Part 1 : "Basic Studies of Magic Squares and Cubes"

Chapter 2 : Basic Study of Magic Squares: Kanji Setsuda

Section 1 : Standard Magic Squares of Order 3

1. Basic Form, Basic Conditions for Semi-Magic Squares 3x3

Let's study about the Semi-Magic Squares of Order 3 first of all, one of the most basic things. Each of them should be built in the Basic Form shown below. It must be made up of the natural numbers {1, 2, 3, 4, ..., 7, 8 and 9} using each strictly once, and they must be placed in the 3x3 regular array. The Basic Form below shows their positions and let's call the variables as "n1, n2, n3, n4, ..., n8 and n9."

```
/* Semi-Magic Squares of Order 3: */
/* Basic Form and
  .--.--.--.   Basic Definitions */
  |n1|n2|n3|   n1+n2+n3=C ... rw1;  n1+n5+n9=C ... d1;
  |--+--+--|   n4+n5+n6=C ... rw2;
  |n4|n5|n6|   n7+n8+n9=C ... rw3;
  |--+--+--|   n1+n4+n7=C ... cl1;
  |n7|n8|n9|   n2+n5+n8=C ... cl2;
  '--'--'--'   n3+n6+n9=C ... cl3;  n3+n5+n7=C ... d2;
```

2. We must suppose every 3 entries on any row and any column in the two directions should add up to the magic constant C, but now we don't yet define anything about every sum of 3 entries on the two primary diagonals (d1, d2).

```
rw1+rw2+rw3
    n1+n2+n3+n4+n5+n6+n7+n8+n9  = 3xC
                1+2+3+4+5+6+7+8+9  = 3xC
                         (1+9)x9/2  = 3xC
                       Therefore  C =  15
```

3. It is natural that there are not a few solutions for this type, for we have only 6 simultaneous equations while we have as many variables as 9.

Let’s dictate such a good program for our computer as to make it calculate and count how many solutions there are in all for this type, shall we?

I would like to show you a little program of mine as listed below.

I actually wrote it as a simple, primitive program here so that all of you, beginners, could understand well. Since it does not use any 'List-forming Inequality Conditions' yet, it should list out the larger set of all the 'primitive' solutions.

```
/** Primitive PPROGRAM for Semi-Magic Squares of Order 3 **/
/** 'SMS33Prmtv.c' built by Kanji Setsuda **/
/**  Nov. 28, 2008; on MacOSX & Xcode 3.0 **/
/**/
#include <stdio.h>
/**/
short int cnt, LSM;
short n1,n2,n3,n4,n5,n6,n7,n8,n9;
short uflg[10];
/**/
/* Main Program */  (to be continued)
```

*** Semi-Magic Squares of Order 3: List of the Primitive Solutions ***

```
[1]
  2  9  4
  6  1  8
  7  5  3
[2]
  2  6  7
  9  1  5
  4  8  3
[3]
  3  8  4
  5  1  9
  7  6  2
[4]
  3  5  7
  8  1  6
  4  9  2
[5]
  4  9  2
  8  1  6
  3  5  7
[6]
  4  8  3
  9  1  5
  2  6  7
[7]
  7  6  2
  5  1  9
  3  8  4
[8]
  7  5  3
  6  1  8
  2  9  4
[9]
  1  9  5
  6  2  7
  8  4  3
[10]
  1  6  8
  9  2  4
  5  7  3
[11]
  3  7  5
  4  2  9
  8  6  1
[12]
  3  4  8
  7  2  6
  5  9  1
[13]
  5  9  1
  7  2  6
  3  4  8
[14]
  5  7  3
  9  2  4
  1  6  8
[15]
  8  6  1
  4  2  9
  3  7  5
[16]
  8  4  3
  6  2  7
  1  9  5
[17]
  1  8  6
  5  3  7
  9  4  2
[18]
  1  5  9
  8  3  4
  6  7  2
[19]
  2  7  6
  4  3  8
  9  5  1
[20]
  2  4  9
  7  3  5
  6  8  1
[21]
  6  8  1
  7  3  5
  2  4  9
[22]
  6  7  2
  8  3  4
  1  5  9
[23]
  9  5  1
  4  3  8
  2  7  6
[24]
  9  4  2
  5  3  7
  1  8  6
[25]
  1  9  5
  8  4  3
  6  2  7
[26]
  1  8  6
  9  4  2
  5  3  7
[27]
  5  9  1
  3  4  8
  7  2  6
[28]
  5  3  7
  9  4  2
  1  8  6
[29]
  6  8  1
  2  4  9
  7  3  5
[30]
  6  2  7
  8  4  3
  1  9  5
[31]
  7  3  5
  2  4  9
  6  8  1
[32]
  7  2  6
  3  4  8
  5  9  1
[33]
  2  9  4
  7  5  3
  6  1  8
[34]
  2  7  6
  9  5  1
  4  3  8
[35]
  4  9  2
  3  5  7
  8  1  6
[36]
  4  3  8
  9  5  1
  2  7  6
[37]
  6  7  2
  1  5  9
  8  3  4
[38]
  6  1  8
  7  5  3
  2  9  4
[39]
  8  3  4
  1  5  9
  6  7  2
[40]
  8  1  6
  3  5  7
  4  9  2
[41]
  3  8  4
  7  6  2
  5  1  9
[42]
  3  7  5
  8  6  1
  4  2  9
[43]
  4  8  3
  2  6  7
  9  1  5
[44]
  4  2  9
  8  6  1
  3  7  5
[45]
  5  7  3
  1  6  8
  9  2  4
[46]
  5  1  9
  7  6  2
  3  8  4
[47]
  9  2  4
  1  6  8
  5  7  3
[48]
  9  1  5
  2  6  7
  4  8  3
[49]
  1  6  8
  5  7  3
  9  2  4
[50]
  1  5  9
  6  7  2
  8  3  4
[51]
  4  3  8
  2  7  6
  9  5  1
[52]
  4  2  9
  3  7  5
  8  6  1
[53]
  8  6  1
  3  7  5
  4  2  9
[54]
  8  3  4
  6  7  2
  1  5  9
[55]
  9  5  1
  2  7  6
  4  3  8
[56]
  9  2  4
  5  7  3
  1  6  8
[57]
  2  6  7
  4  8  3
  9  1  5
[58]
  2  4  9
  6  8  1
  7  3  5
[59]
  5  3  7
  1  8  6
  9  4  2
[60]
  5  1  9
  3  8  4
  7  6  2
[61]
  7  6  2
  3  8  4
  5  1  9
[62]
  7  3  5
  6  8  1
  2  4  9
[63]
  9  4  2
  1  8  6
  5  3  7
[64]
  9  1  5
  4  8  3
  2  6  7
[65]
  3  5  7
  4  9  2
  8  1  6
[66]
  3  4  8
  5  9  1
  7  2  6
[67]
  6  2  7
  1  9  5
  8  4  3
[68]
  6  1  8
  2  9  4
  7  5  3
[69]
  7  5  3
  2  9  4
  6  1  8
[70]
  7  2  6
  5  9  1
  3  4  8
[71]
  8  4  3
  1  9  5
  6  2  7
[72]
  8  1  6
  4  9  2
  3  5  7
[Count = 72]
  OK!
```

```
/* Main Program */  (continues)
int main(){
 short n;
  printf("\n* Semi-Magic Squares 3x3 *\n");
  for(n=0;n<10;n++){uflg[n]=0;}
  LSM=15; cnt=0;
  for(n5=1;n5<10;n5++){
   if(uflg[n5]==0){uflg[n5]=1;
    for(n1=1;n1<10;n1++){
     if(uflg[n1]==0){uflg[n1]=1;
      for(n9=1;n9<10;n9++){
       if(uflg[n9]==0){uflg[n9]=1;
        for(n3=1;n3<10;n3++){
         if(uflg[n3]==0){uflg[n3]=1;
          for(n7=1;n7<10;n7++){
           if(uflg[n7]==0){uflg[n7]=1;
            for(n2=1;n2<10;n2++){
             if((n1+n2+n3==LSM)&&(uflg[n2]==0)){uflg[n2]=1;
              for(n4=1;n4<10;n4++){
               if((n1+n4+n7==LSM)&&(uflg[n4]==0)){uflg[n4]=1;
                for(n6=1;n6<10;n6++){
                 if((n4+n5+n6==LSM)&&(n3+n6+n9==LSM)&&(uflg[n6]==0)){uflg[n6]=1;
                  for(n8=1;n8<10;n8++){
                   if((n2+n5+n8==LSM)&&(n7+n8+n9==LSM)&&(uflg[n8]==0)){
                    uflg[n8]=1;
                    cnt++;
                    printf("[%d]\n",cnt);
                    printf("%3d%3d%3d\n",n1,n2,n3);
                    printf("%3d%3d%3d\n",n4,n5,n6);
                    printf("%3d%3d%3d\n",n7,n8,n9);
                    uflg[n8]=0;}}
                  uflg[n6]=0;}}
                uflg[n4]=0;}}
              uflg[n2]=0;}}
            uflg[n7]=0;}}
          uflg[n3]=0;}}
        uflg[n9]=0;}}
      uflg[n1]=0;}}
    uflg[n5]=0;}}
  printf("[Count = %d]\n",cnt); printf("  OK!\n");
 return 0;
}
/**/
```

4. Would you watch the solution list above and check if it has several sets of 'a little different but essentially the same' solutions? Each of them could be identified with another one. Don't you think every 8 solutions should be classified into a same group? They are really either 'reflected' or 'rotated' patterns of the same solution.

*** Mirror Reflection and Rotations: Conceptual Diagrams ***

```
             1/ -(Rotate)-> 2/ -(Rotate)-> 3/ -(Rotate)-> 4/
 .---.---.---.   .---.---.---.   .---.---.---.   .---.---.---.
 | 2 | 9 | 4 |   | 6 | 7 | 2 |   | 8 | 1 | 6 |   | 4 | 3 | 8 |
 |---+---+---|   |---+---+---|   |---+---+---|   |---+---+---|
 | 7 | 5 | 3 |   | 1 | 5 | 9 |   | 3 | 5 | 7 |   | 9 | 5 | 1 |
 |---+---+---|   |---+---+---|   |---+---+---|   |---+---+---|
 | 6 | 1 | 8 |   | 8 | 3 | 4 |   | 4 | 9 | 2 |   | 2 | 7 | 6 |
 '---'---'---'   '---'---'---'   '---'---'---'   '---'---'---'
       ↓ (Reflect with respect to the primary diagonal {2, 5, 8})
```

```
     ↓    5/ -(Rotate)-> 6/ -(Rotate)-> 7/ -(Rotate)-> 8/
 .---.---.---.  .---.---.---.  .---.---.---.  .---.---.---.
 | 2 | 7 | 6 |  | 4 | 9 | 2 |  | 8 | 3 | 4 |  | 6 | 1 | 8 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 9 | 5 | 1 |  | 3 | 5 | 7 |  | 1 | 5 | 9 |  | 7 | 5 | 3 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 4 | 3 | 8 |  | 8 | 1 | 6 |  | 6 | 7 | 2 |  | 2 | 9 | 4 |
 '---'---'---'  '---'---'---'  '---'---'---'  '---'---'---'
```

5. Why don't you think we could take a representative out of 8 solutions in the same group, and we could get the smart list of only those representative ones?
But, how can we make it?
We can use such the 'List-forming Inequality Conditions' as follows. Let me demonstrate here how they precisely effect when they select those solutions.

```
** The Effect of Inequality Conditions **
* Basic Form & Basic Conditions
  .--.--.--.  for Standard Magic Squares 3x3 *
  |n1|n2|n3|  n1+n2+n3=C; n1+n4+n7=C;
  |--+--+--|  n4+n5+n6=C; n2+n5+n8=C;
  |n4|n5|n6|  n7+n8+n9=C; n3+n6+n9=C;
  |--+--+--|  n1+n5+n9=C; n3+n5+n7=C;
  |n7|n8|n9|  ('C' means the Magic Constant 15.)
  '--'--'--'

** Case 1: Under No Inequality Condition **
       1/          2/          3/          4/
   2  9  4    2  7  6    4  9  2    4  3  8
   7  5  3    9  5  1    3  5  7    9  5  1
   6  1  8    4  3  8    8  1  6    2  7  6
       5/          6/          7/          8/
   6  7  2    6  1  8    8  3  4    8  1  6
   1  5  9    7  5  3    1  5  9    3  5  7
   8  3  4    2  9  4    6  7  2    4  9  2
 [Count = 8]  OK!

 ** Case 2: Select under n1<n9; **
       1/          2/          3/          4/
   2  9  4    2  7  6    4  9  2    4  3  8
   7  5  3    9  5  1    3  5  7    9  5  1
   6  1  8    4  3  8    8  1  6    2  7  6
 [Count = 4]  OK!

 ** Case 3: Select under {n1<n3; n1<n9;} **
       1/          2/          3/
   2  9  4    2  7  6    4  3  8
   7  5  3    9  5  1    9  5  1
   6  1  8    4  3  8    2  7  6
 [Count = 3]  OK!

 ** Case 4: Select under {n1<n3; n1<n7; n1<n9;} **
       1/          2/
   2  9  4    2  7  6
   7  5  3    9  5  1
   6  1  8    4  3  8
 [Count = 2]  OK!

 ** Case 5: Select under {n1<n3; n3<n7; n1<n9;} **
```

```
        1/
   2   9   4
   7   5   3
   6   1   8
 [Count = 1]   OK!
```

```
** Case 6: Select under {n1<n3; n1<n7; n1<n9; n2>n4;} **
        1/
   2   9   4
   7   5   3
   6   1   8
 [Count = 1]   OK!
```

We can use either {n1<n3; n3<n7; n1<n9;} or {n1<n3; n1<n7; n1<n9; n2>n4;} for our purpose, so that we can select the only one 'Standard Solution' out of the 8 'Primitive Solutions' in a same group.

6. Let's have the smart list of 'Standard Solutions' for our object Semi-Magic Squares of Order 3 under the condition set {n1<n3; n1<n7; n1<n9; n2>n4;} right here.

```
** Semi-Magic Squares of Order 3: List of Standard Solutions **
      1/        2/        3/        4/        5/
   2 9 4     1 9 5     1 8 6     1 9 5     2 9 4
   6 1 8     6 2 7     5 3 7     8 4 3     7 5 3
   7 5 3     8 4 3     9 4 2     6 2 7     6 1 8
      6/        7/        8/        9/
   3 8 4     1 6 8     2 6 7     3 5 7
   7 6 2     5 7 3     4 8 3     4 9 2
   5 1 9     9 2 4     9 1 5     8 1 6     [Count = 9]   OK!
```

7. For the next step we must examine these 9 Standard Solutions precisely to check what properties each one has got in itself.

```
[1]
   .---.---.---. Rows & Columns; Pan-diagonals
 4 | 2 | 9 | 4 | 2   2+9+4=15;   2+1+3= 6;
   |---+---+---|     6+1+8=15;   9+8+7=24;
 8 | 6 | 1 | 8 | 6   7+5+3=15;   4+6+5=15;
   |---+---+---|     2+6+7=15;   2+8+5=15;
 3 | 7 | 5 | 3 | 7   9+1+5=15;   9+6+3=18;
   '---'---'---'     4+8+3=15;   4+1+7=12;
* Complementary Pairs:
   2+3=5; 9+5=14;  4+7=11;  6+8=14;  1+1=2;
 ... This is a Semi-Magic Square 3x3!
[2]
   .---.---.---. Rows & Columns; Pan-diagonals
 5 | 1 | 9 | 5 | 1   1+9+5=15;   1+2+3= 6;
   |---+---+---|     6+2+7=15;   9+7+8=24;
 7 | 6 | 2 | 7 | 6   8+4+3=15;   5+6+4=15;
   |---+---+---|     1+6+8=15;   1+7+4=12;
 3 | 8 | 4 | 3 | 8   9+2+4=15;   9+6+3=18;
   '---'---'---'     5+7+3=15;   5+2+8=15;
* Complementary Pairs:
   1+3=4; 9+4=13;  5+8=13;  6+7=13;  2+2=4;
 ... This is a Semi-Magic Square 3x3!
```

[3]

```
   .---.---.---. Rows & Columns; Pan-diagonals
 6 | 1 | 8 | 6 | 1   1+8+6=15;   1+3+2= 6;
   |---+---+---|     5+3+7=15;   8+7+9=24;
 7 | 5 | 3 | 7 | 5   9+4+2=15;   6+5+4=15;
   |---+---+---|     1+5+9=15;   1+7+4=12;
 2 | 9 | 4 | 2 | 9   8+3+4=15;   8+5+2=15;
   '---'---'---'     6+7+2=15;   6+3+9=18;
* Complementary Pairs:
   1+2=3; 8+4=12;  6+9=15;  5+7=12;  3+3=6;
 ... This is a Semi-Magic Square 3x3!
```

[4]

```
   .---.---.---. Rows & Columns; Pan-diagonals
 5 | 1 | 9 | 5 | 1   1+9+5=15;   1+4+7=12;
   |---+---+---|     8+4+3=15;   9+3+6=18;
 3 | 8 | 4 | 3 | 8   6+2+7=15;   5+8+2=15;
   |---+---+---|     1+8+6=15;   1+3+2= 6;
 7 | 6 | 2 | 7 | 6   9+4+2=15;   9+8+7=24;
   '---'---'---'     5+3+7=15;   5+4+6=15;
* Complementary Pairs:
   1+7=8; 9+2=11;  5+6=11;  8+3=11;  4+4=8;
 ... This is a Semi-Magic Square 3x3!
```

[5]

```
   .---.---.---. Rows & Columns; Pan-diagonals
 4 | 2 | 9 | 4 | 2   2+9+4=15;   2+5+8=15;
   |---+---+---|     7+5+3=15;   9+3+6=18;
 3 | 7 | 5 | 3 | 7   6+1+8=15;   4+7+1=12;
   |---+---+---|     2+7+6=15;   2+3+1= 6;
 8 | 6 | 1 | 8 | 6   9+5+1=15;   9+7+8=24;
   '---'---'---'     4+3+8=15;   4+5+6=15;
* Complementary Pairs:
   2+8=10;  9+1=10;  4+6=10;  7+3=10;  5+5=10;
 ... This is a Self-complementary Magic Square 3x3!
```

[6]

```
   .---.---.---. Rows & Columns; Pan-diagonals
 4 | 3 | 8 | 4 | 3   3+8+4=15;   3+6+9=18;
   |---+---+---|     7+6+2=15;   8+2+5=15;
 2 | 7 | 6 | 2 | 7   5+1+9=15;   4+7+1=12;
   |---+---+---|     3+7+5=15;   3+2+1= 6;
 9 | 5 | 1 | 9 | 5   8+6+1=15;   8+7+9=24;
   '---'---'---'     4+2+9=15;   4+6+5=15;
* Complementary Pairs:
   3+9=12;  8+1=9; 4+5=9; 7+2=9; 6+6=12;
 ... This is a Semi-Magic Square 3x3!
```

[7]

```
   .---.---.---. Rows & Columns; Pan-diagonals
 8 | 1 | 6 | 8 | 1   1+6+8=15;   1+7+4=12;
   |---+---+---|     5+7+3=15;   6+3+9=18;
 3 | 5 | 7 | 3 | 5   9+2+4=15;   8+5+2=15;
   |---+---+---|     1+5+9=15;   1+3+2= 6;
 4 | 9 | 2 | 4 | 9   6+7+2=15;   6+5+4=15;
   '---'---'---'     8+3+4=15;   8+7+9=24;
* Complementary Pairs:
   1+4=5; 6+2=8; 8+9=17;  5+3=8; 7+7=14;
 ... This is a Semi-Magic Square 3x3!
```

```
[8]
   .---.---.---. Rows & Columns; Pan-diagonals
 7 | 2 | 6 | 7 | 2  2+6+7=15;   2+8+5=15;
   |---+---+---|    4+8+3=15;   6+3+9=18;
 3 | 4 | 8 | 3 | 4  9+1+5=15;   7+4+1=12;
   |---+---+---|    2+4+9=15;   2+3+1= 6;
 5 | 9 | 1 | 5 | 9  6+8+1=15;   6+4+5=15;
   '---'---'---'    7+3+5=15;   7+8+9=24;
* Complementary Pairs:
   2+5=7; 6+1=7; 7+9=16;  4+3=7; 8+8=16;
 ... This is a Semi-Magic Square 3x3!
[9]
   .---.---.---. Rows & Columns; Pan-diagonals
 7 | 3 | 5 | 7 | 3  3+5+7=15;   3+9+6=18;
   |---+---+---|    4+9+2=15;   5+2+8=15;
 2 | 4 | 9 | 2 | 4  8+1+6=15;   7+4+1=12;
   |---+---+---|    3+4+8=15;   3+2+1= 6;
 6 | 8 | 1 | 6 | 8  5+9+1=15;   5+4+6=15;
   '---'---'---'    7+2+6=15;   7+9+8=24;
* Complementary Pairs:
   3+6=9; 5+1=6; 7+8=15;  4+2=6; 9+9=18;
 ... This is a Semi-Magic Square 3x3!
[Count = 9]  OK!
```

We now know Solution [5] is a special type of Magic Square 3x3, whose 2 primary diagonals equally add up to the magic constant. And any pair situated symmetrically with respect to the central n5(=5) really adds up to 10, another magic constant.
We usually call them "Complementary Pairs of 10": (1, 9), (2, 8), (3, 7), (4, 6)
Yes. It comes up to us as a 'Self-complementary Magic Square of Order 3'.

8. Standard Magic Squares of Order 3

We now come to the step when we study about 'Standard' Magic Squares of Order 3. Two primary diagonals must add up to the same magic constant 15 here.
Adding two basic conditions (1) n1+n5+n9==LSM and (2) n3+n5+n7==LSM, I wrote a new program and made my machine calculate and list out all the primitive solutions.

```
* 'Standard' Magic Squares 3x3: With n1+n5+n9==LSM & n3+n5+n7==LSM *
      1/       2/       3/       4/       5/       6/       7/       8/
   2 9 4    2 7 6    4 9 2    4 3 8    6 7 2    6 1 8    8 3 4    8 1 6
   7 5 3    9 5 1    3 5 7    9 5 1    1 5 9    7 5 3    1 5 9    3 5 7
   6 1 8    4 3 8    8 1 6    2 7 6    8 3 4    2 9 4    6 7 2    4 9 2
 [Count = 8]  OK!
```

9. Self-Complementary Magic Squares of Order 3

Now that we know those 8 solutions above are all Self-complementary type of Magic Squares of Order 3, we want to have such an entirely new program that we could directly make this object, Self-complementary Magic Squares of Order 3.
I added such the new conditions as:(n1+n9)=(n2+n8)=(n3+n7)=(n4+n6)=10 and n5=5

```
/** Sophisticated PROGRAM #2 for **/
/**  Self-Complementary Magic Squares 3x3 **/
/** 'MS33SC2.c' dictated by Kanji Setsuda **/
/** on Nove.28,'08 with MacOSX & Xcode3.0 **/
/**/
```

```c
/*  Basic Form & Simultaneous Equations:
  .--.--.--.  n1+n2+n3=C ... eq1;
  |n1|n2|n3|  n4+n5+n6=C ... eq2;
  |--+--+--|  n7+n8+n9=C ... eq3;
  |n4|n5|n6|  n1+n4+n7=C ... eq4;
  |--+--+--|  n2+n5+n8=C ... eq5;
  |n7|n8|n9|  n3+n6+n9=C ... eq6;
  '--'--'--'  C=15;
* Self-Complementary Conditions:
  n1+n9=n2+n8=n3+n7=n4+n6=n5+n5=10 ... eq7;
 eq7 makes both Primary Diagonals add up to 15.
*/
/**/
#include <stdio.h>
/**/
short cnt, cnt2;
short LSM, CC;
short nm[10], uflg[10];
short tn[5][10];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void recordans(void);
void prans(short x);
/**/
/* Main Program */
int main(){
 short n;
  printf("\n** Self-Complementary Magic Squares of Order 3 **\n");
  for(n=0;n<10;n++){nm[n]=0; uflg[n]=0;}
  LSM=15; CC=10; nm[5]=5; uflg[5]=1;
  cnt=0; cnt2=0;
  stp01(); /* Calculations */
  if(cnt2>0){prans(cnt2);}
  printf(" [Count = %d]  OK!\n",cnt);
 return 0;
}
/* Calculations */
/* Set n1 and n9 */
void stp01(){
 short m,n;
  for(n=1;n<10;n++){m=CC-n;
    if((uflg[n]==0)&&(uflg[m]==0)){
      nm[1]=n; nm[9]=m;
      uflg[n]=1; uflg[m]=1;
      stp02();
      uflg[m]=0; uflg[n]=0;
    }
  }
}
/**/
/* Set n3 & n7 */
void stp02(){
 short m,n;
  for(n=1;n<10;n++){m=CC-n;
    if((uflg[n]==0)&&(uflg[m]==0)){
      nm[3]=n; nm[7]=m;
      uflg[n]=1; uflg[m]=1;
      stp03();
      uflg[m]=0; uflg[n]=0;
```

```
    }
  }
}
/**/
/* Set n2=LSM-n1-n3 & n8=LSM-n9-n7 */
void stp03(){
 short m,n;
  m=LSM-nm[1]-nm[3];
  n=LSM-nm[9]-nm[7];
  if((0<m)&&(m<10)&&(m+n==CC)){
    if((uflg[m]==0)&&(uflg[n]==0)){
      nm[2]=m; nm[8]=n;
      uflg[m]=1; uflg[n]=1;
      stp04();
      uflg[n]=0; uflg[m]=0;
    }
  }
}
/**/
/* Set n4=LSM-n1-n7 & n6=LSM-n9-n3 */
void stp04(){
 short m,n;
  m=LSM-nm[1]-nm[7];
  n=LSM-nm[9]-nm[3];
  if((0<m)&&(m<10)&&(m+n==CC)){
    if((uflg[m]==0)&&(uflg[n]==0)){
      nm[4]=m; nm[6]=n;
      uflg[m]=1; uflg[n]=1;
      recordans();
      uflg[n]=0; uflg[m]=0;
    }
  }
}
/**/
/* Record the Answer to the Table */
void recordans(){
 short n;
  cnt++; tn[cnt2][0]=cnt;
  for(n=1;n<10;n++){tn[cnt2][n]=nm[n];}
  cnt2++; if(cnt2==4){prans(cnt2); cnt2=0;}
}
/**/
/* Print the Answers */
void prans(short x){
 short l,l3,n;
  for(n=0;n<x;n++){printf("%14d/",tn[n][0]);}; printf("\n");
  for(n=0;n<x;n++){printf("   .---.---.---.");}; printf("\n");
  for(l=0;l<3;l++){l3=l*3;
   for(n=0;n<x;n++){
      printf("   |%2d |%2d |%2d |",tn[n][l3+1],tn[n][l3+2],tn[n][l3+3]);}
    printf("\n");
  if(l<2){for(n=0;n<x;n++){printf("   |---+---+---|");}; printf("\n");}
  }
  for(n=0;n<x;n++){printf("   '---'---'---'");}; printf("\n");
}
/**/
```

This new program should always list out the set of 8 primitive solutions of SCMS33.

```
** Self-Complementary Magic Squares of Order 3 **
            1/            2/             3/            4/
 .---.---.---.  .---.---.---.  .---.---.---.  .---.---.---.
 | 2 | 9 | 4 |  | 2 | 7 | 6 |  | 4 | 9 | 2 |  | 4 | 3 | 8 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 7 | 5 | 3 |  | 9 | 5 | 1 |  | 3 | 5 | 7 |  | 9 | 5 | 1 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 6 | 1 | 8 |  | 4 | 3 | 8 |  | 8 | 1 | 6 |  | 2 | 7 | 6 |
 '---'---'---'  '---'---'---'  '---'---'---'  '---'---'---'
            5/            6/             7/            8/
 .---.---.---.  .---.---.---.  .---.---.---.  .---.---.---.
 | 6 | 7 | 2 |  | 6 | 1 | 8 |  | 8 | 3 | 4 |  | 8 | 1 | 6 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 1 | 5 | 9 |  | 7 | 5 | 3 |  | 1 | 5 | 9 |  | 3 | 5 | 7 |
 |---+---+---|  |---+---+---|  |---+---+---|  |---+---+---|
 | 8 | 3 | 4 |  | 2 | 9 | 4 |  | 6 | 7 | 2 |  | 4 | 9 | 2 |
 '---'---'---'  '---'---'---'  '---'---'---'  '---'---'---'
  [Count = 8]  OK!
```

Yes. This is just the same solution set with the one of 'Standard' Magic Type.
It indicates that both Standard MS33 and Self-complementary MS33 are essentially 'equivalent' to each other from the logical viewpoint.
There is only one 'Standard Solution' for each type of those two, of course. I selected it under the List-forming Inequality Conditions: {n1<n9; n1<n3; n1<n7 and n2>n4}.

```
**  Self-Complementary Magic Squares 3x3  **
** List of the Only One Standard Solution **
** Under {n1<n9; n1<n3; n1<n7; and n4<n2} **
             1/
 .---.---.---.
 | 2 | 9 | 4 |
 |---+---+---|
 | 7 | 5 | 3 |
 |---+---+---|
 | 6 | 1 | 8 |
 '---'---'---'    [Count = 1]  OK!
```

Section 2: Standard Magic Squares of Order 4: Kanji Setsuda

1. Basic Form and Basic Conditions defining every row, column and 2 diagonals

Let's study about the 'Standard' Magic Squares of Order 4 at first. Each of them should be built in the Basic Form shown below. It must be made up of the natural numbers {1, 2, 3, 4, 5, 6, ..., 14, 15 and 16} using each strictly once, and they must be placed in the 4x4 regular array. The Basic Form below shows their positions and let's call the variables as "n1, n2, n3, n4, n5, ..., n14, n15 and n16."

2. We must suppose every 4 entries on any row and any column in the two directions should add up to the magic constant C, and we also define every 4 entries on the 2 primary diagonals should add up to the same constant C.

**** Standard Magic Squares of Order 4: ****

```
 15 16 13 14 15 16 13 14  * Basic Form & Basic Equations *
       .--.--.--.--.       n1+n2+n3+n4=C      ... rw1;
  3  4| 1| 2| 3| 4| 1  2   n5+n6+n7+n8=C      ... rw2;
      |--+--+--+--|        n9+n10+n11+n12=C   ... rw3;
  7  8| 5| 6| 7| 8| 5  6   n13+n14+n15+n16=C  ... rw4;
      |--+--+--+--|        n1+n5+n9+n13=C     ... cl1;
 11 12| 9|10|11|12| 9 10   n2+n6+n10+n14=C    ... cl2;
      |--+--+--+--|        n3+n7+n11+n15=C    ... cl3;
 15 16|13|14|15|16|13 14   n4+n8+n12+n16=C    ... cl4;
       '--'--'--'--'       n1+n6+n11+n16=C    ...  d1;
  3  4  1  2  3  4  1  2   n4+n7+n10+n13=C    ...  d2;

rw1+rw2+rw3+rw4
   n1+n2+n3+n4+n5+n6+n7+ ... +n14+n15+n16 = 4xC
             1+2+3+4+5+6+7+ ... +14+15+16 = 4xC
                              (1+16)x16/2 = 4xC
                             Therefore  C = 34
```

3. There are many solutions for this type naturally, for we have only 10 simultaneous equations while we have as many variables as 16.

Let's dictate such a good program for our computer as to make it calculate and count how many solutions there are in all for this type.

I would like to show you a little program of mine as listed below.

I wrote it without any 'List-forming Inequality Conditions' at all. It should list out the large set of all the 'primitive' solutions.

```
/** 'Standard' Magic Squares of Order 4: **/
/** 'MS44StndP.c' built by Kanji Setsuda **/
/**  on /12/08/04; /01/11/06; /11/30/08; **/
/** Working with MacOSX10.5 and Xcode3.0 **/
/**/
#include <stdio.h>
/**/
short cnt, cnt2, cnt3;
short LSM;
short nm[17], uflg[17];
short anm[9][17];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void), stp16(void);
void ansprint(void);
void printans(short x);
/* Main Program */
int main(){
 short n;
  printf("\n** Standard Magic Squares of Order 4:");
  printf(" List of the Primitive Solutions **\n");
  for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
  LSM=34; cnt=0; cnt3=0;
  stp01(); /* Begin the Calculations */
  if(cnt3>0){printans(cnt3);}
  printf(" [Total Count = %d]  OK!\n",cnt);
 return 0;
}
```

```
/* Begin the Calculations */
/* Set N1 */
void stp01(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[1]=a; uflg[a]=1; /cnt2=0;
      stp02();
      uflg[a]=0;}
  }
}
/* Set N16 */
void stp02(){
 short a;
  for(a=16;a>0;a--){
    if(uflg[a]==0){
      nm[16]=a; uflg[a]=1;
      stp03();
      uflg[a]=0;}
  }
}
/* Set N6 */
void stp03(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[6]=a; uflg[a]=1;
      stp04();
      uflg[a]=0;}
  }
}
/* Set n11=LSM-n1-n6-n16 */
void stp04(){
 short a;
  a=LSM-nm[1]-nm[6]-nm[16];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
      nm[11]=a; uflg[a]=1;
      stp05();
      uflg[a]=0;}}
}
/* Set N4 */
void stp05(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[4]=a; uflg[a]=1;
      stp06();
      uflg[a]=0;}
  }
}
/* Set N13 */
void stp06(){
 short a;
  for(a=16;a>0;a--){
    if(uflg[a]==0){
      nm[13]=a; uflg[a]=1;
      stp07();
      uflg[a]=0;}
  }
}
/* Set N7 */
void stp07(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[7]=a; uflg[a]=1;
      stp08();
      uflg[a]=0;}
  }
}
/* Set n10=LSM-n4-n7-n13 */
void stp08(){
 short a;
  a=LSM-nm[4]-nm[7]-nm[13];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
      nm[10]=a; uflg[a]=1;
      stp09();
      uflg[a]=0;}}
}
/* Define Level 2 */
/* Set N2 */
void stp09(){
 short a;
  for(a=16;a>0;a--){
    if(uflg[a]==0){
      nm[2]=a; uflg[a]=1;
      stp10();
      uflg[a]=0;}
  }
}
/* Set n3=LSM-n1-n2-n4 */
void stp10(){
 short a;
  a=LSM-nm[1]-nm[2]-nm[4];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
      nm[3]=a; uflg[a]=1;
      stp11();
      uflg[a]=0;}}
}
/* Set N5 */
void stp11(){
 short a;
  for(a=16;a>0;a--){
    if(uflg[a]==0){
      nm[5]=a; uflg[a]=1; cnt2=0;
      stp12();
      uflg[a]=0;}
  }
}
/* Set n9=LSM-n1-n5-n13 */
void stp12(){
 short a;
  a=LSM-nm[1]-nm[5]-nm[13];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
```

```
      nm[9]=a; uflg[a]=1;
      stp13();
      uflg[a]=0;}}
}
/* Set n8=LSM-n5-n6-n7 */
void stp13(){
 short a;
  a=LSM-nm[5]-nm[6]-nm[7];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
      nm[8]=a; uflg[a]=1;
      stp14();
      uflg[a]=0;}}
}
/* Set n12=LSM-n9-n10-n11 */
void stp14(){
 short a,b;
  a=LSM-nm[9]-nm[10]-nm[11];
  b=LSM-nm[4]-nm[8]-nm[16];
  if((0<a)&&(a<17)){
    if((a==b)&&(uflg[a]==0)){
      nm[12]=a; uflg[a]=1;
      stp15();
      uflg[a]=0;}}
}
/* Set n14=LSM-n2-n6-n10 */
void stp15(){
 short a;
  a=LSM-nm[2]-nm[6]-nm[10];
  if((0<a)&&(a<17)){
    if(uflg[a]==0){
      nm[14]=a; uflg[a]=1;
      stp16();
      uflg[a]=0;}}
}
/* Set n15=LSM-n3-n7-n11 */
void stp16(){
 short a,b;
  a=LSM-nm[3]-nm[7]-nm[11];
  b=LSM-nm[13]-nm[14]-nm[16];
  if((0<a)&&(a<17)){
    if((a==b)&&(uflg[a]==0)){
      nm[15]=a; uflg[a]=1;
      ansprint();
      uflg[a]=0;}}
}
/**/
```

```
/* Print the Answer */
void ansprint(){
 short n;
  cnt++; cnt2++;
  if(cnt2==1){
    anm[cnt3][0]=cnt;
    for(n=1;n<17;n++){anm[cnt3][n]=nm[n];}
    cnt3++; if(cnt3==8){printans(cnt3); cnt3=0;}
  }
}
/* Print the Answers in a line */
void printans(short x){
 short l,l4,m,n;
  for(m=0;m<x;m++){printf("%12d/",anm[m][0]);}
  printf("\n");
  for(l=0;l<4;l++){l4=l*4;
    for(m=0;m<x;m++){printf(" ");
      for(n=1;n<5;n++){printf("%3d",anm[m][l4+n]);}
    }
    printf("\n");
  }
}
/**/
```

**** Standard Magic Squares of Order 4: Compact List of the Primitive Solutions ****

```
          1/          2/          3/          4/          5/          6/          7/          8/
  1 15 12  6   1 14 12  7   1 15  8 10   1 14  8 11   1 15 14  4   1 12 14  7   1 15  8 10   1 12  8 13
 14  4  7  9  15  4  6  9  14  4 11  5  15  4 10  5  12  6  7  9  15  6  4  9  12  6 13  3  15  6 10  3
  8 10 13  3   8 11 13  2  12  6 13  3  12  7 13  2   8 10 11  5   8 13 11  2  14  4 11  5  14  7 11  2
 11  5  2 16  10  5  3 16   7  9  2 16   6  9  3 16  13  3  2 16  10  3  5 16   7  9  2 16   4  9  5 16
          9/         10/         11/         12/         13/         14/         15/         16/
  1 14 15  4   1 12 15  6   1 14  8 11   1 12  8 13   1 15 14  4   1 15 12  6   1  8 14 11   1  8 12 13
 12  7  6  9  14  7  4  9  12  7 13  2  14  7 11  2   8 10 11  5   8 10 13  3  15 10  4  5  15 10  6  3
  8 11 10  5   8 13 10  3  15  4 10  5  15  6 10  3  12  6  7  9  14  4  7  9  12 13  7  2  14 11  7  2
 13  2  3 16  11  2  5 16   6  9  3 16   4  9  5 16  13  3  2 16  11  5  2 16   6  3  9 16   4  5  9 16
```

17/	18/	19/	20/	21/	22/	23/	24/
1 14 15 4	1 14 12 7	1 8 15 10	1 8 12 13	1 12 15 6	1 12 14 7	1 8 15 10	1 8 14 11
8 11 10 5	8 11 13 2	14 11 4 5	14 11 7 2	8 13 10 3	8 13 11 2	12 13 6 3	12 13 7 2
12 7 6 9	15 4 6 9	12 13 6 3	15 10 6 3	14 7 4 9	15 6 4 9	14 11 4 5	15 10 4 5
13 2 3 16	10 5 3 16	7 2 9 16	4 5 9 16	11 2 5 16	10 3 5 16	7 2 9 16	6 3 9 16

25/	26/	27/	28/	29/	31/	33/	34/
1 16 11 6	1 13 12 8	1 16 7 10	1 13 8 12	1 14 11 8	1 16 7 10	1 16 13 4	1 11 14 8
13 4 7 10	16 4 5 9	13 4 11 6	16 4 9 5	16 5 4 9	14 5 12 3	11 6 7 10	16 6 3 9
8 9 14 3	7 11 14 2	12 5 14 3	11 7 14 2	7 12 13 2	11 4 13 6	8 9 12 5	7 13 12 2
12 5 2 15	10 6 3 15	8 9 2 15	6 10 3 15	10 3 6 15	8 9 2 15	14 3 2 15	10 4 5 15

35/	36/	37/	38/	39/	40/	41/	42/
1 16 7 10	1 11 8 14	1 13 16 4	1 11 16 6	1 14 7 12	1 12 7 14	1 16 13 4	1 16 11 6
11 6 13 4	16 6 9 3	12 8 5 9	14 8 3 9	11 8 13 2	13 8 11 2	7 10 11 6	7 10 13 4
14 3 12 5	13 7 12 2	7 11 10 6	7 13 10 4	16 3 10 5	16 5 10 3	12 5 8 9	14 3 8 9
8 9 2 15	4 10 5 15	14 2 3 15	12 2 5 15	6 9 4 15	4 9 6 15	14 3 2 15	12 5 2 15

43/	44/	45/	46/	47/	48/	49/	51/
1 7 14 12	1 7 12 14	1 13 16 4	1 14 11 8	1 7 16 10	1 8 11 14	1 14 11 8	1 7 16 10
16 10 3 5	16 10 5 3	8 12 9 5	7 12 13 2	14 12 3 5	13 12 7 2	7 13 12 2	14 13 4 3
11 13 8 2	13 11 8 2	11 7 6 10	16 3 6 9	11 13 6 4	16 9 6 3	16 4 5 9	11 12 5 6
6 4 9 15	4 6 9 15	14 2 3 15	10 5 4 15	8 2 9 15	4 5 10 15	10 3 6 15	8 2 9 15

53/	54/	55/	56/	57/	58/	59/	60/
1 11 16 6	1 12 13 8	1 7 16 10	1 8 13 12	1 16 10 7	1 13 12 8	1 16 6 11	1 13 8 12
8 14 9 3	7 14 11 2	12 14 5 3	11 14 7 2	13 4 6 11	16 4 5 9	13 4 10 7	16 4 9 5
13 7 4 10	16 5 4 9	13 11 4 6	16 9 4 5	8 9 15 2	6 10 15 3	12 5 15 2	10 6 15 3
12 2 5 15	10 3 6 15	8 2 9 15	6 3 10 15	12 5 3 14	11 7 2 14	8 9 3 14	7 11 2 14

65/	69/	73/	77/	81/	85/	89/	93/
1 13 16 4	1 16 13 4	1 13 16 4	1 16 13 4	1 13 16 4	1 10 16 7	1 15 10 8	1 16 9 8
12 8 5 9	12 9 8 5	12 10 7 5	6 11 10 7	8 12 9 5	8 15 9 2	14 4 5 11	14 5 4 11
6 10 11 7	6 7 10 11	6 8 9 11	12 5 8 9	10 6 7 11	13 6 4 11	7 9 16 2	7 10 15 2
15 3 2 14	15 2 3 14	15 3 2 14	15 2 3 14	15 3 2 14	12 3 5 14	12 6 3 13	12 3 6 13

95/	99/	107/	115/	123/	131/	135/	137/
1 15 10 8	1 15 14 4	1 15 14 4	1 15 14 4	1 15 14 4	1 15 10 8	1 9 16 8	1 11 14 8
16 6 3 9	11 8 5 10	12 9 6 7	10 11 8 5	7 12 9 6	5 14 11 4	7 15 10 2	6 16 9 3
5 11 14 4	6 9 12 7	5 8 11 10	7 6 9 12	10 5 8 11	16 3 6 9	14 4 5 11	15 5 4 10
12 2 7 13	16 2 3 13	16 2 3 13	16 2 3 13	16 2 3 13	12 2 7 13	12 6 3 13	12 2 7 13

141/	145/	149/	153/	157/	161/	165/	169/
1 16 10 7	1 16 11 6	1 11 16 6	1 16 11 6	1 11 16 6	1 10 16 7	1 15 10 8	1 13 12 8
11 6 4 13	10 7 4 13	14 8 3 9	4 13 10 7	8 14 9 3	8 15 9 2	12 6 3 13	16 7 2 9
8 9 15 2	8 9 14 3	4 10 13 7	14 3 8 9	10 4 7 13	11 4 6 13	7 9 16 2	3 10 15 6
14 3 5 12	15 2 5 12	15 5 2 12	15 2 5 12	15 5 2 12	14 5 3 12	14 4 5 11	14 4 5 11

173/	181/	189/	193/	197/	205/	213/	217/
1 15 12 6	1 15 12 6	1 16 9 8	1 9 16 8	1 15 12 6	1 15 12 6	1 13 12 8	1 13 12 8
13 8 3 10	14 9 4 7	15 10 7 2	15 12 5 2	10 13 8 3	7 14 9 4	3 15 10 6	4 16 9 5
4 9 14 7	3 8 13 10	4 5 12 13	4 7 10 13	7 4 9 14	10 3 8 13	16 2 7 9	15 3 6 10
16 2 5 11	16 2 5 11	14 3 6 11	14 6 3 11	16 2 5 11	16 2 5 11	14 4 5 11	14 2 7 11

221/	225/	233/	235/	239/	243/	245/	253/
1 14 11 8	1 14 12 7	1 12 13 8	1 16 9 8	1 9 16 8	1 13 12 8	1 14 12 7	1 13 12 8
12 7 2 13	13 8 2 11	16 9 4 5	14 11 6 3	14 12 5 3	2 14 7 11	6 15 9 4	4 16 9 5
6 9 16 3	4 9 15 6	2 7 14 11	4 5 12 13	4 6 11 13	16 4 9 5	11 2 8 13	14 2 7 11
15 4 5 10	16 3 5 10	15 6 3 10	15 2 7 10	15 7 2 10	15 3 6 10	16 3 5 10	15 3 6 10

257/	265/	273/	281/	289/	293/	297/	301/
1 13 12 8	1 15 10 8	1 15 10 8	1 13 12 8	1 16 6 11	1 16 7 10	1 7 16 10	1 16 7 10
15 10 3 6	14 11 4 5	12 13 6 3	11 14 7 2	7 10 4 13	6 11 4 13	14 12 3 5	4 13 6 11
2 7 14 11	3 6 13 12	5 4 11 14	6 3 10 15	12 5 15 2	12 5 14 3	4 6 13 11	14 3 12 5
16 4 5 9	16 2 7 9	16 2 7 9	16 4 5 9	14 3 9 8	15 2 9 8	15 9 2 8	15 2 9 8

305/	309/	313/	317/	321/	329/	337/	341/
1 7 16 10	1 6 16 11	1 15 6 12	1 13 8 12	1 15 8 10	1 15 8 10	1 13 8 12	1 13 8 12
12 14 5 3	12 15 5 2	8 10 3 13	16 11 2 5	13 12 3 6	11 14 5 4	3 15 6 10	4 16 5 9
6 4 11 13	7 4 10 13	11 5 16 2	3 6 15 10	4 5 14 11	6 3 12 13	16 2 11 5	15 3 10 6
15 9 2 8	14 9 3 8	14 4 9 7	14 4 9 7	16 2 9 7	16 2 9 7	14 4 9 7	14 2 11 7

345/	349/	357/	361/	365/	373/	377/	385/
1 14 7 12	1 14 8 11	1 16 5 12	1 5 16 12	1 14 8 11	1 13 8 12	1 14 7 12	1 14 7 12
8 11 2 13	13 12 2 7	10 13 4 7	10 14 3 7	10 15 5 4	4 16 5 9	11 13 2 8	9 15 4 6
10 5 16 3	4 5 15 10	8 3 14 9	8 4 13 9	7 2 12 13	14 2 11 7	6 4 15 9	8 2 13 11
15 4 9 6	16 3 9 6	15 2 11 6	15 11 2 6	16 3 9 6	15 3 10 6	16 3 10 5	16 3 10 5

393/	397/	405/	413/	417/	421/	425/	429/
1 12 7 14	1 12 8 13	1 12 8 13	1 11 8 14	2 15 12 5	2 14 11 7	2 15 14 3	2 14 15 3
8 13 2 11	11 14 2 7	10 15 3 6	6 16 3 9	14 3 8 9	15 4 5 10	12 5 8 9	11 7 6 10
10 3 16 5	6 3 15 10	7 2 14 11	12 2 13 7	7 10 13 4	8 13 12 1	7 10 11 6	8 12 9 5
15 6 9 4	16 5 9 4	16 5 9 4	15 5 10 4	11 6 1 16	9 3 6 16	13 4 1 16	13 1 4 16

. . . .

But how many primitive solutions are there in all? The next list tells us about it.

```
** Standard Magic Squares of Order 4: Abstract List of the Primitive Solutions **
          1/         417/         817/        1221/        1697/        2129/        2585/        3045/
  1 15 12  6   2 15 12  5   3 14 12  5   4 15 10  5   5 12 14  3   6 15 10  3   7 14 11  2   8 13 12  1
 14  4  7  9  14  3  8  9  15  2  8  9  14  1  8 11  15  2  8  9  12  1  8 13  12  1  8 13  15  3 10  6
  8 10 13  3   7 10 13  4   6 11 13  4   7 12 13  2   4 13 11  6   7 14 11  2   6 15 10  3   2 14  7 11
 11  5  2 16  11  6  1 16  10  7  1 16   9  6  3 16  10  7  1 16   9  4  5 16   9  4  5 16   9  4  5 16
       3521/        3997/        4457/        4913/        5345/        5821/        6225/        6625/
  9  8 14  3  10 15  6  3  11 14  7  2  12 14  7  1  13 12  7  2  14 11  7  2  15 12  5  2  16 11  5  2
 15  2 12  5   8  1 12 13   8  1 12 13  11  2 13  8   8  1 14 11  12  1 13  8  10  1 16  7   6  1 15 12
  4 13  7 10  11 14  7  2  10 15  6  3   6 15  4  9  10 15  4  5   5 16  4  9   6 13  4 11   9 14  4  7
  6 11  1 16   5  4  9 16   5  4  9 16   5  3 10 16   3  6  9 16   3  6 10 15   3  8  9 14   3  8 10 13
 [Total Count = 7040]  OK!
```

What a great count! This list contains every 8 derivative forms of any solution as in either reflected patterns or rotated ones. For instance, the solution 4/ is a reflected pattern of 1/, Solution 3/ is a reflected one of 2/, 8/ is a reflected one of 5/, ...

Solution 4733/ is a rotated pattern of 1/, Solution 7040/ is a rotated one of 4733/, Solution 2308/ is a rotated one of 7040/, ...

Solution 3045/ is a rotated pattern of 264/, 4913/ is a rotated one of 384/, ...

```
** Mirror Reflection and Rotations by 90 Degrees Clockwise **
             1/  -(Rotate)-> 4733/  -(Rotate)-> 7040/  -(Rotate)-> 2308/
 .--.--.--.--.    .--.--.--.--.    .--.--.--.--.    .--.--.--.--.
 | 1|15|12| 6|    |11| 8|14| 1|    |16| 2| 5|11|    | 6| 9| 3|16|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 |14| 4| 7| 9|    | 5|10| 4|15|    | 3|13|10| 8|    |12| 7|13| 2|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 | 8|10|13| 3|    | 2|13| 7|12|    | 9| 7| 4|14|    |15| 4|10| 5|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 |11| 5| 2|16|    |16| 3| 9| 6|    | 6|12|15| 1|    | 1|14| 8|11|
 '--'--'--'--'    '--'--'--'--'    '--'--'--'--'    '--'--'--'--'

          |(Reflect with respect to the diagonal {1,4,13,16})
          V  4/  -(Rotate)-> 2305/  -(Rotate)-> 7037/  -(Rotate)-> 4736/
 .--.--.--.--.    .--.--.--.--.    .--.--.--.--.    .--.--.--.--.
 | 1|14| 8|11|    | 6|12|15| 1|    |16| 3| 9| 6|    |11| 5| 2|16|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 |15| 4|10| 5|    | 9| 7| 4|14|    | 2|13| 7|12|    | 8|10|13| 3|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 |12| 7|13| 2|    | 3|13|10| 8|    | 5|10| 4|15|    |14| 4| 7| 9|
 |--+--+--+--|    |--+--+--+--|    |--+--+--+--|    |--+--+--+--|
 | 6| 9| 3|16|    |16| 2| 5|11|    |11| 8|14| 1|    | 1|15|12| 6|
 '--'--'--'--'    '--'--'--'--'    '--'--'--'--'    '--'--'--'--'
```

4. Why don't we pick up a representative solution from those 8 derivative forms? Let's collect all representatives to make them list out the 'Standard Solutions'.

We can use such the 'List-forming Inequality Conditions' as {n1<n16; n1<n4; n1<n13 and n2>n5} to select them and make our smart list at last.

Here you see a few sample results of my recent calculation.

```
** Standard Magic Squares of Order 4: List of the Standard Solutions **
          1/           2/           3/           4/           5/           6/           7/           8/
  1 15 12  6   1 15  8 10   1 15 14  4   1 15  8 10   1 14 15  4   1 14  8 11   1 15 14  4   1 15 12  6
 14  4  7  9  14  4 11  5  12  6  7  9  12  6 13  3  12  7  6  9  12  7 13  2   8 10 11  5   8 10 13  3
  8 10 13  3  12  6 13  3   8 10 11  5  14  4 11  5   8 11 10  5  15  4 10  5  12  6  7  9  14  4  7  9
 11  5  2 16   7  9  2 16  13  3  2 16   7  9  2 16  13  2  3 16   6  9  3 16  13  3  2 16  11  5  2 16
```

9/	10/	11/	12/	13/	14/	15/	16/
1 14 15 4	1 14 12 7	1 12 15 6	1 12 14 7	1 16 11 6	1 16 7 10	1 16 7 10	1 16 7 10
8 11 10 5	8 11 13 2	8 13 10 3	8 13 11 2	13 4 7 10	13 4 11 6	14 5 12 3	11 5 12 6
12 7 6 9	15 4 6 9	14 7 4 9	15 6 4 9	8 9 14 3	12 5 14 3	11 4 13 6	14 4 13 3
13 2 3 16	10 5 3 16	11 2 5 16	10 3 5 16	12 5 2 15	8 9 2 15	8 9 2 15	8 9 2 15

17/	18/	19/	20/	21/	22/	23/	24/
1 16 13 4	1 16 7 10	1 13 16 4	1 14 7 12	1 16 13 4	1 16 11 6	1 13 16 4	1 14 11 8
11 6 7 10	11 6 13 4	12 8 5 9	11 8 13 2	7 10 11 6	7 10 13 4	8 12 9 5	7 12 13 2
8 9 12 5	14 3 12 5	7 11 10 6	16 3 10 5	12 5 8 9	14 3 8 9	11 7 6 10	16 3 6 9
14 3 2 15	8 9 2 15	14 2 3 15	6 9 4 15	14 3 2 15	12 5 2 15	14 2 3 15	10 5 4 15

25/	26/	27/	28/	29/	30/	31/	32/
1 14 11 8	1 11 14 8	1 11 16 6	1 12 13 8	1 16 10 7	1 16 6 11	1 16 13 4	1 16 6 11
7 13 12 2	7 13 12 2	8 14 9 3	7 14 11 2	13 4 6 11	13 4 10 7	10 7 6 11	10 7 13 4
16 4 5 9	16 4 5 9	13 7 4 10	16 5 4 9	8 9 15 2	12 5 15 2	8 9 12 5	15 2 12 5
10 3 6 15	10 6 3 15	12 2 5 15	10 3 6 15	12 5 3 14	8 9 3 14	15 2 3 14	8 9 3 14

33/	35/	37/	39/	41/	43/	45/	47/
1 13 16 4	1 16 13 4	1 13 16 4	1 16 13 4	1 13 16 4	1 10 16 7	1 15 10 8	1 16 9 8
12 8 5 9	12 9 8 5	12 10 7 5	6 11 10 7	8 12 9 5	8 15 9 2	14 4 5 11	14 5 4 11
6 10 11 7	6 7 10 11	6 8 9 11	12 5 8 9	10 6 7 11	13 6 4 11	7 9 16 2	7 10 15 2
15 3 2 14	15 2 3 14	15 3 2 14	15 2 3 14	15 3 2 14	12 3 5 14	12 6 3 13	12 3 6 13

48/	50/	54/	58/	62/	66/	68/	69/
1 16 5 12	1 15 14 4	1 15 14 4	1 15 14 4	1 15 14 4	1 15 10 8	1 9 16 8	1 11 14 8
15 6 11 2	11 8 5 10	12 9 6 7	10 11 8 5	7 12 9 6	5 14 11 4	7 15 10 2	6 16 9 3
10 3 14 7	6 9 12 7	5 8 11 10	7 6 9 12	10 5 8 11	16 3 6 9	14 4 5 11	15 5 4 10
8 9 4 13	16 2 3 13	16 2 3 13	16 2 3 13	16 2 3 13	12 2 7 13	12 6 3 13	12 2 7 13

71/	73/	75/	77/	79/	81/	83/	85/
1 16 10 7	1 16 11 6	1 15 4 14	1 16 11 6	1 11 16 6	1 10 16 7	1 15 10 8	1 16 3 14
11 6 4 13	10 7 4 13	10 8 11 5	4 13 10 7	8 14 9 3	8 15 9 2	12 6 3 13	13 7 10 4
8 9 15 2	8 9 14 3	16 2 13 3	14 3 8 9	10 4 7 13	11 4 6 13	7 9 16 2	12 2 15 5
14 3 5 12	15 2 5 12	7 9 6 12	15 2 5 12	15 5 2 12	14 5 3 12	14 4 5 11	8 9 6 11

87/	91/	95/	97/	99/	103/	107/	109/
1 15 12 6	1 15 12 6	1 16 9 8	1 9 16 8	1 15 12 6	1 15 12 6	1 13 12 8	1 13 12 8
13 8 3 10	14 9 4 7	15 10 7 2	4 12 5 13	10 13 8 3	7 14 9 4	3 15 10 6	4 16 9 5
4 9 14 7	3 8 13 10	4 5 12 13	15 7 10 2	7 4 9 14	10 3 8 13	16 2 7 9	15 3 6 10
16 2 5 11	16 2 5 11	14 3 6 11	14 6 3 11	16 2 5 11	16 2 5 11	14 4 5 11	14 2 7 11

111/	113/	117/	118/	120/	122/	123/	127/
1 14 11 8	1 14 12 7	1 16 2 15	1 16 9 8	1 9 16 8	1 13 12 8	1 14 12 7	1 13 12 8
12 7 2 13	13 8 2 11	12 9 7 6	14 11 6 3	4 12 5 13	2 14 7 11	6 15 9 4	4 16 9 5
6 9 16 3	4 9 15 6	13 4 14 3	4 5 12 13	14 6 11 3	16 4 9 5	11 2 8 13	14 2 7 11
15 4 5 10	16 3 5 10	8 5 11 10	15 2 7 10	15 7 2 10	15 3 6 10	16 3 5 10	15 3 6 10

129/	133/	137/	141/	145/	147/	149/	151/
1 13 12 8	1 15 10 8	1 15 10 8	1 13 12 8	1 16 6 11	1 16 7 10	1 15 4 14	1 16 7 10
6 10 3 15	14 11 4 5	12 13 6 3	11 14 7 2	7 10 4 13	6 11 4 13	6 12 7 9	4 13 6 11
11 7 14 2	3 6 13 12	5 4 11 14	6 3 10 15	12 5 15 2	12 5 14 3	16 2 13 3	14 3 12 5
16 4 5 9	16 2 7 9	16 2 7 9	16 4 5 9	14 3 9 8	15 2 9 8	11 5 10 8	15 2 9 8

153/	155/	157/	159/	161/	165/	169/	171/
1 15 6 12	1 14 7 12	1 15 6 12	1 16 3 14	1 15 8 10	1 15 8 10	1 13 8 12	1 13 8 12
4 14 7 9	4 15 6 9	8 10 3 13	13 11 6 4	13 12 3 6	11 14 5 4	3 15 6 10	4 16 5 9
16 2 11 5	16 3 10 5	11 5 16 2	8 2 15 9	4 5 14 11	6 3 12 13	16 2 11 5	15 3 10 6
13 3 10 8	13 2 11 8	14 4 9 7	12 5 10 7	16 2 9 7	16 2 9 7	14 4 9 7	14 2 11 7

173/	175/	179/	181/	183/	187/	189/	193/
1 14 7 12	1 14 8 11	1 16 5 12	1 10 8 15	1 14 8 11	1 13 8 12	1 14 7 12	1 14 7 12
8 11 2 13	13 12 2 7	10 13 4 7	5 14 4 11	10 15 5 4	4 16 5 9	11 13 2 8	9 15 4 6
10 5 16 3	4 5 15 10	8 3 14 9	16 3 13 2	7 2 12 13	14 2 11 7	6 4 15 9	8 2 13 11
15 4 9 6	16 3 9 6	15 2 11 6	12 7 9 6	16 3 9 6	15 3 10 6	16 3 10 5	16 3 10 5

197/	199/	203/	207/	209/	211/	213/	215/
1 12 7 14	1 12 8 13	1 12 8 13	1 11 8 14	2 15 12 5	2 15 8 9	2 15 14 3	2 14 15 3
8 13 2 11	11 14 2 7	10 15 3 6	6 16 3 9	14 3 8 9	14 4 13 3	12 5 8 9	11 7 6 10
10 3 16 5	6 3 15 10	7 2 14 11	12 2 13 7	7 10 13 4	11 5 12 6	7 10 11 6	8 12 9 5
15 6 9 4	16 5 9 4	16 5 9 4	15 5 10 4	11 6 1 16	7 10 1 16	13 4 1 16	13 1 4 16

217/	219/	221/	223/	225/	227/	229/	231/
2 15 14 3	2 14 15 3	2 14 11 7	2 12 15 5	2 16 11 5	2 16 13 3	2 13 16 3	2 16 13 3
8 9 12 5	7 11 10 6	8 12 13 1	7 13 10 4	13 3 8 10	11 5 8 10	11 8 5 10	7 9 12 6
11 6 7 10	12 8 5 9	15 5 4 10	14 8 3 9	7 9 14 4	7 9 12 6	7 12 9 6	11 5 8 10
13 4 1 16	13 1 4 16	9 3 6 16	11 1 6 16	12 6 1 15	14 4 1 15	14 1 4 15	14 4 1 15

. . . .

How many standard solutions could we find for our object?

```
** Standard Magic Squares 4x4: Abstract List of the Standard Solutions **
         1/          209/          409/          575/          753/          817/          865/
  1 15 12  6   2 15 12  5   3 15 10  6   4 15 10  5   5 16  4  9   6 16  3  9   7 14  4  9
 14  4  7  9  14  3  8  9  14  2  7 11  14  1  8 11  11  2 14  7  11  1 14  8  12  3 13  6
  8 10 13  3   7 10 13  4   8 12 13  1   7 12 13  2  10  3 15  6  10  4 15  5   5  2 16 11
 11  5  2 16  11  6  1 16   9  5  4 16   9  6  3 16   8 13  1 12   7 13  2 12  10 15  1  8
[Total Count = 880]  OK!
```

The count 880 is so reasonable that we can naturally accept for 880 x 8 = 7040.

5. For the next step we must examine some sample solutions of those Standard Magic Squares 4x4 to know what properties each of them has got in itself.

```
** Standard Magic Squares of Order 4: **
  ** Examine Some Sample Solutions **
            1/        Rows & Columns; Pan-diagonals
  .--.--.--.--.        1+15+12+ 6=34;  1+ 4+13+16=34
 6| 1|15|12| 6| 1 15  14+ 4+ 7+ 9=34;  1+ 9+13+ 5=28
  |--+--+--+--|        8+10+13+ 3=34; 15+ 7+ 3+11=36
 9|14| 4| 7| 9|14  4  11+ 5+ 2+16=34; 15+14+ 3+ 2=34
  |--+--+--+--|        1+14+ 8+11=34; 12+ 9+ 8+ 5=34
 3| 8|10|13| 3| 8 10  15+ 4+10+ 5=34; 12+ 4+ 8+16=40
  |--+--+--+--|       12+ 7+13+ 2=34;  6+14+10+ 2=32
16|11| 5| 2|16|11  5   6+ 9+ 3+16=34;  6+ 7+10+11=34
  '--'--'--'--'       Complementary Pairs:
      1+16=17; 15+ 2=17; 12+ 5=17;  6+11=17;
     14+ 3=17;  4+13=17;  7+10=17;  9+ 8=17;
 ... This is a Self-complementary Magic Square 4x4!

           13/        Rows & Columns; Pan-diagonals
  .--.--.--.--.        1+16+11+ 6=34;  1+ 4+14+15=34
 6| 1|16|11| 6| 1 16  13+ 4+ 7+10=34;  1+10+14+ 5=30
  |--+--+--+--|        8+ 9+14+ 3=34; 16+ 7+ 3+12=38
10|13| 4| 7|10|13  4  12+ 5+ 2+15=34; 16+13+ 3+ 2=34
  |--+--+--+--|        1+13+ 8+12=34; 11+10+ 8+ 5=34
 3| 8| 9|14| 3| 8  9  16+ 4+ 9+ 5=34; 11+ 4+ 8+15=38
  |--+--+--+--|       11+ 7+14+ 2=34;  6+13+ 9+ 2=30
15|12| 5| 2|15|12  5   6+10+ 3+15=34;  6+ 7+ 9+12=34
  '--'--'--'--'       Complementary Pairs:
      1+15=16; 16+ 2=18; 11+ 5=16;  6+12=18;
     13+ 3=16;  4+14=18;  7+ 9=16; 10+ 8=18;
 ... This is a Standard Magic Square 4x4!

           45/        Rows & Columns; Pan-diagonals
  .--.--.--.--.        1+15+10+ 8=34;  1+ 4+16+13=34
 8| 1|15|10| 8| 1 15  14+ 4+ 5+11=34;  1+11+16+ 6=34
  |--+--+--+--|        7+ 9+16+ 2=34; 15+ 5+ 2+12=34
11|14| 4| 5|11|14  4  12+ 6+ 3+13=34; 15+14+ 2+ 3=34
  |--+--+--+--|        1+14+ 7+12=34; 10+11+ 7+ 6=34
 2| 7| 9|16| 2| 7  9  15+ 4+ 9+ 6=34; 10+ 4+ 7+13=34
  |--+--+--+--|       10+ 5+16+ 3=34;  8+14+ 9+ 3=34
13|12| 6| 3|13|12  6   8+11+ 2+13=34;  8+ 5+ 9+12=34
  '--'--'--'--'       Complementary Pairs:
      1+13=14; 15+ 3=18; 10+ 6=16;  8+12=20;
     14+ 2=16;  4+16=20;  5+ 9=14; 11+ 7=18;
 ... This is a Pan-diagonal Magic Square 4x4!

 . . . .
```

** Standard Magic Squares 4x4: List of Standard Solutions with /Type **

** /SC: Self-Complementary, /PD: Pan-Diagonal, /ST: Standard **

			1/SC				2/SC				3/SC				4/SC				5/SC				6/SC
1	15	12	6	1	15	8	10	1	15	14	4	1	15	8	10	1	14	15	4	1	14	8	11
14	4	7	9	14	4	11	5	12	6	7	9	12	6	13	3	12	7	6	9	12	7	13	2
8	10	13	3	12	6	13	3	8	10	11	5	14	4	11	5	8	11	10	5	15	4	10	5
11	5	2	16	7	9	2	16	13	3	2	16	7	9	2	16	13	2	3	16	6	9	3	16

			7/SC				8/SC				9/SC				10/SC				11/SC				12/SC
1	15	14	4	1	15	12	6	1	14	15	4	1	14	12	7	1	12	15	6	1	12	14	7
8	10	11	5	8	10	13	3	8	11	10	5	8	11	13	2	8	13	10	3	8	13	11	2
12	6	7	9	14	4	7	9	12	7	6	9	15	4	6	9	14	7	4	9	15	6	4	9
13	3	2	16	11	5	2	16	13	2	3	16	10	5	3	16	11	2	5	16	10	3	5	16

			13/ST				15/ST				17/ST				19/ST				21/ST				23/ST
1	16	11	6	1	16	7	10	1	16	13	4	1	13	16	4	1	16	13	4	1	13	16	4
13	4	7	10	14	5	12	3	11	6	7	10	12	8	5	9	7	10	11	6	8	12	9	5
8	9	14	3	11	4	13	6	8	9	12	5	7	11	10	6	12	5	8	9	11	7	6	10
12	5	2	15	8	9	2	15	14	3	2	15	14	2	3	15	14	3	2	15	14	2	3	15

			25/ST				27/ST				29/ST				31/ST				33/ST				35/ST
1	14	11	8	1	11	16	6	1	16	10	7	1	16	13	4	1	13	16	4	1	16	13	4
7	13	12	2	8	14	9	3	13	4	6	11	10	7	6	11	12	8	5	9	12	9	8	5
16	4	5	9	13	7	4	10	8	9	15	2	8	9	12	5	6	10	11	7	6	7	10	11
10	3	6	15	12	2	5	15	12	5	3	14	15	2	3	14	15	3	2	14	15	2	3	14

			37/ST				39/ST				41/ST				43/ST				45/PD				47/ST
1	13	16	4	1	16	13	4	1	13	16	4	1	10	16	7	1	15	10	8	1	16	9	8
12	10	7	5	6	11	10	7	8	12	9	5	8	15	9	2	14	4	5	11	14	5	4	11
6	8	9	11	12	5	8	9	10	6	7	11	13	6	4	11	7	9	16	2	7	10	15	2
15	3	2	14	15	2	3	14	15	3	2	14	12	3	5	14	12	6	3	13	12	3	6	13

			48/ST				50/ST				54/ST				58/ST				62/ST				66/ST
1	16	5	12	1	15	14	4	1	15	14	4	1	15	14	4	1	15	14	4	1	15	10	8
15	6	11	2	11	8	5	10	12	9	6	7	10	11	8	5	7	12	9	6	5	14	11	4
10	3	14	7	6	9	12	7	5	8	11	10	7	6	9	12	10	5	8	11	16	3	6	9
8	9	4	13	16	2	3	13	16	2	3	13	16	2	3	13	16	2	3	13	12	2	7	13

			68/ST				69/ST				71/ST				73/ST				75/ST				77/ST
1	9	16	8	1	11	14	8	1	16	10	7	1	16	11	6	1	15	4	14	1	16	11	6
7	15	10	2	6	16	9	3	11	6	4	13	10	7	4	13	10	8	11	5	4	13	10	7
14	4	5	11	15	5	4	10	8	9	15	2	8	9	14	3	16	2	13	3	14	3	8	9
12	6	3	13	12	2	7	13	14	3	5	12	15	2	5	12	7	9	6	12	15	2	5	12

			79/ST				81/ST				83/PD				85/ST				87/ST				91/ST
1	11	16	6	1	10	16	7	1	15	10	8	1	16	3	14	1	15	12	6	1	15	12	6
8	14	9	3	8	15	9	2	12	6	3	13	13	7	10	4	13	8	3	10	14	9	4	7
10	4	7	13	11	4	6	13	7	9	16	2	12	2	15	5	4	9	14	7	3	8	13	10
15	5	2	12	14	5	3	12	14	4	5	11	8	9	6	11	16	2	5	11	16	2	5	11

			95/ST				97/ST				99/ST				103/ST				107/ST				109/ST
1	16	9	8	1	9	16	8	1	15	12	6	1	15	12	6	1	13	12	8	1	13	12	8
15	10	7	2	4	12	5	13	10	13	8	3	7	14	9	4	3	15	10	6	4	16	9	5
4	5	12	13	15	7	10	2	7	4	9	14	10	3	8	13	16	2	7	9	15	3	6	10
14	3	6	11	14	6	3	11	16	2	5	11	16	2	5	11	14	4	5	11	14	2	7	11

			111/PD				129/ST				145/ST				157/PD				173/PD				189/ST
1	14	11	8	1	13	12	8	1	16	6	11	1	15	6	12	1	14	7	12	1	14	7	12
12	7	2	13	6	10	3	15	7	10	4	13	8	10	3	13	8	11	2	13	11	13	2	8
6	9	16	3	11	7	14	2	12	5	15	2	11	5	16	2	10	5	16	3	6	4	15	9
15	4	5	10	16	4	5	9	14	3	9	8	14	4	9	7	15	4	9	6	16	3	10	5

			197/PD				209/ST				225/SC				237/PD				253/ST				269/ST
1	12	7	14	2	15	12	5	2	16	11	5	2	16	9	7	2	15	9	8	2	15	10	7
8	13	2	11	14	3	8	9	13	3	8	10	13	3	6	12	14	3	5	12	11	4	5	14
10	3	16	5	7	10	13	4	7	9	14	4	8	10	15	1	7	10	16	1	8	9	16	1
15	6	9	4	11	6	1	16	12	6	1	15	11	5	4	14	11	6	4	13	13	6	3	12

			291/ST				303/ST				319/ST				337/PD				353/ST				365/ST
2	15	9	8	2	13	4	15	2	13	11	8	2	16	5	11	2	15	5	12	2	13	4	15
12	5	3	14	11	6	9	8	12	7	1	14	7	9	4	14	8	9	3	14	7	10	5	12
7	10	16	1	14	3	16	1	5	10	16	3	12	6	15	1	11	6	16	1	14	3	16	1
13	4	6	11	7	12	5	10	15	4	6	9	13	3	10	8	13	4	10	7	11	8	9	6

```
     373/ST       389/ST       397/ST       409/ST       417/PD       425/SC
 2 13  7 12   2 11  8 13   2 11  7 14   3 15 10  6   3 16  9  6   3 16 10  5
 8 11  1 14   7 12  1 14   8 13  1 12  14  2  7 11  13  2  7 12  13  2  8 11
 9  6 16  3  10  5 16  3   9  4 16  5   8 12 13  1   8 11 14  1   6  9 15  4
15  4 10  5  15  6  9  4  15  6 10  3   9  5  4 16  10  5  4 15  12  7  1 14
     433/ST       441/PD       457/ST       473/ST       485/ST       505/ST
 3 15 10  6   3 16  9  6   3 13 12  6   3 14  9  8   3 13 10  8   3 14  4 13
14  2  7 11  10  5  4 15  10  4  5 15  12  5  2 15  12  6  1 15   6  7  9 12
 5  9 16  4   8 11 14  1   7  9 16  2   6 11 16  1   5 11 16  2  15  2 16  1
12  8  1 13  13  2  7 12  14  8  1 11  13  4  7 10  14  4  7  9  10 11  5  8
     523/ST       539/ST       551/ST       567/ST       575/PD       585/ST
 3 15  6 10   3 14  5 12   3 13  6 12   3 10  7 14   4 15 10  5   4 16  9  5
13  8  1 12   8  9  2 15   8 10  1 15   6 11  2 15  14  1  8 11  13  1  8 12
 4  9 16  5  10  7 16  1   9  7 16  2  12  5 16  1   7 12 13  2   7 11 14  2
14  2 11  7  13  4 11  6  14  4 11  5  13  8  9  4   9  6  3 16  10  6  3 15
     593/ST       601/SC       609/ST       625/ST       641/ST       663/ST
 4 16  9  5   4 15  9  6   4 15 10  5   4 15  9  6   4 14  9  7   4 15  1 14
13  1  8 12  14  1  7 12  14  2  7 11  10  5  3 16  11  5  2 16  10  5 11  8
 6 10 15  3   5 10 16  3   3  9 16  6   7 12 14  1   6 12 15  1  13  2 16  3
11  7  2 14  11  8  2 13  13  8  1 12  13  2  8 11  13  3  8 10   7 12  6  9
     675/ST       699/ST       717/ST       741/ST       753/SC       757/ST
 4 14  7  9   4 13  3 14   4 14  5 11   4 15  1 14   5 16  4  9   5 15  4 10
12  6  1 15   5  8 10 11  10  8  1 15   6  9  7 12  11  2 14  7  12  2 13  7
 5 11 16  2  16  1 15  2   7  9 16  2  13  2 16  3  10  3 15  6   9  3 16  6
13  3 10  8   9 12  6  7  13  3 12  6  11  8 10  5   8 13  1 12   8 14  1 11
     761/ST       765/ST       773/PD       785/ST       801/ST       817/ST
 5 14  4 11   5 15  2 12   5 16  3 10   5 11  8 10   5 10  8 11   6 16  3  9
12  3 13  6  11  4 13  6   4  9  6 15   4  6  9 15   4  7  9 14  11  1 14  8
 9  2 16  7  10  1 16  7  14  7 12  1  13  3 16  2  13  2 16  3  10  4 15  5
 8 15  1 10   8 14  3  9  11  2 13  8  12 14  1  7  12 15  1  6   7 13  2 12
     821/SC       825/ST       833/ST       837/ST       853/ST       865/ST
 6 15  3 10   6 16  1 11   6 15  1 12   6 15  2 11   6 15  3 10   7 14  4  9
12  1 13  8  12  3 14  5  10  3 13  8  12  4 13  5   4  9  5 16  12  3 13  6
 9  4 16  5   9  2 15  8  11  2 16  5   7  1 16 10  13  8 12  1   5  2 16 11
 7 14  2 11   7 13  4 10   7 14  4  9   9 14  3  8  11  2 14  7  10 15  1  8
[Type Count: /SC=48, /PD=48, /ST=784] [Total Count = 880]  OK!
```

We have come to know there are 48 Self-complementary and 48 Pan-diagonal type of solutions among all the 880 Standard Magic Squares of Order 4.
We are looking forward to study those two special types of MS44 for the next stage.

6. Right here let me decompose some of those standard solutions by the 4-th increment number system and make the list with /D4i diagrams for each.
We want to know whether our MS44 are really the 'Greco-Latin Squares' or not.

```
** List of the Standard Solutions with /D4i **
 Classical Notation with Decomposition Diagrams:
       1/SC  /D4i H/       L/          13/ST  /D4i H/       L/
  1 15 12  6  0 3 2 1  0 2 3 1    1 16 11  6  0 3 2 1  0 3 2 1
 14  4  7  9  3 0 1 2  1 3 2 0   13  4  7 10  3 0 1 2  0 3 2 1
  8 10 13  3  1 2 3 0  3 1 0 2    8  9 14  3  1 2 3 0  3 0 1 2
 11  5  2 16  2 1 0 3  2 0 1 3   12  5  2 15  2 1 0 3  3 0 1 2
      71/ST  /D4i H/       L/          83/PD  /D4i H/       L/
  1 16 10  7  0 3 2 1  0 3 1 2    1 15 10  8  0 3 2 1  0 2 1 3
 11  6  4 13  2 1 0 3  2 1 3 0   12  6  3 13  2 1 0 3  3 1 2 0
  8  9 15  2  1 2 3 0  3 0 2 1    7  9 16  2  1 2 3 0  2 0 3 1
 14  3  5 12  3 0 1 2  1 2 0 3   14  4  5 11  3 0 1 2  1 3 0 2
```

When you are not very well used to discuss about this style of solution list, would you please go to Part 4, Chapter 4, Section 1 and read the article of mine, "Notation and 'Euler Squares'", though I am afraid it might be bothering you quite a little?
You should have your habit of checking always if every row and column consists of {0, 1, 2 and 3} in both high layer and low of Decomposition Diagrams. You don't have to care about the permutation of 4 numbers, but you have to care for the only constant combination. Each of {0, 1, 2 and 3} must be used strictly once in a line.
Watch the solution 13/ in the list above, especially the low layer. Each column fails to consist of {0, 1, 2 and 3}. It is not a "Latin Square" at all.
But the other three solutions are good enough to be called "Greco-Latin Squares".
On top of that, two solutions 71/ and 83/ are perfect "Greco-Latin Squares" even provided with their two primary diagonals consisting of {0, 1, 2 and 3} beside every rows and columns.

How many 'Greco-Latin' squares are there among the 880 standard MS44?
I tried to count (1) Simple Greco-Latin type without any definition to the two primary diagonals at first, and (2) Perfect Greco-Latin type with the two primary diagonals consisting of {0, 1, 2 and 3} finally.

```
** Standard Magic Squares of Order 4 with /Type and /D4i **
** Abstract List 1 of the Standard Solutions Type/SC: Self-Complementary,
     /PD: Pan-Diagonal, /ST: Standard; /D4i  E: Euler type. N: Non-Euler type **
        1/SC  /D4i  E        E        13/ST  /D4i  E        N        29/ST  /D4i  E        N
  1 15 12  6  0 3 2 1  0 2 3 1    1 16 11  6  0 3 2 1  0 3 2 1    1 16 10  7  0 3 2 1  0 3 1 2
 14  4  7  9  3 0 1 2  1 3 2 0   13  4  7 10  3 0 1 2  0 3 2 1   13  4  6 11  3 0 1 2  0 3 1 2
  8 10 13  3  1 2 3 0  3 1 0 2    8  9 14  3  1 2 3 0  3 0 1 2    8  9 15  2  1 2 3 0  3 0 2 1
 11  5  2 16  2 1 0 3  2 0 1 3   12  5  2 15  2 1 0 3  3 0 1 2   12  5  3 14  2 1 0 3  3 0 2 1
       45/PD  /D4i  E        E        71/ST  /D4i  E  P.Euler        83/PD  /D4i  E  P.Euler
  1 15 10  8  0 3 2 1  0 2 1 3    1 16 10  7  0 3 2 1  0 3 1 2    1 15 10  8  0 3 2 1  0 2 1 3
 14  4  5 11  3 0 1 2  1 3 0 2   11  6  4 13  2 1 0 3  2 1 3 0   12  6  3 13  2 1 0 3  3 1 2 0
  7  9 16  2  1 2 3 0  2 0 3 1    8  9 15  2  1 2 3 0  3 0 2 1    7  9 16  2  1 2 3 0  2 0 3 1
 12  6  3 13  2 1 0 3  3 1 2 0   14  3  5 12  3 0 1 2  1 2 0 3   14  4  5 11  3 0 1 2  1 3 0 2
      111/PD  /D4i  E  P.Euler       129/ST  /D4i  E        N       145/ST  /D4i  E  P.Euler
  1 14 11  8  0 3 2 1  0 1 2 3    1 13 12  8  0 3 2 1  0 0 3 3    1 16  6 11  0 3 1 2  0 3 1 2
 12  7  2 13  2 1 0 3  3 2 1 0    6 10  3 15  1 2 0 3  1 1 2 2    7 10  4 13  1 2 0 3  2 1 3 0
  6  9 16  3  1 2 3 0  1 0 3 2   11  7 14  2  2 1 3 0  2 2 1 1   12  5 15  2  2 1 3 0  3 0 2 1
 15  4  5 10  3 0 1 2  2 3 0 1   16  4  5  9  3 0 1 2  3 3 0 0   14  3  9  8  3 0 2 1  1 2 0 3
      157/PD  /D4i  E  P.Euler       173/PD  /D4i  E  P.Euler       189/ST  /D4i  N        N
  1 15  6 12  0 3 1 2  0 2 1 3    1 14  7 12  0 3 1 2  0 1 2 3    1 14  7 12  0 3 1 2  0 1 2 3
  8 10  3 13  1 2 0 3  3 1 2 0    8 11  2 13  1 2 0 3  3 2 1 0   11 13  2  8  2 3 0 1  2 0 1 3
 11  5 16  2  2 1 3 0  2 0 3 1   10  5 16  3  2 1 3 0  1 0 3 2    6  4 15  9  1 0 3 2  1 3 2 0
 14  4  9  7  3 0 2 1  1 3 0 2   15  4  9  6  3 0 2 1  2 3 0 1   16  3 10  5  3 0 2 1  3 2 1 0
      197/PD  /D4i  E        E       209/ST  /D4i  E        N       225/SC  /D4i  E        E
  1 12  7 14  0 2 1 3  0 3 2 1    2 15 12  5  0 3 2 1  1 2 3 0    2 16 11  5  0 3 2 1  1 3 2 0
  8 13  2 11  1 3 0 2  3 0 1 2   14  3  8  9  3 0 1 2  1 2 3 0   13  3  8 10  3 0 1 2  0 2 3 1
 10  3 16  5  2 0 3 1  1 2 3 0    7 10 13  4  1 2 3 0  2 1 0 3    7  9 14  4  1 2 3 0  2 0 1 3
 15  6  9  4  3 1 2 0  2 1 0 3   11  6  1 16  2 1 0 3  2 1 0 3   12  6  1 15  2 1 0 3  3 1 0 2
      237/PD  /D4i  E        E       253/ST  /D4i  E        N       269/ST  /D4i  E        N
  2 16  9  7  0 3 2 1  1 3 0 2    2 15  9  8  0 3 2 1  1 2 0 3    2 15 10  7  0 3 2 1  1 2 1 2
 13  3  6 12  3 0 1 2  0 2 1 3   14  3  5 12  3 0 1 2  1 2 0 3   11  4  5 14  2 0 1 3  2 3 0 1
  8 10 15  1  1 2 3 0  3 1 2 0    7 10 16  1  1 2 3 0  2 1 3 0    8  9 16  1  1 2 3 0  3 0 3 0
 11  5  4 14  2 1 0 3  2 0 3 1   11  6  4 13  2 1 0 3  2 1 3 0   13  6  3 12  3 1 0 2  0 1 2 3
      291/ST  /D4i  E  P.Euler       303/ST  /D4i  N        N       319/ST  /D4i  E  P.Euler
  2 15  9  8  0 3 2 1  1 2 0 3    2 13  4 15  0 3 0 3  1 0 3 2    2 13 11  8  0 3 2 1  1 0 2 3
 12  5  3 14  2 1 0 3  3 0 2 1   11  6  9  8  2 1 2 1  2 1 0 3   12  7  1 14  2 1 0 3  3 2 0 1
  7 10 16  1  1 2 3 0  2 1 3 0   14  3 16  1  3 0 3 0  1 2 3 0    5 10 16  3  1 2 3 0  0 1 3 2
 13  4  6 11  3 0 1 2  0 3 1 2    7 12  5 10  1 2 1 2  2 3 0 1   15  4  6  9  3 0 1 2  2 3 1 0
      337/PD  /D4i  E  P.Euler       353/ST  /D4i  E  P.Euler       365/ST  /D4i  N        N
  2 16  5 11  0 3 1 2  1 3 0 2    2 15  5 12  0 3 1 2  1 2 0 3    2 13  4 15  0 3 0 3  1 0 3 2
  7  9  4 14  1 2 0 3  2 0 3 1    8  9  3 14  1 2 0 3  3 0 2 1    7 10  5 12  1 2 1 2  2 1 0 3
 12  6 15  1  2 1 3 0  3 1 2 0   11  6 16  1  2 1 3 0  2 1 3 0   14  3 16  1  3 0 3 0  1 2 3 0
 13  3 10  8  3 0 2 1  0 2 1 3   13  4 10  7  3 0 2 1  0 3 1 2   11  8  9  6  2 1 2 1  2 3 0 1
```

373/ST	/D4i E	P.Euler	389/ST	/D4i N	N	397/ST	/D4i E	N
2 13 7 12	0 3 1 2	1 0 2 3	2 11 8 13	0 2 1 3	1 2 3 0	2 11 7 14	0 2 1 3	1 2 2 1
8 11 1 14	1 2 0 3	3 2 0 1	7 12 1 14	1 2 0 3	2 3 0 1	8 13 1 12	1 3 0 2	3 0 0 3
9 6 16 3	2 1 3 0	0 1 3 2	10 5 16 3	2 1 3 0	1 0 3 2	9 4 16 5	2 0 3 1	0 3 3 0
15 4 10 5	3 0 2 1	2 3 1 0	15 6 9 4	3 1 2 0	2 1 0 3	15 6 10 3	3 1 2 0	2 1 1 2
409/ST	/D4i E	N	417/PD	/D4i E	E	425/SC	/D4i E	E
3 15 10 6	0 3 2 1	2 2 1 1	3 16 9 6	0 3 2 1	2 3 0 1	3 16 10 5	0 3 2 1	2 3 1 0
14 2 7 11	3 0 1 2	1 1 2 2	13 2 7 12	3 0 1 2	0 1 2 3	13 2 8 11	3 0 1 2	0 1 3 2
8 12 13 1	1 2 3 0	3 3 0 0	8 11 14 1	1 2 3 0	3 2 1 0	6 9 15 4	1 2 3 0	1 0 2 3
9 5 4 16	2 1 0 3	0 0 3 3	10 5 4 15	2 1 0 3	1 0 3 2	12 7 1 14	2 1 0 3	3 2 0 1
433/ST	/D4i E	N	441/PD	/D4i E	P.Euler	457/ST	/D4i E	N
3 15 10 6	0 3 2 1	2 2 1 1	3 16 9 6	0 3 2 1	2 3 0 1	3 13 12 6	0 3 2 1	2 0 3 1
14 2 7 11	3 0 1 2	1 1 2 2	10 5 4 15	2 1 0 3	1 0 3 2	10 4 5 15	2 0 1 3	1 3 0 2
5 9 16 4	1 2 3 0	0 0 3 3	8 11 14 1	1 2 3 0	3 2 1 0	7 9 16 2	1 2 3 0	2 0 3 1
12 8 1 13	2 1 0 3	3 3 0 0	13 2 7 12	3 0 1 2	0 1 2 3	14 8 1 11	3 1 0 2	1 3 0 2
473/ST	/D4i E	P.Euler	485/ST	/D4i E	P.Euler	505/ST	/D4i N	N
3 14 9 8	0 3 2 1	2 1 0 3	3 13 10 8	0 3 2 1	2 0 1 3	3 14 4 13	0 3 0 3	2 1 3 0
12 5 2 15	2 1 0 3	3 0 1 2	12 6 1 15	2 1 0 3	3 1 0 2	6 7 9 12	1 1 2 2	1 2 0 3
6 11 16 1	1 2 3 0	1 2 3 0	5 11 16 2	1 2 3 0	0 2 3 1	15 2 16 1	3 0 3 0	2 1 3 0
13 4 7 10	3 0 1 2	0 3 2 1	14 4 7 9	3 0 1 2	1 3 2 0	10 11 5 8	2 2 1 1	1 2 0 3
523/ST	/D4i N	N	539/ST	/D4i E	P.Euler	551/ST	/D4i E	P.Euler
3 15 6 10	0 3 1 2	2 2 1 1	3 14 5 12	0 3 1 2	2 1 0 3	3 13 6 12	0 3 1 2	2 0 1 3
13 8 1 12	3 1 0 2	0 3 0 3	8 9 2 15	1 2 0 3	3 0 1 2	8 10 1 15	1 2 0 3	3 1 0 2
4 9 16 5	0 2 3 1	3 0 3 0	10 7 16 1	2 1 3 0	1 2 3 0	9 7 16 2	2 1 3 0	0 2 3 1
14 2 11 7	3 0 2 1	1 1 2 2	13 4 11 6	3 0 2 1	0 3 2 1	14 4 11 5	3 0 2 1	1 3 2 0
567/ST	/D4i N	N	575/PD	/D4i E	E	585/ST	/D4i E	N
3 10 7 14	0 2 1 3	2 1 2 1	4 15 10 5	0 3 2 1	3 2 1 0	4 16 9 5	0 3 2 1	3 3 0 0
6 11 2 15	1 2 0 3	1 2 1 2	14 1 8 11	3 0 1 2	1 0 3 2	13 1 8 12	3 0 1 2	0 0 3 3
12 5 16 1	2 1 3 0	3 0 3 0	7 12 13 2	1 2 3 0	2 3 0 1	7 11 14 2	1 2 3 0	2 2 1 1
13 8 9 4	3 1 2 0	0 3 0 3	9 6 3 16	2 1 0 3	0 1 2 3	10 6 3 15	2 1 0 3	1 1 2 2
593/ST	/D4i E	N	601/SC	/D4i E	E	609/ST	/D4i N	N
4 16 9 5	0 3 2 1	3 3 0 0	4 15 9 6	0 3 2 1	3 2 0 1	4 15 10 5	0 3 2 1	3 2 1 0
13 1 8 12	3 0 1 2	0 0 3 3	14 1 7 12	3 0 1 2	1 0 2 3	14 2 7 11	3 0 1 2	1 1 2 2
6 10 15 3	1 2 3 0	1 1 2 2	5 10 16 3	1 2 3 0	0 1 3 2	3 9 16 6	0 2 3 1	2 0 3 1
11 7 2 14	2 1 0 3	2 2 1 1	11 8 2 13	2 1 0 3	2 3 1 0	13 8 1 12	3 1 0 2	0 3 0 3
625/ST	/D4i E	P.Euler	641/ST	/D4i E	P.Euler	663/ST	/D4i N	E
4 15 9 6	0 3 2 1	3 2 0 1	4 14 9 7	0 3 2 1	3 1 0 2	4 15 1 14	0 3 0 3	3 2 0 1
10 5 3 16	2 1 0 3	1 0 2 3	11 5 2 16	2 1 0 3	2 0 1 3	10 5 11 8	2 1 2 1	1 0 2 3
7 12 14 1	1 2 3 0	2 3 1 0	6 12 15 1	1 2 3 0	1 3 2 0	13 2 16 3	3 0 3 0	0 1 3 2
13 2 8 11	3 0 1 2	0 1 3 2	13 3 8 10	3 0 1 2	0 2 3 1	7 12 6 9	1 2 1 2	2 3 1 0
675/ST	/D4i E	N	699/ST	/D4i N	N	717/ST	/D4i E	N
4 14 7 9	0 3 1 2	3 1 2 0	4 13 3 14	0 3 0 3	3 0 2 1	4 14 5 11	0 3 1 2	3 1 0 2
12 6 1 15	2 1 0 3	3 1 0 2	5 8 10 11	1 1 2 2	0 3 1 2	10 8 1 15	2 1 0 3	1 3 0 2
5 11 16 2	1 2 3 0	0 2 3 1	16 1 15 2	3 0 3 0	3 0 2 1	7 9 16 2	1 2 3 0	2 0 3 1
13 3 10 8	3 0 2 1	0 2 1 3	9 12 6 7	2 2 1 1	0 3 1 2	13 3 12 6	3 0 2 1	0 2 3 1
741/ST	/D4i N	E	753/SC	/D4i N	N	757/ST	/D4i N	N
4 15 1 14	0 3 0 3	3 2 0 1	5 16 4 9	1 3 0 2	0 3 3 0	5 15 4 10	1 3 0 2	0 2 3 1
6 9 7 12	1 2 1 2	1 0 2 3	11 2 14 7	2 0 3 1	2 1 1 2	12 2 13 7	2 0 3 1	3 1 0 2
13 2 16 3	3 0 3 0	0 1 3 2	10 3 15 6	2 0 3 1	1 2 2 1	9 3 16 6	2 0 3 1	0 2 3 1
11 8 10 5	2 1 2 1	2 3 1 0	8 13 1 12	1 3 0 2	3 0 0 3	8 14 1 11	1 3 0 2	3 1 0 2
761/ST	/D4i N	N	765/ST	/D4i N	E	773/PD	/D4i E	E
5 14 4 11	1 3 0 2	0 1 3 2	5 15 2 12	1 3 0 2	0 2 1 3	5 16 3 10	1 3 0 2	0 3 2 1
12 3 13 6	2 0 3 1	3 2 0 1	11 4 13 6	2 0 3 1	2 3 0 1	4 9 6 15	0 2 1 3	3 0 1 2
9 2 16 7	2 0 3 1	0 1 3 2	10 1 16 7	2 0 3 1	1 0 3 2	14 7 12 1	3 1 2 0	1 2 3 0
8 15 1 10	1 3 0 2	3 2 0 1	8 14 3 9	1 3 0 2	3 1 2 0	11 2 13 8	2 0 3 1	2 1 0 3
785/ST	/D4i N	N	801/ST	/D4i N	N	817/ST	/D4i N	N
5 11 8 10	1 2 1 2	0 2 3 1	5 10 8 11	1 2 1 2	0 1 3 2	6 16 3 9	1 3 0 2	1 3 2 0
4 6 9 15	0 1 2 3	3 1 0 2	4 7 9 14	0 1 2 3	3 2 0 1	11 1 14 8	2 0 3 1	2 0 1 3
13 3 16 2	3 0 3 0	0 2 3 1	13 2 16 3	3 0 3 0	0 1 3 2	10 4 15 5	2 0 3 1	1 3 2 0
12 14 1 7	2 3 0 1	3 1 0 2	12 15 1 6	2 3 0 1	3 2 0 1	7 13 2 12	1 3 0 2	2 0 1 3
821/SC	/D4i N	N	825/ST	/D4i N	E	833/ST	/D4i N	N
6 15 3 10	1 3 0 2	1 2 2 1	6 16 1 11	1 3 0 2	1 3 0 2	6 15 1 12	1 3 0 2	1 2 0 3
12 1 13 8	2 0 3 1	3 0 0 3	12 3 14 5	2 0 3 1	3 2 1 0	10 3 13 8	2 0 3 1	1 2 0 3
9 4 16 5	2 0 3 1	0 3 3 0	9 2 15 8	2 0 3 1	0 1 2 3	11 2 16 5	2 0 3 1	2 1 3 0
7 14 2 11	1 3 0 2	2 1 1 2	7 13 4 10	1 3 0 2	2 0 3 1	7 14 4 9	1 3 0 2	2 1 3 0
837/ST	/D4i N	N	853/ST	/D4i E	N	865/ST	/D4i N	N
6 15 2 11	1 3 0 2	1 2 1 2	6 15 3 10	1 3 0 2	1 2 2 1	7 14 4 9	1 3 0 2	2 1 3 0
12 4 13 5	2 0 3 1	3 3 0 0	4 9 5 16	0 2 1 3	3 0 0 3	12 3 13 6	2 0 3 1	3 2 0 1
7 1 16 10	1 0 3 2	2 0 3 1	13 8 12 1	3 1 2 0	0 3 3 0	5 2 16 11	1 0 3 2	0 1 3 2
9 14 3 8	2 3 0 1	0 1 2 3	11 2 14 7	2 0 3 1	2 1 1 2	10 15 1 8	2 3 0 1	1 2 0 3

[Counts: 48/SC, 48/PD, 784/ST, 144/P.Euler, 192/Euler, 880/Total]

There are 192 solutions of that type among 880 Standard MS44.

** Perfect Greco-Latin Type of Standard Magic Squares of Order 4: **
** Abstract List 2 of the Standard Solutions with /Type and /D4i **

[/SC: Self-Complementary, /PD: Pan-Diagonal, /ST: Standard]

```
       4/SC  /D4i H/       L/        18/ST  /D4i H/       L/        32/ST  /D4i H/       L/
  1 15  8 10  0 3 1 2  0 2 3 1    1 16  7 10  0 3 1 2  0 3 2 1    1 16  6 11  0 3 1 2  0 3 1 2
 12  6 13  3  2 1 3 0  3 1 0 2   11  6 13  4  2 1 3 0  2 1 0 3   10  7 13  4  2 1 3 0  1 2 0 3
 14  4 11  5  3 0 2 1  1 3 2 0   14  3 12  5  3 0 2 1  1 2 3 0   15  2 12  5  3 0 2 1  2 1 3 0
  7  9  2 16  1 2 0 3  2 0 1 3    8  9  2 15  1 2 0 3  3 0 1 2    8  9  3 14  1 2 0 3  3 0 2 1
      71/ST  /D4i H/       L/        83/PD  /D4i H/       L/       111/PD  /D4i H/       L/
  1 16 10  7  0 3 2 1  0 3 1 2    1 15 10  8  0 3 2 1  0 2 1 3    1 14 11  8  0 3 2 1  0 1 2 3
 11  6  4 13  2 1 0 3  2 1 3 0   12  6  3 13  2 1 0 3  3 1 2 0   12  7  2 13  2 1 0 3  3 2 1 0
  8  9 15  2  1 2 3 0  3 0 2 1    7  9 16  2  1 2 3 0  2 0 3 1    6  9 16  3  1 2 3 0  1 0 3 2
 14  3  5 12  3 0 1 2  1 2 0 3   14  4  5 11  3 0 1 2  1 3 0 2   15  4  5 10  3 0 1 2  2 3 0 1
     145/ST  /D4i H/       L/       157/PD  /D4i H/       L/       173/PD  /D4i H/       L/
  1 16  6 11  0 3 1 2  0 3 1 2    1 15  6 12  0 3 1 2  0 2 1 3    1 14  7 12  0 3 1 2  0 1 2 3
  7 10  4 13  1 2 0 3  2 1 3 0    8 10  3 13  1 2 0 3  3 1 2 0    8 11  2 13  1 2 0 3  3 2 1 0
 12  5 15  2  2 1 3 0  3 0 2 1   11  5 16  2  2 1 3 0  2 0 3 1   10  5 16  3  2 1 3 0  1 0 3 2
 14  3  9  8  3 0 2 1  1 2 0 3   14  4  9  7  3 0 2 1  1 3 0 2   15  4  9  6  3 0 2 1  2 3 0 1
     214/ST  /D4i H/       L/       228/SC  /D4i H/       L/       256/ST  /D4i H/       L/
  2 15  8  9  0 3 1 2  1 2 3 0    2 16  7  9  0 3 1 2  1 3 2 0    2 16  5 11  0 3 1 2  1 3 0 2
 12  5 14  3  2 1 3 0  3 0 1 2   11  5 14  4  2 1 3 0  2 0 1 3    9  7 14  4  2 1 3 0  0 2 1 3
 13  4 11  6  3 0 2 1  0 3 2 1   13  3 12  6  3 0 2 1  0 2 3 1   15  1 12  6  3 0 2 1  2 0 3 1
  7 10  1 16  1 2 0 3  2 1 0 3    8 10  1 15  1 2 0 3  3 1 0 2    8 10  3 13  1 2 0 3  3 1 2 0
     270/PD  /D4i H/       L/       291/ST  /D4i H/       L/       319/ST  /D4i H/       L/
  2 16  9  7  0 3 2 1  1 3 0 2    2 15  9  8  0 3 2 1  1 2 0 3    2 13 11  8  0 3 2 1  1 0 2 3
 11  5  4 14  2 1 0 3  2 0 3 1   12  5  3 14  2 1 0 3  3 0 2 1   12  7  1 14  2 1 0 3  3 2 0 1
  8 10 15  1  1 2 3 0  3 1 2 0    7 10 16  1  1 2 3 0  2 1 3 0    5 10 16  3  1 2 3 0  0 1 3 2
 13  3  6 12  3 0 1 2  0 2 1 3   13  4  6 11  3 0 1 2  0 3 1 2   15  4  6  9  3 0 1 2  2 3 1 0
     337/PD  /D4i H/       L/       353/ST  /D4i H/       L/       373/ST  /D4i H/       L/
  2 16  5 11  0 3 1 2  1 3 0 2    2 15  5 12  0 3 1 2  1 2 0 3    2 13  7 12  0 3 1 2  1 0 2 3
  7  9  4 14  1 2 0 3  2 0 3 1    8  9  3 14  1 2 0 3  3 0 2 1    8 11  1 14  1 2 0 3  3 2 0 1
 12  6 15  1  2 1 3 0  3 1 2 0   11  6 16  1  2 1 3 0  2 1 3 0    9  6 16  3  2 1 3 0  0 1 3 2
 13  3 10  8  3 0 2 1  0 2 1 3   13  4 10  7  3 0 2 1  0 3 1 2   15  4 10  5  3 0 2 1  2 3 1 0
     411/ST  /D4i H/       L/       427/SC  /D4i H/       L/       435/ST  /D4i H/       L/
  3 14  8  9  0 3 1 2  2 1 3 0    3 16  6  9  0 3 1 2  2 3 1 0    3 16  5 10  0 3 1 2  2 3 0 1
 12  5 15  2  2 1 3 0  3 0 2 1   10  5 15  4  2 1 3 0  1 0 2 3    9  6 15  4  2 1 3 0  0 1 2 3
 13  4 10  7  3 0 2 1  0 3 1 2   13  2 12  7  3 0 2 1  0 1 3 2   14  1 12  7  3 0 2 1  1 0 3 2
  6 11  1 16  1 2 0 3  1 2 0 3    8 11  1 14  1 2 0 3  3 2 0 1    8 11  2 13  1 2 0 3  3 2 1 0
     441/PD  /D4i H/       L/       473/ST  /D4i H/       L/       485/ST  /D4i H/       L/
  3 16  9  6  0 3 2 1  2 3 0 1    3 14  9  8  0 3 2 1  2 1 0 3    3 13 10  8  0 3 2 1  2 0 1 3
 10  5  4 15  2 1 0 3  1 0 3 2   12  5  2 15  2 1 0 3  3 0 1 2   12  6  1 15  2 1 0 3  3 1 0 2
  8 11 14  1  1 2 3 0  3 2 1 0    6 11 16  1  1 2 3 0  1 2 3 0    5 11 16  2  1 2 3 0  0 2 3 1
 13  2  7 12  3 0 1 2  0 1 2 3   13  4  7 10  3 0 1 2  0 3 2 1   14  4  7  9  3 0 1 2  1 3 2 0
     506/PD  /D4i H/       L/       539/ST  /D4i H/       L/       551/ST  /D4i H/       L/
  3 16  5 10  0 3 1 2  2 3 0 1    3 14  5 12  0 3 1 2  2 1 0 3    3 13  6 12  0 3 1 2  2 0 1 3
  6  9  4 15  1 2 0 3  1 0 3 2    8  9  2 15  1 2 0 3  3 0 1 2    8 10  1 15  1 2 0 3  3 1 0 2
 12  7 14  1  2 1 3 0  3 2 1 0   10  7 16  1  2 1 3 0  1 2 3 0    9  7 16  2  2 1 3 0  0 2 3 1
 13  2 11  8  3 0 2 1  0 1 2 3   13  4 11  6  3 0 2 1  0 3 2 1   14  4 11  5  3 0 2 1  1 3 2 0
     587/ST  /D4i H/       L/       595/ST  /D4i H/       L/       603/SC  /D4i H/       L/
  4 14  7  9  0 3 1 2  3 1 2 0    4 15  6  9  0 3 1 2  3 2 1 0    4 15  5 10  0 3 1 2  3 2 0 1
 11  5 16  2  2 1 3 0  2 0 3 1   10  5 16  3  2 1 3 0  1 0 3 2    9  6 16  3  2 1 3 0  0 1 3 2
 13  3 10  8  3 0 2 1  0 2 1 3   13  2 11  8  3 0 2 1  0 1 2 3   14  1 11  8  3 0 2 1  1 0 2 3
  6 12  1 15  1 2 0 3  1 3 0 2    7 12  1 14  1 2 0 3  2 3 0 1    7 12  2 13  1 2 0 3  2 3 1 0
     625/ST  /D4i H/       L/       641/ST  /D4i H/       L/       665/ST  /D4i H/       L/
  4 15  9  6  0 3 2 1  3 2 0 1    4 14  9  7  0 3 2 1  3 1 0 2    4 13 10  7  0 3 2 1  3 0 1 2
 10  5  3 16  2 1 0 3  1 0 2 3   11  5  2 16  2 1 0 3  2 0 1 3   11  6  1 16  2 1 0 3  2 1 0 3
  7 12 14  1  1 2 3 0  2 3 1 0    6 12 15  1  1 2 3 0  1 3 2 0    5 12 15  2  1 2 3 0  0 3 2 1
 13  2  8 11  3 0 1 2  0 1 3 2   13  3  8 10  3 0 1 2  0 2 3 1   14  3  8  9  3 0 1 2  1 2 3 0
     700/ST  /D4i H/       L/       721/ST  /D4i H/       L/       743/ST  /D4i H/       L/
  4 15  5 10  0 3 1 2  3 2 0 1    4 14  5 11  0 3 1 2  3 1 0 2    4 13  6 11  0 3 1 2  3 0 1 2
  6  9  3 16  1 2 0 3  1 0 2 3    7  9  2 16  1 2 0 3  2 0 1 3    7 10  1 16  1 2 0 3  2 1 0 3
 11  8 14  1  2 1 3 0  2 3 1 0   10  8 15  1  2 1 3 0  1 3 2 0    9  8 15  2  2 1 3 0  0 3 2 1
 13  2 12  7  3 0 2 1  0 1 3 2   13  3 12  6  3 0 2 1  0 2 3 1   14  3 12  5  3 0 2 1  1 2 3 0
```

```
[Type Counts: SC=16, PD=16, ST=112] [Total Count = 880]  OK!
```

There are 144 solutions of that perfect type among 880 Standard MS44.

```
** Pan-diagonal Magic Squares of Order 4: List of Standard Solutions
 with /D4i and Euler Check(E: Euler, N: Non-Euler, P.Euler: Perfect Euler Type) **
       45/PD  /D4i  E         E          46/PD  /D4i  E         E          83/PD  /D4i  E  P.Euler
  1 15 10  8  0 3 2 1  0 2 1 3    1 15  6 12  0 3 1 2  0 2 1 3    1 15 10  8  0 3 2 1  0 2 1 3
 14  4  5 11  3 0 1 2  1 3 0 2   14  4  9  7  3 0 2 1  1 3 0 2   12  6  3 13  2 1 0 3  3 1 2 0
  7  9 16  2  1 2 3 0  2 0 3 1   11  5 16  2  2 1 3 0  2 0 3 1    7  9 16  2  1 2 3 0  2 0 3 1
 12  6  3 13  2 1 0 3  3 1 2 0    8 10  3 13  1 2 0 3  3 1 2 0   14  4  5 11  3 0 1 2  1 3 0 2
       84/PD  /D4i  N         N         111/PD  /D4i  E  P.Euler        112/PD  /D4i  N         N
  1 15  4 14  0 3 0 3  0 2 3 1    1 14 11  8  0 3 2 1  0 1 2 3    1 14  4 15  0 3 0 3  0 1 3 2
 12  6  9  7  2 1 2 1  3 1 0 2   12  7  2 13  2 1 0 3  3 2 1 0   12  7  9  6  2 1 2 1  3 2 0 1
 13  3 16  2  3 0 3 0  0 2 3 1    6  9 16  3  1 2 3 0  1 0 3 2   13  2 16  3  3 0 3 0  0 1 3 2
  8 10  5 11  1 2 1 2  3 1 0 2   15  4  5 10  3 0 1 2  2 3 0 1    8 11  5 10  1 2 1 2  3 2 0 1
      157/PD  /D4i  E  P.Euler        158/PD  /D4i  N         N         173/PD  /D4i  E  P.Euler
  1 15  6 12  0 3 1 2  0 2 1 3    1 15  4 14  0 3 0 3  0 2 3 1    1 14  7 12  0 3 1 2  0 1 2 3
  8 10  3 13  1 2 0 3  3 1 2 0    8 10  5 11  1 2 1 2  3 1 0 2    8 11  2 13  1 2 0 3  3 2 1 0
 11  5 16  2  2 1 3 0  2 0 3 1   13  3 16  2  3 0 3 0  0 2 3 1   10  5 16  3  2 1 3 0  1 0 3 2
 14  4  9  7  3 0 2 1  1 3 0 2   12  6  9  7  2 1 2 1  3 1 0 2   15  4  9  6  3 0 2 1  2 3 0 1
      174/PD  /D4i  N         N         197/PD  /D4i  E         E         198/PD  /D4i  E         E
  1 14  4 15  0 3 0 3  0 1 3 2    1 12  7 14  0 2 1 3  0 3 2 1    1 12  6 15  0 2 1 3  0 3 1 2
  8 11  5 10  1 2 1 2  3 2 0 1    8 13  2 11  1 3 0 2  3 0 1 2    8 13  3 10  1 3 0 2  3 0 2 1
 13  2 16  3  3 0 3 0  0 1 3 2   10  3 16  5  2 0 3 1  1 2 3 0   11  2 16  5  2 0 3 1  2 1 3 0
 12  7  9  6  2 1 2 1  3 2 0 1   15  6  9  4  3 1 2 0  2 1 0 3   14  7  9  4  3 1 2 0  1 2 0 3
      237/PD  /D4i  E         E         238/PD  /D4i  E         E         270/PD  /D4i  E  P.Euler
  2 16  9  7  0 3 2 1  1 3 0 2    2 16  5 11  0 3 1 2  1 3 0 2    2 16  9  7  0 3 2 1  1 3 0 2
 13  3  6 12  3 0 1 2  0 2 1 3   13  3 10  8  3 0 2 1  0 2 1 3   11  5  4 14  2 1 0 3  2 0 3 1
  8 10 15  1  1 2 3 0  3 1 2 0   12  6 15  1  2 1 3 0  3 1 2 0    8 10 15  1  1 2 3 0  3 1 2 0
 11  5  4 14  2 1 0 3  2 0 3 1    7  9  4 14  1 2 0 3  2 0 3 1   13  3  6 12  3 0 1 2  0 2 1 3
      271/PD  /D4i  N         N         323/PD  /D4i  E  P.Euler        324/PD  /D4i  N         N
  2 16  3 13  0 3 0 3  1 3 2 0    2 13 12  7  0 3 2 1  1 0 3 2    2 13  3 16  0 3 0 3  1 0 2 3
 11  5 10  8  2 1 2 1  2 0 1 3   11  8  1 14  2 1 0 3  2 3 0 1   11  8 10  5  2 1 2 1  2 3 1 0
 14  4 15  1  3 0 3 0  1 3 2 0    5 10 15  4  1 2 3 0  0 1 2 3   14  1 15  4  3 0 3 0  1 0 2 3
  7  9  6 12  1 2 1 2  2 0 1 3   16  3  6  9  3 0 1 2  3 2 1 0    7 12  6  9  1 2 1 2  2 3 1 0
      337/PD  /D4i  E  P.Euler        338/PD  /D4i  N         N         377/PD  /D4i  E  P.Euler
  2 16  5 11  0 3 1 2  1 3 0 2    2 16  3 13  0 3 0 3  1 3 2 0    2 13  8 11  0 3 1 2  1 0 3 2
  7  9  4 14  1 2 0 3  2 0 3 1    7  9  6 12  1 2 1 2  2 0 1 3    7 12  1 14  1 2 0 3  2 3 0 1
 12  6 15  1  2 1 3 0  3 1 2 0   14  4 15  1  3 0 3 0  1 3 2 0    9  6 15  4  2 1 3 0  0 1 2 3
 13  3 10  8  3 0 2 1  0 2 1 3   11  5 10  8  2 1 2 1  2 0 1 3   16  3 10  5  3 0 2 1  3 2 1 0
      378/PD  /D4i  N         N         401/PD  /D4i  E         E         402/PD  /D4i  E         E
  2 13  3 16  0 3 0 3  1 0 2 3    2 11  8 13  0 2 1 3  1 2 3 0    2 11  5 16  0 2 1 3  1 2 0 3
  7 12  6  9  1 2 1 2  2 3 1 0    7 14  1 12  1 3 0 2  2 1 0 3    7 14  4  9  1 3 0 2  2 1 3 0
 14  1 15  4  3 0 3 0  1 0 2 3    9  4 15  6  2 0 3 1  0 3 2 1   12  1 15  6  2 0 3 1  3 0 2 1
 11  8 10  5  2 1 2 1  2 3 1 0   16  5 10  3  3 1 2 0  3 0 1 2   13  8 10  3  3 1 2 0  0 3 1 2
      417/PD  /D4i  E         E         418/PD  /D4i  E         E         441/PD  /D4i  E  P.Euler
  3 16  9  6  0 3 2 1  2 3 0 1    3 16  5 10  0 3 1 2  2 3 0 1    3 16  9  6  0 3 2 1  2 3 0 1
 13  2  7 12  3 0 1 2  0 1 2 3   13  2 11  8  3 0 2 1  0 1 2 3   10  5  4 15  2 1 0 3  1 0 3 2
  8 11 14  1  1 2 3 0  3 2 1 0   12  7 14  1  2 1 3 0  3 2 1 0    8 11 14  1  1 2 3 0  3 2 1 0
 10  5  4 15  2 1 0 3  1 0 3 2    6  9  4 15  1 2 0 3  1 0 3 2   13  2  7 12  3 0 1 2  0 1 2 3
      442/PD  /D4i  N         N         491/PD  /D4i  E  P.Euler        492/PD  /D4i  N         N
  3 16  2 13  0 3 0 3  2 3 1 0    3 13 12  6  0 3 2 1  2 0 3 1    3 13  2 16  0 3 0 3  2 0 1 3
 10  5 11  8  2 1 2 1  1 0 2 3   10  8  1 15  2 1 0 3  1 3 0 2   10  8 11  5  2 1 2 1  1 3 2 0
 15  4 14  1  3 0 3 0  2 3 1 0    5 11 14  4  1 2 3 0  0 2 1 3   15  1 14  4  3 0 3 0  2 0 1 3
  6  9  7 12  1 2 1 2  1 0 2 3   16  2  7  9  3 0 1 2  3 1 2 0    6 12  7  9  1 2 1 2  1 3 2 0
      506/PD  /D4i  E  P.Euler        507/PD  /D4i  N         N         557/PD  /D4i  E  P.Euler
  3 16  5 10  0 3 1 2  2 3 0 1    3 16  2 13  0 3 0 3  2 3 1 0    3 13  8 10  0 3 1 2  2 0 3 1
  6  9  4 15  1 2 0 3  1 0 3 2    6  9  7 12  1 2 1 2  1 0 2 3    6 12  1 15  1 2 0 3  1 3 0 2
 12  7 14  1  2 1 3 0  3 2 1 0   15  4 14  1  3 0 3 0  2 3 1 0    9  7 14  4  2 1 3 0  0 2 1 3
 13  2 11  8  3 0 2 1  0 1 2 3   10  5 11  8  2 1 2 1  1 0 2 3   16  2 11  5  3 0 2 1  3 1 2 0
      558/PD  /D4i  N         N         575/PD  /D4i  E         E         576/PD  /D4i  E         E
  3 13  2 16  0 3 0 3  2 0 1 3    4 15 10  5  0 3 2 1  3 2 1 0    4 15  6  9  0 3 1 2  3 2 1 0
  6 12  7  9  1 2 1 2  1 3 2 0   14  1  8 11  3 0 1 2  1 0 3 2   14  1 12  7  3 0 2 1  1 0 3 2
 15  1 14  4  3 0 3 0  2 0 1 3    7 12 13  2  1 2 3 0  2 3 0 1   11  8 13  2  2 1 3 0  2 3 0 1
 10  8 11  5  2 1 2 1  1 3 2 0    9  6  3 16  2 1 0 3  0 1 2 3    5 10  3 16  1 2 0 3  0 1 2 3
```

```
      629/PD  /D4i  E  P.Euler        630/PD  /D4i  N        N        647/PD  /D4i  E  P.Euler
 4 15 10  5  0 3 2 1  3 2 1 0    4 15  1 14  0 3 0 3  3 2 0 1    4 14 11  5  0 3 2 1  3 1 2 0
 9  6  3 16  2 1 0 3  0 1 2 3    9  6 12  7  2 1 2 1  0 1 3 2    9  7  2 16  2 1 0 3  0 2 1 3
 7 12 13  2  1 2 3 0  2 3 0 1   16  3 13  2  3 0 3 0  3 2 0 1    6 12 13  3  1 2 3 0  1 3 0 2
14  1  8 11  3 0 1 2  1 0 3 2    5 10  8 11  1 2 1 2  0 1 3 2   15  1  8 10  3 0 1 2  2 0 3 1
      648/PD  /D4i  N        N        704/PD  /D4i  E  P.Euler        705/PD  /D4i  N        N
 4 14  1 15  0 3 0 3  3 1 0 2    4 15  6  9  0 3 1 2  3 2 1 0    4 15  1 14  0 3 0 3  3 2 0 1
 9  7 12  6  2 1 2 1  0 2 3 1    5 10  3 16  1 2 0 3  0 1 2 3    5 10  8 11  1 2 1 2  0 1 3 2
16  2 13  3  3 0 3 0  3 1 0 2   11  8 13  2  2 1 3 0  2 3 0 1   16  3 13  2  3 0 3 0  3 2 0 1
 5 11  8 10  1 2 1 2  0 2 3 1   14  1 12  7  3 0 2 1  1 0 3 2    9  6 12  7  2 1 2 1  0 1 3 2
      727/PD  /D4i  E  P.Euler        728/PD  /D4i  N        N        773/PD  /D4i  E        E
 4 14  7  9  0 3 1 2  3 1 2 0    4 14  1 15  0 3 0 3  3 1 0 2    5 16  3 10  1 3 0 2  0 3 2 1
 5 11  2 16  1 2 0 3  0 2 1 3    5 11  8 10  1 2 1 2  0 2 3 1    4  9  6 15  0 2 1 3  3 0 1 2
10  8 13  3  2 1 3 0  1 3 0 2   16  2 13  3  3 0 3 0  3 1 0 2   14  7 12  1  3 1 2 0  1 2 3 0
15  1 12  6  3 0 2 1  2 0 3 1    9  7 12  6  2 1 2 1  0 2 3 1   11  2 13  8  2 0 3 1  2 1 0 3
      774/PD  /D4i  E        E        857/PD  /D4i  E        E        858/PD  /D4i  E        E
 5 16  2 11  1 3 0 2  0 3 1 2    6 15  4  9  1 3 0 2  1 2 3 0    6 15  1 12  1 3 0 2  1 2 0 3
 4  9  7 14  0 2 1 3  3 0 2 1    3 10  5 16  0 2 1 3  2 1 0 3    3 10  8 13  0 2 1 3  2 1 3 0
15  6 12  1  3 1 2 0  2 1 3 0   13  8 11  2  3 1 2 0  0 3 2 1   16  5 11  2  3 1 2 0  3 0 2 1
10  3 13  8  2 0 3 1  1 2 0 3   12  1 14  7  2 0 3 1  3 0 1 2    9  4 14  7  2 0 3 1  0 3 1 2
[Counts: 48/SC, 48/PD, 784/ST, 16/P.Euler, 32/Euler, 880/Total]
```

As in the list above I found all the 16 Pan-diagonal MS44 of the perfect Greco-Latin type having the two primary diagonals consisting of {0, 1, 2 and 3}.
How beautiful these decomposed diagrams are! Don't you think so?

7. But, what do you think it means by these miraculous "Greco-Latin Squares"?
It does not seem that Greco-Latin Squares explains very well about the secrets of any one of such the three types as Standard, Self-complementary or Pan-diagonal. Because only a part of solutions of any type could be the Greco-Latin Squares.
All the 48 Self-complementary MS44 are not always the Greco-Latin Squares.
All the 48 Pan-diagonal MS44 are not always the Greco-Latin Squares.
If you try to define all the 8 pan-diagonals must consist of {0, 1, 2 and 3}, no answer would come. There are no 'Complete Euler Squares' for the Pan-diagonal MS44, only when decomposed by the 4-th increment number system.
When you want to have the true 'Complete Euler Squares' for Pan-diagonal MS44, you must use the Binary Number System instead of Base 4.
Read another articles of mine in Part 4, Chapter 4, will you please?

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