Part 3: "New Algebraic Study of Magic Squares"

Chapter 5: Outlines of Magic Squares of Order 7 : **Kanji Setsuda**

In this chapter we are going to study only about three types of magic squares of order 7: Self-Complementary, Pan-Diagonal and Simultaneous of both S-C and P-D.

I intend to present you a simple guide-book that explains only about the outlines of typical magic squares of order 7 here.

Section 1: Self-Complementary Magic Squares of Order 7

#1. Definition of the Self-Complementary Type

First of all, let's study about Self-Complementary Magic Squares of order 7. See the diagram below and the simultaneous equations we assume. Complementary pairs of 50 are all located symmetrically with respect to n25(Geometric center of our object).

```
   ** Basic Form **        ** List of Basic Conditions: C=175 **
 .--.--.--.--.--.--.--.     n1+n2+n3+n4+n5+n6+n7=C          ... rw1;
 | 1| 2| 3| 4| 5| 6| 7|     n8+n9+n10+n11+n12+n13+n14=C     ... rw2;
 |--+--+--+--+--+--+--|     n15+n16+n17+n18+n19+n20+n21=C   ... rw3;
 | 8| 9|10|11|12|13|14|     n22+n23+n24+n25+n26+n27+n28=C   ... rw4;
 |--+--+--+--+--+--+--|     n29+n30+n31+n32+n33+n34+n35=C   ... rw5;
 |15|16|17|18|19|20|21|     n36+n37+n38+n39+n40+n41+n42=C   ... rw6;
 |--+--+--+--+--+--+--|     n43+n44+n45+n46+n47+n48+n49=C   ... rw7;
 |22|23|24|25|26|27|28|     n1+n8+n15+n22+n29+n36+n43=C     ... cl1;
 |--+--+--+--+--+--+--|     n2+n9+n16+n23+n30+n37+n44=C     ... cl2;
 |29|30|31|32|33|34|35|     n3+n10+n17+n24+n31+n38+n45=C    ... cl3;
 |--+--+--+--+--+--+--|     n4+n11+n18+n25+n32+n39+n46=C    ... cl4;
 |36|37|38|39|40|41|42|     n5+n12+n19+n26+n33+n40+n47=C    ... cl5;
 |--+--+--+--+--+--+--|     n6+n13+n20+n27+n34+n41+n48=C    ... cl6;
 |43|44|45|46|47|48|49|     n7+n14+n21+n28+n35+n42+n49=C    ... cl7;
 '--'--'--'--'--'--'--'     n1+n9+n17+n25+n33+n41+n49=C     ... pd1;
                            n7+n13+n19+n25+n31+n37+n43=C    ... pb7;
** Self-complementary Conditions: CC=50 **
  n1+n49=n2+n48=n3+n47=n4+n46=n5+n45=n6+n44=n7+n43=
  n8+n42=n9+n41=n10+n40=n11+n39=n12+n38=n13+n37=n14+n36=
  n15+n35=n16+n34=n17+n33=n18+n32=n19+n31=n20+n30=n21+n29=
  n22+n28=n23+n27=n24+n26=n25+n25=CC ... scc
```

To the multi-equations of Self-complementary Conditions we can expect:

```
  n49=C-n1; n48=C-n2; n47=C-n3; n46=C-n4; n45=C-n5; n44=C-n6; n43=C-n7;
  n42=C-n8; n41=C-n9; .... n27=C-n23; n26=C-n24; and n25=C-n25=25
```

If you put any temporary value to n1, then n49 is defined according to C-n1 at the same time. If you define n2, then n48 is defined automatically. ... Therefore we must really deal with about half of all entries.

#2. New Notation for Complementary Pairs

We know there are too many solutions for us to calculate and count up through.

Though we have to deal with 49 numbers, we have only 17 equations in all.

Under these conditions we can only draw the rough outline of our object. We can only get some guide-lines to know its structure.

But when you adopt the next new notation for the Complementary Pairs, you may probably have a new approach to the structural study of this type.

```
n1=C-n1=n49; n2=C-n2=n48; n3=C-n3=n47; n4=C-n4=n46; n5=C-n5=n45;
n6=C-n6=n44; n7=C-n7=n43; n8=C-n8=n42; n9=C-n9=n41; n10=C-n10=n40; ....;
n23=C-n23=n27; n24=C-n24=n26; and n25=C-n25=n25=25
```

```
 ** New Basic Form **    ** List of Basic Conditions: C=175 **
.--.--.--.--.--.--.--.    n1+n2+n3+n4+n5+n6+n7=C          ... rw1;
|n1|n2|n3|n4|n5|n6|n7|    n8+n9+n10+n11+n12+n13+n14=C     ... rw2;
|--+--+--+--+--+--+--|    n15+n16+n17+n18+n19+n20+n21=C   ... rw3;
|n8|n9|10|11|12|13|14|    n22+n23+n24+n25+n24+n23+n22=C   ... rw4;
|--+--+--+--+--+--+--|    n21+n20+n19+n18+n17+n16+n15=C   ... rw5;
|15|16|17|18|19|20|21|    n14+n13+n12+n11+n10+n9+n8=C     ... rw6;
|--+--+--+--+--+--+--|    n7+n6+n5+n4+n3+n2+n1=C          ... rw7;
|22|23|24|25|24|23|22|    n1+n8+n15+n22+n21+n14+n7=C      ... cl1;
|--+--+--+--+--+--+--|    n2+n9+n16+n23+n20+n13+n6=C      ... cl2;
|21|20|19|18|17|16|15|    n3+n10+n17+n24+n19+n12+n5=C     ... cl3;
|--+--+--+--+--+--+--|    n4+n11+n18+n25+n18+n11+n4=C     ... cl4;
|14|13|12|11|10|n9|n8|    n5+n12+n19+n24+n17+n10+n3=C     ... cl5;
|--+--+--+--+--+--+--|    n6+n13+n20+n23+n16+n9+n2=C      ... cl6;
|n7|n6|n5|n4|n3|n2|n1|    n7+n14+n21+n22+n15+n8+n1=C      ... cl7;
'--'--'--'--'--'--'--'    n1+n9+n17+n25+n17+n9+n1=C       ... pd1;
                          n7+n13+n19+n25+n19+n13+n7=C     ... pb7;
** Self-complementary Conditions: CC=50 **
  n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7=
  n8+n8=n9+n9=n10+n10=n11+n11=n12+n12=n13+n13=n14+n14=
  n15+n15=n16+n16=n17+n17=n18+n18=n19+n19=n20+n20=n21+n21=
  n22+n22=n23+n23=n24+n24=n25+n25=CC ... scc
```

You only have to draw the next diagrams like these below to examine the contents of any row, column, or primary diagonal. You will be able to see which rows or columns you can exchange with each other, without any calculations. Even if you exchange such a couple of rows and columns as below, you can still have your Self-complementary MS77 unspoiled, while you will make a different solution from it.

```
 ** New Basic Form **    ** Results of Row/Column Exchange **
                                                   *           *
.--.--.--.--.--.--.--.    .--.--.--.--.--.--.--.    .--.--.--.--.--.--.--.
|n1|n2|n3|n4|n5|n6|n7|    |n1|n2|n3|n4|n5|n6|n7|    |n1|n6|n3|n4|n5|n2|n7|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|n8|n9|10|11|12|13|14|    |n8|n9|10|11|12|13|14|    |n8|13|10|11|12|n9|14|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|15|16|17|18|19|20|21|   *|21|20|19|18|17|16|15|*   |15|20|17|18|19|16|21|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|22|23|24|25|24|23|22|    |22|23|24|25|24|23|22|    |22|23|24|25|24|23|22|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|21|20|19|18|17|16|15|   *|15|16|17|18|19|20|21|*   |21|16|19|18|17|20|15|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|14|13|12|11|10|n9|n8|    |14|13|12|11|10|n9|n8|    |14|n9|12|11|10|13|n8|
|--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|    |--+--+--+--+--+--+--|
|n7|n6|n5|n4|n3|n2|n1|    |n7|n6|n5|n4|n3|n2|n1|    |n7|n2|n5|n4|n3|n6|n1|
'--'--'--'--'--'--'--'    '--'--'--'--'--'--'--'    '--'--'--'--'--'--'--'
                                                   *           *
```

This implies that you can make 16 different transformations in all for each solution of the fundamental type. (2x2x2x2=16 including the original)

And in the case when you want to have the set of fundamental solutions, you have to take such the list-forming inequality conditions as follows, as you can easily guess.

```
n1=1; n2>n8; n2>n6; n3<n5; n8>n36; and n15<n29;
```

Let me show you the compact lists of fundamental solutions of our object here.

** Self-Complementary Magic Squares of Order 7: Fundamental Solutions with n1=1 **

```
                  1/              1298448/              2752583/              3834145/
   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28
  43 11 26 37 36 12 10  43 11 27 37 36 12  9  43 10 30 37 34 12  9  43  9 29 37 36 11 10
   8 16 35 32 31 23 30   8 18 33 34 21 31 30   8 26 27 18 31 36 29   8 16 35 32 31 26 27
  41 29 33 25 17 21  9  40 35 24 25 26 15 10  39 33 35 25 15 17 11  38 33 30 25 20 17 12
  20 27 19 18 15 34 42  20 19 29 16 17 32 42  21 14 19 32 23 24 42  23 24 19 18 15 34 42
  40 38 14 13 24 39  7  41 38 14 13 23 39  7  41 38 16 13 20 40  7  40 39 14 13 21 41  7
  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49
            5165688/              6172186/              7276495/              8067454/
   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28
  43 10 29 38 35 11  9  43 10 27 38 37 11  9  43  9 27 38 37 11 10  43 10 27 38 37 11  9
   8 24 30 19 33 34 27   8 17 34 31 29 30 26   8 19 33 29 32 30 24   8 17 36 29 31 30 24
  37 32 36 25 14 18 13  36 35 32 25 18 15 14  35 34 36 25 14 16 15  34 35 32 25 18 15 16
  23 16 17 31 20 26 42  24 20 21 19 16 33 42  26 20 18 21 17 31 42  26 20 19 21 14 33 42
  41 39 15 12 21 40  7  41 39 13 12 23 40  7  40 39 13 12 23 41  7  41 39 13 12 23 40  7
  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49
            9124439/              9860391/             10699608/             11417766/
   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28
  43 10 29 38 35 11  9  43  9 27 38 37 11 10  43 10 27 38 37 11  9  43  9 27 38 37 11 10
   8 16 37 31 36 24 23   8 24 34 35 33 20 21   8 18 34 35 33 26 21   8 24 36 21 34 33 19
  33 30 32 25 18 20 17  32 19 36 25 14 31 18  31 30 36 25 14 20 19  30 32 35 25 15 18 20
  27 26 14 19 13 34 42  29 30 17 15 16 26 42  29 24 17 15 16 32 42  31 17 16 29 14 26 42
  41 39 15 12 21 40  7  40 39 13 12 23 41  7  41 39 13 12 23 40  7  40 39 13 12 23 41  7
  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49
           12259890/             12973650/             13553321/             14230504/
   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28
  43 10 27 38 37 11  9  43 10 29 38 35 11  9  43 10 27 38 37 11  9  43 10 27 38 37 11  9
   8 18 36 35 33 26 19   8 26 37 31 36 20 17   8 21 33 35 32 30 16   8 34 35 33 19 32 14
  29 30 34 25 16 20 21  27 16 32 25 18 34 23  26 31 36 25 14 19 24  24 20 21 25 29 30 26
  31 24 17 15 14 32 42  33 30 14 19 13 24 42  34 20 18 15 17 29 42  36 18 31 17 15 16 42
  41 39 13 12 23 40  7  41 39 15 12 21 40  7  41 39 13 12 23 40  7  41 39 13 12 23 40  7
  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49
           14749321/             15037003/             15170081/             15342180/
   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28   1 48  3 46  5 44 28
  43 10 29 38 35 11  9  43 10 30 35 36 12  9  43 11 27 37 36 12  9  43 11 26 37 36 12 10
   8 24 31 33 34 32 13   8 27 34 26 32 37 11   8 26 33 34 29 35 10   9 23 35 30 29 17 32
  23 30 36 25 14 20 27  21 33 31 25 19 17 29  20 31 32 25 18 19 30  42 16 31 25 19 34  8
  37 18 16 17 19 26 42  39 13 18 24 16 23 42  40 15 21 16 17 24 42  18 33 21 20 15 27 41
  41 39 15 12 21 40  7  41 38 14 15 20 40  7  41 38 14 13 23 39  7  40 38 14 13 24 39  7
  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49  22  6 45  4 47  2 49
```

. . . . . .

** Self-Complementary Magic Squares of Order 7: Fundamental Solutions with n4=1 **

```
                  1/               131425/               247579/               463841/
   2 46 34  1 44 45  3   2 46 34  1 44 45  3   2 46 34  1 44 45  3   2 46 34  1 44 45  3
  42 11 17 29 36 12 28  42 10 18 37 27 12 29  42 10 17 29 36 11 30  42 10 14 38 29 11 31
   9 20 35 27 18 26 40   9 33 28 15 20 31 39   9 26 37 19 18 28 38   9 27 33 24 15 30 37
  43 31 37 25 13 19  7  43 24 36 25 14 26  7  43 27 35 25 15 23  7  43 28 32 25 18 22  7
  10 24 32 23 15 30 41  11 19 30 35 22 17 41  12 22 32 31 13 24 41  13 20 35 26 17 23 41
  22 38 14 21 33 39  8  21 38 23 13 32 40  8  20 39 14 21 33 40  8  19 39 21 12 36 40  8
  47  5  6 49 16  4 48  47  5  6 49 16  4 48  47  5  6 49 16  4 48  47  5  6 49 16  4 48
             655928/               948454/              1207374/              1368906/
   2 46 34  1 44 45  3   2 46 34  1 44 45  3   2 46 34  1 44 45  3   2 46 34  1 44 45  3
  42 10 15 38 27 11 32  42 10 14 38 27 11 33  42 10 14 37 26 11 35  42 10 13 35 28 11 36
   9 20 37 24 21 28 36   9 28 37 18 19 29 35   9 27 31 21 22 32 33   9 26 33 23 21 31 32
  43 33 31 25 19 17  7  43 26 30 25 20 24  7  43 30 38 25 12 20  7  43 30 38 25 12 20  7
  14 22 29 26 13 30 41  15 21 31 32 13 22 41  17 18 28 29 19 23 41  18 19 29 27 17 24 41
  18 39 23 12 35 40  8  17 39 23 12 36 40  8  15 39 24 13 36 40  8  14 39 22 15 37 40  8
  47  5  6 49 16  4 48  47  5  6 49 16  4 48  47  5  6 49 16  4 48  47  5  6 49 16  4 48
```

```
             1598454/                 1823644/                 2113109/                 2380888/
   2 46 34  1 44 45  3    2 46 34  1 44 45  3    2 46 34  1 44 45  3    2 46 34  1 44 45  3
  42 10 12 36 27 11 37   42 10 13 33 28 11 38   42 10 13 36 23 12 39   42 11 13 36 21 12 40
   9 26 33 28 18 30 31    9 26 36 27 15 32 30    9 33 35 20 18 31 29    9 27 32 24 20 35 28
  43 29 35 25 15 21  7   43 31 29 25 21 19  7   43 24 28 25 22 26  7   43 33 31 25 19 17  7
  19 20 32 22 17 24 41   20 18 35 23 14 24 41   21 19 32 30 15 17 41   22 15 30 26 18 23 41
  13 39 23 14 38 40  8   12 39 22 17 37 40  8   11 38 27 14 37 40  8   10 38 29 14 37 39  8
  47  5  6 49 16  4 48   47  5  6 49 16  4 48   47  5  6 49 16  4 48   47  5  6 49 16  4 48
             2707286/                 3071058/                 6836309/                11080870/
   2 46 34  1 44 45  3    2 46 34  1 44 45  3    2 46 34  1 44 45  3    2 46 34  1 44 45  3
  42 11 13 36 20 12 41   41 11 17 37 31 12 26   40 11 18 37 33 12 24   39 10 19 37 35 12 23
  10 27 35 18 26 31 28    8 18 35 29 22 23 40    8 23 35 19 21 28 41    8 24 36 21 17 28 41
  43 29 33 25 17 21  7   43 30 36 25 14 20  7   43 30 36 25 14 20  7   43 30 32 25 18 20  7
  22 19 24 32 15 23 40   10 27 28 21 15 32 42    9 22 29 31 15 27 42    9 22 33 29 14 26 42
   9 38 30 14 37 39  8   24 38 19 13 33 39  9   26 38 17 13 32 39 10   27 38 15 13 31 40 11
  47  5  6 49 16  4 48   47  5  6 49 16  4 48   47  5  6 49 16  4 48   47  5  6 49 16  4 48

  . . . . . .
```

** Self-Complementary Magic Squares 7x7: Fundamental Solutions with /D7i **

```
                   1/  /D7i                                  7994/  /D7i
   1 48  3 46  5 44 28  0606063 0523416     1 48  3 46  5 44 28  0606063 0523416
  43 11 26 37 36 12 10  6135511 0341042    43 12 26 37 36 11 10  6135511 0441032
   8 16 35 32 31 23 30  1244434 0163211     8 17 31 34 23 32 30  1244344 0225131
  41 29 33 25 17 21  9  5443221 5043261    41 35 29 25 21 15  9  5443221 5603601
  20 27 19 18 15 34 42  2322245 5543056    20 18 27 16 19 33 42  2232245 5351446
  40 38 14 13 24 39  7  5511350 4265236    40 39 14 13 24 38  7  5511350 4365226
  22  6 45  4 47  2 49  3060606 0523416    22  6 45  4 47  2 49  3060606 0523416
               17069/  /D7i                                 25852/  /D7i
   1 48  3 46  5 44 28  0606063 0523416     1 48  3 46  5 44 28  0606063 0523416
  43 13 24 38 36 11 10  6135511 0522032    43 14 24 38 35 11 10  6135411 0622632
   8 18 33 31 21 34 30  1244244 0342651     8 16 37 31 32 21 30  1254424 0112361
  41 35 27 25 23 15  9  5433321 5653101    41 23 33 25 17 27  9  5343231 5143251
  20 16 29 19 17 32 42  2242245 5104236    20 29 18 19 13 34 42  2422145 5034556
  40 39 14 12 26 37  7  5511350 4364416    40 39 15 12 26 36  7  5521350 4304406
  22  6 45  4 47  2 49  3060606 0523416    22  6 45  4 47  2 49  3060606 0523416
               34027/  /D7i                                 42190/  /D7i
   1 48  3 46  5 44 28  0606063 0523416     1 48  3 46  5 44 28  0606063 0523416
  43 15 21 38 37 11 10  6225511 0062132    43 16 21 38 36 11 10  6225511 0162032
   8 32 33 27 14 31 30  1443144 0345621     8 15 37 27 26 32 30  1253344 0015431
  41 16 24 25 26 34  9  5233341 5123451    41 33 31 25 19 17  9  5443221 5423421
  20 19 36 23 17 18 42  2253225 5401236    20 18 24 23 13 35 42  2233145 5321566
  40 39 13 12 29 35  7  5511440 4354066    40 39 14 12 29 34  7  5511440 4364056
  22  6 45  4 47  2 49  3060606 0523416    22  6 45  4 47  2 49  3060606 0523416
               51192/  /D7i                                 59136/  /D7i
   1 48  3 46  5 44 28  0606063 0523416     1 48  3 46  5 44 28  0606063 0523416
  43 17 19 38 37 11 10  6225511 0242132    43 18 19 38 36 11 10  6225511 0342032
   8 24 34 35 21 23 30  1344234 0256611     8 16 37 35 26 23 30  1254334 0116411
  41 14 32 25 18 36  9  5143251 5633301    41 21 33 25 17 29  9  5243241 5643201
  20 27 29 15 16 26 42  2342235 5500146    20 27 24 15 13 34 42  2332145 5520556
  40 39 13 12 31 33  7  5511440 4354246    40 39 14 12 31 32  7  5511440 4364236
  22  6 45  4 47  2 49  3060606 0523416    22  6 45  4 47  2 49  3060606 0523416

  ..............
```

Section 2: Pan-diagonal Magic Squares of Order 7

#1. Definitions of 'Extended Space' and the Pan-diagonals

The next type we study about is so called "Pan-diagonal" magic squares of order 7.

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**  Basic Form & List of Basic Equations  **

 47 48 49 43 44 45 46 47 48 49 43 44 45 46
         .--.--.--.--.--.--.--.
  5  6  7|  1|  2|  3|  4|  5|  6|  7|  1  2  3  4
         |--+--+--+--+--+--+--|
 12 13 14|  8|  9|10|11|12|13|14|  8  9 10 11
         |--+--+--+--+--+--+--|
 19 20 21|15|16|17|18|19|20|21|15 16 17 18
         |--+--+--+--+--+--+--|
 26 27 28|22|23|24|25|26|27|28|22 23 24 25
         |--+--+--+--+--+--+--|
 33 34 35|29|30|31|32|33|34|35|29 30 31 32
         |--+--+--+--+--+--+--|
 40 41 42|36|37|38|39|40|41|42|36 37 38 39
         |--+--+--+--+--+--+--|
 47 48 49|43|44|45|46|47|48|49|43 44 45 46
         '--'--'--'--'--'--'--'
  5  6  7  1  2  3  4  5  6  7  1  2  3  4
            [Extended Space]

** Basic Conditions: C=175 **
  n1+n2+n3+n4+n5+n6+n7=C          ... rw1;| n1+n8+n15+n22+n29+n36+n43=C   ... cl1;
  n8+n9+n10+n11+n12+n13+n14=C     ... rw2;| n2+n9+n16+n23+n30+n37+n44=C   ... cl2;
  n15+n16+n17+n18+n19+n20+n21=C   ... rw3;| n3+n10+n17+n24+n31+n38+n45=C  ... cl3;
  n22+n23+n24+n25+n26+n27+n28=C   ... rw4;| n4+n11+n18+n25+n32+n39+n46=C  ... cl4;
  n29+n30+n31+n32+n33+n34+n35=C   ... rw5;| n5+n12+n19+n26+n33+n40+n47=C  ... cl5;
  n36+n37+n38+n39+n40+n41+n42=C   ... rw6;| n6+n13+n20+n27+n34+n41+n48=C  ... cl6;
  n43+n44+n45+n46+n47+n48+n49=C   ... rw7;| n7+n14+n21+n28+n35+n42+n49=C  ... cl7;
** Pan-diagonal Conditions: C=175 **
  n1+n9+n17+n25+n33+n41+n49=C     ... pd1;| n1+n14+n20+n26+n32+n38+n44=C  ... pb1;
  n2+n10+n18+n26+n34+n42+n43=C    ... pd2;| n2+n8+n21+n27+n33+n39+n45=C   ... pb2;
  n3+n11+n19+n27+n35+n36+n44=C    ... pd3;| n3+n9+n15+n28+n34+n40+n46=C   ... pb3;
  n4+n12+n20+n28+n29+n37+n45=C    ... pd4;| n4+n10+n16+n22+n35+n41+n47=C  ... pb4;
  n5+n13+n21+n22+n30+n38+n46=C    ... pd5;| n5+n11+n17+n23+n29+n42+n48=C  ... pb5;
  n6+n14+n15+n23+n31+n39+n47=C    ... pd6;| n6+n12+n18+n24+n30+n36+n49=C  ... pb6;
  n7+n8+n16+n24+n32+n40+n48=C     ... pd7;| n7+n13+n19+n25+n31+n37+n43=C  ... pb7;
```

The Basic form for 'Extended Space' is invented to make you easily find all pan-diagonals in straight lines in place of those notorious "broken wrapping" lines.

When you deal with 49 numbers, you only have 14+14 equations in all. You will surely have comparatively fewer solutions this time, although you will really take more time to calculate and find true solutions. Even if you use your super personal computer, you will surely have to wait and wait for a long time.

#2. Algebraic Study of this Type

We must study the structure of this type in advance by some 'Algebraic' calculations. We must know the secret keys to improve our clever calculation.

Why don't you look for various combinations of 6 numbers which should add up to the same sums?

[Sums of 6 Numbers Combined into a Triangle on the Corners]

```
  n1 n2 n3 n4 n5 n6 n7       n1+ n2+ n3+ n8+ n9+n15 = P ...(1)
  n8 n9 10 11 12 13 14       n5+ n6+ n7+n13+n14+n21 = Q ...(2)
  15 16 17 18 19 20 21      n29+n36+n37+n43+n44+n45 = R ...(3)
  22 23 24 25 26 27 28      n35+n41+n42+n47+n48+n49 = S ...(4)
  29 30 31 32 33 34 35
  36 37 38 39 40 41 42       P = S; Q = R; are expected.
  43 44 45 46 47 48 49
```

```
rw1+rw2+pb3+pd5
    n1+ n2+ n3+ n4+ n5+ n6+ n7 = K ... rw1
    n8+ n9+n10+n11+n12+n13+n14 = K ... rw2
    n3+ n9+n15+n28+n34+n40+n46 = K ... pb3
 +) n5+n13+n21+n22+n30+n38+n46 = K ... pd5
  ----------------------------------------
n1+n2+n3+n3+n4+n5+n5+n6+n7+n8+n9+n9+n10+n11+n12+n13+n13+n14
 +n15+n21+n22+n28+n30+n34+n38+n40+n46+n46 = 4K
(n1+n2+n3+n8+n9+n15)+(n5+n6+n7+n13+n14+n21)+(n4+n22+n28+n46)+
 (n3+n5+n9+n10+n11+n12+n13+n30+n34+n38+n40+n46) = 4K
Put (n4+n22+n28+n46) = W
  P+Q+W+(n3+n5+n9+n10+n11+n12+n13+n30+n34+n38+n40+n46) = 4K      ....(30)

cl6+cl7+pb4+pd5
    n6+n13+n20+n27+n34+n41+n48 = K ... cl6
    n7+n14+n21+n28+n35+n42+n49 = K ... cl7
    n4+n10+n16+n22+n35+n41+n47 = K ... pb4
 +) n5+n13+n21+n22+n30+n38+n46 = K ... pd5
  ----------------------------------------
n4+n5+n6+n7+n10+n13+n13+n14+n16+n20+n21+n21+n22+n22+n27+n28
 +n30+n34+n35+n35+n38+n41+n41+n42+n46+n47+n48+n49 = 4K
(n5+n6+n7+n13+n14+n21)+(n35+n41+n42+n47+n48+n49)+(n4+n22+n28+n46)+
 (n10+n13+n16+n20+n21+n22+n27+n30+n34+n35+n38+n41) = 4K
  Q+S+W+(n10+n13+n16+n20+n21+n22+n27+n30+n34+n35+n38+n41) = 4K   ....(31)

(30)-(31)
  P-S = (n16+n20+n21+n22+n27+n35+n41)-(n3+n5+n9+n11+n12+n40+n46)
  P+(n3+n5+n9+n11+n12+n40+n46) = S+(n16+n20+n21+n22+n27+n35+n41)   ..(32)
This conclusion means that if you want P = S, then you must also accept
  (n3+n5+n9+n11+n12+n40+n46) = (n16+n20+n21+n22+n27+n35+n41)

In the same way
    n1+n8+n15+n22+n29+n36+n43 = K ... cl1
    n2+n9+n16+n23+n30+n37+n44 = K ... cl2
    n3+n9+n15+n28+n34+n40+n46 = K ... pb3
 +)n4+n12+n20+n28+n29+n37+n45 = K ... pd4
  ----------------------------------------
 (n1+n2+n3+n8+n9+n15)+(n29+n36+n37+n43+n44+n45)+(n4+n22+n28+n46)+
 (n9+n12+n15+n16+n20+n23+n28+n29+n30+n34+n37+n40) = 4K
  P+R+W+(n9+n12+n15+n16+n20+n23+n28+n29+n30+n34+n37+n40) = 4K    ....(33)

    n36+n37+n38+n39+n40+n41+n42 = K ... rw6
    n43+n44+n45+n46+n47+n48+n49 = K ... rw7
     n4+n12+n20+n28+n29+n37+n45 = K ... pd4
  +) n4+n10+n16+n22+n35+n41+n47 = K ... pb4
  -------------------------------------------
 (n29+n36+n37+n43+n44+n45)+(n35+n41+n42+n47+n48+n49)+(n4+n22+n28+n46)+
(n4+n10+n12+n16+n20+n37+n38+n39+n40+n41+n45+n47) = 4K
  R+S+W+(n4+n10+n12+n16+n20+n37+n38+n39+n40+n41+n45+n47) = 4K    ....(34)

(33)-(34)
P+(n9+n15+n23+n28+n29+n30+n34) = S+(n4+n10+n38+n39+n41+n45+n47)   ...(35)
```

```
(32)+(35)
     P+n3+n5+n9+n11+n12+n40+n46 = S+n16+n20+n21+n22+n27+n35+n41   ...(32)
+) P+n9+n15+n23+n28+n29+n30+n34 = S+n4+n10+n38+n39+n41+n45+n47   ....(35)
  ------------------------------------------------------------------------
 2P+(n3+n9+n15+n28+n34+n40+n46)+(n5+n9+n11+n12+n23+n29+n30)
   =  2S+(n4+n10+n16+n22+n35+n41+n47)+(n20+n21+n27+n38+n39+n41+n45)
But   n3+n9+n15+n28+n34+n40+n46 = K  ...(20)
      n4+n10+n16+n22+n35+n41+n47 = K ...(22)
 2P+K+(n5+n9+n11+n12+n23+n29+n30) =  2S+K+(n20+n21+n27+n38+n39+n41+n45)
Therefore
 2P+(n5+n9+n11+n12+n23+n29+n30) = 2S+(n20+n21+n27+n38+n39+n41+n45) ..(36)

If you want P = S, then you must accept
 (n5+n9+n11+n12+n23+n29+n30) = (n20+n21+n27+n38+n39+n41+n45)

Add (n2+n8+n17+n33+n42+n48) to both sides of (36) and calculate.
 2P+{(n5+n11+n17+n23+n29+n42+n48)+(n2+n8+n9+n12+n30+n33)}
 =  2S+{(n2+n8+n21+n27+n33+n39+n45)+(n17+n20+n38+n41+n42+n48)}
But   n5+n11+n17+n23+n29+n42+n48 = K ...(24)
      n2+n8+n21+n27+n33+n39+n45 = K  ...(18)
 2P+K+(n2+n8+n9+n12+n30+n33) = 2S+K+(n17+n20+n38+n41+n42+n48)
Therefore
 2P+(n2+n8+n9+n12+n30+n33) = 2S+(n17+n20+n38+n41+n42+n48)        ....(37)
This conclusion is a little bit shorter than (36)

Add (n1+n3+n15+n35+n47+n49) to both sides.
 2P+(n1+n2+n3+n8+n9+n15)+(n12+n30+n33+n35+n47+n49)
 =  2S+(n35+n41+n42+n47+n48+n49)+(n1+n3+n15+n17+n20+n38)
 3P+(n12+n30+n33+n35+n47+n49) = 3S+(n1+n3+n15+n17+n20+n38)       ....(38)

   n1+ n2+ n3+ n8+ n9+n15 = n35+n41+n42+n47+n48+n49 ....(T6.1)
   n5+ n6+ n7+n13+n14+n21 = n29+n36+n37+n43+n44+n45 ....(T6.2)
   n4+n10+n11+n16+n17+n18 = n26+n27+n28+n33+n34+n40 ....(T6.3)
   n4+n11+n12+n18+n19+n20 = n22+n23+n24+n30+n31+n38 ....(T6.4)
   n2+ n8+ n9+n12+n30+n33 = n17+n20+n38+n41+n42+n48 ....(T6.5)
   n6+n10+n13+n14+n31+n34 = n16+n19+n36+n37+n40+n44 ....(T6.6)
   n1+ n3+n15+n17+n20+n38 = n12+n30+n33+n35+n47+n49 ....(T6.7)
   n5+ n7+n16+n19+n21+n40 = n10+n29+n31+n34+n43+n45 ....(T6.8)
```

[6 Entries Combined into a Triangle on the Courner]

```
 n1 n2 n3 n4 n5 n6 n7      n1+ n2+ n3+ n8+ n9+n15 = P ...(1)
 n8 n9 10 11 12 13 14      n5+ n6+ n7+n13+n14+n21 = Q ...(2)
 15 16 17 18 19 20 21     n29+n36+n37+n43+n44+n45 = R ...(3)
 22 23 24 25 26 27 28     n35+n41+n42+n47+n48+n49 = S ...(4)
 29 30 31 32 33 34 35
 36 37 38 39 40 41 42      If you want P = S, then you must
 43 44 45 46 47 48 49      accept Q = R, (T6.5), and (T6.6).
```

[Other 6 Entries Combined]

```
 n1 n2 n3 n4 n5 n6 n7      n1 n2 n3 n4 n5 n6 n7
 n8 n9 10 11 12 13 14      n8 n9 10 11 12 13 14
 15 16 17 18 19 20 21      15 16 17 18 19 20 21
 22 23 24 25 26 27 28      22 23 24 25 26 27 28
 29 30 31 32 33 34 35      29 30 31 32 33 34 35
 36 37 38 39 40 41 42      36 37 38 39 40 41 42
 43 44 45 46 47 48 49      43 44 45 46 47 48 49

 n2+n8+n9+n12+n30+n33 =     n1+n3+n15+n17+n20+n38 =
  n17+n20+n38+n41+n42+n48     n12+n30+n33+n35+n47+n49
```

Why don't you take these conclusions into your program coding positively and replace some independent variables for some subordinate ones?

But it is such a hard work for you to calculate solutions of order 7 that you should know it is practically impossible to count them up to the last. When I once tried to do it, I had to wait and wait for a long time every time. I got almost sick.

Let me show you some sample sets of fundamental solutions of Pan-diagonal magic squares 7x7 with the Decomposition layers by the number system of 7-th increment.

```
* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #1 *
                    1/  /D7i                                     2/  /D7i
  1 48 39 30 28 19 10  0654321 0531642    1 48 39 23 35 19 10  0653421 0531642
 18  9  7 47 38 29 27  2106543 3164205   18  9  7 47 38 22 34  2106534 3164205
 35 26 17  8  6 46 37  4321065 6420531   28 33 17  8  6 46 37  3421065 6420531
 45 36 34 25 16 14  5  6543210 2053164   45 36 27 32 16 14  5  6534210 2053164
 13  4 44 42 33 24 15  1065432 5316420   13  4 44 42 26 31 15  1065342 5316420
 23 21 12  3 43 41 32  3210654 1642053   30 21 12  3 43 41 25  4210653 1642053
 40 31 22 20 11  2 49  5432106 4205316   40 24 29 20 11  2 49  5342106 4205316
                    3/  /D7i                                     4/  /D7i
  1 48 39 30 21 26 10  0654231 0531642    1 48 39 16 35 26 10  0652431 0531642
 25  9  7 47 38 29 20  3106542 3164205   25  9  7 47 38 15 34  3106524 3164205
 35 19 24  8  6 46 37  4231065 6420531   21 33 24  8  6 46 37  2431065 6420531
 45 36 34 18 23 14  5  6542310 2053164   45 36 20 32 23 14  5  6524310 2053164
 13  4 44 42 33 17 22  1065423 5316420   13  4 44 42 19 31 22  1065243 5316420
 16 28 12  3 43 41 32  2310654 1642053   30 28 12  3 43 41 18  4310652 1642053
 40 31 15 27 11  2 49  5423106 4205316   40 17 29 27 11  2 49  5243106 4205316
                    5/  /D7i                                     6/  /D7i
  1 48 39 23 21 33 10  0653241 0531642    1 48 39 16 28 33 10  0652341 0531642
 32  9  7 47 38 22 20  4106532 3164205   32  9  7 47 38 15 27  4106523 3164205
 28 19 31  8  6 46 37  3241065 6420531   21 26 31  8  6 46 37  2341065 6420531
 45 36 27 18 30 14  5  6532410 2053164   45 36 20 25 30 14  5  6523410 2053164
 13  4 44 42 26 17 29  1065324 5316420   13  4 44 42 19 24 29  1065234 5316420
 16 35 12  3 43 41 25  2410653 1642053   23 35 12  3 43 41 18  3410652 1642053
 40 24 15 34 11  2 49  5324106 4205316   40 17 22 34 11  2 49  5234106 4205316

  . . . . .
```

```
* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #2 *
                    1/  /D7i                                     2/  /D7i
  1 45 40 35 23 18 13  0654321 0246135    1 45 40 28 30 18 13  0653421 0246135
 21  9  4 48 36 31 26  2106543 6135024   21  9  4 48 36 24 33  2106534 6135024
 34 22 17 12  7 44 39  4321065 5024613   27 29 17 12  7 44 39  3421065 5024613
 47 42 30 25 20  8  3  6543210 4613502   47 42 23 32 20  8  3  6534210 4613502
 11  6 43 38 33 28 16  1065432 3502461   11  6 43 38 26 35 16  1065342 3502461
 24 19 14  2 46 41 29  3210654 2461350   31 19 14  2 46 41 22  4210653 2461350
 37 32 27 15 10  5 49  5432106 1350246   37 25 34 15 10  5 49  5342106 1350246
                    3/  /D7i                                     4/  /D7i
  1 45 40 35 16 25 13  0654231 0246135    1 45 40 21 30 25 13  0652431 0246135
 28  9  4 48 36 31 19  3106542 6135024   28  9  4 48 36 17 33  3106524 6135024
 34 15 24 12  7 44 39  4231065 5024613   20 29 24 12  7 44 39  2431065 5024613
 47 42 30 18 27  8  3  6542310 4613502   47 42 16 32 27  8  3  6524310 4613502
 11  6 43 38 33 21 23  1065423 3502461   11  6 43 38 19 35 23  1065243 3502461
 17 26 14  2 46 41 29  2310654 2461350   31 26 14  2 46 41 15  4310652 2461350
 37 32 20 22 10  5 49  5423106 1350246   37 18 34 22 10  5 49  5243106 1350246
```

```
                     5/  /D7i                                        6/  /D7i
  1 45 40 28 16 32 13  0653241 0246135    1 45 40 21 23 32 13  0652341 0246135
 35  9  4 48 36 24 19  4106532 6135024   35  9  4 48 36 17 26  4106523 6135024
 27 15 31 12  7 44 39  3241065 5024613   20 22 31 12  7 44 39  2341065 5024613
 47 42 23 18 34  8  3  6532410 4613502   47 42 16 25 34  8  3  6523410 4613502
 11  6 43 38 26 21 30  1065324 3502461   11  6 43 38 19 28 30  1065234 3502461
 17 33 14  2 46 41 22  2410653 2461350   24 33 14  2 46 41 15  3410652 2461350
 37 25 20 29 10  5 49  5324106 1350246   37 18 27 29 10  5 49  5234106 1350246

 . . . . . .
```

* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #3 *

```
                     1/  /D7i                                        2/  /D7i
  1 49 41 33 25 17  9  0654321 0654321    1 49 41 26 32 17  9  0653421 0654321
 40 32 24 16  8  7 48  5432106 4321065   40 25 31 16  8  7 48  5342106 4321065
 23 15 14  6 47 39 31  3210654 1065432   30 15 14  6 47 39 24  4210653 1065432
 13  5 46 38 30 22 21  1065432 5432106   13  5 46 38 23 29 21  1065342 5432106
 45 37 29 28 20 12  4  6543210 2106543   45 37 22 35 20 12  4  6534210 2106543
 35 27 19 11  3 44 36  4321065 6543210   28 34 19 11  3 44 36  3421065 6543210
 18 10  2 43 42 34 26  2106543 3210654   18 10  2 43 42 27 33  2106534 3210654
                     3/  /D7i                                        4/  /D7i
  1 49 41 33 18 24  9  0654231 0654321    1 49 41 19 32 24  9  0652431 0654321
 40 32 17 23  8  7 48  5423106 4321065   40 18 31 23  8  7 48  5243106 4321065
 16 22 14  6 47 39 31  2310654 1065432   30 22 14  6 47 39 17  4310652 1065432
 13  5 46 38 30 15 28  1065423 5432106   13  5 46 38 16 29 28  1065243 5432106
 45 37 29 21 27 12  4  6542310 2106543   45 37 15 35 27 12  4  6524310 2106543
 35 20 26 11  3 44 36  4231065 6543210   21 34 26 11  3 44 36  2431065 6543210
 25 10  2 43 42 34 19  3106542 3210654   25 10  2 43 42 20 33  3106524 3210654
                     5/  /D7i                                        6/  /D7i
  1 49 41 26 18 31  9  0653241 0654321    1 49 41 19 25 31  9  0652341 0654321
 40 25 17 30  8  7 48  5324106 4321065   40 18 24 30  8  7 48  5234106 4321065
 16 29 14  6 47 39 24  2410653 1065432   23 29 14  6 47 39 17  3410652 1065432
 13  5 46 38 23 15 35  1065324 5432106   13  5 46 38 16 22 35  1065234 5432106
 45 37 22 21 34 12  4  6532410 2106543   45 37 15 28 34 12  4  6523410 2106543
 28 20 33 11  3 44 36  3241065 6543210   21 27 33 11  3 44 36  2341065 6543210
 32 10  2 43 42 27 19  4106532 3210654   32 10  2 43 42 20 26  4106523 3210654

 . . . . . .
```

Section 3: Simultaneous Magic Squares of Order 7: Both Self-Complementary and Pan-Diagonal

#1. Definition of Simultaneous Magic Squares 7x7: Both S-C and P-D Type

The next goal we are trying to build is the Simultaneous type of magic squares of order 7: both Self-complementary and Pan-diagonal.

All conditions we mentioned above must be used here and be made active.

```
**  Basic Form & List of Basic Equations  **

 47 48 49 43 44 45 46 47 48 49 43 44 45 46
         .--.--.--.--.--.--.--.
  5  6  7| 1| 2| 3| 4| 5| 6| 7| 1  2  3  4
         |--+--+--+--+--+--+--|
 12 13 14| 8| 9|10|11|12|13|14| 8  9 10 11
         |--+--+--+--+--+--+--|
 19 20 21|15|16|17|18|19|20|21|15 16 17 18
         |--+--+--+--+--+--+--|
 26 27 28|22|23|24|25|26|27|28|22 23 24 25
         |--+--+--+--+--+--+--|
 33 34 35|29|30|31|32|33|34|35|29 30 31 32
         |--+--+--+--+--+--+--|
 40 41 42|36|37|38|39|40|41|42|36 37 38 39
         |--+--+--+--+--+--+--|
 47 48 49|43|44|45|46|47|48|49|43 44 45 46
         '--'--'--'--'--'--'--'
  5  6  7  1  2  3  4  5  6  7  1  2  3  4

** Basic Conditions: C=175 **
  n1+n2+n3+n4+n5+n6+n7=C          ... rw1;| n1+n8+n15+n22+n29+n36+n43=C   ... cl1;
  n8+n9+n10+n11+n12+n13+n14=C     ... rw2;| n2+n9+n16+n23+n30+n37+n44=C   ... cl2;
  n15+n16+n17+n18+n19+n20+n21=C   ... rw3;| n3+n10+n17+n24+n31+n38+n45=C  ... cl3;
  n22+n23+n24+n25+n26+n27+n28=C   ... rw4;| n4+n11+n18+n25+n32+n39+n46=C  ... cl4;
  n29+n30+n31+n32+n33+n34+n35=C   ... rw5;| n5+n12+n19+n26+n33+n40+n47=C  ... cl5;
  n36+n37+n38+n39+n40+n41+n42=C   ... rw6;| n6+n13+n20+n27+n34+n41+n48=C  ... cl6;
  n43+n44+n45+n46+n47+n48+n49=C   ... rw7;| n7+n14+n21+n28+n35+n42+n49=C  ... cl7;
** Pan-diagonal Conditions: C=175 **
  n1+n9+n17+n25+n33+n41+n49=C     ... pd1;| n1+n14+n20+n26+n32+n38+n44=C  ... pb1;
  n2+n10+n18+n26+n34+n42+n43=C    ... pd2;| n2+n8+n21+n27+n33+n39+n45=C   ... pb2;
  n3+n11+n19+n27+n35+n36+n44=C    ... pd3;| n3+n9+n15+n28+n34+n40+n46=C   ... pb3;
  n4+n12+n20+n28+n29+n37+n45=C    ... pd4;| n4+n10+n16+n22+n35+n41+n47=C  ... pb4;
  n5+n13+n21+n22+n30+n38+n46=C    ... pd5;| n5+n11+n17+n23+n29+n42+n48=C  ... pb5;
  n6+n14+n15+n23+n31+n39+n47=C    ... pd6;| n6+n12+n18+n24+n30+n36+n49=C  ... pb6;
  n7+n8+n16+n24+n32+n40+n48=C     ... pd7;| n7+n13+n19+n25+n31+n37+n43=C  ... pb7;
** Self-complementary Conditions: CC=50 **
  n1+n49=n2+n48=n3+n47=n4+n46=n5+n45=n6+n44=n7+n43=
  n8+n42=n9+n41=n10+n40=n11+n39=n12+n38=n13+n37=n14+n36=
  n15+n35=n16+n34=n17+n33=n18+n32=n19+n31=n20+n30=n21+n29=
  n22+n28=n23+n27=n24+n26=n25+n25=CC ... scc
```

The last set of equations appears again to define Complementary Pairs of CC. Place all pairs symmetrically with respect to n25(Geometric center of our object).

This makes our object a Self-complementary type as well as a Pan-diagonal one.

#2. Powerful Combinations of Properties

For the Simultaneous type you can combine the properties of Pan-diagonal with those of Self-complementary. For instance, various sums of 6 numbers combined could be more clearly defined like this:

```
 [6 Entries Combined in a Triangle on the Corner]

  n1 n2 n3 n4 n5 n6 n7       n1+ n2+ n3+ n8+ n9+n15 = P ...(1)
  n8 n9 10 11 12 13 14       n5+ n6+ n7+n13+n14+n21 = Q ...(2)
  15 16 17 18 19 20 21      n29+n36+n37+n43+n44+n45 = R ...(3)
  22 23 24 25 26 27 28      n35+n41+n42+n47+n48+n49 = S ...(4)
  29 30 31 32 33 34 35
  36 37 38 39 40 41 42       If you want P = S, then you must
  43 44 45 46 47 48 49       accept Q = R, (T6.5), and (T6.6).

 n1+n49=50; n2+n48=50; n3+n47=50; n8+n42=50; n9+n41=50; n15+n35=50;
 P+S=(n1+n49)+(n2+n48)+(n3+n47)+(n8+n42)+(n9+n41)+(n15+n35)= 50 x 6 = 300;
If P==S, then P=S=300/2=150
 n1+ n2+ n3+ n8+ n9+n15 = n35+n41+n42+n47+n48+n49 = 150;
In the same way Q=R=150
 n5+ n6+ n7+n13+n14+n21 = n29+n36+n37+n43+n44+n45 = 150;
 n2+ n8+ n9+n12+n30+n33 = n17+n20+n38+n41+n42+n48 = 150;
 n1+ n3+n15+n17+n20+n38 = n12+n30+n33+n35+n47+n49 = 150;
```

You may well assume these conditions positively even at the definition stage, although I could not yet prove the first single equation P=S logically.

You might even take (P=S)&(Q=R) positively into your program coding. These new conditions will help you a lot. Because they are the guide-lines we have expected. And that would neither spoil your object, nor lose any solutions you should have found.

That would also save your time to calculate and count them up far. This may be the best relief to you.

```
 ** Simultaneous Magic Squares 7x7: Both Self-Complementary and Pan-Diagonal **
                    1/                4563/                9768/               14218/
  1 48  6 27 46  9 38   1 48  7 33 41 10 35   1 48  7 23 42 10 44   1 48  8 20 43 10 45
 47  5 34 39 17 19 14  46  3 29 36  8 26 27  45  3 31 41 17 18 20  44  3 38 36 21 16 17
 43  8 37 15 18 30 24  45 12 37 22 30 18 11  46 14 39 21 15 24 16  46  9 39 13 22 27 19
 10 22 29 25 21 28 40   6 16 31 25 19 34 44  13 12 22 25 28 38 37  15 18 26 25 24 32 35
 26 20 32 35 13 42  7  39 32 20 28 13 38  5  34 26 35 29 11 36  4  31 23 28 37 11 41  4
 36 31 33 11 16 45  3  23 24 42 14 21 47  4  30 32 33  9 19 47  5  33 34 29 14 12 47  6
 12 41  4 23 44  2 49  15 40  9 17 43  2 49   6 40  8 27 43  2 49   5 40  7 30 42  2 49
                19292/               23906/               28582/               32996/
  1 48  9 27 42 10 38   1 48 10 18 44 20 34   1 48 11 21 45  7 42   1 48 12 30 43 15 26
 43  3 35 29 11 28 26  42  3 36 38 17 15 24  41  3 40 28 20 17 26  40  3 36 44  8 21 23
 46 16 37 19 33 18  6  46 23 41 11 22 19 13  46  6 35 14 34 27 13  46 11 17 13 34 32 22
  5 14 30 25 20 36 45   7  5 21 25 29 45 43  18 19 38 25 12 31 32   9 31 45 25  5 19 41
 44 32 17 31 13 34  4  37 31 28 39  9 27  4  37 23 16 36 15 44  4  28 18 16 37 33 39  4
 24 22 39 21 15 47  7  26 35 33 12 14 47  8  24 33 30 22 10 47  9  27 29 42  6 14 47 10
 12 40  8 23 41  2 49  16 30  6 32 40  2 49   8 43  5 29 39  2 49  24 35  7 20 38  2 49
                37665/               41830/               46374/               50541/
  1 48 13 22 43  6 42   1 48 14 34 45  6 27   1 48 15 36 44  8 23   1 48 16  9 44 17 40
 39  3 32 35 30 26 10  38  3 26 28 21 20 39  37  3 26 29 10 32 38  36  3 43 38 11 23 21
 46 19 41 12  5 29 23  46  9 43 15 32 17 13  46 20 31 22 34 17  5  46 30 13 22 24 35  5
 14 16 17 25 33 34 36  19  8 40 25 10 42 31   7 11 41 25  9 39 43   8 19 32 25 18 31 42
 27 21 45 38  9 31  4  37 33 18 35  7 41  4  45 33 16 28 19 30  4  45 15 26 28 37 20  4
 40 24 20 15 18 47 11  11 30 29 22 24 47 12  12 18 40 21 24 47 13  29 27 39 12  7 47 14
  8 44  7 28 37  2 49  23 44  5 16 36  2 49  27 42  6 14 35  2 49  10 33  6 41 34  2 49
  . . . . . .
```

How many solutions could we find? The computation result was very surprising.
We have got too many solutions to count them up through, taking too much time.
See the sample lists of fundamental solutions above and below. Simultaneous type of squares 7x7 prove to be no longer rare, precious jewels at all.
When Prof. Mutsumi Suzuki discovered all of order 5, he predicted that there must exist any Simultaneous magic squares: both Self-complementary and Pan-diagonal of orders 7, 9, 11, 13, ...
I really found many examples of those 'miraculous' solutions of order 7 and 9 for Prof. M. Suzuki.

```
** Simultaneous Magic Squares 7x7: Both Self-complementary and Pan-diagonal **
                  1/TC  /D7i                               45/TC  /D7i
  1 48  3 45 39 27 12  0606531 0522354    1 48  4 43 39 23 17  0606532 0530312
 18 46 17 37  6 36 15  2625052 3321500   18 45 22 26  8 47  9  2633161 3204041
 34 26 28  7 19 40 21  4330252 5466446   34 30 31  6 21 38 15  4440252 5125620
 20  8 41 25  9 42 30  2153154 5053161   13 10 36 25 14 40 37  1153155 5203641
 29 10 31 43 22 24 16  4146332 0220021   35 12 29 44 19 20 16  4146222 6401451
 35 14 44 13 33  4 32  4161404 6615433   41  3 42 24 28  5 32  5053304 5262643
 38 23 11  5 47  2 49  5310606 2134416   33 27 11  7 46  2 49  4310606 4536316
                 94/TC  /D7i                              143/TC  /D7i
  1 48  5 38 20 37 26  0605253 0542514    1 48  6 38 39 28 15  0605532 0552360
 18 47 14 33  7 46 10  2614061 3464632   18 47 24 17  8 40 21  2632152 3422046
 31 34 41  6 23 29 11  4450341 2555103   30 27 37  4 36 34  7  4350540 1513056
 22  8 15 25 35 42 28  3123453 0003666   19  5 41 25  9 45 31  2053164 4453122
 39 21 27 44  9 16 19  5236122 3651114   43 16 14 46 13 23 20  6216132 0163515
 40  4 43 17 36  3 32  5062504 4302023   29 10 42 33 26  3 32  4154304 0264423
 24 13 30 12 45  2 49  3141606 2514216   35 22 11 12 44  2 49  4311606 6034116
               210/TC                264/TC                323/TC                372/TC
  1 48  7 46 37 17 19   1 48  8 36 34 38 10   1 48  9 40 30 39  8   1 48 10 36 15 37 28
 18 47  8 26 22 34 20  18 47 11 45 13 26 15  18 47  6 43  5 34 22  18 46 16 38  8 30 19
 29 15 45  9 14 40 23  28 31 33  4  9 43 27  27 36 31 12 15 37 17  27  9 26  5 43 44 21
 39  6 38 25 12 44 11  30  6 29 25 21 44 20  26  4 29 25 21 46 24  47 33 39 25 11 17  3
 27 10 36 41  5 35 21  23  7 41 46 17 19 22  33 13 35 38 19 14 23  29  6  7 45 24 41 23
 30 16 28 24 42  3 32  35 24 37  5 39  3 32  28 16 45  7 44  3 32  31 20 42 12 34  4 32
 31 33 13  4 43  2 49  40 12 16 14 42  2 49  42 11 20 10 41  2 49  22 13 35 14 40  2 49
               438/TC                521/TC                614/TC                678/TC
  1 48 11 47 36 23  9   1 48 12 35 36 34  9   1 48 13 35 31 41  6   1 48 14  8 19 45 40
 18 46  5 33 20 31 22  18 47 21 17  5 40 27  18 47 14 30  4 45 17  18 47 21 30 17 26 16
 26 16 38  7 15 44 29  24 30 31  7 42 37  4  23 43 29  8 24 38 10  22 35 37 11 23 43  4
 40 13 42 25  8 37 10  22 11 44 25  6 39 28  16 11 28 25 22 39 34  44  9 12 25 38 41  6
 21  6 35 43 12 34 24  46 13  8 43 19 20 26  40 12 26 42 21  7 27  46  7 27 39 13 15 28
 28 19 30 17 45  4 32  23 10 45 33 29  3 32  33  5 46 20 36  3 32  34 24 33 20 29  3 32
 41 27 14  3 39  2 49  41 16 14 15 38  2 49  44  9 19 15 37  2 49  10  5 31 42 36  2 49
               747/TC                821/TC                908/TC                996/TC
  1 48 15 36 28 38  9   1 48 16 28 39 38  5   1 48 17 29 44 23 13   1 48 19 41 40 22  4
 18 47  4 34  7 45 20  18 47 24 17  8 40 21  18 47 15 11 26 42 16  18 47  6 14 21 43 26
 21 44 31 17 13 39 10  20 37 27  4 36 44  7  19 36 43  9 10 46 12  17 27 39 12 20 45 15
 24  8 23 25 27 42 26  19 15 41 25  9 35 31  28  5 30 25 20 45 22  34 13 42 25  8 37 16
 40 11 37 33 19  6 29  43  6 14 46 23 13 30  38  4 40 41  7 14 31  35  5 30 38 11 23 33
 30  5 43 16 46  3 32  29 10 42 33 26  3 32  34  8 24 39 35  3 32  24  7 29 36 44  3 32
 41 12 22 14 35  2 49  45 12 11 22 34  2 49  37 27  6 21 33  2 49  46 28 10  9 31  2 49
 . . . . . .
```

I haven't yet known how many solutions there exist in all. Magic squares of higher orders than 6 have too many solutions for us to count piece by piece up to the last.
It might be meaningless to know that, I myself think, unless you could divide the

total count into some meaningful factors that indicate some structural necessities of our object.

#3. About 'Complete Euler Squares' 7x7 for this Type

Give your kind look at the next list. It shows some special solutions accompanied with decomposition layers by the number system of 7-th increment for each.

This is the complete list of all solutions I got when I defined the two primary diagonals strictly like these:

```
n1=1; n9=9; n17=17; n25=25; n33=33; n41=41; n49=49;
{n7, n13, n19, n31, n37, n43}={7, 13, 19, 31, 37, 43}
```

The latter definition means each variable on the left side could take any value only of the six values listed on the right.

```
** Simultaneous Magic Squares 7x7: Both Self-Complementary and Pan-Diagonal **
    **  n1=1; n9=9; n17=17; n25=25; n33=33; n41=41; n49=49;  **
    ** {n7, n13, n19, n31, n37, n43}={7, 13, 19, 31, 37, 43} **
                      1/  /D7i                                   2/  /D7i
  1 48 39 14 44 22  7  0651630 0536106    1 30 40 45 48  4  7  0456600 0142536
 18  9 23 34 38 13 40  2134515 3115254   26  9 22 29 34 13 42  3134415 4100556
 35 29 17  5 19 46 24  4420263 6024432   44 12 17 15 19 32 36  6122245 1420430
 42 20 47 25  3 30  8  5263041 6543210   39 23 47 25  3 27 11  5363031 3143253
 26  4 31 45 33 21 15  3046422 4322460   14 18 31 35 33 38  6  1244450 6326425
 10 37 12 16 27 41 32  1512354 2141553    8 37 16 21 28 41 24  1522353 0116652
 43 28  6 36 11  2 49  6305106 0650316   43 46  2  5 10 20 49  6600126 0314256
                      3/  /D7i                                   4/  /D7i
  1 30 48 35 42 12  7  0464510 0156646    1 44 47 23 24 29  7  0663340 0141206
 16  9 26 27 44 13 40  2133615 1145154   38  9  2 28 16 37 45  5103256 2116112
 46 22 17  3 19 32 36  6320245 3022430   11 32 17 36 31 40  8  1425451 3320240
 45 21 39 25 11 29  5  6253140 2633304   35 46 30 25 20  4 15  4643202 6313530
 14 18 31 47 33 28  4  1246430 6324463   42 10 19 14 33 18 39  5121425 6246433
 10 37  6 23 24 41 34  1503354 2151255    5 13 34 22 48 41 12  0143651 4550554
 43 38  8 15  2 20 49  6512026 0200156   43 21 26 27  3  6 49  6233006 0645256
                      5/  /D7i                                   6/  /D7i
  1 36 34 38 29 24 13  0545431 0052025    1 39 46 21 47  8 13  0562611 0336405
 30  9 22 48 27  7 32  4136304 1105563   27  9 12 24 36 19 48  3113526 5142045
 40  4 17  3 31 35 45  5020446 4322262   28 30 17 34  7 44 15  3424062 6125610
 44 42 39 25 11  8  6  6553110 1633305   45 18 40 25 10 32  5  6253140 2343234
  5 15 19 47 33 46 10  0226461 4044432   35  6 43 16 33 20 22  4062423 6501450
 18 43 23  2 28 41 20  2630352 3011655    2 31 14 26 38 41 23  0413553 1264251
 37 26 21 12 16 14 49  5321216 1464166   37 42  3 29  4 11 49  5504016 1620336
                      7/  /D7i                                   8/  /D7i
  1 45 40 27 47  2 13  0653601 0245415    1 45 40 35 23 18 13  0654321 0246135
 21  9 12 24 36 31 42  2113545 6142026   21  9  4 48 36 31 26  2106543 6135024
 34 30 17 28  7 44 15  4423062 5126610   34 22 17 12  7 44 39  4321065 5024613
 39 18 46 25  4 32 11  5263041 3333333   47 42 30 25 20  8  3  6543210 4613502
 35  6 43 22 33 20 16  4063422 6500451   11  6 43 38 33 28 16  1065432 3502461
  8 19 14 26 38 41 29  1213554 0464250   24 19 14  2 46 41 29  3210654 2461350
 37 48  3 23 10  5 49  5603106 1521246   37 32 27 15 10  5 49  5432106 1350246
                      9/  /D7i                                  10/  /D7i
  1 39 28 45 47  2 13  0536601 0362415    1 30 48 38 23 22 13  0465331 0152105
 27  9 12 36 18 31 42  3115245 5140326   18  9  5 40 26 31 46  2105346 3144423
 46 30 17 34  7 26 15  6424032 3125640   44 47 17 29  7 16 15  6624022 1420610
 21  6 40 25 10 44 29  2053164 6543210   36  8 11 25 39 42 14  5113551 0033366
 35 24 43 16 33 20  4  4362420 6201453   35 34 43 21 33  3  6  4462400 6506425
  8 19 32 14 38 41 23  1241553 0436251    4 19 24 10 45 41 32  0231654 3422253
 37 48  3  5 22 11 49  5600316 1524036   37 28 27 12  2 20 49  5331026 1654156
```

						11/	/D7i	
1	45	46	27	35	8	13	0663411	0235605
21	9	6	18	30	43	48	2102466	6153105
28	36	17	34	19	38	3	3524250	6025422
39	24	40	25	10	26	11	5353131	3243243
47	12	31	16	33	14	22	6142413	4421460
2	7	20	32	44	41	29	0024654	1653150
37	42	15	23	4	5	49	5523006	1601346

						12/	/D7i	
1	39	28	10	48	30	19	0531642	0362514
27	9	47	29	18	7	38	3164205	5140362
46	35	17	6	37	26	8	6420531	3625140
16	5	36	25	14	45	34	2053164	1403625
42	24	13	44	33	15	4	5316420	6251403
12	43	32	21	3	41	23	1642053	4036251
31	20	2	40	22	11	49	4205316	2514036

						13/	/D7i	
1	40	38	29	45	3	19	0554602	0420224
20	9	32	22	44	37	11	2143651	5130113
42	26	17	2	7	46	35	5320064	6421636
27	36	34	25	16	14	23	3543213	5053161
15	4	43	48	33	24	8	2066431	0305420
39	13	6	28	18	41	30	5103254	3556351
31	47	5	21	12	10	49	4602116	2446426

						14/	/D7i	
1	42	46	27	30	10	19	0563412	0635124
14	9	22	11	34	37	48	1131456	6103515
38	24	17	15	43	32	6	5322640	2220035
45	29	47	25	3	21	5	6463020	2043264
44	18	7	35	33	26	12	6204431	1366444
2	13	16	39	28	41	36	0125355	1513650
31	40	20	23	4	8	49	4523016	2451306

						15/	/D7i	
1	39	46	10	48	12	19	0561612	0332544
27	9	35	5	36	43	20	3140562	5164005
28	47	17	18	13	44	8	3622161	6423510
16	29	24	25	26	21	34	2433324	1023465
42	6	37	32	33	3	22	5054403	6513420
30	7	14	45	15	41	23	4016253	1662051
31	38	2	40	4	11	49	4505016	2214336

						16/	/D7i	
1	39	34	22	48	12	19	0543612	0350544
27	9	47	5	24	43	20	3160362	5144205
40	35	17	6	37	32	8	5420541	4625130
4	29	36	25	14	21	46	0453126	3003663
42	18	13	44	33	15	10	5216421	6351402
30	7	26	45	3	41	23	4036053	1642251
31	38	2	28	16	11	49	4503216	2216136

						17/	/D7i	
1	46	22	21	14	40	31	0632154	0306642
24	9	34	45	6	37	20	3146052	2152515
48	38	17	3	43	18	8	6520621	5222030
11	27	15	25	35	23	39	1323435	3503613
42	32	7	47	33	12	2	5406410	6364441
30	13	44	5	16	41	26	4160253	1514154
19	10	36	29	28	4	49	2154306	4200636

						18/	/D7i	
1	40	34	21	2	46	31	0542064	0456132
30	9	22	45	6	37	26	4136053	1102514
36	38	17	15	43	18	8	5522621	0220030
23	39	3	25	47	11	27	3503613	1323435
42	32	7	35	33	12	14	5404411	6366446
24	13	44	5	28	41	20	3160352	2514655
19	4	48	29	16	10	49	2064216	4350126

						19/	/D7i	
1	45	40	4	42	12	31	0650514	0243642
21	9	35	11	36	43	20	2141562	6163005
34	47	17	18	13	44	2	4622160	5423511
22	23	24	25	26	27	28	3333333	0123456
48	6	37	32	33	3	16	6054402	5513421
30	7	14	39	15	41	29	4015254	1663050
19	38	8	46	10	5	49	2516106	4203246

						20/	/D7i	
1	45	40	12	18	28	31	0651234	0244362
21	9	27	35	36	43	4	2134560	6156003
34	39	17	2	13	44	26	4520163	5321514
30	47	8	25	42	3	20	4613502	1403625
24	6	37	48	33	11	16	3056412	2515431
46	7	14	15	23	41	29	6012354	3660150
19	22	32	38	10	5	49	2345106	4032246

						21/	/D7i	
1	27	48	47	11	10	31	0366114	0554322
21	9	12	34	36	43	20	2114562	6145005
44	22	17	18	13	26	35	6322134	1023546
45	46	8	25	42	4	5	6613500	2303634
15	24	37	32	33	28	6	2354430	0213465
30	7	14	16	38	41	29	4012554	1661250
19	40	39	3	2	23	49	2550036	4432116

						22/	/D7i	
1	48	30	38	11	16	31	0645124	0512312
18	9	5	40	14	43	46	2105166	3144603
44	23	17	29	37	22	3	6324530	1120102
42	26	35	25	15	24	8	5343231	6463020
47	28	13	21	33	27	6	6312430	4656455
4	7	36	10	45	41	32	0051654	3602253
19	34	39	12	20	2	49	2451206	4534516

						23/	/D7i	
1	45	28	16	42	12	31	0632514	0261642
21	9	47	11	24	43	20	2161362	6143205
46	35	17	6	37	32	2	6420540	3625131
10	23	36	25	14	27	40	1353135	2103654
48	18	13	44	33	15	4	6216420	5351403
30	7	26	39	3	41	29	4035054	1643250
19	38	8	34	22	5	49	2514306	4205046

						24/	/D7i	
1	45	28	24	6	40	31	0633054	0262542
21	9	27	47	12	43	16	2136162	6154401
46	39	17	2	37	20	14	6520521	3321156
18	35	8	25	42	15	32	2413524	3603603
36	30	13	48	33	11	4	5416410	0155433
34	7	38	3	23	41	29	4050354	5622150
19	10	44	26	22	5	49	2163306	4214046

						25/	/D7i	
1	39	34	21	35	8	37	0542415	0356601
27	9	12	36	24	19	48	3115326	5140245
40	30	17	46	7	32	3	5426040	4123632
45	6	28	25	22	44	5	6033360	2563014
47	18	43	4	33	20	10	6260421	4303452
2	31	26	14	38	41	23	0431553	1246251
13	42	15	29	16	11	49	1524216	5600136

						26/	/D7i	
1	45	28	27	35	2	37	0633405	0265611
21	9	12	36	24	31	42	2115345	6140226
46	30	17	40	7	32	3	6425040	3124632
39	6	34	25	16	44	11	5043261	3553113
47	18	43	10	33	20	4	6261420	4302453
8	19	26	14	38	41	29	1231554	0446250
13	48	15	23	22	5	49	1623306	5501046

```
                27/  /D7i                                         28/  /D7i
 1 47 30 27 29  4 37  0643405 0415031    1 42 36 38 11 10 37  0555115 0602321
39  9 14 22 16 43 32  5113264 3160103   18  9  5 34 20 43 46  2104266 3145503
24 12 17 42 31 44  5  3125460 2426214   44 23 17 35 31 22  3  6324430 1126202
35 48 40 25 10  2 15  4653102 6543210   48 26 29 25 21 24  2  6343230 5403621
45  6 19  8 33 38 26  6021453 2540424   47 28 19 15 33 27  6  6322430 4640455
18  7 34 28 36 41 11  2043551 3656053    4  7 30 16 45 41 32  0042654 3611253
13 46 21 23 20  3 49  1623206 5361526   13 40 39 12 14  8 49  1551116 5434606
                29/  /D7i                                         30/  /D7i
 1 30 48 22 21 10 43  0463216 0150620    1 46 42  2 27 14 43  0650316 0361560
16  9 36 18 47 37 12  2152651 1103414   20  9 34 12 39 37 24  2141553 5154312
46 26 17  5 19 35 27  6320243 3424465   32 28 17 15 31 47  5  4322460 3620244
44 42 11 25 39  8  6  6513510 1633305   44 40 29 25 21 10  6  6543210 1403625
23 15 31 45 33 24  4  3246430 1022423   45  3 19 35 33 22 18  6024432 2246403
38 13  3 32 14 41 34  5104154 2523655   26 13 11 38 16 41 30  3115254 4532151
 7 40 29 28  2 20 49  0543026 6406156    7 36 23 48  8  4 49  0536106 6015036
                31/  /D7i                                         32/  /D7i
 1 44 32  3 38 14 43  0640516 0132260    1 42 27 44 14  4 43  0536106 0651630
34  9 46 11 28 37 10  4161351 5133612   24  9 11 40 20 37 34  3115254 2134515
30 26 17 15 31 48  8  4322461 1420250   47  5 17 29 31 28 18  6024432 4420263
21 45 27 25 23  5 29  2633304 6253140   48 38 35 25 15 12  2  6543210 5263041
42  2 19 35 33 24 20  5024432 6146425   32 22 19 21 33 45  3  4322460 3046422
40 13 22 39  4 41 16  5135052 4503351   16 13 30 10 39 41 26  2141553 1512354
 7 36 12 47 18  6 49  0516206 6044356    7 46 36  6 23  8 49  0650316 6305106
                33/  /D7i
 1 38 26 14 21 32 43  0531246 0246630
28  9 46 34 15 37  6  3164250 6135015
48 40 17  5 31 23 11  6520431 5424213
 8 30  3 25 47 20 42  1403625 0123456
39 27 19 45 33 10  2  5326410 3542421
44 13 35 16  4 41 22  6142053 1561350
 7 18 29 36 24 12 49  0245316 6300246   [Count = 33]  OK!
```

In the list above each solution has its decomposition layers by 7th increment, in separate forms for higher position and lower.

Find such solutions that should contain the next special properties:

[Definitions of "Complete Euler Squares" of Order 7]

(1) Every row and every column must consist of {0, 1, 2, 3, 4, 5 and 6} using each strictly once even in both high and low layers.

(2) Every pan-diagonal must consist of {0, 1, 2, 3, 4, 5 and 6} using each strictly once even in both high and low layers.

(3) Suppose you combine each value of the corresponding positions high and low, and calculate the value of each variable in the next equation: Vn=Vh*7+Vl.

The result must always be any one of integers from 00 to 66(N7i) appearing strictly once. Neither repetition nor drop-off of a certain integer must be found.

I would like to call them the "Complete Euler Squares" from now on.

The sum of each row, each column and each pandiagonal is always calculated in the same way as:

$(0+1+2+3+4+5+6) \times 7^1 + (0+1+2+3+4+5+6) \times 7^0 = 168$(Decimal; equivalent to 175 in the classical style)

Once you know of this property, you don't have to calculate any longer to know the magic sum. You have to see only the constant pattern of the contents.

Could you find and pick up any solutions of this kind among those above?

Yes. There are two such solutions: No.8 and No.12.

There are only two solutions among 33, really rare, precious jewels.

You must be surprised that 31 non-'Euler Squares' exist among 33.

In the case of order 5, Simultaneous type of magic squares are all 'Complete Euler Squares,' and nothing else. On the other hand in the case of order 7, 'non-Euler Squares' are found far more than Euler ones. It is really surprising.

What do we have to do with these 'non-Euler Squares'? How embarrassing it is!

#4. How many 'Complete Euler Squares' 7x7 could we find?

The next job we have to do is to calculate every 'Complete Euler Squares' of order 7 for our Simultaneous MS77: both Self-complementary and Pan-diagonal and to get the complete list of standard solutions of them. But how can we do that?

Guess if we have to pick up to select them out of all solutions of Simultaneous type. But, we have not yet counted the latter through. It is terrible we imagine how long it should take a time for us to do that job?

We have to make our objects in any other direct method of composition. But how?

I had to invent anything new about effective ways of compositions to do that job.

I wrote some articles and reported of them in some sections of Chapter 4, Part 4.

Let me tell you only about the conclusions here as follows:

*** Complete Euler Squares for Simultaneous MS77: Both S-C. & P-D. ***

```
    1/H     9/L                      1#       1/H     10/L                    2#
0641352 0516234   1 48 30 14 24 39 19    0641352 0516432   1 48 30 14 26 39 17
5206413 6234051  42 17  4 47 29 13 23    5206413 6432051  42 19  4 45 29 13 23
1352064 4051623  12 22 41 16  7 45 32    1352064 2051643  10 22 41 16  7 47 32
6413520 1623405  44 35 10 25 40 15  6    6413520 1643205  44 35 12 25 38 15  6
2064135 3405162  18  5 43 34  9 28 38    2064135 3205164  18  3 43 34  9 28 40
3520641 5162340  27 37 21  3 46 33  8    3520641 5164320  27 37 21  5 46 31  8
4135206 2340516  31 11 26 36 20  2 49    4135206 4320516  33 11 24 36 20  2 49
    2/H     9/L                      3#       2/H     10/L                    4#
0621354 0516234   1 48 16 14 24 39 33    0621354 0516432   1 48 16 14 26 39 31
5406213 6234051  42 31  4 47 15 13 23    5406213 6432051  42 33  4 45 15 13 23
1354062 4051623  12 22 41 30  7 45 18    1354062 2051643  10 22 41 30  7 47 18
6213540 1623405  44 21 10 25 40 29  6    6213540 1643205  44 21 12 25 38 29  6
4062135 3405162  32  5 43 20  9 28 38    4062135 3205164  32  3 43 20  9 28 40
3540621 5162340  27 37 35  3 46 19  8    3540621 5164320  27 37 35  5 46 17  8
2135406 2340516  17 11 26 36 34  2 49    2135406 4320516  19 11 24 36 34  2 49
    1/H    11/L                      5#       3/H      9/L                    9#
0641352 0531624   1 48 32  9 28 38 19    0652341 0516234   1 48 37 21 24 32 12
5206413 3162405  39 16  7 45 33  8 27    4106523 6234051  35 10  4 47 36 20 23
1352064 6240531  14 24 40 15  6 46 30    2341065 4051623  19 22 34  9  7 45 39
6413520 4053162  47 29 13 25 37 21  3    6523410 1623405  44 42 17 25 33  8  6
2064135 5316240  20  4 44 35 10 26 36    1065234 3405162  11  5 43 41 16 28 31
3520641 1624053  23 42 17  5 43 34 11    3410652 5162340  27 30 14  3 46 40 15
4135206 2405316  31 12 22 41 18  2 49    5234106 2340516  38 18 26 29 13  2 49
    3/H    10/L                     10#       4/H      9/L                   11#
0652341 0516432   1 48 37 21 26 32 10    0612345 0516234   1 48  9 21 24 32 40
4106523 6432051  35 12  4 45 36 20 23    4506123 6234051  35 38  4 47  8 20 23
2341065 2051643  17 22 34  9  7 47 39    2345061 4051623  19 22 34 37  7 45 11
6523410 1643205  44 42 19 25 31  8  6    6123450 1623405  44 14 17 25 33 36  6
1065234 3205164  11  3 43 41 16 28 33    5061234 3405162  39  5 43 13 16 28 31
3410652 5164320  27 30 14  5 46 38 15    3450612 5162340  27 30 42  3 46 12 15
5234106 4320516  40 18 24 29 13  2 49    1234506 2340516  10 18 26 29 41  2 49
    4/H    10/L                     12#       3/H     11/L                   13#
0612345 0516432   1 48  9 21 26 32 38    0652341 0531624   1 48 39 16 28 31 12
4506123 6432051  35 40  4 45  8 20 23    4106523 3162405  32  9  7 45 40 15 27
2345061 2051643  17 22 34 37  7 47 11    2341065 6240531  21 24 33  8  6 46 37
6123450 1643205  44 14 19 25 31 36  6    6523410 4053162  47 36 20 25 30 14  3
5061234 3205164  39  3 43 13 16 28 33    1065234 5316240  13  4 44 42 17 26 29
3450612 5164320  27 30 42  5 46 10 15    3410652 1624053  23 35 10  5 43 41 18
1234506 4320516  12 18 24 29 41  2 49    5234106 2405316  38 19 22 34 11  2 49
```

5/H	9/L							17#
0654321	0516234	1	48	37	35	24	18	12
2106543	6234051	21	10	4	47	36	34	23
4321065	4051623	33	22	20	9	7	45	39
6543210	1623405	44	42	31	25	19	8	6
1065432	3405162	11	5	43	41	30	28	17
3210654	5162340	27	16	14	3	46	40	29
5432106	2340516	38	32	26	15	13	2	49

5/H	10/L							18#
0654321	0516432	1	48	37	35	26	18	10
2106543	6432051	21	12	4	45	36	34	23
4321065	2051643	31	22	20	9	7	47	39
6543210	1643205	44	42	33	25	17	8	6
1065432	3205164	11	3	43	41	30	28	19
3210654	5164320	27	16	14	5	46	38	29
5432106	4320516	40	32	24	15	13	2	49

6/H	9/L							19#
0614325	0516234	1	48	9	35	24	18	40
2506143	6234051	21	38	4	47	8	34	23
4325061	4051623	33	22	20	37	7	45	11
6143250	1623405	44	14	31	25	19	36	6
5061432	3405162	39	5	43	13	30	28	17
3250614	5162340	27	16	42	3	46	12	29
1432506	2340516	10	32	26	15	41	2	49

6/H	10/L							20#
0614325	0516432	1	48	9	35	26	18	38
2506143	6432051	21	40	4	45	8	34	23
4325061	2051643	31	22	20	37	7	47	11
6143250	1643205	44	14	33	25	17	36	6
5061432	3205164	39	3	43	13	30	28	19
3250614	5164320	27	16	42	5	46	10	29
1432506	4320516	12	32	24	15	41	2	49

5/H	11/L							21#
0654321	0531624	1	48	39	30	28	17	12
2106543	3162405	18	9	7	45	40	29	27
4321065	6240531	35	24	19	8	6	46	37
6543210	4053162	47	36	34	25	16	14	3
1065432	5316240	13	4	44	42	31	26	15
3210654	1624053	23	21	10	5	43	41	32
5432106	2405316	38	33	22	20	11	2	49

7/H	9/L							25#
0645312	0516234	1	48	30	42	24	11	19
1206453	6234051	14	17	4	47	29	41	23
5312064	4051623	40	22	13	16	7	45	32
6453120	1623405	44	35	38	25	12	15	6
2064531	3405162	18	5	43	34	37	28	10
3120645	5162340	27	9	21	3	46	33	36
4531206	2340516	31	39	26	8	20	2	49

7/H	10/L							26#
0645312	0516432	1	48	30	42	26	11	17
1206453	6432051	14	19	4	45	29	41	23
5312064	2051643	38	22	13	16	7	47	32
6453120	1643205	44	35	40	25	10	15	6
2064531	3205164	18	3	43	34	37	28	12
3120645	5164320	27	9	21	5	46	31	36
4531206	4320516	33	39	24	8	20	2	49

8/H	9/L							27#
0625314	0516234	1	48	16	42	24	11	33
1406253	6234051	14	31	4	47	15	41	23
5314062	4051623	40	22	13	30	7	45	18
6253140	1623405	44	21	38	25	12	29	6
4062531	3405162	32	5	43	20	37	28	10
3140625	5162340	27	9	35	3	46	19	36
2531406	2340516	17	39	26	8	34	2	49

8/H	10/L							28#
0625314	0516432	1	48	16	42	26	11	31
1406253	6432051	14	33	4	45	15	41	23
5314062	2051643	38	22	13	30	7	47	18
6253140	1643205	44	21	40	25	10	29	6
4062531	3205164	32	3	43	20	37	28	12
3140625	5164320	27	9	35	5	46	17	36
2531406	4320516	19	39	24	8	34	2	49

7/H	11/L							29#
0645312	0531624	1	48	32	37	28	10	19
1206453	3162405	11	16	7	45	33	36	27
5312064	6240531	42	24	12	15	6	46	30
6453120	4053162	47	29	41	25	9	21	3
2064531	5316240	20	4	44	35	38	26	8
3120645	1624053	23	14	17	5	43	34	39
4531206	2405316	31	40	22	13	18	2	49

1/H	13/L	33#				
1	47	31	14	23	39	20
42	16	4	48	29	12	24
13	22	40	17	7	44	32
45	35	9	25	41	15	5
18	6	43	33	10	28	37
26	38	21	2	46	34	8
30	11	27	36	19	3	49

1/H	15/L	37#				
1	47	32	10	28	37	20
39	17	7	44	34	8	26
14	23	41	15	5	46	31
48	29	12	25	38	21	2
19	4	45	35	9	27	36
24	42	16	6	43	33	11
30	13	22	40	18	3	49

3/H	13/L	41#				
1	47	38	21	23	32	13
35	9	4	48	36	19	24
20	22	33	10	7	44	39
45	42	16	25	34	8	5
11	6	43	40	17	28	30
26	31	14	2	46	41	15
37	18	27	29	12	3	49

3/H	15/L	45#				
1	47	39	17	28	30	13
32	10	7	44	41	15	26
21	23	34	8	5	46	38
48	36	19	25	31	14	2
12	4	45	42	16	27	29
24	35	9	6	43	40	18
37	20	22	33	11	3	49

5/H	13/L	49#				
1	47	38	35	23	18	13
21	9	4	48	36	33	24
34	22	19	10	7	44	39
45	42	30	25	20	8	5
11	6	43	40	31	28	16
26	17	14	2	46	41	29
37	32	27	15	12	3	49

5/H	15/L	53#				
1	47	39	31	28	16	13
18	10	7	44	41	29	26
35	23	20	8	5	46	38
48	36	33	25	17	14	2
12	4	45	42	30	27	15
24	21	9	6	43	40	32
37	34	22	19	11	3	49

7/H	13/L	57#				
1	47	31	42	23	11	20
14	16	4	48	29	40	24
41	22	12	17	7	44	32
45	35	37	25	13	15	5
18	6	43	33	38	28	9
26	10	21	2	46	34	36
30	39	27	8	19	3	49

7/H	15/L	61#				
1	47	32	38	28	9	20
11	17	7	44	34	36	26
42	23	13	15	5	46	31
48	29	40	25	10	21	2
19	4	45	35	37	27	8
24	14	16	6	43	33	39
30	41	22	12	18	3	49

1/H	17/L	65#				
1	46	35	10	27	37	19
41	16	5	43	32	14	24
11	28	38	20	2	47	29
44	33	8	25	42	17	6
21	3	48	30	12	22	39
26	36	18	7	45	34	9
31	13	23	40	15	4	49

1/H	19/L	69#				
1	46	35	9	26	38	20
40	17	6	43	32	14	23
11	28	37	19	3	48	29
45	34	8	25	42	16	5
21	2	47	31	13	22	39
27	36	18	7	44	33	10
30	12	24	41	15	4	49

1/H	21/L	73#				
1	46	35	9	24	40	20
38	19	6	43	32	14	23
11	28	37	17	5	48	29
47	34	8	25	42	16	3
21	2	45	33	13	22	39
27	36	18	7	44	31	12
30	10	26	41	15	4	49

1/H	23/L	77#				
1	46	35	10	23	41	19
37	20	5	43	32	14	24
11	28	38	16	6	47	29
48	33	8	25	42	17	2
21	3	44	34	12	22	39
26	36	18	7	45	30	13
31	9	27	40	15	4	49

3/H 17/L 81#

1	46	42	17	27	30	12
34	9	5	43	39	21	24
18	28	31	13	2	47	36
44	40	15	25	35	10	6
14	3	48	37	19	22	32
26	29	11	7	45	41	16
38	20	23	33	8	4	49

3/H 19/L 85#

1	46	42	16	26	31	13
33	10	6	43	39	21	23
18	28	30	12	3	48	36
45	41	15	25	35	9	5
14	2	47	38	20	22	32
27	29	11	7	44	40	17
37	19	24	34	8	4	49

3/H 21/L 89#

1	46	42	16	24	33	13
31	12	6	43	39	21	23
18	28	30	10	5	48	36
47	41	15	25	35	9	3
14	2	45	40	20	22	32
27	29	11	7	44	38	19
37	17	26	34	8	4	49

3/H 23/L 93#

1	46	42	17	23	34	12
30	13	5	43	39	21	24
18	28	31	9	6	47	36
48	40	15	25	35	10	2
14	3	44	41	19	22	32
26	29	11	7	45	37	20
38	16	27	33	8	4	49

5/H 17/L 97#

1	46	42	31	27	16	12
20	9	5	43	39	35	24
32	28	17	13	2	47	36
44	40	29	25	21	10	6
14	3	48	37	33	22	18
26	15	11	7	45	41	30
38	34	23	19	8	4	49

5/H 19/L 101#

1	46	42	30	26	17	13
19	10	6	43	39	35	23
32	28	16	12	3	48	36
45	41	29	25	21	9	5
14	2	47	38	34	22	18
27	15	11	7	44	40	31
37	33	24	20	8	4	49

5/H 21/L 105#

1	46	42	30	24	19	13
17	12	6	43	39	35	23
32	28	16	10	5	48	36
47	41	29	25	21	9	3
14	2	45	40	34	22	18
27	15	11	7	44	38	33
37	31	26	20	8	4	49

5/H 23/L 109#

1	46	42	31	23	20	12
16	13	5	43	39	35	24
32	28	17	9	6	47	36
48	40	29	25	21	10	2
14	3	44	41	33	22	18
26	15	11	7	45	37	34
38	30	27	19	8	4	49

7/H 17/L 113#

1	46	35	38	27	9	19
13	16	5	43	32	42	24
39	28	10	20	2	47	29
44	33	36	25	14	17	6
21	3	48	30	40	22	11
26	8	18	7	45	34	37
31	41	23	12	15	4	49

7/H 19/L 117#

1	46	35	37	26	10	20
12	17	6	43	32	42	23
39	28	9	19	3	48	29
45	34	36	25	14	16	5
21	2	47	31	41	22	11
27	8	18	7	44	33	38
30	40	24	13	15	4	49

7/H 21/L 121#

1	46	35	37	24	12	20
10	19	6	43	32	42	23
39	28	9	17	5	48	29
47	34	36	25	14	16	3
21	2	45	33	41	22	11
27	8	18	7	44	31	40
30	38	26	13	15	4	49

7/H 23/L 125#

1	46	35	38	23	13	19
9	20	5	43	32	42	24
39	28	10	16	6	47	29
48	33	36	25	14	17	2
21	3	44	34	40	22	11
26	8	18	7	45	30	41
31	37	27	12	15	4	49

1/H 25/L 129#

1	45	33	14	23	39	20
42	16	4	48	29	10	26
13	22	38	19	7	44	32
47	35	9	25	41	15	3
18	6	43	31	12	28	37
24	40	21	2	46	34	8
30	11	27	36	17	5	49

1/H 27/L 133#

1	45	32	12	28	37	20
39	19	7	44	34	8	24
14	23	41	15	3	46	33
48	29	10	25	40	21	2
17	4	47	35	9	27	36
26	42	16	6	43	31	11
30	13	22	38	18	5	49

3/H 25/L 137#

1	45	40	21	23	32	13
35	9	4	48	36	17	26
20	22	31	12	7	44	39
47	42	16	25	34	8	3
11	6	43	38	19	28	30
24	33	14	2	46	41	15
37	18	27	29	10	5	49

3/H 27/L 141#

1	45	39	19	28	30	13
32	12	7	44	41	15	24
21	23	34	8	3	46	40
48	36	17	25	33	14	2
10	4	47	42	16	27	29
26	35	9	6	43	38	18
37	20	22	31	11	5	49

5/H 25/L 145#

1	45	40	35	23	18	13
21	9	4	48	36	31	26
34	22	17	12	7	44	39
47	42	30	25	20	8	3
11	6	43	38	33	28	16
24	19	14	2	46	41	29
37	32	27	15	10	5	49

5/H 27/L 149#

1	45	39	33	28	16	13
18	12	7	44	41	29	24
35	23	20	8	3	46	40
48	36	31	25	19	14	2
10	4	47	42	30	27	15
26	21	9	6	43	38	32
37	34	22	17	11	5	49

7/H 25/L 153#

1	45	33	42	23	11	20
14	16	4	48	29	38	26
41	22	10	19	7	44	32
47	35	37	25	13	15	3
18	6	43	31	40	28	9
24	12	21	2	46	34	36
30	39	27	8	17	5	49

7/H 27/L 157#

1	45	32	40	28	9	20
11	19	7	44	34	36	24
42	23	13	15	3	46	33
48	29	38	25	12	21	2
17	4	47	35	37	27	8
26	14	16	6	43	31	39
30	41	22	10	18	5	49

1/H 29/L 161#

1	44	34	14	24	39	19
42	17	4	47	29	9	27
12	22	37	20	7	45	32
48	35	10	25	40	15	2
18	5	43	30	13	28	38
23	41	21	3	46	33	8
31	11	26	36	16	6	49

9/H 1/L 193#

1	42	26	9	46	20	31
27	10	43	21	33	2	39
44	18	34	3	36	28	12
35	5	37	25	13	45	15
38	22	14	47	16	32	6
11	48	17	29	7	40	23
19	30	4	41	24	8	49

9/H 9/L 209#

1	41	23	14	45	18	33
28	10	46	19	29	6	37
47	15	34	2	42	24	11
30	7	38	25	12	43	20
39	26	8	48	16	35	3
13	44	21	31	4	40	22
17	32	5	36	27	9	49

9/H 13/L 213#

1	40	24	14	44	18	34
28	9	46	20	29	5	38
48	15	33	3	42	23	11
31	7	37	25	13	43	19
39	27	8	47	17	35	2
12	45	21	30	4	41	22
16	32	6	36	26	10	49

9/H 17/L 217#

1	39	28	10	48	16	33
27	9	47	15	32	7	38
46	21	31	6	37	26	8
30	5	36	25	14	45	20
42	24	13	44	19	29	4
12	43	18	35	3	41	23
17	34	2	40	22	11	49

9/H 25/L 233#

1	38	26	14	44	18	34
28	9	46	20	29	3	40
48	15	31	5	42	23	11
33	7	37	25	13	43	17
39	27	8	45	19	35	2
10	47	21	30	4	41	22
16	32	6	36	24	12	49

9/H 29/L 237#

1	37	27	14	45	18	33
28	10	46	19	29	2	41
47	15	30	6	42	24	11
34	7	38	25	12	43	16
39	26	8	44	20	35	3
9	48	21	31	4	40	22
17	32	5	36	23	13	49

11/H 1/L 241#

1	35	26	16	46	13	38
27	17	43	14	40	2	32
44	11	41	3	29	28	19
42	5	30	25	20	45	8
31	22	21	47	9	39	6
18	48	10	36	7	33	23
12	37	4	34	24	15	49

11/H 9/L 257#

1	34	23	21	45	11	40
28	17	46	12	36	6	30
47	8	41	2	35	24	18
37	7	31	25	19	43	13
32	26	15	48	9	42	3
20	44	14	38	4	33	22
10	39	5	29	27	16	49

11/H 13/L 261#

1	33	24	21	44	11	41
28	16	46	13	36	5	31
48	8	40	3	35	23	18
38	7	30	25	20	43	12
32	27	15	47	10	42	2
19	45	14	37	4	34	22
9	39	6	29	26	17	49

11/H 17/L 265#

1	32	28	17	48	9	40
27	16	47	8	39	7	31
46	14	38	6	30	26	15
37	5	29	25	21	45	13
35	24	20	44	12	36	4
19	43	11	42	3	34	23
10	41	2	33	22	18	49

11/H 25/L 281#

1	31	26	21	44	11	41
28	16	46	13	36	3	33
48	8	38	5	35	23	18
40	7	30	25	20	43	10
32	27	15	45	12	42	2
17	47	14	37	4	34	22
9	39	6	29	24	19	49

11/H 29/L 285#

1	30	27	21	45	11	40
28	17	46	12	36	2	34
47	8	37	6	35	24	18
41	7	31	25	19	43	9
32	26	15	44	13	42	3
16	48	14	38	4	33	22
10	39	5	29	23	20	49

13/H 1/L 289#

1	28	47	9	18	34	38
20	31	36	7	26	44	11
23	46	13	17	29	42	5
35	40	2	25	48	10	15
45	8	21	33	37	4	27
39	6	24	43	14	19	30
12	16	32	41	3	22	49

13/H 9/L 321#

1	27	44	14	17	32	40
21	31	39	5	22	48	9
26	43	13	16	35	38	4
30	42	3	25	47	8	20
46	12	15	34	37	7	24
41	2	28	45	11	19	29
10	18	33	36	6	23	49

13/H 13/L 337#

1	26	45	14	16	32	41
21	30	39	6	22	47	10
27	43	12	17	35	37	4
31	42	2	25	48	8	19
46	13	15	33	38	7	23
40	3	28	44	11	20	29
9	18	34	36	5	24	49

13/H 25/L 353#

1	24	47	14	16	32	41
21	30	39	6	22	45	12
27	43	10	19	35	37	4
33	42	2	25	48	8	17
46	13	15	31	40	7	23
38	5	28	44	11	20	29
9	18	34	36	3	26	49

13/H 29/L 369#

1	23	48	14	17	32	40
21	31	39	5	22	44	13
26	43	9	20	35	38	4
34	42	3	25	47	8	16
46	12	15	30	41	7	24
37	6	28	45	11	19	29
10	18	33	36	2	27	49

[Count(n1=1/All) = 384/3456] OK!

I have counted all the 3456 standard solutions of Complete Euler Squares for our Simultaneous MS77: Self-complementary and Pan-diagonal, independently in three different ways of compositions.

I have also counted the 38102400 standard solutions of Complete Euler Squares for Pan-diagonal Magic Squares of Order 7.

I am afraid they may look very many, but they are really rare, precious jewels, theoretically speaking. There are indeed far more 'Non-Euler Squares' of those two types. I cannot even imagine how many solutions of them there are in all.

(First Written on August 10, 2001 by Kanji Setsuda with MacOS9;
Revised on April 3, 2007 by Kanji Setsuda with MacOSX & Xcode2.2)

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