
 135204 230451    9 22 31 17  6 26    135240 250314    9 24 31 16 26  5
 504312 154032   32  6 29 19 10 15    504312 435021   35  4 30 19  9 14
 421035 302145   28 13  9  2 23 36    421035 103245   26 13 10  3 23 36
     3/   2314/                259#        4/    506/                 315#
 012453 051432    1 12 14 29 34 21    012453 015234    1  8 18 27 34 23
 340512 542103   24 29  3 32  7 16    340512 143520   20 29  4 36  9 13
 153240 403521   11 31 22 18 27  2    153204 532041   12 34 21 13  5 26
 235104 314250   16 20 35  9  6 25    235140 401352   17 19 32 10 30  3
 504321 235014   33  4 30 19 14 11    504321 250413   33  6 25 23 14 10
 421035 120345   26 15  7  4 23 36    421035 324105   28 15 11  2 19 36
     5/    511/                377#        6/   2311/                 439#
 012453 015234    1  8 18 27 34 23    012453 051432    1 12 14 29 34 21
 540321 240513   33 29  1 24 14 10    540321 235104   33 28  6 20 13 11
 253140 431052   17 34 20  7 30  3    253104 314520   16 32 23 12  3 25
 135204 502341   12 19 33 16  5 26    135240 403251   11 19 34 15 30  2
 304512 153420   20  6 28 35  9 13    304512 542013   24  5 27 31  8 16
 421035 324105   28 15 11  2 19 36    421035 120345   26 15  7  4 23 36
     7/     87/                493#        8/    172/                 557#
 012453 014532    1  8 17 30 34 21    012453 013524    1  8 16 30 33 23
 540312 432051   35 28  3 19 12 14    540312 324051   34 27  5 19 12 14
 153240 251304    9 36 20 16 25  5    153204 541203   12 35 20 15  1 28
 235104 520413   18 21 31 11  2 28    235140 450312   17 24 31 10 26  3
 304521 345120   22  5 30 32 15  7    304521 235140   21  4 30 32 17  7
 421035 103245   26 13 10  3 23 36    421035 102435   26 13  9  5 22 36
     9/     95/                753#       10/     77/                 815#
 014523 014352    1  8 29 34 18 21    014523 014352    1  8 29 34 18 21
 143052 540231   12 29 19  3 34 14    243051 325104   16 27 24  2 31 11
 532104 253410   33 24 16 11  2 25    532104 401523   35 19 14 12  3 28
 250341 425103   17 33  6 20 25 10    150342 253410    9 36  4 23 26 13
 325410 301524   22 13 32 30  9  5    325410 542031   24 17 33 25 10  2
 401235 132045   26  4  9 13 23 36    401235 130245   26  4  7 15 23 36
    11/    161/                879#       12/    655/                 941#
 014523 013254    1  8 28 33 18 23    014523 024351    1  9 29 34 18 20
 325410 245103   21 17 36 26  7  4    325410 541032   24 17 32 25 10  3
 243051 301542   16 25 20  6 35  9    143052 153420    8 30 22  5 33 13
 532104 452310   35 24 15 10  2 25    532104 415203   35 20 18  9  1 28
 150342 534021   12 34  5 19 27 14    250341 302514   16 31  3 24 26 11
 401235 120435   26  3  7 17 22 36    401235 230145   27  4  7 14 23 36
```

```
   13/   101/               1001#
014253 015432   1  8 30 17 34 21
345021 341250  22 29 32  3 18  7
132540 253014   9 24 16 31 26  5
251304 524103  18 33 11 20  1 28
503412 430521  35  4 19 30  9 14
420135 102345  26 13  3 10 23 36

   14/    61/               1061#
014253 013452   1  8 28 17 36 21
345021 321504  22 27 32  6 13 11
132504 254310   9 24 17 34  2 25
251340 405123  17 31 12 20 27  4
503412 540231  36  5 19 27 10 14
420135 132045  26 16  3  7 23 36

   15/   385/               1259#
014253 012534   1  8 27 18 34 23
345012 145320  20 29 36  4  9 13
231540 403152  17 19 10 32 30  3
152304 530241  12 34 13 21  5 26
503421 254013  33  6 23 25 14 10
420135 321405  28 15  2 11 19 36

   16/  2433/               1321#
014253 054132   1 12 29 14 34 21
345012 541203  24 29 32  3  7 16
231504 312450  16 20  9 35  6 25
152340 405321  11 31 18 22 27  2
503421 230514  33  4 19 30 14 11
420135 123045  26 15  4  7 23 36

   17/  2432/               1377#
014253 054132   1 12 29 14 34 21
543021 231504  33 28 20  6 13 11
132540 402351  11 19 15 34 30  2
251304 315420  16 32 12 23  3 25
305412 540213  24  5 31 27  8 16
420135 123045  26 15  4  7 23 36

   18/   392/               1431#
014253 012534   1  8 27 18 34 23
543021 245013  33 29 24  1 14 10
132504 503241  12 19 16 33  5 26
251340 430152  17 34  7 20 30  3
305412 154320  20  6 35 28  9 13
420135 321405  28 15  2 11 19 36

   19/   243/               1493#
014253 015324   1  8 30 16 33 23
543012 320451  34 27 19  5 12 14
231540 453012  17 24 10 31 26  3
152304 542103  12 35 15 20  1 28
305421 231540  21  4 32 30 17  7
420135 104235  26 13  5  9 22 36

   20/   107/               1689#
014253 015432   1  8 30 17 34 21
543012 430251  35 28 19  3 12 14
231504 254013  15 24 11 31  2 28
152340 523104  12 33 16 20 25  5
305421 341520  22  5 32 30 15  7
420135 102345  26 13  3 10 23 36

 1001/   513/              94001#
043521 015243   1 26 24 33 17 10
125340 120354   8 15 31 22 30  5
514203 453021  35 12 28 13  3 20
430152 501432  30 19  2 11 34 15
251034 342510  16 35  9  6 20 25
302415 234105  21  4 17 26  7 36

 1002/     1/              94055#
043521 012453   1 26 21 35 18 10
125340 231504   9 16 32 24 25  5
514203 354210  34 12 29 15  2 19
230154 405132  17 19  6  8 34 27
451032 543021  30 35 10  1 21 14
302415 120345  20  3 13 28 11 36

 1003/   514/              94251#
043521 015243   1 26 24 33 17 10
230154 342510  16 23  3 12 32 25
451032 501432  30 31  8  5 22 15
514203 420351  35  9 25 16  6 20
125340 153024   8 18 34 19 27  5
302415 234105  21  4 17 26  7 36

 1004/     2/              94307#
043521 012453   1 26 21 35 18 10
430152 431502  29 22  2 12 31 15
251034 354210  16 36 11  3 20 25
514203 205134  33  7 30 14  4 23
125340 543021  12 17 34 19 27  2
302415 120345  20  3 13 28 11 36

 1005/    17/              94505#
043152 012534   1 26 21 12 34 17
124503 120453   8 15 25 35  6 22
251340 354021  16 36 11 19 27  2
305421 541302  24  5 32 28 13  9
512034 435210  35 10 18  3 20 25
430215 203145  27 19  4 14 11 36

 1006/    19/              94567#
043152 012534   1 26 21 12 34 17
324501 435210  23 16 30 33  2  7
251340 541302  18 35  8 22 25  3
105423 320451  10  3 31 29 18 20
512034 154023  32 12 17  1 21 28
430215 203145  27 19  4 14 11 36

 1007/    19/              94627#
043152 012534   1 26 21 12 34 17
512034 435210  35 10 18  3 20 25
324501 541302  24 17 26 34  1  9
251340 320451  16 33  7 23 30  2
105423 154023   8  6 35 25 15 22
430215 203145  27 19  4 14 11 36

 1008/    17/              94689#
043152 012534   1 26 21 12 34 17
512034 120453  32  9 13  5 24 28
124503 354021  10 18 29 31  3 20
251340 541302  18 35  8 22 25  3
305421 435210  23  4 36 27 14  7
430215 203145  27 19  4 14 11 36
```

```
 1009/    67/             94753#    1010/    65/             94813#
043521 014253   1 26 23 33 18 10   043521 014253   1 26 23 33 18 10
125340 543021  12 17 34 19 27  2   125340 235104   9 16 36 20 25  5
431052 352410  28 24  9  5 32 13   231054 401532  17 19  8  6 34 27
512403 435102  35 10 18 26  1 21   512403 352410  34 12 15 29  2 19
250134 201534  15 31  2 12 22 29   450132 543021  30 35  4  7 21 14
304215 120345  20  3 25 16 11 36   304215 120345  20  3 25 16 11 36
 . . . . . . .
```

Although all Latin Squares certainly belong to the Euler type, any combination of the two units picked up out of them could not make any correct answer at all. It is because we could not avoid repeating usage of certain numbers and missing of some others in any composed solution, while we must obey another Euler's Condition every time: We must use each one of the serial integers {00, 01, 02, 03, 04, 05, 10, 11, ..., 53, 54, 55(N6i)} strictly once, and must not use any number twice or more often.

In fact we could not compose any Greco-Latin Square of order 6, even when we want to make them for the basic type of standard magic squares 6x6.

That means all the solutions in page 9-10 are really the "Non-Euler type" of them and there are no Euler Squares among the solution set of standard magic squares of order 6. What a conclusion it is! I have had no idea about that.

But, we cannot help accepting it, however embarrassing this conclusion might be.

It is in the order 6 that we have got any "Non-Euler type" of magic squares for the first time. We now know there exist Complete Euler Squares and nothing else among the solution set of pan-diagonal magic squares of order 4 and 5. We also know there exist both the Complete Euler Squares and many Non-Euler ones among the solution sets of pan-diagonal magic squares of orders 7, 8, 9, ...

It is in the case of order 6 when anything new like 'Non-Euler type' was born. It is also in the case of order 6 when 'nothingness' begins to exist. Why!

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