Part 1 : "Basic Studies of Magic Squares and Cubes"
Chapter 3 : "Algebraic Study of Magic Squares and Cubes (2)"
Section 1 : Standard Magic Squares of Order 5: Kanji Setsuda

1. Basic Form, Basic Conditions for Standard Magic Squares 5x5

Let's study about the standard magic squares of order 5 at first. Each of them should be built in the basic form shown below. It must be made up of the natural numbers {1, 2, 3, 4, 5, 6, ..., 22, 23, 24 and 25} using each strictly once, and they must be placed in the 5x5 regular array. The Basic Form below shows their positions and let's call the variables as "n1, n2, n3, n4, ..., n24 and n25."

```
** Basic Form & Basic Conditions for MS55 **
  .--.--.--.--.--.    n1+n2+n3+n4+n5=C          ...rw1;
  |n1|n2|n3|n4|n5|    n6+n7+n8+n9+n10=C         ...rw2;
  |--+--+--+--+--|    n11+n12+n13+n14+n15=C     ...rw3;
  |n6|n7|n8|n9|10|    n16+n17+n18+n19+n20=C     ...rw4;
  |--+--+--+--+--|    n21+n22+n23+n24+n25=C     ...rw5;
  |11|12|13|14|15|    n1+n6+n11+n16+n21=C       ...cl1;
  |--+--+--+--+--|    n2+n7+n12+n17+n22=C       ...cl2;
  |16|17|18|19|20|    n3+n8+n13+n18+n23=C       ...cl3;
  |--+--+--+--+--|    n4+n9+n14+n19+n24=C       ...cl4;
  |21|22|23|24|25|    n5+n10+n15+n20+n25=C      ...cl5;
  '--'--'--'--'--'    n1+n7+n13+n19+n25=C       ... d1;
                      n5+n9+n13+n17+n21=C       ... d2;
** List-forming Inequality Conditions for MS55 **
  n1<n5, n5<n21 and n1<n25                      ...lic;
```

2. We must assume every 5 entries on any row and any column in the two directions should add up to the magic constant C, and even every 5 entries on the two primary diagonals should equally add up to the same sum.

```
rw1+rw2+rw3+rw4+rw5
  n1+n2+n3+n4+n5+n6+n7+n8+...+n23+n24+n25  = 5xC
            1+2+3+4+5+6+7+8+...+23+24+25  = 5xC
                              (1+25)x25/2  = 5xC
                Therefore                C  = 65
```

3. There are many solutions for this type naturally, I would say 'too many', for we have only 12 simultaneous equations while we have as many variables as 25.

Let's dictate such a good program for our computer as to make it calculate and count how many standard solutions there are in all for this type, shall we?

```c
/** Standard Magic Squares of Order 5 **/
/** 'MS55St.c' built by Kanji Setsuda **/
/** Nov.12, 2008 with MacOSX & Xcode3 **/
/**/
#include <stdio.h>
/**/
long int cnt, cnt2;
long cntr[5];
short int cnt3;
short LSM;
short nm[26], uflg[26];
```

```
short ntb[6][26];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void), stp16(void);
void stp17(void), stp18(void), stp19(void), stp20(void);
void stp21(void), stp22(void), stp23(void), stp24(void);
void stp25(void);
void ansprint(void);
void prans(short x);
/**/
/* Main Program */
int main(){
 short n;
  printf("\n** Standard Magic Squares of Order 5:");
  printf(" List 1 of Standard Solutions **\n");
  for(n=0;n<26;n++){nm[n]=0; uflg[n]=0;}
  LSM=65; cnt=0; cnt3=0;
  stp01(); //Begin The Search
  if(cnt3>0){prans(cnt3);}
  printf(" [Total Count = %d]  OK!\n",cnt);
 return 0;
}
/* Begin The Search */
/* Define Level 1: */
/* Set N1 */
void stp01(){
 short a;
  for(a=1;a<26;a++){
    if(uflg[a]==0){nm[1]=a; //cnt2=0;
      uflg[a]=1; stp02();
      uflg[a]=0;}
  }
}
/* Set N25(>N1) */
void stp02(){
 short a;
  for(a=25;a>nm[1];a--){
    if(uflg[a]==0){nm[25]=a; //cnt2=0;
      uflg[a]=1; stp03();
      uflg[a]=0;}
  }
}
/* Set N7 */
void stp03(){
 short a;
  for(a=1;a<26;a++){
    if(uflg[a]==0){nm[7]=a; //cnt2=0;
      uflg[a]=1; stp04();
      uflg[a]=0;}
  }
}
/* Set N19 */
void stp04(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[19]=a;
      uflg[a]=1; stp05();
```

```
      uflg[a]=0;}
  }
}
/* Set n13=C-n1-n7-n19-n25 */
void stp05(){
 short a;
  a=LSM-nm[1]-nm[7]-nm[19]-nm[25];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[13]=a; cnt2=0;
      uflg[a]=1; stp06();
      uflg[a]=0;}
  }
}
/* Set N5(>N1) */
void stp06(){
 short a;
  for(a=nm[1]+1;a<26;a++){
    if(uflg[a]==0){nm[5]=a;
      uflg[a]=1; stp07();
      uflg[a]=0;}
  }
}
/* Set N21(>N5) */
void stp07(){
 short a;
  for(a=25;a>nm[5];a--){
    if(uflg[a]==0){nm[21]=a;
      uflg[a]=1; stp08();
      uflg[a]=0;}
  }
}
/* Set N17 */
void stp08(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[17]=a;
      uflg[a]=1; stp09();
      uflg[a]=0;}
  }
}
/* Set n9=C-n5-n13-n17-n21 */
void stp09(){
 short a;
  a=LSM-nm[5]-nm[13]-nm[17]-nm[21];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[9]=a;
      uflg[a]=1; stp10();
      uflg[a]=0;}
  }
}
/**/
/* Define Level 2: */
/* Set N2 */
void stp10(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[2]=a;
      uflg[a]=1; stp11();
      uflg[a]=0;}
```

```
  }
}
/* Set N4 */
void stp11(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[4]=a;
      uflg[a]=1; stp12();
      uflg[a]=0;}
  }
}
/* Set n3=C-n1-n2-n4-n5 */
void stp12(){
 short a;
  a=LSM-nm[1]-nm[2]-nm[4]-nm[5];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[3]=a;
      uflg[a]=1; stp13();
      uflg[a]=0;}
  }
}
/* Set N22 */
void stp13(){
 short a;
  for(a=1;a<26;a++){
    if(uflg[a]==0){nm[22]=a;
      uflg[a]=1; stp14();
      uflg[a]=0;}
  }
}
/* Set n12=C-n2-n7-n17-n22 */
void stp14(){
 short a;
  a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[12]=a;
      uflg[a]=1; stp15();
      uflg[a]=0;}
  }
}
/* Set N24 */
void stp15(){
 short a;
  for(a=1;a<26;a++){
    if(uflg[a]==0){nm[24]=a;
      uflg[a]=1; stp16();
      uflg[a]=0;}
  }
}
/* Set n23=C-n21-n22-n24-n25 */
void stp16(){
 short a;
  a=LSM-nm[21]-nm[22]-nm[24]-nm[25];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[23]=a;
      uflg[a]=1; stp17();
      uflg[a]=0;}
  }
}
```

```
/* Set n14=C-n4-n9-n19-n24 */
void stp17(){
 short a;
  a=LSM-nm[4]-nm[9]-nm[19]-nm[24];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[14]=a;
      uflg[a]=1; stp18();
      uflg[a]=0;}
  }
}
/**/
/* Define Level 3: */
/* Set N8 */
void stp18(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[8]=a;
      uflg[a]=1; stp19();
      uflg[a]=0;}
  }
}
/* Set n18=C-n3-n8-n13-n23 */
void stp19(){
 short a;
  a=LSM-nm[3]-nm[8]-nm[13]-nm[23];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[18]=a;
      uflg[a]=1; stp20();
      uflg[a]=0;}
  }
}
/* Set N11 */
void stp20(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[11]=a;
      uflg[a]=1; stp21();
      uflg[a]=0;}
  }
}
/* Set n15=C-n11-n12-n13-n14 */
void stp21(){
 short a;
  a=LSM-nm[11]-nm[12]-nm[13]-nm[14];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[15]=a;
      uflg[a]=1; stp22();
      uflg[a]=0;}
  }
}
/* Set N6 */
void stp22(){
 short a;
  for(a=25;a>0;a--){
    if(uflg[a]==0){nm[6]=a;
      uflg[a]=1; stp23();
      uflg[a]=0;}
  }
}
```

```
/* Set n10=C-n6-n7-n8-n9 */
void stp23(){
 short a;
  a=LSM-nm[6]-nm[7]-nm[8]-nm[9];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[10]=a;
      uflg[a]=1; stp24();
      uflg[a]=0;}
  }
}
/* Set n16=C-n1-n6-n11-n21 */
void stp24(){
 short a;
  a=LSM-nm[1]-nm[6]-nm[11]-nm[21];
  if((0<a)&&(a<26)){
    if(uflg[a]==0){nm[16]=a;
      uflg[a]=1; stp25();
      uflg[a]=0;}
  }
}
/* Set n20=C-n16-n17-n18-n19 */
void stp25(){
 short a,b;
  a=LSM-nm[16]-nm[17]-nm[18]-nm[19];
  b=LSM-nm[5]-nm[10]-nm[15]-nm[25];
  if((0<a)&&(a<26)&&(a==b)){
    if(uflg[a]==0){nm[20]=a;
      uflg[a]=1;
      ansprint();
      uflg[a]=0;}
  }
}
/**/
/* Print the Answers */
void ansprint(){
 short n;
  cnt++; cnt2++;
  if(cnt2==1){
    for(n=1;n<26;n++){ntb[cnt3][n]=nm[n];}
    cntr[cnt3]=cnt;
    cnt3++; if(cnt3==5){prans(cnt3); cnt3=0;}}
}
/**/
/* Print the Answers */
void prans(short x){
 short l,l5,m,n;
  for(n=0;n<x;n++){
    printf("%14d/",cntr[n]);
    if(n<(x-1)){printf("  ");}}
  printf("\n");
  for(l=0;l<5;l++){l5=l*5;
    for(m=0;m<x;m++){
      printf("%3d%3d%3d%3d%3d",
        ntb[m][l5+1],ntb[m][l5+2],ntb[m][l5+3],ntb[m][l5+4],ntb[m][l5+5]);
      if(m<(x-1)){printf("  ");}
    }
    printf("\n");}
}
/**/
```

**** Standard Magic Squares of Order 5: List Part 1 of Standard Solutions ****

```
             1/            5112/            8586/           12671/           16631/
 1 22 21 18  3   1 24 20 17  3   1 23 17 21  3   1 24 15 22  3   1 24 16 21  3
20  2 15  9 19  19  2 15  8 21  18  2 16  9 20  19  2 18  6 20  18  2 14  9 22
14 16 13 10 12  12 16 14 13 10  12 19 15  8 11  13 14 16 12 10   8 19 17 11 10
 7 17 11 24  6  11 18  7 23  6  10 14 13 22  6   9 17 11 21  7  15 13 12 20  5
23  8  5  4 25  22  5  9  4 25  24  7  4  5 25  23  8  5  4 25  23  7  6  4 25
        21163/           25491/           29987/           33724/           36615/
 1 24 17 20  3   1 24 16 21  3   1 23 14 24  3   1 24 15 22  3   1 23 14 24  3
22  2 16 10 15  22  2 15  9 17  22  2 16  6 19  23  2 18  9 13  18  2 16  8 21
 5 21 18 12  9   7 20 19 13  6  12 18 20 10  5   4 19 21 11 10   7 17 22 13  6
14 11  8 19 13  12 11 10 18 14   9 15 11 17 13  17 12  6 16 14  19 12  9 15 10
23  7  6  4 25  23  8  5  4 25  21  7  4  8 25  20  8  5  7 25  20 11  4  5 25
        38818/           40414/           41093/           44567/           51728/
 1 24 17 20  3   1 23 18 20  3   1 23 21 18  2   1 22 21 19  2   1 23 19 20  2
21  2 12  8 22  22  2 10 15 16  22  3 15 14 11  18  3 20  9 15  15  3 17  9 21
 6 15 23 16  5   7 19 24 11  4  13 16 12  5 19  14 16 13 10 12  12 18 14 10 11
19 13  9 14 10  21  9  5 13 17   9 17  7 24  8   8 17  6 23 11  13 16  8 22  6
18 11  4  7 25  14 12  8  6 25  20  6 10  4 25  24  7  5  4 25  24  5  7  4 25
        57062/           61341/           66314/           70937/           75211/
 1 23 20 19  2   1 23 18 21  2   1 24 18 20  2   1 24 16 22  2   1 24 17 21  2
22  3 17 10 13  22  3 13  8 19  21  3 12  7 22  21  3 18  9 14  23  3 12  8 19
12 18 15 11  9  12 17 16 11  9   9 14 17 15 10   5 20 19 10 11   9 18 20 14  4
 6 14  8 21 16   6 15 14 20 10  11 16 13 19  6  15 12  8 17 13  10 13 11 16 15
24  7  5  4 25  24  7  4  5 25  23  8  5  4 25  23  6  4  7 25  22  7  5  6 25
        79059/           82034/           84549/           86125/           86874/
 1 22 17 23  2   1 24 15 23  2   1 24 16 22  2   1 23 18 21  2   1 22 21 19  2
24  3 14  6 18  20  3 17  9 16  21  3 14  9 18  22  3 11  9 20  14  4 20 12 15
 8 19 21 10  7   4 18 22 11 10   7 20 23 10  5   6 16 24 15  4  18 16 11  7 13
12 16  9 15 13  21 13  5 14 12  17 12  8 13 15  19 13  7 12 14   9 17  5 24 10
20  5  4 11 25  19  7  6  8 25  19  6  4 11 25  17 10  5  8 25  23  6  8  3 25
  . . . .
```

4. Here you see several abstract lists of the object solutions I have recently got.
I tried to have some solutions accompanied with the Decomposition diagrams by the 5-th increment number system in the following list.

**** Standard Magic Squares of Order 5: List Part 2 of Standard Solutions ****

```
             1/  /D5i:  H/          L/        2014025/  /D5i:  H/          L/
 1 22 21 18  3  0 4 4 3 0  0 1 0 2 2   1 21 22 18  3  0 4 4 3 0  0 0 1 2 2
20  2 15  9 19  3 0 2 1 3  4 1 4 3 3  20  2 14 10 19  3 0 2 1 3  4 1 3 4 3
14 16 13 10 12  2 3 2 1 2  3 0 2 4 1  15 17 13  8 12  2 3 2 1 2  4 1 2 2 1
 7 17 11 24  6  1 3 2 4 1  1 1 0 3 0   6 16 11 25  7  1 3 2 4 1  0 0 0 4 1
23  8  5  4 25  4 1 0 0 4  2 2 4 3 4  23  9  5  4 24  4 1 0 0 4  2 3 4 3 3
       4013925/  /D5i:  H/          L/        5996665/  /D5i:  H/          L/
 1 24 20 17  3  0 4 3 3 0  0 3 4 1 2   1 23 17 21  3  0 4 3 4 0  0 2 1 0 2
19  2 21  8 15  3 0 4 1 2  3 1 0 2 4  16  2 20  9 18  3 0 3 1 3  0 1 4 3 2
16 12 14 10 13  3 2 2 1 2  0 1 3 4 2  13 19 15  6 12  2 3 2 1 2  2 3 4 0 1
 7 18  4 25 11  1 3 0 4 2  1 2 3 4 0  11 14  5 25 10  2 2 0 4 1  0 3 4 4 4
22  9  6  5 23  4 1 1 0 4  1 3 0 4 2  24  7  8  4 22  4 1 1 0 4  3 1 2 3 1
       8055937/  /D5i:  H/          L/        9916517/  /D5i:  H/          L/
 1 22 20 19  3  0 4 3 3 0  0 1 4 3 2   1 22 16 23  3  0 4 3 4 0  0 1 0 2 2
23  2 15  8 17  4 0 2 1 3  2 1 4 2 1  15  2 19  8 21  2 0 3 1 4  4 1 3 2 0
12 18 16  6 13  2 3 3 1 2  1 2 0 0 2  14 18 17  4 12  2 3 3 0 2  3 2 1 3 1
 5 14 10 25 11  0 2 1 4 2  4 3 4 4 0  11 13  7 25  9  2 2 1 4 1  0 2 1 4 3
24  9  4  7 21  4 1 0 1 4  3 3 3 1 0  24 10  6  5 20  4 1 1 0 3  3 4 0 4 4
      11927511/  /D5i:  H/          L/       13840403/  /D5i:  H/          L/
 1 23 21 17  3  0 4 4 3 0  0 2 0 1 2   1 23 17 21  3  0 4 3 4 0  0 2 1 0 2
20  2 15  6 22  3 0 2 1 4  4 1 4 0 1  20  2 16  5 22  3 0 3 0 4  4 1 0 4 1
11 16 18 12  8  2 3 3 2 1  0 0 2 1 2  13 15 19  6 12  2 2 3 1 2  2 4 3 0 1
 9 14  4 25 13  1 2 0 4 2  3 3 3 4 2   7 14  9 25 10  1 2 1 4 1  1 3 3 4 4
24 10  7  5 19  4 1 1 0 3  3 4 1 4 3  24 11  4  8 18  4 2 0 1 3  3 0 3 2 2
```

```
      15729139/   /D5i:  H/         L/        17498665/   /D5i:  H/         L/
 1 21 18 22  3    0 4 3 4 0  0 0 2 1 2    1 22 19 20  3    0 4 3 3 0  0 1 3 4 2
19  2 14  7 23    3 0 2 1 4  3 1 3 1 2   17  2 15  8 23    3 0 2 1 4  1 1 4 2 2
13 16 20  6 10    2 3 3 1 1  2 0 4 0 4   11 18 21  5 10    2 3 4 0 1  0 2 0 4 4
 8 11  9 25 12    1 2 1 4 2  2 0 3 4 1   12  9  6 25 13    2 1 1 4 2  1 3 0 4 2
24 15  4  5 17    4 2 0 0 3  3 4 3 4 1   24 14  4  7 16    4 2 0 1 3  3 3 3 1 0
      19262329/       20945487/       22650789/       24256015/       25849695/
 1 24 17 20  3    1 24 18 19  3    1 23 16 21  4    1 23 19 18  4    1 23 18 20  3
21  2 13 10 19   21  2 15  7 20   19  2 12 10 22   20  3 14  7 21   17  4 15  7 22
11 14 22  6 12   12 13 23  6 11   14 15 24  3  9    9 16 24  5 11   14  9 24  5 13
 9  7  8 25 16    9 10  4 25 17   11  7  5 25 17   13  8  2 25 17   12 10  2 25 16
23 18  5  4 15   22 16  5  8 14   20 18  8  6 13   22 15  6 10 12   21 19  6  8 11
      27246415/       28752703/       30012441/       31280461/       32357217/
 1 23 20 18  3    1 22 19 20  3    1 22 18 20  4    1 20 19 21  4    1 21 20 19  4
19  5 13  7 21   18  6 13  5 23   17  7 15  5 21   16  8 14  5 22   16  9  7 10 23
 8 12 24  4 17   15  8 24  4 14   14 11 24  3 13   13 10 24  3 15   11 13 24  3 14
15  9  2 25 14   10 12  2 25 16   10  9  2 25 19   12  9  2 25 17   15  5  2 25 18
22 16  6 11 10   21 17  7 11  9   23 16  6 12  8   23 18  6 11  7   22 17 12  8  6
      33313773/       34050313/       34727611/       35163051/       35472327/
 1 23 19 18  4    1 23 16 20  5    1 21 19 18  6    1 22 15 20  7    2 22 21 17  3
17 10 11  6 21   18 11  6  8 22   16 12 11  4 22   18 13 10  3 21   20  1 15 10 19
12  7 24  2 20   13 10 24  3 15   15  7 24  5 14   14  5 24  6 16   14 18 13  9 11
13  9  3 25 15   12  7  2 25 19   10  8  2 25 20    9  8  4 25 19    6 16 12 24  7
22 16  8 14  5   21 14 17  9  4   23 17  9 13  3   23 17 12 11  2   23  8  4  5 25
 . . . .
```

5. We must now know about the total count of the standard solutions for MS55.

```
** Standard Magic Squares of Order 5: List Part 3 of Standard Solutions **
             1/       35472327/       72448255/      109794755/      143903959/
 1 22 21 18  3    2 22 21 17  3    3 21 20 17  4    4 21 19 16  5    5 22 15 17  6
20  2 15  9 19   20  1 15 10 19   19  1 22  8 15   14  1 22  8 20   21  1 16  8 19
14 16 13 10 12   14 18 13  9 11   13 16 12 14 10   17 13 11 15  9    9 20 10 14 12
 7 17 11 24  6    6 16 12 24  7    7 18  5 24 11    7 18 10 24  6    7 18 13 24  3
23  8  5  4 25   23  8  4  5 25   23  9  6  2 25   23 12  3  2 25   23  4 11  2 25
     174813433/      201422209/      223435493/      240804015/      253537291/
 6 15 17 20  7    7 14 18 17  9    8 18 20 10  9    9 20 14 12 10   10 20 17  7 11
21  1 22  8 13   20  1 22  6 16   13  1 17 12 22   13  1 19 11 21   15  1 19 12 18
10 19  9 11 16   12 21  8 13 11   16 21  7 15  6   17 22  6 16  4   13 22  5 16  9
 5 18 14 24  4    3 19 15 24  4    5 14 19 24  3    3 15 18 24  5    4 14 21 24  2
23 12  3  2 25   23 10  2  5 25   23 11  2  4 25   23  7  8  2 25   23  8  3  6 25
     262175289/      267791141/      271388851/      273683093/      274792651/
11 18 14 10 12   12 19 16  5 13   13 19 15  4 14   14 18 13  5 15   15 14 13  6 17
17  1 23  8 16   17  1 23 10 14   17  1 22 16  9   20  1 24 12  8   20  1 23 12  9
13 21  4 20  7    9 20  3 22 11    6 24  3 20 12    9 22  2 21 11    7 24  2 21 11
 2 19 15 24  5    6 18 15 24  2    8 11 18 23  5    3 17 16 23  6    5 16 19 22  3
22  6  9  3 25   21  7  8  4 25   21 10  7  2 25   19  7 10  4 25   18 10  8  4 25
     275181411/      275279597/      275301177/
16 14 12  6 17   17 15  9  6 18   18 10  5 13 19
19  1 23 13  9   14  2 25 11 13   15  6 23  7 14
 7 24  2 22 10   12 24  1 21  7   11 25  3 24  2
 5 15 20 21  4    3 16 20 22  4    1 16 22 17  9
18 11  8  3 25   19  8 10  5 23   20  8 12  4 21    [Total Count = 275305224]  OK!
```

Though we have known about this count for a long time, I just verified it by my recent calculation, getting as many solutions as 275,305,224 for this type of MS55.

What a great count it is! There is only one solution for the standard type of MS33 and there are 880 for the standard MS44, we know. It was so impressive that we would often call it the "notorious inflation of solution counts" naturally.

The next list is made up by starting to define the value of variable **n13** in the center.

**** Standard Magic Squares of Order 5: List Part 4 of Standard Solutions ****

```
            1/         4365793/         9830509/        17490445/        25325793/
 2 19 22 12 10     1 14 23 17 10     1 19 23 15  7     1 21 22 15  6     1 19 21 18  6
11 13 18 17  6    15 13 20 12  5    10 12 17 18  8    16 11 17 14  7    12 10 20 14  9
21 16  1  7 20    21 16  2  8 18    22 14  3  6 20    20 13  4  9 19    16 15  5  7 22
 8 14 15 24  4     6 19  9 24  7    11 16  9 24  5     5 18 10 24  8    13 17  8 24  3
23  3  9  5 25    22  3 11  4 25    21  4 13  2 25    23  2 12  3 25    23  4 11  2 25
     35053017/        45456533/        57524057/        69972701/        83862861/
 1 23 21 16  4     1 22 21 18  3     1 22 20 19  3     1 22 23 17  2     1 23 21 18  2
17  9 14 15 10    12  8 16 15 14    13  7 18 15 12    15  6 18 14 12    15  5 17 12 16
20 12  6  8 19    20 13  7  6 19    17 14  8  5 21    20 13  9  7 16    20 14 10  8 13
 5 18 11 24  7     9 17 11 24  4    11 16 10 24  4     8 19  4 24 10     7 19  6 24  9
22  3 13  2 25    23  5 10  2 25    23  6  9  2 25    21  5 11  3 25    22  4 11  3 25
     97238997/       112974269/       128112741/       147192485/       162330957/
 1 22 21 19  2     1 23 21 18  2     1 22 21 18  3     1 24 20 17  3     1 23 17 21  3
14  4 20 12 15    22  3 15 14 11    20  2 15  9 19    19  2 15  8 21    18  2 16  9 20
18 16 11  7 13    13 16 12  5 19    14 16 13 10 12    12 16 14 13 10    12 19 15  8 11
 9 17  5 24 10     9 17  7 24  8     7 17 11 24  6    11 18  7 23  6    10 14 13 22  6
23  6  8  3 25    20  6 10  4 25    23  8  5  4 25    22  5  9  4 25    24  7  4  5 25
    178066229/       191442365/       205332525/       217781169/       229848693/
 1 24 15 22  3     1 24 16 21  3     1 24 17 20  3     1 24 16 21  3     1 23 14 24  3
19  2 18  6 20    18  2 14  9 22    22  2 16 10 15    22  2 15  9 17    22  2 16  6 19
13 14 16 12 10     8 19 17 11 10     5 21 18 12  9     7 20 19 13  6    12 18 20 10  5
 9 17 11 21  7    15 13 12 20  5    14 11  8 19 13    12 11 10 18 14     9 15 11 17 13
23  8  5  4 25    23  7  6  4 25    23  7  6  4 25    23  8  5  4 25    21  7  4  8 25
    240252209/       249979433/       257814781/       265474717/       270939433/
 1 24 15 22  3     1 23 14 24  3     1 24 17 20  3     1 23 18 20  3     1 21 17 23  3
23  2 18  9 13    18  2 16  8 21    21  2 12  8 22    22  2 10 15 16    22  2 11 12 18
 4 19 21 11 10     7 17 22 13  6     6 15 23 16  5     7 19 24 11  4     6 19 25 10  5
17 12  6 16 14    19 12  9 15 10    19 13  9 14 10    21  9  5 13 17    20  9  8 13 15
20  8  5  7 25    20 11  4  5 25    18 11  4  7 25    14 12  8  6 25    16 14  4  7 24
 [Total Count = 275305224]  OK!
```

6. This vast count of solutions means hopeless to be analyzed and classified into any small groups. It is too big. They mean nothing special any more.

We can not help wanting to study about anything rarer and more precious like (1) Self-complementary type, (2) Pan-diagonal one of MS55, or anything like those.

By the way, do you know how many solutions there are for the 'Euler Squares' of standard MS55, that should be the 'Greco-Latin' squares when decomposed by the 5-th increment number system, even on the 2 primary diagonals?

**** Abstract List of 'Euler Squares' 5x5 of the Standard Magic Type ****

```
            1/  /D5i                      73/  /D5i                     145/  /D5i
 1 18 22 15  9  03421 02143    1 18 22 14 10  03421 02134    1 19 22 13 10  03421 03124
14  7 20 21  3  21340 31402   15  7 19 21  3  21340 41302   15  7 18 21  4  21340 41203
10 24 13  2 16  14203 43210    9 25 13  2 16  14203 34210    8 25 14  2 16  14203 24310
23  5  6 19 12  40132 24031   23  4  6 20 12  40132 23041   24  3  6 20 12  40132 32041
17 11  4  8 25  32014 10324   17 11  5  8 24  32014 10423   17 11  5  9 23  32014 10432
          217/  /D5i                     289/  /D5i                     361/  /D5i
 1 19 23 12 10  03421 03214    1 18 25 12  9  03421 02413    1 18 24 12 10  03421 02314
15  8 17 21  4  21340 42103   15  7  4 16 23  21034 41302   14  7  5 16 23  21034 31402
 7 25 14  3 16  14203 14320   19 21 13 10  2  34210 30241   20 21 13  9  2  34210 40231
24  2  6 20 13  40132 31042    8  5 17 24 11  10342 24130    8  4 17 25 11  10342 23140
18 11  5  9 22  32014 20431   22 14  6  3 20  42103 13024   22 15  6  3 19  42103 14023
          433/  /D5i                     505/  /D5i                     577/  /D5i
 1 19 23 12 10  03421 03214    1 19 22 13 10  03421 03124    1 13 25 17  9  02431 02413
13  7  5 16 24  21034 21403   12  8  5 16 24  21034 12403   20  7  4 11 23  31024 41302
20 21 14  8  2  34210 40321   20 21 14  7  3  34210 40312   14 21 18 10  2  24310 30241
 9  3 17 25 11  10342 32140    9  2 18 25 11  10342 31240    8  5 12 24 16  10243 24130
22 15  6  4 18  42103 14032   23 15  6  4 17  42103 24031   22 19  6  3 15  43102 13024
```

```
            649/  /D5i                 721/  /D5i                 793/  /D5i
 1 13 24 17 10  02431 02314   1 14 23 17 10  02431 03214   1 14 22 18 10  02431 03124
19  7  5 11 23  31024 31402  18  7  5 11 24  31024 21403  17  8  5 11 24  31024 12403
15 21 18  9  2  24310 40231  15 21 19  8  2  24310 40321  15 21 19  7  3  24310 40312
 8  4 12 25 16  10243 23140   9  3 12 25 16  10243 32140   9  2 13 25 16  10243 31240
22 20  6  3 14  43102 14023  22 20  6  4 13  43102 14032  23 20  6  4 12  43102 24031
            865/  /D5i                 937/  /D5i                1009/  /D5i
 1  8 25 17 14  01432 02413   1  8 24 17 15  01432 02314   1  9 23 17 15  01432 03214
20 12  4  6 23  32014 41302  19 12  5  6 23  32014 31402  18 12  5  6 24  32014 21403
 9 21 18 15  2  14320 30241  10 21 18 14  2  14320 40231  10 21 19 13  2  14320 40321
13  5  7 24 16  20143 24130  13  4  7 25 16  20143 23140  14  3  7 25 16  20143 32140
22 19 11  3 10  43201 13024  22 20 11  3  9  43201 14023  22 20 11  4  8  43201 14032
           1081/  /D5i                1153/  /D5i                1225/  /D5i
 1  9 22 18 15  01432 03124   2 18 21 15  9  03421 12043   2 18 21 14 10  03421 12034
17 13  5  6 24  32014 12403  14  6 20 22  3  21340 30412  15  6 19 22  3  21340 40312
10 21 19 12  3  14320 40312  10 24 13  1 17  14203 43201   9 25 13  1 17  14203 34201
14  2  8 25 16  20143 31240  23  5  7 19 11  40132 24130  23  4  7 20 11  40132 23140
23 20 11  4  7  43201 24031  16 12  4  8 25  32014 01324  16 12  5  8 24  32014 01423
           1297/  /D5i                1369/  /D5i                1441/  /D5i
 2 19 21 13 10  03421 13024   2 19 23 11 10  03421 13204   2 18 25 11  9  03421 12403
15  6 18 22  4  21340 40213  15  8 16 22  4  21340 42013  15  6  4 17 23  21034 40312
 8 25 14  1 17  14203 24301   6 25 14  3 17  14203 04321  19 22 13 10  1  34210 31240
24  3  7 20 11  40132 32140  24  1  7 20 13  40132 30142   8  5 16 24 12  10342 24031
16 12  5  9 23  32014 01432  18 12  5  9 21  32014 21430  21 14  7  3 20  42103 03124
           1513/  /D5i                1585/  /D5i                1657/  /D5i
 2 18 24 11 10  03421 12304   2 19 23 11 10  03421 13204   2 19 21 13 10  03421 13024
14  6  5 17 23  21034 30412  13  6  5 17 24  21034 20413  11  8  5 17 24  21034 02413
20 22 13  9  1  34210 41230  20 22 14  8  1  34210 41320  20 22 14  6  3  34210 41302
 8  4 16 25 12  10342 23041   9  3 16 25 12  10342 32041   9  1 18 25 12  10342 30241
21 15  7  3 19  42103 04123  21 15  7  4 18  42103 04132  23 15  7  4 16  42103 24130
           1729/  /D5i                1801/  /D5i                1873/  /D5i
 2 13 25 16  9  02431 12403   2 13 24 16 10  02431 12304   2 14 23 16 10  02431 13204
20  6  4 12 23  31024 40312  19  6  5 12 23  31024 30412  18  6  5 12 24  31024 20413
14 22 18 10  1  24310 31240  15 22 18  9  1  24310 41230  15 22 19  8  1  24310 41320
 8  5 11 24 17  10243 24031   8  4 11 25 17  10243 23041   9  3 11 25 17  10243 32041
21 19  7  3 15  43102 03124  21 20  7  3 14  43102 04123  21 20  7  4 13  43102 04132
           1945/  /D5i                2017/  /D5i                2089/  /D5i
 2 14 21 18 10  02431 13024   2  8 25 16 14  01432 12403   2  8 24 16 15  01432 12304
16  8  5 12 24  31024 02413  20 11  4  7 23  32014 40312  19 11  5  7 23  32014 30412
15 22 19  6  3  24310 41302   9 22 18 15  1  14320 31240  10 22 18 14  1  14320 41230
 9  1 13 25 17  10243 30241  13  5  6 24 17  20143 24031  13  4  6 25 17  20143 23041
23 20  7  4 11  43102 24130  21 19 12  3 10  43201 03124  21 20 12  3  9  43201 04123
           2161/  /D5i                2233/  /D5i                2305/  /D5i
 2  9 23 16 15  01432 13204   2  9 21 18 15  01432 13024   3 17 21 15  9  03421 21043
18 11  5  7 24  32014 20413  16 13  5  7 24  32014 02413  14  6 20 23  2  21340 30421
10 22 19 13  1  14320 41320  10 22 19 11  3  14320 41302  10 24 12  1 18  14203 43102
14  3  6 25 17  20143 32041  14  1  8 25 17  20143 30241  22  5  8 19 11  40132 14230
21 20 12  4  8  43201 04132  23 20 12  4  6  43201 24130  16 13  4  7 25  32014 02314
           3457/  /D5i                4609/  /D5i                5761/  /D5i
 4 17 21 15  8  03421 31042   5 17 21 14  8  03421 41032   6 18 22  5 14  13402 02143
13  6 20 24  2  21340 20431  13  6 19 25  2  21340 20341   4 12 20 21  8  02341 31402
10 23 12  1 19  14203 42103   9 23 12  1 20  14203 32104  15 24  3  7 16  24013 43210
22  5  9 18 11  40132 14320  22  4 10 18 11  40132 13420  23 10 11 19  2  41230 24031
16 14  3  7 25  32014 03214  16 15  3  7 24  32014 04213  17  1  9 13 25  30124 10324
           6049/  /D5i                6337/  /D5i                6625/  /D5i
 7 18 21  5 14  13402 12043   8 17 21  5 14  13402 21043   9 17 21  5 13  13402 31042
 4 11 20 22  8  02341 30412   4 11 20 23  7  02341 30421   3 11 20 24  7  02341 20431
15 24  3  6 17  24013 43201  15 24  2  6 18  24013 43102  15 23  2  6 19  24013 42103
23 10 12 19  1  41230 24130  22 10 13 19  1  41230 14230  22 10 14 18  1  41230 14320
16  2  9 13 25  30124 01324  16  3  9 12 25  30124 02314  16  4  8 12 25  30124 03214
```

```
          6913/  /D5i
10 17 21  4 13  13402 41032
 3 11 19 25  7  02341 20341
14 23  2  6 20  24013 32104
22  9 15 18  1  41230 13420
16  5  8 12 24  30124 04213    [Count = 7200]  OK!
```

7. How small! It counts just twice as many as Pan-diagonal magic squares 5x5.
If you take no such restriction to the contents of those two primary diagonals but arithmetic sums, you will have a little more solutions as shown below.

```
** Simple 'Euler Squares' 5x5 of the Standard Magic Type **
             1/  /D5i                    2435/  /D5i                    4999/  /D5i
 1 20 22 14  8  03421 04132   2 20 14 21  8  03241 14302   3 22 10 19 11  04132 21430
24  3 10 17 11  40132 32410  15  3 22  9 16  20413 42130  20  1 24 12  8  30421 40312
 7 21 13  5 19  14203 10243   6 24 18  5 12  14302 03241  21  9 13  5 17  41203 03241
15  9 16 23  2  21340 43021  23 11 10 17  4  42130 20413   7 15 16 23  4  12340 14023
18 12  4  6 25  32014 21304  19  7  1 13 25  31024 31024  14 18  2  6 25  23014 32104
          8765/  /D5i                   11233/  /D5i                   13577/  /D5i
 4 20  8 22 11  03142 34210   5  8 22 19 11  01432 42130   6 20  4 23 12  13042 04321
10  1 24 13 17  10423 40321  23  1 20 12  9  40321 20413  18  2 21 15  9  30421 21043
12 23 16  5  9  24301 12043  16 24 13 10  2  34210 03241   5 24 13  7 16  04213 43210
21  7 15 19  3  41230 01432   7 15  4 21 18  12043 14302  22 11 10 19  3  42130 10432
18 14  2  6 25  32014 23104  14 17  6  3 25  23104 31024  14  8 17  1 25  21304 32104
         14539/  /D5i                   15589/  /D5i                   16963/  /D5i
 7 20 21  4 13  13402 14032   8 20  4 22 11  13042 24310   9 20  3 22 11  13042 34210
23  1 17 15  9  40321 20143  17  1 23 15  9  30421 10243  17  1 24 15  8  30421 10342
11 24 10 18  2  24130 03421   5 24 12  6 18  04213 43102   5 23 12  6 19  04213 42103
 5  8 14 22 16  01243 42310  21 13 10 19  2  42130 02431  21 14 10 18  2  42130 03421
19 12  3  6 25  32014 31204  14  7 16  3 25  21304 31024  13  7 16  4 25  21304 21034
         17897/  /D5i                   18747/  /D5i                   19007/  /D5i
10 19  3 21 12  13042 43201  11 23 10  4 17  24103 02431  12 23  9  5 16  24103 12340
22  6 20 13  4  41320 10423  22  9 16 15  3  41320 13042  24  6 20 13  2  41320 30421
14 23  7  5 16  24103 32140   8 20  2 21 14  13042 24103  10 17  3 21 14  13042 41203
 1 15 24 17  8  02431 04312   5 12 24 18  6  02431 41320   1 15 22 19  8  02431 04132
18  2 11  9 25  30214 21034  19  1 13  7 25  30214 30214  18  4 11  7 25  30214 23014
         19205/  /D5i                   19341/  /D5i                   19417/  /D5i
13 24  5  7 16  24013 23410  14 23  5  7 16  24013 32410  15 22  4  8 16  24013 41320
21  2 18 15  9  40321 01243  21  2 19 15  8  40321 01342  21  3 20 14  7  40321 02431
 4 20  6 23 12  03142 34021   3 20  6 24 12  03142 24031   2 19  6 25 13  03142 13042
10 11 22 19  3  12430 40132  10 11 22 18  4  12430 40123   9 11 23 17  5  12430 30214
17  8 14  1 25  31204 12304  17  9 13  1 25  31204 13204  18 10 12  1 24  31204 24103
 [Count = 19436]  OK!
```

Section 2 : Standard Magic Cubes of Order 3: Kanji Setsuda

1. Basic Form and Basic Conditions defining every row, column and pillars

```
1-----10-----19       n1+n2+n3=C;     | n1+n10+n19=C;  | n1+n4+n7=C;
| 2      11     |20   n4+n5+n6=C;     | n2+n11+n20=C;  | n2+n5+n8=C;
|   3-----12-----21   n7+n8+n9=C;     | n3+n12+n21=C;  | n3+n6+n9=C;
4   | 13      22   |  n10+n11+n12=C;  | n4+n13+n22=C;  | n10+n13+n16=C;
| 5 |    14     |23 | n13+n14+n15=C;  | n5+n14+n23=C;  | n11+n14+n17=C;
|   6      15   |  24 n16+n17+n18=C;  | n6+n15+n24=C;  | n12+n15+n18=C;
7---|-16-----25    |  n19+n20+n21=C;  | n7+n16+n25=C;  | n19+n22+n25=C;
  8 |    17      26 | n22+n23+n24=C;  | n8+n17+n26=C;  | n20+n23+n26=C;
    9-----18-----27   n25+n26+n27=C;  | n9+n18+n27=C;  | n21+n24+n27=C;
```

```
Primary Triagonals:    n1+n14+n27=C;  | n3+n14+n25=C;  |
                       n7+n14+n21=C;  | n9+n14+n19=C;  |
```

2. We only define the 4 Primary Triagonals should add up to the same constant sum just as above and define nothing about any diagonals on any plane in the three directions: from top to bottom, from left to right, and from back to front.

We must make it up of the natural numbers {1, 2, 3, 4, 5, ..., 25, 26 and 27} using each strictly once, and we must neither use any number twice or more often there, nor un-use any number. It is one of the basic promises that we should usually keep.

We can determine the value C in advance as follows:

```
n1+n2+n3+n4+n5+n6+ ... +n25+n26+n27 = 9xC
         1+2+3+4+5+6+ ... +25+26+27 = 9xC
                        (27+1)x27/2 = 9xC
                  Therefore       C = 42
```

3. Let's actually build this type of MC333 by dictating a computer program only under these definitions and making our machine calculate and count.

Here you see an abstract list of 'Primitive' solutions for the standard type of MC333, the result of my recent calculation. Here I did not yet use any 'List-forming Inequality Conditions' at all.

**** Standard Magic Cubes of Order 3: List of 'Primitive' Solutions ****

```
 1/                     2/                     3/                     4/
  1-----15-----26         1-----17-----24         1-----15-----26         1-----23-----18
  |23      7    |12       |23      3    |16       |17     19    | 6       |17      3    |22
  |  18-----20----- 4     |  18-----22----- 2     |  24----- 8-----10     |  24-----16----- 2
 17   | 19      6   |    15   | 19      8   |    23   |  7     12   |    15   |  7     20   |
  | 3 |   14    |25 |     | 7 |   14    |21 |     | 3 |   14    |25 |     |19 |   14    | 9 |
  |  22      9  |  11     |  20      9  |  13     |  16     21  |   5     |   8     21  |  13
 24---|- 8-----10   |    26---|- 6-----10   |    18---|-20----- 4   |    26---|-12----- 4   |
   16 |   21      5 |      12 |   25      5 |      22 |    9     11 |       6 |   25     11 |
      2-----13-----27         4-----11-----27         2-----13-----27        10----- 5-----27
 5/                     6/                     7/                     8/
  1-----17-----24         1-----23-----18         2-----15-----25         2-----18-----22
  |15     19    | 8       |15      7    |20       |24      7    |11       |24      1    |17
  |  26----- 6-----10     |  26-----12----- 4     |  16-----20----- 6     |  16-----23----- 3
 23   |  3     16   |    17   |  3     22   |    18   | 19      5   |    15   | 19      8   |
  | 7 |   14    |21 |     |19 |   14    | 9 |     | 1 |   14    |27 |     | 7 |   14    |21 |
  |  12     25  |   5     |   6     25  |  11     |  23      9  |  10     |  20      9  |  13
 18---|-22----- 2   |    24---|-16----- 2   |    22---|- 8-----12   |    25---|- 5-----12   |
   20 |    9     13 |       8 |   21     13 |      17 |   21      4 |      11 |   27      4 |
      4-----11-----27        10----- 5-----27         3-----13-----26         6-----10-----26
 9/                    10/                    11/                    12/
  2-----13-----27         2-----16-----24         2-----15-----25         2-----24-----16
  |22      9    |11       |22      3    |17       |18     19    | 5       |18      1    |23
  |  18-----20----- 4     |  18-----23----- 1     |  22----- 8-----12     |  22-----17----- 3
 16   | 21      5   |    13   | 21      8   |    24   |  7     11   |    15   |  7     20   |
  | 3 |   14    |25 |     | 9 |   14    |19 |     | 1 |   14    |27 |     |19 |   14    | 9 |
  |  23      7  |  12     |  20      7  |  15     |  17     21  |   4     |   8     21  |  13
 24---|- 8-----10   |    27---|- 5-----10   |    16---|-20----- 6   |    25---|-11----- 6   |
   17 |   19      6 |      11 |   25      6 |      23 |    9     10 |       5 |   27     10 |
      1-----15-----26         4-----12-----26         3-----13-----26        12----- 4-----26
13/                    14/                    15/                    16/
  2-----13-----27         2-----22-----18         2-----18-----22         2-----24-----16
  |16     21    | 5       |16      3    |23       |15     19    | 8       |15      7    |20
  |  24----- 8-----10     |  24-----17----- 1     |  25----- 5-----12     |  25-----11----- 6
 22   |  9     11   |    13   |  9     20   |    24   |  1     17   |    18   |  1     23   |
  | 3 |   14    |25 |     |21 |   14    | 7 |     | 7 |   14    |21 |     |19 |   14    | 9 |
  |  17     19  |   6     |   8     19  |  15     |  11     27  |   4     |   5     27  |  10
 18---|-20----- 4   |    27---|-11----- 4   |    16---|-23----- 3   |    22---|-17----- 3   |
   23 |    7     12 |       5 |   25     12 |      20 |    9     13 |       8 |   21     13 |
      1-----15-----26        10----- 6-----26         6-----10-----26        12----- 4-----26
```

17/
```
  2-----16-----24
  |13     21    | 8
  |  27----- 5-----10
 22   |  3     17   |
  | 9 |    14    |19 |
  |  11     25   |   6
 18---|-23----- 1   |
   20 |    7     15 |
      4-----12-----26
```

18/
```
  2-----22-----18
  |13      9    |20
  |  27-----11----- 4
 16   |  3     23   |
  |21 |    14    | 7 |
  |   5     25   |  12
 24---|-17----- 1   |
    8 |   19     15 |
     10----- 6-----26
```

19/
```
  3-----13-----26
  |23      9    |10
  |  16-----20----- 6
 17   | 21      4   |
  | 1 |    14    |27 |
  |  24      7   |  11
 22---|- 8-----12   |
   18 |   19      5 |
      2-----15-----25
```

20/
```
  3-----17-----22
  |23      1    |18
  |  16-----24----- 2
 13   | 21      8   |
  | 9 |    14    |19 |
  |  20      7   |  15
 26---|- 4-----12   |
   10 |   27      5 |
      6-----11-----25
```

21/
```
  3-----13-----26
  |17     21    | 4
  |  22----- 8-----12
 23   |  9     10   |
  | 1 |    14    |27 |
  |  18     19   |   5
 16---|-20----- 6   |
   24 |    7     11 |
      2-----15-----25
```

22/
```
  3-----23-----16
  |17      1    |24
  |  22-----18----- 2
 13   |  9     20   |
  |21 |    14    | 7 |
  |   8     19   |  15
 26---|-10----- 6   |
    4 |   27     11 |
     12----- 5-----25
```

23/
```
  3-----17-----22
  |13     21    | 8
  |  26----- 4-----12
 23   |  1     18   |
  | 9 |    14    |19 |
  |  10     27   |   5
 16---|-24----- 2   |
   20 |    7     15 |
      6-----11-----25
```

24/
```
  3-----23-----16
  |13      9    |20
  |  26-----10----- 6
 17   |  1     24   |
  |21 |    14    | 7 |
  |   4     27   |  11
 22---|-18----- 2   |
    8 |   19     15 |
     12----- 5-----25
```

25/
```
  4-----17-----21
  |26      3    |13
  |  12-----22----- 8
 18   | 19      5   |
  | 1 |    14    |27 |
  |  23      9   |  10
 20---|- 6-----16   |
   15 |   25      2 |
      7-----11-----24
```

26/
```
  4-----18-----20
  |26      1    |15
  |  12-----23----- 7
 17   | 19      6   |
  | 3 |    14    |25 |
  |  22      9   |  11
 21---|- 5-----16   |
   13 |   27      2 |
      8-----10-----24
```

27/
```
  4-----11-----27
  |20      9    |13
  |  18-----22----- 2
 12   | 25      5   |
  | 7 |    14    |21 |
  |  23      3   |  16
 26---|- 6-----10   |
   15 |   19      8 |
      1-----17-----24
```

28/
```
  4-----12-----26
  |20      7    |15
  |  18-----23----- 1
 11   | 25      6   |
  | 9 |    14    |19 |
  |  22      3   |  17
 27---|- 5-----10   |
   13 |   21      8 |
      2-----16-----24
```

29/
```
  4-----17-----21
  |18     19    | 5
  |  20----- 6-----16
 26   |  3     13   |
  | 1 |    14    |27 |
  |  15     25   |   2
 12---|-22----- 8   |
   23 |    9     10 |
      7-----11-----24
```

30/
```
  4-----26-----12
  |18      1    |23
  |  20-----15----- 7
 17   |  3     22   |
  |19 |    14    | 9 |
  |   6     25   |  11
 21---|-13----- 8   |
    5 |   27     10 |
     16----- 2-----24
```

31/
```
  4-----18-----20
  |17     19    | 6
  |  21----- 5-----16
 26   |  1     15   |
  | 3 |    14    |25 |
  |  13     27   |   2
 12---|-23----- 7   |
   22 |    9     11 |
      8-----10-----24
```

32/
```
  4-----26-----12
  |17      3    |22
  |  21-----13----- 8
 18   |  1     23   |
  |19 |    14    | 9 |
  |   5     27   |  10
 20---|-15----- 7   |
    6 |   25     11 |
     16----- 2-----24
```

33/
```
  4-----11-----27
  |12     25    | 5
  |  26----- 6-----10
 20   |  9     13   |
  | 7 |    14    |21 |
  |  15     19   |   8
 18---|-22----- 2   |
   23 |    3     16 |
      1-----17-----24
```

34/
```
  4-----20-----18
  |12      7    |23
  |  26-----15----- 1
 11   |  9     22   |
  |25 |    14    | 3 |
  |   6     19   |  17
 27---|-13----- 2   |
    5 |   21     16 |
     10----- 8-----24
```

35/
```
  4-----12-----26
  |11     25    | 6
  |  27----- 5-----10
 20   |  7     15   |
  | 9 |    14    |19 |
  |  13     21   |   8
 18---|-23----- 1   |
   22 |    3     17 |
      2-----16-----24
```

36/
```
  4-----20-----18
  |11      9    |22
  |  27-----13----- 2
 12   |  7     23   |
  |25 |    14    | 3 |
  |   5     21   |  16
 26---|-15----- 1   |
    6 |   19     17 |
     10----- 8-----24
```

37/
```
  6-----16-----20
  |26      3    |13
  |  10-----23----- 9
 17   | 21      4   |
  | 1 |    14    |27 |
  |  24      7   |  11
 19---|- 5-----18   |
   15 |   25      2 |
      8-----12-----22
```

49/
```
  7-----11-----24
  |23      9    |10
  |  12-----22----- 8
 15   | 25      2   |
  | 1 |    14    |27 |
  |  26      3   |  13
 20---|- 6-----16   |
   18 |   19      5 |
      4-----17-----21
```

55/
```
  8-----12-----22
  |24      7    |11
  |  10-----23----- 9
 15   | 25      2   |
  | 1 |    14    |27 |
  |  26      3   |  13
 19---|- 5-----18   |
   17 |   21      4 |
      6-----16-----20
```

67/
```
  9-----11-----22
  |23      7    |12
  |  10-----24----- 8
 13   | 27      2   |
  | 3 |    14    |25 |
  |  26      1   |  15
 20---|- 4-----18   |
   16 |   21      5 |
      6-----17-----19
```

73/
```
 10-----23----- 9
  |26      3    |13
  |    6-----16-----20
 24   |  7     11   |
  | 1 |    14    |27 |
  |  17     21   |   4
  8---|-12-----22   |
   15 |   25      2 |
     19----- 5-----18
```

85/
```
 12-----22----- 8
  |26      3    |13
  |    4-----17-----21
 23   |  9     10   |
  | 1 |    14    |27 |
  |  18     19   |   5
  7---|-11-----24   |
   15 |   25      2 |
     20----- 6-----16
```

97/
```
 16-----20----- 6
  |24      7    |11
  |    2-----15-----25
 23   |  9     10   |
  | 1 |    14    |27 |
  |  18     19   |   5
  3---|-13-----26   |
   17 |   21      4 |
     22----- 8-----12
```

109/
```
 18-----20----- 4
  |23      7    |12
  |    1-----15-----26
 22   |  9     11   |
  | 3 |    14    |25 |
  |  17     19   |   6
  2---|-13-----27   |
   16 |   21      5 |
     24----- 8-----10
```

```
121/
  19----- 5-----18
   |17     21    | 4
   |    6-----16-----20
  15   | 25      2   |
   | 1 |   14    |27 |
   |  26      3  |  13
   8---|-12-----22    |
    24 |   7     11 |
      10-----23----- 9

127/
  20----- 6-----16
   |18     19    | 5
   |    4-----17-----21
  15   | 25      2   |
   | 1 |   14    |27 |
   |  26      3  |  13
   7---|-11-----24    |
    23 |   9     10 |
      12-----22----- 8

139/
  21----- 5-----16
   |17     19    | 6
   |    4-----18-----20
  13   | 27      2   |
   | 3 |   14    |25 |
   |  26      1  |  15
   8---|-10-----24    |
    22 |   9     11 |
      12-----23----- 7

145/
  22----- 8-----12
   |18     19    | 5
   |    2-----15-----25
  17   | 21      4   |
   | 1 |   14    |27 |
   |  24      7  |  11
   3---|-13-----26    |
    23 |   9     10 |
      16-----20----- 6

157/
  24----- 8-----10
   |17     19    | 6
   |    1-----15-----26
  16   | 21      5   |
   | 3 |   14    |25 |
   |  23      7  |  12
   2---|-13-----27    |
    22 |   9     11 |
      18-----20----- 4

169/
  25----- 5-----12
   |15     19    | 8
   |    2-----18-----22
  11   | 27      4   |
   | 7 |   14    |21 |
   |  24      1  |  17
   6---|-10-----26    |
    20 |   9     13 |
      16-----23----- 3

175/
  26----- 6-----10
   |15     19    | 8
   |    1-----17-----24
  12   | 25      5   |
   | 7 |   14    |21 |
   |  23      3  |  16
   4---|-11-----27    |
    20 |   9     13 |
      18-----22----- 2

187/
  27----- 5-----10
   |13     21    | 8
   |    2-----16-----24
  11   | 25      6   |
   | 9 |   14    |19 |
   |  22      3  |  17
   4---|-12-----26    |
    20 |   7     15 |
      18-----23----- 1

[Count = 192]  OK!
```

4. You could probably find some solutions which are 'a little different but almost the same' as each other among the above list, couldn't you? Yes. You are right.

For instance, all the solutions any of whose corner top is occupied by the number 1 are really all the same. Each of them is considered to be the reflected pattern, rotated one or the one simply rolled around.

All of them should be identified with the only representative 'Standard Solution', and they should be thought as the derivative forms of supposed standard one.

How many derivative forms are there in all, then? How many standard solutions in all could you finally find for that type?

The first 6 solutions above here show the rotated patterns around the diagonal axis {1, 14, 27}. The difference seems to come up with the permutations of {15, 17, 23}, that means $_3P_3$ = 3x2x1= 6.

On the other hand in any cube there are 8 corner tops, where the number 1 can be placed. The differences come from this fact in 8 ways.

Therefore, we can imagine 6x8 derivative forms in all from the standard solution, including the representative itself. And we can reasonably find how many standard solutions in all for that, as many as 4 = 192/48.

5. The next job for us is to list out the four correct 'Standard Solutions' directly.

But, how? Why don't we use the 'List-forming Inequality Conditions' for that?

```
1-----10-----19          n1+n2+n3=C;     | n1+n10+n19=C;  | n1+n4+n7=C;
| 2      11    |20       n4+n5+n6=C;     | n2+n11+n20=C;  | n2+n5+n8=C;
|    3-----12-----21     n7+n8+n9=C;     | n3+n12+n21=C;  | n3+n6+n9=C;
4    | 13      22   |    n10+n11+n12=C;  | n4+n13+n22=C;  | n10+n13+n16=C;
| 5 |    14    |23 |     n13+n14+n15=C;  | n5+n14+n23=C;  | n11+n14+n17=C;
|    6      15   |  24   n16+n17+n18=C;  | n6+n15+n24=C;  | n12+n15+n18=C;
7---|-16-----25    |     n19+n20+n21=C;  | n7+n16+n25=C;  | n19+n22+n25=C;
  8 |   17     26 |      n22+n23+n24=C;  | n8+n17+n26=C;  | n20+n23+n26=C;
    9-----18-----27      n25+n26+n27=C;  | n9+n18+n27=C;  | n21+n24+n27=C;

Primary Triagonals:      n1+n14+n27=C;  | n3+n14+n25=C;  |
                         n7+n14+n21=C;  | n9+n14+n19=C;  |

List-forming Inequality Conditions:
 n1<n3; n1<n7; n1<n9; n1<n19; n1<n21; n1<n25; n1<n27; n2>n4>n10;

/**  Magic Cubes 3*3*3: Standard Type  **/
```

```
/** 'MC333St.c' made by Kanji Setsuda  **/
/**  on 05/01/05; 07/03/05; 08/11/17   **/
/** Working with MacOSX10.5 & Xcode3.0 **/
/**/
#include <stdio.h>
/**/
short LSM;
short cnt, cnt2, cnt3;
short nm[28], uflg[28];
short anm[3][28];
/**/
void step01(void), step02(void), step03(void), step04(void);
void step05(void), step06(void), step07(void), step08(void);
void step09(void), step10(void), step11(void), step12(void);
void step13(void), step14(void), step15(void), step16(void);
void step17(void), step18(void), step19(void), step20(void);
void step21(void), step22(void), step23(void), step24(void);
void step25(void), step26(void), step27(void);
void anssave(void);
void print2sol(void);
/**/
int main(){
 short n;
  printf("\n** Standard Magic Cubes of Order 3:\n");
  printf("  List of the Four Standard Solutions **\n");
  for(n=0;n<28;n++){nm[n]=0; uflg[n]=0;}
  LSM=42; cnt=0; cnt2=0;
  step01();  //Begin The Search 2
  if(cnt2>0){print2sol();}
  printf("[Count = %d]  OK!\n",cnt);
 return 0;
}
/* Begin The Search 2 */
/* Set N1 */
void step01(){
 short n;
  for(n=1;n<28;n++){
    if(uflg[n]==0){nm[1]=n; cnt3=0;
      uflg[n]=1; step02();
      uflg[n]=0;}
   }
}
/* Set N27(>N1) */
void step02(){
 short n;
  for(n=27;n>nm[1];n--){
    if(uflg[n]==0){nm[27]=n;
      uflg[n]=1; step03();
      uflg[n]=0;}
   }
}
/* Set n14=LSM-n1-n27 */
void step03(){
 short n;
  n=LSM-nm[1]-nm[27];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[14]=n;
      uflg[n]=1; step04();
      uflg[n]=0;}}
```

```
}
/* Set N3(>N1) */
void step04(){
 short n;
  for(n=nm[1]+1;n<28;n++){
    if(uflg[n]==0){nm[3]=n;
      uflg[n]=1; step05();
      uflg[n]=0;}
   }
}
/* Set n25=LSM-n3-n14 & N1<n25 */
void step05(){
 short n;
  n=LSM-nm[3]-nm[14];
  if((nm[1]<n)&&(n<28)){
    if(uflg[n]==0){nm[25]=n;
      uflg[n]=1; step06();
      uflg[n]=0;}}
}
/* Set n2=LSM-n1-n3 */
void step06(){
 short n;
  n=LSM-nm[1]-nm[3];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[2]=n;
      uflg[n]=1; step07();
      uflg[n]=0;}}
}
/* Set n26=LSM-n25-n27 */
void step07(){
 short n;
  n=LSM-nm[25]-nm[27];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[26]=n;
      uflg[n]=1; step08();
      uflg[n]=0;}}
}
/* Set N7(>N1) */
void step08(){
 short n;
  for(n=nm[1]+1;n<28;n++){
    if(uflg[n]==0){nm[7]=n;
      uflg[n]=1; step09();
      uflg[n]=0;}
   }
}
/* Set n4=LSM-n1-n7 & n2>n4 */
void step09(){
 short n;
  n=LSM-nm[1]-nm[7];
  if((0<n)&&(n<nm[2])){
    if(uflg[n]==0){nm[4]=n;
      uflg[n]=1; step10();
      uflg[n]=0;}}
}
/* Set n21=LSM-n7-n14 & N1<n21 */
void step10(){
 short n;
  n=LSM-nm[7]-nm[14];
```

```
  if((nm[1]<n)&&(n<28)){
    if(uflg[n]==0){nm[21]=n;
      uflg[n]=1; step11();
      uflg[n]=0;}}
}
/* Set n24=LSM-n21-n27 */
void step11(){
 short n;
  n=LSM-nm[21]-nm[27];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[24]=n;
      uflg[n]=1; step12();
      uflg[n]=0;}}
}
/* Set n12=LSM-n3-n21 */
void step12(){
 short n;
  n=LSM-nm[3]-nm[21];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[12]=n;
      uflg[n]=1; step13();
      uflg[n]=0;}}
}
/* Set n16=LSM-n7-n25 */
void step13(){
 short n;
  n=LSM-nm[7]-nm[25];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[16]=n;
      uflg[n]=1; step14();
      uflg[n]=0;}}
}
/* Set N9(>N1) */
void step14(){
 short n;
  for(n=nm[1]+1;n<28;n++){
    if(uflg[n]==0){nm[9]=n;
      uflg[n]=1; step15();
      uflg[n]=0;}
   }
}
/* Set n19=LSM-n9-n14 & N1<n19 */
void step15(){
 short n;
  n=LSM-nm[9]-nm[14];
  if((nm[1]<n)&&(n<28)){
    if(uflg[n]==0){nm[19]=n;
      uflg[n]=1; step16();
      uflg[n]=0;}}
}
/* Set n6=LSM-n3-n9 */
void step16(){
 short n;
  n=LSM-nm[3]-nm[9];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[6]=n;
      uflg[n]=1; step17();
      uflg[n]=0;}}
}
```

```
/* Set n22=LSM-n19-n25 */
void step17(){
 short n;
  n=LSM-nm[19]-nm[25];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[22]=n;
      uflg[n]=1; step18();
      uflg[n]=0;}}
}
/* Set n8=LSM-n7-n9 */
void step18(){
 short n;
  n=LSM-nm[7]-nm[9];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[8]=n;
      uflg[n]=1; step19();
      uflg[n]=0;}}
}
/* Set n20=LSM-n19-n21 */
void step19(){
 short n;
  n=LSM-nm[19]-nm[21];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[20]=n;
      uflg[n]=1; step20();
      uflg[n]=0;}}
}
/* Set n5=LSM-n4-n6 */
void step20(){
 short n,m;
  n=LSM-nm[4]-nm[6];
  m=LSM-nm[2]-nm[8];
  if((0<n)&&(n<28)&&(n==m)){
    if(uflg[n]==0){nm[5]=n;
      uflg[n]=1; step21();
      uflg[n]=0;}}
}
/* Set n23=LSM-n22-n24 */
void step21(){
 short n,m,p;
  n=LSM-nm[22]-nm[24];
  m=LSM-nm[20]-nm[26];
  p=LSM-nm[5]-nm[14];
  if((0<n)&&(n<28)&&(n==m)){
    if((n==p)&&(uflg[n]==0)){nm[23]=n;
      uflg[n]=1; step22();
      uflg[n]=0;}}
}
/* Set n10=LSM-n1-n19 & n4>n10 */
void step22(){
 short n;
  n=LSM-nm[1]-nm[19];
  if((0<n)&&(n<nm[4])){
    if(uflg[n]==0){nm[10]=n;
      uflg[n]=1; step23();
      uflg[n]=0;}}
}
/* Set n18=LSM-n9-n27 */
void step23(){
```

```
 short n;
  n=LSM-nm[9]-nm[27];
  if((0<n)&&(n<28)){
    if(uflg[n]==0){nm[18]=n;
      uflg[n]=1; step24();
      uflg[n]=0;}}
}
/* Set n11=LSM-n10-n12 */
void step24(){
 short n,m;
  n=LSM-nm[10]-nm[12];
  m=LSM-nm[2]-nm[20];
  if((0<n)&&(n<28)&&(n==m)){
    if(uflg[n]==0){nm[11]=n;
      uflg[n]=1; step25();
      uflg[n]=0;}}
}
/* Set n13=LSM-n10-n16 */
void step25(){
 short n,m;
  n=LSM-nm[10]-nm[16];
  m=LSM-nm[4]-nm[22];
  if((0<n)&&(n<28)&&(n==m)){
    if(uflg[n]==0){nm[13]=n;
      uflg[n]=1; step26();
      uflg[n]=0;}}
}
/* Set n15=LSM-n13-n14 */
void step26(){
 short n,m,p;
  n=LSM-nm[13]-nm[14];
  m=LSM-nm[12]-nm[18];
  p=LSM-nm[6]-nm[24];
  if((0<n)&&(n<28)&&(n==m)){
    if((n==p)&&(uflg[n]==0)){nm[15]=n;
      uflg[n]=1; step27();
      uflg[n]=0;}}
}
/* Set n17=LSM-n16-n18 */
void step27(){
 short n,m,p;
  n=LSM-nm[16]-nm[18];
  m=LSM-nm[11]-nm[14];
  p=LSM-nm[8]-nm[26];
  if((0<n)&&(n<28)&&(n==m)){
    if((n==p)&&(uflg[n]==0)){nm[17]=n;
      uflg[n]=1; anssave();
      uflg[n]=0;}}
}
/* Save the Answer */
void anssave(){
 short n;
  cnt++;
  anm[cnt2][0]=cnt;
  for(n=1;n<28;n++){anm[cnt2][n]=nm[n];}
  cnt2++;
  if(cnt2==2){print2sol(); cnt2=0;}
}
/**/
```

```
/* Print 2 Solutions */
void print2sol(){
  printf("%2d/%22d/\n",anm[0][0],anm[1][0]);
  printf("%4d-----%2d-----%2d%9d-----%2d-----%2d\n",
    anm[0][1],anm[0][10],anm[0][19],anm[1][1],anm[1][10],anm[1][19]);
  printf("   |%2d%7d    |%2d      |%2d%7d    |%2d\n",
    anm[0][2],anm[0][11],anm[0][20],anm[1][2],anm[1][11],anm[1][20]);
  printf("   |%4d-----%2d-----%2d    |%4d-----%2d-----%2d\n",
    anm[0][3],anm[0][12],anm[0][21],anm[1][3],anm[1][12],anm[1][21]);
  printf("%4d   | %2d%7d   |%5d   | %2d%7d   |\n",
    anm[0][4],anm[0][13],anm[0][22],anm[1][4],anm[1][13],anm[1][22]);
  printf("   |%2d |%5d    |%2d |     |%2d |%5d    |%2d |\n",
    anm[0][5],anm[0][14],anm[0][23],anm[1][5],anm[1][14],anm[1][23]);
  printf("   |%4d%7d  |%4d    |%4d%7d  |%4d\n",
    anm[0][6],anm[0][15],anm[0][24],anm[1][6],anm[1][15],anm[1][24]);
  printf("%4d---|-%2d-----%2d   |%5d---|-%2d-----%2d   |\n",
    anm[0][7],anm[0][16],anm[0][25],anm[1][7],anm[1][16],anm[1][25]);
  printf("%6d |%5d%7d |%7d |%5d%7d |\n",
    anm[0][8],anm[0][17],anm[0][26],anm[1][8],anm[1][17],anm[1][26]);
  printf("%8d-----%2d-----%2d%9d-----%2d-----%2d\n",
    anm[0][9],anm[0][18],anm[0][27],anm[1][9],anm[1][18],anm[1][27]);
}
/**/
```

Here you see the complete list of 4 Standard Solutions of our object as follows.

```
** Standard Magic Cubes of Order 3:
  List of the Four Standard Solutions **
 1/                       2/
   1-----15-----26          2-----15-----25
   |23      7    |12        |24      7    |11
   |  18-----20----- 4      |  16-----20----- 6
  17   | 19       6   |    18   | 19       5   |
   | 3 |    14    |25 |     | 1 |    14    |27 |
   |  22       9  |   11    |  23       9  |   10
  24---|- 8-----10    |    22---|- 8-----12    |
    16 |    21       5 |     17 |    21       4 |
        2-----13-----27          3-----13-----26
 3/                       4/
   4-----17-----21          6-----16-----20
   |26      3    |13        |26      3    |13
   |  12-----22----- 8      |  10-----23----- 9
  18   | 19       5   |    17   | 21       4   |
   | 1 |    14    |27 |     | 1 |    14    |27 |
   |  23       9  |   10    |  24       7  |   11
  20---|- 6-----16    |    19---|- 5-----18    |
    15 |    25       2 |     15 |    25       2 |
        7-----11-----24          8-----12-----22   [Count = 4]   OK!
```

6. Watch and check where the number 1 is placed in each solution in the above list. Three standard solutions among four have the number 1 placed in **n5**.

This implies a new possibility of listing style of solutions.

Why don't we start defining **n5** at first?

Here you see the new style of listing I have recently got as follows.

```
** Standard Magic Cubes of Order 3:  List 2 of the Primitive Solutions **
```

1/
```
 6-----16-----20
 |26       3    |13
 |  10-----23----- 9
17   | 21       4   |
 | 1 |    14    |27 |
 |  24       7  |  11
19---|- 5-----18   |
   15 |   25       2 |
       8-----12-----22
```
2/
```
 4-----17-----21
 |26       3    |13
 |  12-----22----- 8
18   | 19       5   |
 | 1 |    14    |27 |
 |  23       9  |  10
20---|- 6-----16   |
   15 |   25       2 |
       7-----11-----24
```
3/
```
12-----22----- 8
 |26       3    |13
 |   4-----17-----21
23   |  9      10   |
 | 1 |    14    |27 |
 |  18      19  |   5
 7---|-11-----24   |
   15 |   25       2 |
      20----- 6-----16
```
4/
```
10-----23----- 9
 |26       3    |13
 |   6-----16-----20
24   |  7      11   |
 | 1 |    14    |27 |
 |  17      21  |   4
 8---|-12-----22   |
   15 |   25       2 |
      19----- 5-----18
```
5/
```
 8-----12-----22
 |24       7    |11
 |  10-----23----- 9
15   | 25       2   |
 | 1 |    14    |27 |
 |  26       3  |  13
19---|- 5-----18   |
   17 |   21       4 |
       6-----16-----20
```
6/
```
 2-----15-----25
 |24       7    |11
 |  16-----20----- 6
18   | 19       5   |
 | 1 |    14    |27 |
 |  23       9  |  10
22---|- 8-----12   |
   17 |   21       4 |
       3-----13-----26
```
7/
```
16-----20----- 6
 |24       7    |11
 |   2-----15-----25
23   |  9      10   |
 | 1 |    14    |27 |
 |  18      19  |   5
 3---|-13-----26   |
   17 |   21       4 |
      22----- 8-----12
```
8/
```
10-----23----- 9
 |24       7    |11
 |   8-----12-----22
26   |  3      13   |
 | 1 |    14    |27 |
 |  15      25  |   2
 6---|-16-----20   |
   17 |   21       4 |
      19----- 5-----18
```
9/
```
 7-----11-----24
 |23       9    |10
 |  12-----22----- 8
15   | 25       2   |
 | 1 |    14    |27 |
 |  26       3  |  13
20---|- 6-----16   |
   18 |   19       5 |
       4-----17-----21
```
10/
```
 3-----13-----26
 |23       9    |10
 |  16-----20----- 6
17   | 21       4   |
 | 1 |    14    |27 |
 |  24       7  |  11
22---|- 8-----12   |
   18 |   19       5 |
       2-----15-----25
```
11/
```
16-----20----- 6
 |23       9    |10
 |   3-----13-----26
24   |  7      11   |
 | 1 |    14    |27 |
 |  17      21  |   4
 2---|-15-----25   |
   18 |   19       5 |
      22----- 8-----12
```
12/
```
12-----22----- 8
 |23       9    |10
 |   7-----11-----24
26   |  3      13   |
 | 1 |    14    |27 |
 |  15      25  |   2
 4---|-17-----21   |
   18 |   19       5 |
      20----- 6-----16
```
13/
```
20----- 6-----16
 |18      19    | 5
 |   4-----17-----21
15   | 25       2   |
 | 1 |    14    |27 |
 |  26       3  |  13
 7---|-11-----24   |
   23 |    9      10 |
      12-----22----- 8
```
14/
```
22----- 8-----12
 |18      19    | 5
 |   2-----15-----25
17   | 21       4   |
 | 1 |    14    |27 |
 |  24       7  |  11
 3---|-13-----26   |
   23 |    9      10 |
      16-----20----- 6
```
15/
```
 2-----15-----25
 |18      19    | 5
 |  22----- 8-----12
24   |  7      11   |
 | 1 |    14    |27 |
 |  17      21  |   4
16---|-20----- 6   |
   23 |    9      10 |
       3-----13-----26
```
16/
```
 4-----17-----21
 |18      19    | 5
 |  20----- 6-----16
26   |  3      13   |
 | 1 |    14    |27 |
 |  15      25  |   2
12---|-22----- 8   |
   23 |    9      10 |
       7-----11-----24
```
17/
```
19----- 5-----18
 |17      21    | 4
 |   6-----16-----20
15   | 25       2   |
 | 1 |    14    |27 |
 |  26       3  |  13
 8---|-12-----22   |
   24 |    7      11 |
      10-----23----- 9
```
21/
```
19----- 5-----18
 |15      25    | 2
 |   8-----12-----22
17   | 21       4   |
 | 1 |    14    |27 |
 |  24       7  |  11
 6---|-16-----20   |
   26 |    3      13 |
      10-----23----- 9
```
25/
```
 6-----17-----19
 |26       1    |15
 |  10-----24----- 8
16   | 21       5   |
 | 3 |    14    |25 |
 |  23       7  |  12
20---|- 4-----18   |
   13 |   27       2 |
       9-----11-----22
```
29/
```
 9-----11-----22
 |23       7    |12
 |  10-----24----- 8
13   | 27       2   |
 | 3 |    14    |25 |
 |  26       1  |  15
20---|- 4-----18   |
   16 |   21       5 |
       6-----17-----19
```
33/
```
 8-----10-----24
 |22       9    |11
 |  12-----23----- 7
13   | 27       2   |
 | 3 |    14    |25 |
 |  26       1  |  15
21---|- 5-----16   |
   17 |   19       6 |
       4-----18-----20
```
37/
```
21----- 5-----16
 |17      19    | 6
 |   4-----18-----20
13   | 27       2   |
 | 3 |    14    |25 |
 |  26       1  |  15
 8---|-10-----24   |
   22 |    9      11 |
      12-----23----- 7
```
41/
```
20----- 4-----18
 |16      21    | 5
 |   6-----17-----19
13   | 27       2   |
 | 3 |    14    |25 |
 |  26       1  |  15
 9---|-11-----22   |
   23 |    7      12 |
      10-----24----- 8
```
45/
```
20----- 4-----18
 |13      27    | 2
 |   9-----11-----22
16   | 21       5   |
 | 3 |    14    |25 |
 |  23       7  |  12
 6---|-17-----19   |
   26 |    1      15 |
      10-----24----- 8
```
49/
```
 8-----15-----19
 |24       1    |17
 |  10-----26----- 6
12   | 25       5   |
 | 7 |    14    |21 |
 |  23       3  |  16
22---|- 2-----18   |
   11 |   27       4 |
       9-----13-----20
```
53/
```
 9-----13-----20
 |23       3    |16
 |  10-----26----- 6
11   | 27       4   |
 | 7 |    14    |21 |
 |  24       1  |  17
22---|- 2-----18   |
   12 |   25       5 |
       8-----15-----19
```
57/
```
 6-----10-----26
 |20       9    |13
 |  16-----23----- 3
11   | 27       4   |
 | 7 |    14    |21 |
 |  24       1  |  17
25---|- 5-----12   |
   15 |   19       8 |
       2-----18-----22
```
61/
```
25----- 5-----12
 |15      19    | 8
 |   2-----18-----22
11   | 27       4   |
 | 7 |    14    |21 |
 |  24       1  |  17
 6---|-10-----26   |
   20 |    9      13 |
      16-----23----- 3
```

```
65/                           69/                           73/                           77/
 22----- 2-----18              22----- 2-----18               7-----15-----20               8-----13-----21
  |12     25    | 5             |11     27    | 4             |23      1     |18            |22      3     |17
  |   8-----15-----19           |   9-----13-----20           |  12-----26----- 4           |  12-----26----- 4
 11   | 27      4   |          12   | 25      5   |          11   | 25      6   |          10   | 27      5   |
  | 7 |    14    |21 |          | 7 |    14    |21 |          | 9 |    14    |19 |          | 9 |    14    |19 |
  |  24       1  |  17          |  23       3  |  16          |  22       3  |  17          |  23       1  |  18
  9---|-13-----20   |           8---|-15-----19   |          24---|- 2-----16   |          24---|- 2-----16   |
   23 |    3     16 |            24 |    1     17 |            10 |   27      5 |            11 |   25      6 |
     10-----26----- 6             10-----26----- 6               8-----13-----21               7-----15-----20
81/                           85/                           89/                           93/
  6-----11-----25              26----- 4-----12              24----- 2-----16              24----- 2-----16
  |20      7    |15             |13     21    | 8             |11     25    | 6             |10     27    | 5
  |  16-----24----- 2           |   3-----17-----22           |   7-----15-----20           |   8-----13-----21
 10   | 27      5   |          10   | 27      5   |          10   | 27      5   |          11   | 25      6   |
  | 9 |    14    |19 |          | 9 |    14    |19 |          | 9 |    14    |19 |          | 9 |    14    |19 |
  |  23       1  |  18          |  23       1  |  18          |  23       1  |  18          |  22       3  |  17
 26---|- 4-----12   |           6---|-11-----25   |           8---|-13-----21   |           7---|-15-----20   |
   13 |   21      8 |            20 |    7     15 |            22 |    3     17 |            23 |    1     18 |
      3-----17-----22             16-----24----- 2              12-----26----- 4              12-----26----- 4
97/                          121/                          145/                          169/
 20-----15----- 7              19-----15----- 8              19-----17----- 6              20-----16----- 6
  |18      1    |23             |17      1    |24             |15      1     |26            |13      3     |26
  |   4-----26-----12           |   6-----26-----10           |   8-----24-----10           |   9-----23-----10
  6   | 25     11   |           5   | 25     12   |           5   | 21     16   |           4   | 21     17   |
  |19 |    14    | 9 |          |21 |    14    | 7 |          |25 |    14    | 3 |          |27 |    14    | 1 |
  |  17       3  |  22          |  16       3  |  23          |  12       7  |  23          |  11       7  |  24
 16---|- 2-----24   |          18---|- 2-----22   |          18---|- 4-----20   |          18---|- 5-----19   |
    5 |   27     10 |             4 |   27     11 |             2 |   27     13 |             2 |   25     15 |
     21-----13----- 8             20-----13----- 9              22-----11----- 9              22-----12----- 8
[Count = 192]  OK!
```

**** Standard Magic Cubes of Order 3:**
List 2 of the Standard Four Solutions **

```
1/                        2/
  6-----16-----20           4-----17-----21
  |26      3     |13        |26      3     |13
  |  10-----23----- 9       |  12-----22----- 8
 17   | 21      4   |      18   | 19      5   |
  | 1 |    14    |27 |      | 1 |    14    |27 |
  |  24       7  |  11      |  23       9  |  10
 19---|- 5-----18   |      20---|- 6-----16   |
   15 |   25      2 |        15 |   25      2 |
      8-----12-----22           7-----11-----24
3/                        4/
  2-----15-----25           1-----15-----26
  |24      7     |11        |23      7     |12
  |  16-----20----- 6       |  18-----20----- 4
 18   | 19      5   |      17   | 19      6   |
  | 1 |    14    |27 |      | 3 |    14    |25 |
  |  23       9  |  10      |  22       9  |  11
 22---|- 8-----12   |      24---|- 8-----10   |
   17 |   21      4 |        16 |   21      5 |
      3-----13-----26           2-----13-----27    [Count = 4]  OK!
```

List-forming Inequality Conditions used for the last case are:
`n5<n11, n5<n13, n5<n15, n5<n17, n5<n23, n11<n17 and n11<n15<n13`

7. Will you watch in the list above if **n14** in the cubic center is always occupied by the number 14, and check whether all the 13 pairs situated symmetrically with respect to the cubic center **n14** always add up to the same sum 28 or not?

1+27=2+26=3+25=4+24=5+23=6+22=7+21=8+20=9+19+10+18=11+17=12+16=13+15=28

Do you remember this is just the characteristic property of 'Self-complementary' magic things? Yes. All solutions are just the 'Self-complementary' magic cubes 3x3x3.

What made them so? For we built them as the Standard MC333 under our normal conditions before all. We did never take **n14**=14 yet in the first definition stage.

8. Let's study why no other type than 'Self-complementary' can be made. Did our first definition necessarily mean the property of 'Self-complementary' MC333?

Let me show you some algebraic calculations, though I am afraid it may be tiresome.

```
* Basic Form and Basic Conditions for Standard MC333: C=42 *

 1-----10-----19        n1+n2+n3=C;     | n1+n10+n19=C;  | n1+n4+n7=C;
 | 2      11     |20     n4+n5+n6=C;     | n2+n11+n20=C;  | n2+n5+n8=C;
 |   3-----12-----21     n7+n8+n9=C;     | n3+n12+n21=C;  | n3+n6+n9=C;
 4    | 13      22    |   n10+n11+n12=C; | n4+n13+n22=C;  | n10+n13+n16=C;
 | 5 |    14     |23 |   n13+n14+n15=C; | n5+n14+n23=C;  | n11+n14+n17=C;
 |   6      15    |  24   n16+n17+n18=C; | n6+n15+n24=C;  | n12+n15+n18=C;
 7---|-16-----25    |    n19+n20+n21=C; | n7+n16+n25=C;  | n19+n22+n25=C;
   8 |    17      26 |   n22+n23+n24=C; | n8+n17+n26=C;  | n20+n23+n26=C;
     9-----18-----27     n25+n26+n27=C; | n9+n18+n27=C;  | n21+n24+n27=C;

 Primary Triagonals:    n1+n14+n27=C;  | n3+n14+n25=C;  |
                        n7+n14+n21=C;  | n9+n14+n19=C;  |
```

All the equations that have **n14** within can be transformed like these:

```
  n1+n27=C-n14;   | n3+n25=C-n14; | n5+n23=C-n14; | n7+n21=C-n14; |
  n9+n19=C-n14;   | n11+n17=C-n14; | n13+n15=C-n14; |

      n1+n2+n3=C;                        n1+n4+n7=C;
 +) n25+n26+n27=C;                   +)n21+n24+n27=C;
------------------------------------------------------------------------
(n1+n27)+(n2+n26)+(n3+n25)=2*C;      (n1+n27)+(n4+n24)+(n7+n21)=2*C;
 C-n14   +(n2+n26)+ C-n14   =2*C;      C-n14   +(n4+n24)+ C-n14   =2*C;
            n2+n26=2*C-2*(C-n14)                  n4+n24=2*C-2*(C-n14)
Therefore   n2+n26=2*n14;             Therefore   n4+n24=2*n14;
In the same way n6+n22=2*n14; n8+n20=2*n14; n10+n18=2*n14; n12+n16=2*n14;
```

It tells us there are two groups of Complementary Pairs according to their sums.

```
 n1+n27=n3+n25=n5+n23=n7+n21=n9+n19=n11+n17=n13+n15=C-n14 ... cc;
 n2+n26=n4+n24=n6+n22=n8+n20=n10+n18=n12+n16=2*n14         ... dd;
```

We need all those 13 pairs just as above. What pairs can be actually combined according to the value of **n14**, then? Of course, any pair must be made up of the natural numbers {1, 2, 3, 4, 5, 6, ..., 25, 26 and 27} then, using each strictly once. No number must be used twice nor more often and no one must be un-used, either.

```
** Make the CD Tables for Standard MC333:(NC denotes n14) **
 * Is it possible to make all the 13 Complementary Pairs? *
NC= 2 (cc= 4, dd=40):
 C1( 1, 3),D1{27,13};   ... Impossible!
NC= 3 (cc= 6, dd=39):
 C1( 1, 5),D1{27,12}; C2( 2, 4),D2{26,13};  ... Impossible!
NC= 4 (cc= 8, dd=38):
 C1( 1, 7),D1{27,11}; C2( 2, 6),D2{26,12}; C3( 3, 5),D3{25,13};
   ... Impossible!
```

```
NC= 5 (cc=10, dd=37):
 C1( 1, 9),D1{27,10}; C2( 2, 8),D2{26,11}; C3( 3, 7),D3{25,12};
 C4( 4, 6),D4{24,13};   ... Impossible!
NC= 6 (cc=12, dd=36):
 C1( 1,11),D1{27, 9}; C2( 2,10),D2{24,12}; C3( 4, 8),D3{23,13};
 C4( 5, 7),D4{22,14};   ... Impossible!
NC= 7 (cc=14, dd=35):
 C1( 1,13),D1{27, 8}; C2( 2,12),D2{26, 9}; C3( 3,11),D3{25,10};
    ... Impossible!
NC= 8 (cc=16, dd=34):
 C1( 1,15),D1{27, 7}; C2( 2,14),D2{25, 9}; C3( 3,13),D3{24,10};
 C4( 4,12),D4{23,11};   ... Impossible!
NC= 9 (cc=18, dd=33):
 C1( 1,17),D1{27, 6}; C2( 2,16),D2{26, 7}; C3( 3,15),D3{25, 8};
 C4( 4,14),D4{23,10}; C5( 5,13),D5{22,11};  ... Impossible!
NC=10 (cc=20, dd=32):
 C1( 1,19),D1{27, 5}; C2( 2,18),D2{26, 6}; C3( 3,17),D3{25, 7};
 C4( 4,16),D4{24, 8}; C5( 9,11),  ... Impossible!
NC=11 (cc=22, dd=31):
 C1( 1,21),D1{27, 4}; C2( 2,20),D2{26, 5}; C3( 3,19),D3{25, 6};
 C4( 7,15),D4{18,13}; C5( 8,14),  ... Impossible!
NC=12 (cc=24, dd=30):
 C1( 1,23),D1{27, 3}; C2( 2,22),D2{26, 4}; C3( 5,19),D3{21, 9};
 C4( 6,18),D4{20,10}; C5( 7,17),D5{16,14}; C6(11,13),   ... Impossible!
NC=13 (cc=26, dd=29):
 C1( 1,25),D1{27, 2}; C2( 3,23),D2{24, 5}; C3( 4,22),D3{21, 8};
 C4( 6,20),D4{19,10}; C5( 9,17),D5{18,11}; C6(12,14),   ... Impossible!
NC=14 (cc=28, dd=28):
 C1( 1,27),D1{26, 2}; C2( 3,25),D2{24, 4}; C3( 5,23),D3{22, 6};
 C4( 7,21),D4{20, 8}; C5( 9,19),D5{18,10}; C6(11,17),D6{16,12};
 C7(13,15) ** Possible! **
NC=15 (cc=30, dd=27):
 C1( 3,27),D1{25, 2}; C2( 4,26),D2{22, 5}; C3( 6,24),D3{20, 7};
 C4( 9,21),D4{19, 8}; C5(12,18),D5{16,11}; C6(13,17),   ... Impossible!
NC=16 (cc=32, dd=26):
 C1( 5,27),D1{23, 3}; C2( 6,26),D2{22, 4}; C3( 7,25),D3{18, 8};
 C4(11,21),D4{17, 9}; C5(12,20),  ... Impossible!
NC=17 (cc=34, dd=25):
 C1( 7,27),D1{21, 4}; C2( 8,26),D2{20, 5}; C3( 9,25),D3{19, 6};
 C4(10,24),D4{14,11}; C5(12,22),  ... Impossible!
NC=18 (cc=36, dd=24):
 C1( 9,27),D1{19, 5}; C2(10,26),D2{17, 7}; C3(11,25),D3{16, 8};
 C4(12,24),   ... Impossible!
NC=19 (cc=38, dd=23):
 C1(11,27),D1{17, 6}; C2(12,26),D2{16, 7}; C3(13,25),D3{15, 8};
 C4(14,24),   ... Impossible!
NC=20 (cc=40, dd=22):
 C1(13,27),D1{15, 7}; C2(14,26),D2{12,10}; C3(16,24),   ... Impossible!
NC=21 (cc=42, dd=21):
 C1(15,27),D1{13, 8}; C2(16,26),D2{12, 9}; C3(17,25),D3{11,10};
 C4(18,24),   ... Impossible!
NC=22 (cc=44, dd=20):
 C1(17,27),D1{11, 9}; C2(18,26),  ... Impossible!
NC=23 (cc=46, dd=19):
 C1(19,27),   ... Impossible!
NC=24 (cc=48, dd=18):
 C1(21,27),   ... Impossible!
NC=25 (cc=50, dd=17):
 C1(23,27),   ... Impossible!
```

```
NC=26 (cc=52, dd=16):
 C1(25,27),  ... Impossible!
[ OK! ]
```

Did you notice that it is the only one case when **n14** takes 14 is possible? And all the complementary pairs of 28 must be situated symmetrically with respect to the cubic center **n14**. This means every standard MC333 should necessarily be nothing but the Self-complementary MC333.

9. Now that we know this fact, we can guess when we make the standard solutions of Self-complementary MC333 originally, we can surely have got just the same solution set with our old standard type. Let's verify our reasoning by making it actually here.

```
/** Self-Complementary Magic Cubes 333 **/
/** 'MC333SCn5.c' by Kanji Setsuda  **/
/** on 05/01/04; 07/03/05; 08/11/19 **/
/** Working with MacOSX & Xcode 3.0 **/
/**/
/* Basic Form and Basic Conditions for MC333SC: C=42; CC=28 *
  1-----10-----19       n1+n2+n3=C;     | n1+n10+n19=C; | n1+n4+n7=C;
  | 2     11    |20     n4+n5+n6=C;     | n2+n11+n20=C; | n2+n5+n8=C;
  |   3-----12-----21   n7+n8+n9=C;     | n3+n12+n21=C; | n3+n6+n9=C;
  4   | 13     22   |   n10+n11+n12=C;  | n4+n13+n22=C; | n10+n13+n16=C;
  | 5 |   14    |23 |   n13+n14+n15=C;  | n5+n14+n23=C; | n11+n14+n17=C;
  |   6     15  |  24   n16+n17+n18=C;  | n6+n15+n24=C; | n12+n15+n18=C;
  7---|-16-----25   |   n19+n20+n21=C;  | n7+n16+n25=C; | n19+n22+n25=C;
    8 |   17     26 |   n22+n23+n24=C;  | n8+n17+n26=C; | n20+n23+n26=C;
      9-----18-----27   n25+n26+n27=C;  | n9+n18+n27=C; | n21+n24+n27=C;
* Self-complementary Conditions *
  n1+n27=n2+n26=n3+n25=n4+n24=n5+n23=n6+n22=n7+n21=n8+n20
  =n9+n19=n10+n18=n11+n17=n12+n16=n13+n15=n14+n14=CC
* List-forming Inequality Conditions used for the case *
  n5<n11, n5<n13, n5<n15, n5<n17, n5<n23, n11<n17 and n11<n15<n13
*/
#include <stdio.h>
/**/
short LSM, CC;
short cnt, cnt2;
short nm[28], uflg[28];
short anm[3][28];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void);
void stp12(void), stp13(void), stp14(void);
void ansrecord(void), print2sol(void);
/**/
int main(){
 short n;
  printf("\n** Self-Complementary Magic Cubes of Order 3 **\n");
  printf("** Complete List of the Standard 4 Solutions **\n");
  for(n=0;n<28;n++){nm[n]=0; uflg[n]=0;}
  LSM=42; CC=28; cnt=0; cnt2=0;
  nm[14]=14; uflg[14]=1;
  stp01(); //Begin The Search
  uflg[14]=0;
  if(cnt2>0){print2sol();}
  printf(" [Count = %d]  OK!\n",cnt);
```

```
 return 0;
}
/* Begin The Search */
/* Set n5 & n23 & n5<n23 */
void stp01(){
 short m,n;
  for(m=1;m<14;m++){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[5]=m; nm[23]=n;
      stp02();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n11(>n5) & n17(>n11) */
void stp02(){
 short m,n;
  for(m=nm[5]+1;m<14;m++){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[11]=m; nm[17]=n;
      stp03();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n15(>n11) & n13(>n15) */
void stp03(){
 short m,n;
  for(m=nm[11]+1;m<14;m++){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[15]=m; nm[13]=n;
      stp04();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n1 & n27 */
void stp04(){
 short m,n;
  for(m=1;m<28;m++){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[1]=m; nm[27]=n;
      stp05();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n2 & n26 */
void stp05(){
 short m,n;
  for(m=27;m>0;m--){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[2]=m; nm[26]=n;
      stp06();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n3=LSM-n1-n2 & n25 */
```

```
void stp06(){
 short m,n;
  m=LSM-nm[1]-nm[2];
  n=LSM-nm[27]-nm[26];
  if((0<m)&&(m<28)&&(m+n==CC)){
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[3]=m; nm[25]=n;
      stp07();
      uflg[n]=0; uflg[m]=0;}}
}
/* Set n4 & n24 */
void stp07(){
 short m,n;
  for(m=27;m>0;m--){n=CC-m;
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[4]=m; nm[24]=n;
      stp08();
      uflg[n]=0; uflg[m]=0;}
   }
}
/* Set n7=LSM-n1-n4 & n21 */
void stp08(){
 short m,n;
  m=LSM-nm[1]-nm[4];
  n=LSM-nm[27]-nm[24];
  if((0<m)&&(m<28)&&(m+n==CC)){
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[7]=m; nm[21]=n;
      stp09();
      uflg[n]=0; uflg[m]=0;}}
}
/* Set n6=LSM-n4-n5 & n22 */
void stp09(){
 short m,n,o,p;
  m=LSM-nm[4]-nm[5];
  n=LSM-nm[24]-nm[23];
  o=LSM-nm[15]-nm[24];
  p=LSM-nm[13]-nm[4];
  if((0<m)&&(m<28)&&(0<n)&&(n<28)){
   if((m+n==CC)&&(m==o)&&(n==p)){
    if((uflg[m]==0)&&(uflg[n]==0)){
      uflg[m]=1; uflg[n]=1;
      nm[6]=m; nm[22]=n;
      stp10();
      uflg[n]=0; uflg[m]=0;}}}
}
/* Set n8=LSM-n2-n5 & n20 */
void stp10(){
 short m,n,o,p;
  m=LSM-nm[2]-nm[5];
  n=LSM-nm[26]-nm[23];
  o=LSM-nm[17]-nm[26];
  p=LSM-nm[11]-nm[2];
  if((0<m)&&(m<28)&&(0<n)&&(n<28)){
   if((m+n==CC)&&(m==o)&&(n==p)){
    if((uflg[m]==0)&&(uflg[n]==0)){
```

```
        uflg[m]=1; uflg[n]=1;
        nm[8]=m; nm[20]=n;
        stp11();
        uflg[n]=0; uflg[m]=0;}}}
}
/* Set n9=LSM-n7-n8 & n19 */
void stp11(){
 short m,n,o,p;
  m=LSM-nm[7]-nm[8];
  n=LSM-nm[21]-nm[20];
  o=LSM-nm[3]-nm[6];
  p=LSM-nm[25]-nm[22];
  if((0<m)&&(m<28)&&(0<n)&&(n<28)){
   if((m+n==CC)&&(m==o)&&(n==p)){
    if((uflg[m]==0)&&(uflg[n]==0)){
        uflg[m]=1; uflg[n]=1;
        nm[9]=m; nm[19]=n;
        stp12();
        uflg[n]=0; uflg[m]=0;}}}
}
/* Set n10=LSM-n1-n19 & n18 */
void stp12(){
 short m,n;
  m=LSM-nm[1]-nm[19];
  n=LSM-nm[27]-nm[9];
  if((0<m)&&(m<28)&&(m+n==CC)){
    if((uflg[m]==0)&&(uflg[n]==0)){
        uflg[m]=1; uflg[n]=1;
        nm[10]=m; nm[18]=n;
        stp13();
        uflg[n]=0; uflg[m]=0;}}
}
/* Set n12=LSM-n10-n11 & n16 */
void stp13(){
 short m,n,o,p;
  m=LSM-nm[10]-nm[11];
  n=LSM-nm[18]-nm[17];
  o=LSM-nm[3]-nm[21];
  p=LSM-nm[25]-nm[7];
  if((0<m)&&(m<28)&&(0<n)&&(n<28)){
   if((m+n==CC)&&(m==o)&&(n==p)){
    if((uflg[m]==0)&&(uflg[n]==0)){
        uflg[m]=1; uflg[n]=1;
        nm[12]=m; nm[16]=n;
        stp14();
        uflg[n]=0; uflg[m]=0;}}}
}
/* The Last Checks */
void stp14(){
 short sm1,sm2;
  sm1=nm[12]+nm[15]+nm[18];
  sm2=nm[16]+nm[13]+nm[10];
  if((sm1==LSM)&&(sm2==LSM)){ansrecord();}
}
/* Record the Answer */
void ansrecord(){
 short n;
  cnt++;
  anm[cnt2][0]=cnt;
```

```
  for(n=1;n<28;n++){anm[cnt2][n]=nm[n];}
  cnt2++;
  if(cnt2==2){print2sol(); cnt2=0;
    anm[1][0]=0;
    for(n=1;n<28;n++){anm[1][n]=0;}
  }
}
/**/
/* Print 2 Solutions */
void print2sol(){
  printf("%3d/%22d/\n",anm[0][0],anm[1][0]);
  printf("%4d-----%2d-----%2d%9d-----%2d-----%2d\n",
    anm[0][1],anm[0][10],anm[0][19],anm[1][1],anm[1][10],anm[1][19]);
  printf("    |%2d%7d     |%2d       |%2d%7d     |%2d\n",
    anm[0][2],anm[0][11],anm[0][20],anm[1][2],anm[1][11],anm[1][20]);
  printf("    |%4d-----%2d-----%2d     |%4d-----%2d-----%2d\n",
    anm[0][3],anm[0][12],anm[0][21],anm[1][3],anm[1][12],anm[1][21]);
  printf("%4d     | %2d%7d    |%5d     | %2d%7d    |\n",
    anm[0][4],anm[0][13],anm[0][22],anm[1][4],anm[1][13],anm[1][22]);
  printf("    |%2d |%5d     |%2d |     |%2d |%5d     |%2d |\n",
    anm[0][5],anm[0][14],anm[0][23],anm[1][5],anm[1][14],anm[1][23]);
  printf("    |%4d%7d   |%4d     |%4d%7d   |%4d\n",
    anm[0][6],anm[0][15],anm[0][24],anm[1][6],anm[1][15],anm[1][24]);
  printf("%4d---|-%2d-----%2d     |%5d---|-%2d-----%2d     |\n",
    anm[0][7],anm[0][16],anm[0][25],anm[1][7],anm[1][16],anm[1][25]);
  printf("%6d |%5d%7d |%7d |%5d%7d |\n",
    anm[0][8],anm[0][17],anm[0][26],anm[1][8],anm[1][17],anm[1][26]);
  printf("%8d-----%2d-----%2d%9d-----%2d-----%2d\n",
    anm[0][9],anm[0][18],anm[0][27],anm[1][9],anm[1][18],anm[1][27]);
}
/**/
```

```
** Self-Complementary Magic Cubes of Order 3 **
** Complete List of the Standard 4 Solutions **
  1/                        2/
   6-----16-----20           4-----17-----21
   |26       3     |13        |26       3     |13
   |   10-----23----- 9       |   12-----22----- 8
  17    | 21       4    |    18    | 19       5    |
   | 1 |    14     |27 |      | 1 |    14     |27 |
   |   24       7   |   11    |   23       9   |   10
  19---|- 5-----18      |    20---|- 6-----16      |
    15 |    25       2 |       15 |    25       2 |
        8-----12-----22            7-----11-----24
  3/                        4/
   2-----15-----25           1-----15-----26
   |24       7     |11        |23       7     |12
   |   16-----20----- 6       |   18-----20----- 4
  18    | 19       5    |    17    | 19       6    |
   | 1 |    14     |27 |      | 3 |    14     |25 |
   |   23       9   |   10    |   22       9   |   11
  22---|- 8-----12      |    24---|- 8-----10      |
    17 |    21       4 |       16 |    21       5 |
        3-----13-----26            2-----13-----27    [Count = 4]   OK!
```

10. At the end of this section I would like to propose you a way of composition of mine: How to compose a standard solution of Self-complementary Magic Cube 3x3x3

transforming from the usual Basic Form.
Let me demonstrate it in such the step diagrams as follows:

**** Standard MC333: Step Diagrams: How to Transform Basic Form into Object? ****

```
0/                          1/                          2/                          3/
1-----10-----19             1----- 0----- 0             1----- 0-----26             1----- 0-----26
| 2      11   |20           | 0      0    | 0           | 0      0    | 0           | 0      7    | 0
|   3-----12-----21         |   0----- 0----- 0         |   0----- 0----- 0         |   0----- 0----- 4
4   | 13     22   |         0   | 0      0    |         0   | 0      0    |         0   | 0      0    |
| 5 |    14   |23 |         | 0 |    14   | 0 |         | 3 |    14   |25 |         | 3 |    14   |25 |
|   6      15 |  24         |   0      0  |   0         |   0      0  |   0         |   0      0  |   0
7---|-16-----25   |         0---|- 0----- 0   |         0---|- 0----- 0   |         24--|- 0----- 0   |
  8 |   17     26 |           0 |    0     0 |           0 |    0     0 |           0 |   21     0 |
    9-----18-----27             0----- 0-----27             2----- 0-----27             2----- 0-----27

4/                          5/                          6/                          7/
1----- 0-----26             1----- 0-----26             1----- 0-----26             1-----15-----26
| 0      7    | 0           |23      7    |12           |23      7    |12           |23      7    |12
|  18----- 0----- 4         |  18----- 0----- 4         |  18----- 0----- 4         |  18-----20----- 4
0   | 19     0    |         0   | 19     0    |         17  | 19     6    |         17  | 19     6    |
| 3 |    14   |25 |         | 3 |    14   |25 |         | 3 |    14   |25 |         | 3 |    14   |25 |
|   0      9  |   0         |   0      9  |   0         |  22      9  |  11         |  22      9  |  11
24--|- 0-----10   |         24--|- 0-----10   |         24--|- 0-----10   |         24--|- 8-----10   |
  0 |   21     0 |          16 |   21     5 |          16 |   21     5 |          16 |   21     5 |
    2----- 0-----27             2----- 0-----27             2----- 0-----27             2-----13-----27
```

[Step 0] Prepare the Basic Form at the starting point

[Step 1] Copy the most primary triagonal {1,14,27} of the Basic Form and paste it into the same position of the Object Form.

[Step 2] Copy the first line {1, 2, 3} as {1, 3, 2} and put it into the diagonal positions of the left plane of Object, and place every complementary number of each position on the right .

[Step 3] Copy the second line {1, 4, 7} as {1, 7, 4} and put it into a diagonal of the top plane of Object, and place every complementary number of each position on the bottom.

[Step 4] Copy the third line {1,10, 19} as {1,19,10} and put it into a diagonal of the back plane of Object, and place every complementary number of each position on the front.

[Step 5-7] Values of rest positions can be easily calculated as in **nb=42-na-nc** in any line:
23=42-1-18; 5=42-27-10(or =28-23); 16=42-24-2; 12=42-4-26(or =28-16);
17=42-1-24; 11=42-27-4(or =28-17); 22=42-18-2; 6=42-10-26(or =28-22);
15=42-1-26; 13=42-27-2(or =28-15); 20=42-18-4; 8=42-10-24(or =28-20);

[Step 7] The last form is just what we have wanted to make for our object goal. It is certainly one of the standard solutions of Self-complementary Magic Cubes of Order 3.

(Written originally in Japanese in 2000; Revised English Version written in November 21, 2008 working with MacOSX and Xcode3.0 by Kanji Setsuda)

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