

Part 3: "New Algebraic Studies of Magic Squares": **Kanji Setsuda**
Chapter 2: New Algebraic Study of Magic Squares of Order 4
Section 1: Self-Complementary Magic Squares of Order 4

#0. First Definition

What I am studying intensively now is concentrated to some specific types of magic squares and cubes: One is **Self-Complementary** type, and another is **Pan-Diagonal** one.

The former is sometimes called in a different way such as 'Self-Similar' type or 'Symmetrical type' (with respect to the geometric center), etc.

The concept of **Self-Complementary** type is described like this:

Let's take an example of order 4 and use those diagrams below.

1. When you rotate the object in 180 degrees, add the result to the original, and you will find every sum of the two numbers in corresponding positions is always equal to the magic constant $C(=17)$.

When you move every number to the symmetrical position, you will get the same result. Add this to the original, and you will find every sum of the two numbers in corresponding positions is always equal to 17.

2. When you transform the square by some simple rules of exchanging every Complementary Pairs, rotate the result in 180 degrees, and you will find the similar square to the original, say, the same with itself.

*** Self-Complementary Magic Square 4*4 ***

[1]

1	15	14	4	16	2	3	13	17	17	17	17
12	6	7	9	5	11	10	8	17	17	17	17
8	10	11	5	9	7	6	12	17	17	17	17
13	3	2	16	4	14	15	1	17	17	17	17

1	15	14	4	16	2	3	13	1	15	14	4
12	6	7	9	5	11	10	8	12	6	7	9
8	10	11	5	9	7	6	12	8	10	11	5
13	3	2	16	4	14	15	1	13	3	2	16

[2]

1	15	12	6	16	2	5	11	17	17	17	17
14	4	7	9	3	13	10	8	17	17	17	17
8	10	13	3	9	7	4	14	17	17	17	17
11	5	2	16	6	12	15	1	17	17	17	17

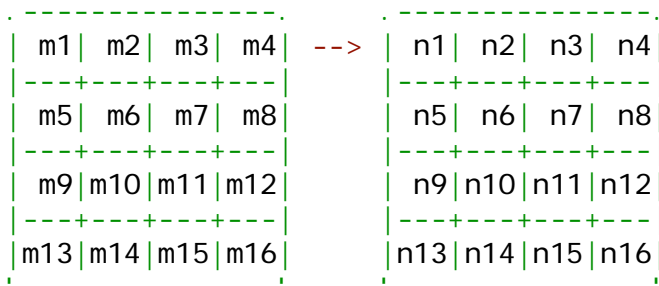
1	15	12	6	16	2	5	11	1	15	12	6
14	4	7	9	3	13	10	8	14	4	7	9
8	10	13	3	9	7	4	14	8	10	13	3
11	5	2	16	6	12	15	1	11	5	2	16

[3]

1	14	12	7	16	3	5	10	17	17	17	17
15	4	6	9	2	13	11	8	17	17	17	17
8	11	13	2	9	6	4	15	17	17	17	17
10	5	3	16	7	12	14	1	17	17	17	17

1	14	12	7	16	3	5	10	1	14	12	7
15	4	6	9	2	13	11	8	15	4	6	9
8	11	13	2	9	6	4	15	8	11	13	2
10	5	3	16	7	12	14	1	10	5	3	16

[Compl ementary Transformati on]



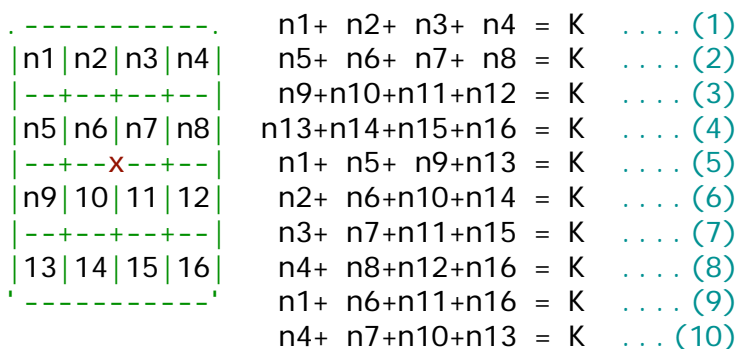
[n1=17-m1; n2=17-m2; n3=17-m3; n4=17-m4;
n5=17-m5; n6=17-m6; n7=17-m7; n8=17-m8;
n9=17-m9; n10=17-m10; n11=17-m11; n12=17-m12;
n13=17-m13; n14=17-m14; n15=17-m15; n16=17-m16;]

Because both tell us the same concept, I call this type of magic square 'Self-Complementary type' (with all Complementary Pairs located symmetrically with respect to the geometric center), and at the first stage of mathematical argument I define it as shown below:

#1. Basic Form and Basic Conditions

We prepare the basic diagram and basic equations for our Magic Square 4x4.

[Figure 1: Basic Form and Basic Conditions]



and n1+n16=n2+n15=n3+n14=n4+n13
=n5+n12+n6+n11=n7+n10=n8+n9=C ... (11)

The eight pairs (n1, n16), (n2, n15), (n3, n14), (n4, n13), (n5, n12), (n6, n11), (n7, n10), and (n8, n9) are usually called "Complementary Pairs of C."

When m+n=C(onstant), we say "(m, n) is one of the Complementary Pairs of C." They must be placed symmetrically with respect to geometric center 'x' of the square.

The constants C and K could be determined in advance.

(1)+(2)+(3)+(4)

n1+n2+n3+...+n15+n16 = 4K
(16+1) x 16 / 2 = 4 x K
136 = 4 x K

Therefore K = 136/4 = 34

(9) $n1+n6+n11+n16=(n1+n16)+(n6+n11)= CC + CC = K = 34$
 Therefore $CC = K / 2 = 34/2 = 17$

#2. Complementary Pairs and New Notations

Let me introduce you a new notation of Complementary Pairs:

$$n1+n1=n2+n2=n3+n3=...=n8+n8=...=n15+n15=n16+n16=C$$

These new notations mean $n1=C-n1=n16$, $n2=C-n2=n15$,

$$n3=C-n3=n14, \dots, n8=C-n8=n9 (\dots, n16=C-n16=n1)$$

Of course $C=17$

We can rewrite the Basic Form 1 by these notations as follows:

[Figure 2: Complementary Pairs in their Notations]

n1 n2 n3 n4	$n1+ n2+ n3+ n4 = K \dots (1)'$
n5 n6 n7 n8	$n5+ n6+ n7+ n8 = K \dots (2)'$
n8 n7 n6 n5	$n8+ n7+ n6+ n5 = K \dots (3)'$
n5 n6 n7 n8	$n4+ n3+ n2+ n1 = K \dots (4)'$
n8 n7 n6 n5	$n1+ n5+ n8+ n4 = K \dots (5)'$
n4 n3 n2 n1	$n2+ n6+ n7+ n3 = K \dots (6)'$
n8 n7 n6 n5	$n3+ n7+ n6+ n2 = K \dots (7)'$
n4 n3 n2 n1	$n4+ n8+ n5+ n1 = K \dots (8)'$
n8 n7 n6 n5	$n1+ n6+ n6+ n1 = K \dots (9)'$
n4 n3 n2 n1	$n4+ n7+ n7+ n4 = K \dots (10)'$

$$n1+n2+n2+n1=(n1+n1)+(n2+n2)=34(=K)$$

$$n1+n2+n3+n4 = K \dots (1)'$$

Therefore $n2+n1 = n3+n4 = K-(n1+n2)$

That is $n3+n4 = n15+n16$

In the same way $n7+n8 = n6+n5 = n11+n12$ is worked out.

$$n1+n2 = n4+n3 = n13+n14; n5+n6 = n8+n7 = n9+n10;$$

$$n1+n5 = n4+n8; n2+n6 = n3+n7; n9+n13 = n12+n16; n10+n14 = n11+n15$$

#3. Block-Sums and their Relations

Calculate four block-sums below and compare the results.

Block A	Block B	[Figure 2(again)]
n1 n2 n3 n4	n5 n6 n7 n8	$n1+ n2+ n3+ n4 = K \dots (1)'$
n5 n6 n7 n8	n8 n7 n6 n5	$n5+ n6+ n7+ n8 = K \dots (2)'$
n8 n7 n6 n5	n4 n3 n2 n1	$n8+ n7+ n6+ n5 = K \dots (3)'$
n5 n6 n7 n8	n8 n7 n6 n5	$n4+ n3+ n2+ n1 = K \dots (4)'$
n8 n7 n6 n5	n1 n5 n8 n4	$n1+ n5+ n8+ n4 = K \dots (5)'$
n4 n3 n2 n1	n2 n6 n7 n3	$n2+ n6+ n7+ n3 = K \dots (6)'$
n8 n7 n6 n5	n3 n7 n6 n2	$n3+ n7+ n6+ n2 = K \dots (7)'$
n4 n3 n2 n1	n4 n8 n5 n1	$n4+ n8+ n5+ n1 = K \dots (8)'$
n8 n7 n6 n5	n1 n6 n6 n1	$n1+ n6+ n6+ n1 = K \dots (9)'$
n4 n3 n2 n1	n4 n7 n7 n4	$n4+ n7+ n7+ n4 = K \dots (10)'$

Block C Block D

$$\{Block A\} + \{Block B\} = (n1+n2+n3+n4) + (n5+n6+n7+n8) = 34+34=68$$

$$\{Block A\} + \{Block D\} = (n1+n1) + (n2+n2) + (n5+n5) + (n6+n6) = 4 \times 17 = 68$$

$$\{Block B\} + \{Block D\} = (n3+n7+n6+n2) + (n4+n8+n5+n1) = 34+34=68$$

$$\{Block\ B\} + \{Block\ C\} = (\underbrace{n3+n3}_{17}) + (\underbrace{n4+n4}_{17}) + (\underbrace{n7+n7}_{17}) + (\underbrace{n8+n8}_{17}) = 4 \times 17 = 68$$

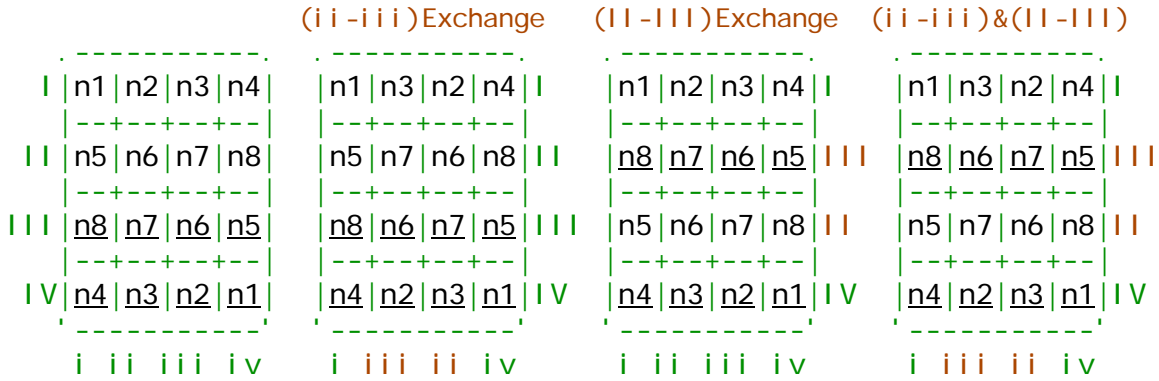
Therefore $\{Block\ A\} = \{Block\ B\} = \{Block\ C\} = \{Block\ D\} = 34$

#4. Which Lines Can We Exchange with Each Other?

New notation of Complementary Pairs is very useful for you to find which rows and columns you can exchange with each other.

[Figure 2 (again):

Which rows and columns can we exchange?]



Every row and every column adds up to the same constant, because the members of each row and column are not changed but just for their permutations.

The primary diagonals are partially changed, but the sums of new diagonals are all equal to the magic constant.

$$n1+n6+n6+n1 = K; \quad n1+n7+n7+n1 = K; \quad n1+n7+n7+n1 = K; \quad n1+n6+n6+n1 = K;$$

$$n4+n7+n7+n4 = K; \quad n4+n6+n6+n4 = K; \quad n4+n6+n6+n4 = K; \quad n4+n7+n7+n4 = K;$$

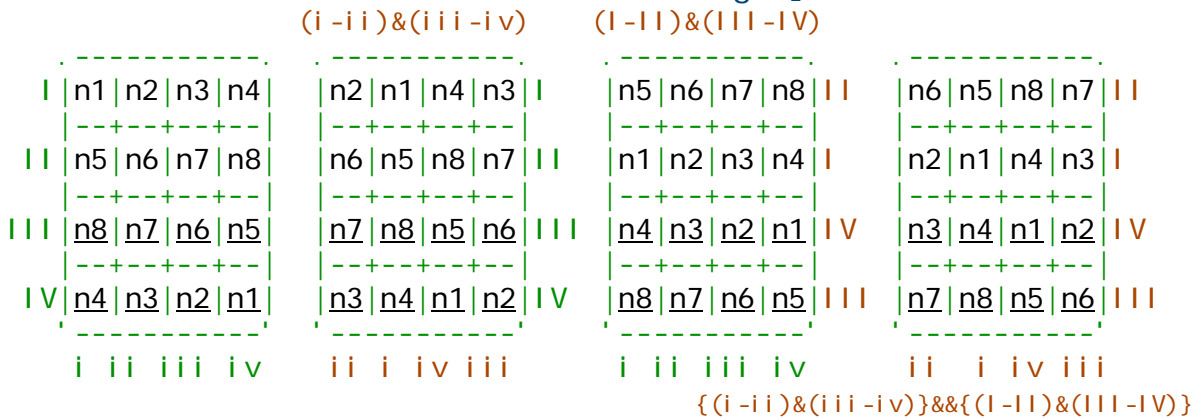
We can exchange (2nd ↔ 3rd) rows/columns with each other.

In the same way (1st ↔ 4th) rows/columns are always exchangeable.

Next diagrams will tell you another way how to exchange lines.

[Figure 3:

Which rows and columns can we exchange?]



If you want to exchange 1st ↔ 2nd rows/columns, then you must 3rd ↔ 4th rows/columns each other at the same time.

#5. How many Solutions can we find?

Let's make all the self-complementary magic squares of order 4 actually with these properties. How many solutions in all can we make for them?

There are three levels of solution sets for any object of any order:

- (1) 'Primitive Solutions' mean all object forms without any selection.
- (2) 'Standard Solutions' mean the selected ones removing the other same forms when transformed by some simple reflection and repeating rotations.
- (3) 'Fundamental Solutions' mean the fewest ones, selected strictly by some reasonable transformation system. They usually have their n1=1 on the left top.

Let me show you the list of 'primitive solutions' of our object at first.

** Self-Complementary Magic Squares of Order 4: **

** List of the 384 'Primitive' Solutions (Part) **

1/		2/		3/		4/		5/		6/	
1 8 15 10	1 8 14 11	1 8 15 10	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13
12 13 6 3	12 13 7 2	14 11 4 5	14 11 7 2	15 10 4 5	15 10 4 5	15 10 6 3	12 13 7 2	14 11 7 2	15 10 6 3	12 13 7 2	14 11 7 2
14 11 4 5	15 10 4 5	12 13 6 3	15 10 6 3	12 13 7 2	14 11 7 2	14 11 7 2	15 10 6 3	12 13 7 2	14 11 7 2	14 11 7 2	15 10 6 3
7 2 9 16	6 3 9 16	7 2 9 16	4 5 9 16	6 3 9 16	4 5 9 16	4 5 9 16	6 3 9 16	4 5 9 16	6 3 9 16	4 5 9 16	6 3 9 16
7/		8/		9/		10/		11/		12/	
1 12 15 6	1 12 14 7	1 12 15 6	1 12 8 13	1 12 14 7	1 12 8 13	1 12 14 7	1 12 8 13	1 12 14 7	1 12 8 13	1 12 14 7	1 12 8 13
8 13 10 3	8 13 11 2	14 7 4 9	14 7 11 2	15 6 4 9	15 6 4 9	15 6 10 3	8 13 11 2	14 7 11 2	15 6 10 3	8 13 11 2	14 7 11 2
14 7 4 9	15 6 4 9	8 13 10 3	15 6 10 3	8 13 11 2	14 7 11 2	14 7 11 2	15 6 10 3	8 13 11 2	14 7 11 2	14 7 11 2	15 6 10 3
11 2 5 16	10 3 5 16	11 2 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 3 5 16	4 9 5 16
13/		14/		15/		16/		17/		18/	
1 14 15 4	1 14 12 7	1 14 15 4	1 14 8 11	1 14 12 7	1 14 8 11	1 14 12 7	1 14 8 11	1 14 12 7	1 14 8 11	1 14 12 7	1 14 8 11
8 11 10 5	8 11 13 2	12 7 6 9	12 7 13 2	15 4 6 9	15 4 6 9	15 4 10 5	8 11 13 2	12 7 13 2	15 4 10 5	8 11 13 2	12 7 13 2
12 7 6 9	15 4 6 9	8 11 10 5	15 4 10 5	8 11 13 2	12 7 13 2	12 7 13 2	15 4 10 5	8 11 13 2	12 7 13 2	12 7 13 2	15 4 10 5
13 2 3 16	10 5 3 16	13 2 3 16	6 9 3 16	10 5 3 16	6 9 3 16	10 5 3 16	6 9 3 16	10 5 3 16	6 9 3 16	10 5 3 16	6 9 3 16
19/		20/		21/		22/		23/		24/	
1 15 14 4	1 15 12 6	1 15 14 4	1 15 8 10	1 15 12 6	1 15 8 10	1 15 12 6	1 15 8 10	1 15 12 6	1 15 8 10	1 15 12 6	1 15 8 10
8 10 11 5	8 10 13 3	12 6 7 9	12 6 13 3	14 4 7 9	14 4 11 5	8 10 13 3	12 6 13 3	14 4 7 9	14 4 11 5	8 10 13 3	12 6 13 3
12 6 7 9	14 4 7 9	8 10 11 5	14 4 11 5	8 10 13 3	12 6 13 3	12 6 13 3	14 4 11 5	8 10 13 3	12 6 13 3	12 6 13 3	14 4 11 5
13 3 2 16	11 5 2 16	13 3 2 16	7 9 2 16	11 5 2 16	7 9 2 16	11 5 2 16	7 9 2 16	11 5 2 16	7 9 2 16	11 5 2 16	7 9 2 16
25/		26/		27/		28/		29/		30/	
2 7 16 9	2 7 13 12	2 7 16 9	2 7 11 14	2 7 13 12	2 7 11 14	2 7 13 12	2 7 11 14	2 7 13 12	2 7 11 14	2 7 13 12	2 7 11 14
11 14 5 4	11 14 8 1	13 12 3 6	13 12 8 1	16 9 3 6	16 9 5 4	16 9 3 6	16 9 5 4	11 14 8 1	13 12 8 1	16 9 5 4	16 9 3 6
13 12 3 6	16 9 3 6	11 14 5 4	16 9 5 4	11 14 8 1	13 12 8 1	13 12 8 1	16 9 5 4	11 14 8 1	13 12 8 1	16 9 3 6	16 9 5 4
8 1 10 15	5 4 10 15	8 1 10 15	3 6 10 15	5 4 10 15	3 6 10 15	5 4 10 15	3 6 10 15	5 4 10 15	3 6 10 15	5 4 10 15	3 6 10 15
31/		32/		33/		34/		35/		36/	
2 11 16 5	2 11 13 8	2 11 16 5	2 11 7 14	2 11 13 8	2 11 7 14	2 11 13 8	2 11 7 14	2 11 13 8	2 11 7 14	2 11 13 8	2 11 7 14
7 14 9 4	7 14 12 1	13 8 3 10	13 8 12 1	16 5 3 10	16 5 9 4	16 5 3 10	16 5 9 4	7 14 12 1	13 8 12 1	16 5 9 4	16 5 3 10
13 8 3 10	16 5 3 10	7 14 9 4	16 5 9 4	7 14 12 1	13 8 12 1	13 8 12 1	16 5 9 4	7 14 12 1	13 8 12 1	16 5 3 10	16 5 9 4
12 1 6 15	9 4 6 15	12 1 6 15	3 10 6 15	9 4 6 15	3 10 6 15	9 4 6 15	3 10 6 15	9 4 6 15	3 10 6 15	9 4 6 15	3 10 6 15
37/		38/		39/		40/		41/		42/	
2 13 16 3	2 13 11 8	2 13 16 3	2 13 7 12	2 13 11 8	2 13 7 12	2 13 11 8	2 13 7 12	2 13 11 8	2 13 7 12	2 13 11 8	2 13 7 12
7 12 9 6	7 12 14 1	11 8 5 10	11 8 14 1	16 3 5 10	16 3 9 6	16 3 5 10	16 3 9 6	7 12 14 1	11 8 14 1	16 3 9 6	16 3 5 10
11 8 5 10	16 3 5 10	7 12 9 6	16 3 9 6	7 12 14 1	11 8 14 1	11 8 14 1	16 3 9 6	7 12 14 1	11 8 14 1	16 3 5 10	16 3 9 6
14 1 4 15	9 6 4 15	14 1 4 15	5 10 4 15	9 6 4 15	5 10 4 15	9 6 4 15	5 10 4 15	9 6 4 15	5 10 4 15	9 6 4 15	5 10 4 15
43/		44/		45/		46/		47/		48/	
2 16 13 3	2 16 11 5	2 16 13 3	2 16 7 9	2 16 11 5	2 16 7 9	2 16 11 5	2 16 7 9	2 16 11 5	2 16 7 9	2 16 11 5	2 16 7 9
7 9 12 6	7 9 14 4	11 5 8 10	11 5 14 4	13 3 8 10	13 3 12 6	13 3 8 10	13 3 12 6	7 9 14 4	11 5 14 4	13 3 12 6	13 3 8 10
11 5 8 10	13 3 8 10	7 9 12 6	13 3 12 6	7 9 14 4	11 5 14 4	11 5 14 4	13 3 12 6	7 9 14 4	11 5 14 4	13 3 8 10	13 3 12 6
14 4 1 15	12 6 1 15	14 4 1 15	8 10 1 15	12 6 1 15	8 10 1 15	12 6 1 15	8 10 1 15	12 6 1 15	8 10 1 15	12 6 1 15	8 10 1 15
49/		50/		51/		52/		53/		54/	
3 6 16 9	3 6 13 12	3 6 16 9	3 6 10 15	3 6 13 12	3 6 10 15	3 6 13 12	3 6 10 15	3 6 13 12	3 6 10 15	3 6 13 12	3 6 10 15
10 15 5 4	10 15 8 1	13 12 2 7	13 12 8 1	16 9 2 7	16 9 5 4	16 9 2 7	16 9 5 4	10 15 8 1	13 12 8 1	16 9 5 4	16 9 2 7
13 12 2 7	16 9 2 7	10 15 5 4	16 9 5 4	10 15 8 1	13 12 8 1	13 12 8 1	16 9 5 4	10 15 8 1	13 12 8 1	16 9 2 7	16 9 5 4
8 1 11 14	5 4 11 14	8 1 11 14	2 7 11 14	5 4 11 14	2 7 11 14	5 4 11 14	2 7 11 14	5 4 11 14	2 7 11 14	5 4 11 14	2 7 11 14
55/		56/		57/		58/		59/		60/	

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3 10 16 5   3 10 13 8   3 10 16 5   3 10 6 15   3 10 13 8   3 10 6 15
6 15 9 4   6 15 12 1 13 8 2 11 13 8 12 1 16 5 2 11 16 5 9 4
13 8 2 11 16 5 2 11 6 15 9 4 16 5 9 4 6 15 12 1 13 8 12 1
12 1 7 14 9 4 7 14 12 1 7 14 2 11 7 14 9 4 7 14 2 11 7 14
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3 13 16 2   3 13 10 8   3 13 16 2   3 13 6 12   3 13 10 8   3 13 6 12
6 12 9 7   6 12 15 1 10 8 5 11 10 8 15 1 16 2 5 11 16 2 9 7
10 8 5 11 16 2 5 11 6 12 9 7 16 2 9 7 6 12 15 1 10 8 15 1
15 1 4 14 9 7 4 14 15 1 4 14 5 11 4 14 9 7 4 14 5 11 4 14
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3 16 13 2   3 16 10 5   3 16 13 2   3 16 6 9   3 16 10 5   3 16 6 9
6 9 12 7   6 9 15 4 10 5 8 11 10 5 15 4 13 2 8 11 13 2 12 7
10 5 8 11 13 2 8 11 6 9 12 7 13 2 12 7 6 9 15 4 10 5 15 4
15 4 1 14 12 7 1 14 15 4 1 14 8 11 1 14 12 7 1 14 8 11 1 14
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4 5 15 10 4 5 14 11 4 5 15 10 4 5 9 16 4 5 14 11 4 5 9 16
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7 2 12 13 6 3 12 13 7 2 12 13 1 8 12 13 6 3 12 13 1 8 12 13

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      361/      362/      363/      364/      365/      366/
16 2 9 7 16 2 5 11 16 2 9 7 16 2 3 13 16 2 5 11 16 2 3 13
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3 6 10 15 5 4 10 15 2 7 11 14 5 4 11 14 2 7 13 12 3 6 13 12
13 12 8 1 11 14 8 1 13 12 8 1 10 15 8 1 11 14 8 1 10 15 8 1

```

* Counts according to the value of n1 *
[1: 24] [2: 24] [3: 24] [4: 24] [5: 24] [6: 24] [7: 24] [8: 24]
[9: 24] [10: 24] [11: 24] [12: 24] [13: 24] [14: 24] [15: 24] [16: 24]
* Total Count = 384 *

If you want to have the 'standard solutions' of our object, you have to put the inequality conditions {n1<n4; n1<n13; n1<n16; and n2<n5}, for instance, to prevent you from counting any reflection or rotation double.

Let me show you (1) a sample list of my recent computer program, and (2) the list of standard solutions calculated and printed out by that program.

```

/* Self-Complementary Magic Squares of Order 4 */
** File: 'MS44SC48.c' built by Kanji Setsuda **/
** on Aug. 10, 2002; Dec. 13, 2005; **/

```

```

/** Worked on MacOSX and Xcode 1.5 */
/*
.----- [Basic Conditions]
| 1| 2| 3| 4| n1+n2+n3+n4=C ... rw1;
|--+-----| n5+n6+n7+n8=C ... rw2;
| 5| 6| 7| 8| n9+n10+n11+n12=C ... rw3;
|--+-----| n13+n14+n15+n16=C ... rw4;
| 9|10|11|12| n1+n5+n9+n13=C ... cl 1;
|--+-----| n2+n6+n10+n14=C ... cl 2;
|13|14|15|16| n3+n7+n11+n15=C ... cl 3;
'-----' n4+n8+n12+n16=C ... cl 4;

[Self-Complementary Conditions]
n1+n16=CC ... Cmp1; n2+n15=CC ... Cmp2;
n3+n14=CC ... Cmp3; n4+n13=CC ... Cmp4;
n5+n12=CC ... Cmp5; n6+n11=CC ... Cmp6;
n7+n10=CC ... Cmp7; n8+n9=CC ... Cmp8;
*/
/**/
#include <stdio.h>
/**/
short cnt, cnt3;
short LSM, PSM;
short nm[17], uflg[17];
short anm[9][17];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void ansprint(void);
void printans(short x);
/**/
/* Main Program */
int main(){
short n;
printf("\n** Self-Complementary Magic Squares of Order 4: **\n");
printf("** List of the 48 'Standard' Solutions **\n");
for(n=0; n<17; n++){nm[n]=0; uflg[n]=0;}
LSM=34; PSM=17; cnt=0; cnt3=0;
stp01(); /* Begin the Calculations */
if(cnt3>0){printans(cnt3);}
printf(" [Count = %d]\n", cnt);
printf(" OK!\n");
return 0;
}
/* Begin the Calculations */
/* Set n1 & n16=PSM-n1 & n1<n16 */
void stp01(){
short a, b;
for(a=1; a<9; a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[1]=a; nm[16]=b;
uflg[a]=1; uflg[b]=1;
stp02();
uflg[b]=0; uflg[a]=0;}
}
}
/* Set n2 & n15=PSM-n2 */
void stp02(){
short a, b;
for(a=1; a<17; a++){b=PSM-a;

```

```

    if((ufl g[a]==0)&&(ufl g[b]==0)){
        nm[2]=a; nm[15]=b;
        ufl g[a]=1; ufl g[b]=1;
        stp03();
        ufl g[b]=0; ufl g[a]=0; }
}
}
/* Set n5 & n12=PSM-n5 & n5>n2 */
void stp03(){
    short a, b;
    for(a=nm[2]+1; a<17; a++){b=PSM-a;
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[5]=a; nm[12]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp04();
            ufl g[b]=0; ufl g[a]=0; }
    }
}
/* Set n6 & n11=PSM-n6 */
void stp04(){
    short a, b;
    for(a=1; a<17; a++){b=PSM-a;
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[6]=a; nm[11]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp05();
            ufl g[b]=0; ufl g[a]=0; }
    }
}
/* Set n4 & n13=PSM-n4 & n1<n4 & n1<n13 */
void stp05(){
    short a, b;
    for(a=nm[1]+1; a<17; a++){b=PSM-a;
        if((b>nm[1])&&(ufl g[a]==0)&&(ufl g[b]==0)){
            nm[4]=a; nm[13]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp06();
            ufl g[b]=0; ufl g[a]=0; }
    }
}
/* Set n3=LSM-n1-n2-n4 & n14=PSM-n3 */
void stp06(){
    short a, b;
    a=LSM-nm[1]-nm[2]-nm[4]; b=PSM-a;
    if((0<a)&&(a<17)){
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[3]=a; nm[14]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp07();
            ufl g[b]=0; ufl g[a]=0; }}
}
/* Set n9=LSM-n1-n5-n13 & n8=LSM-n4-n12-n16 */
/* & Check n8+n9==PSM? */
void stp07(){
    short a, b;
    a=LSM-nm[1]-nm[5]-nm[13];
    b=LSM-nm[4]-nm[12]-nm[16];
    if((0<a)&&(a<17)&&(a+b==PSM)){
        if((ufl g[a]==0)&&(ufl g[b]==0)){

```

```

        nm[9]=a; nm[8]=b;
        uflg[a]=1; uflg[b]=1;
        stp08();
        uflg[b]=0; uflg[a]=0; }}
}
/* Set n7=LSM-n5-n6-n8 & n10=LSM-n2-n6-n14 */
/* & Check n7+n10==PSM? */
void stp08(){
    short a, b;
    a=LSM-nm[5]-nm[6]-nm[8];
    b=LSM-nm[2]-nm[6]-nm[14];
    if((0<a)&&(a<17)&&(a+b==PSM)){
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[7]=a; nm[10]=b;
            uflg[a]=1; uflg[b]=1;
            ansprint();
            uflg[b]=0; uflg[a]=0; }}
}
/* Print the Answers */
void ansprint(){
    short n;
    cnt++;
    anm[cnt3][0]=cnt;
    for(n=1; n<17; n++){anm[cnt3][n]=nm[n];}
    cnt3++; if(cnt3==6){printans(6); cnt3=0;}
}
/* Print the Answers */
void printans(short x){
    short m, n;
    for(m=0; m<x; m++){printf("%12d/", anm[m][0]);}
    printf("\n");
    for(m=0; m<x; m++){
        printf("%4d%3d%3d%3d", anm[m][1], anm[m][2], anm[m][3], anm[m][4]);}
    printf("\n");
    for(m=0; m<x; m++){
        printf("%4d%3d%3d%3d", anm[m][5], anm[m][6], anm[m][7], anm[m][8]);}
    printf("\n");
    for(m=0; m<x; m++){
        printf("%4d%3d%3d%3d", anm[m][9], anm[m][10], anm[m][11], anm[m][12]);}
    printf("\n");
    for(m=0; m<x; m++){
        printf("%4d%3d%3d%3d", anm[m][13], anm[m][14], anm[m][15], anm[m][16]);}
    printf("\n");
}
/**/

```

**** Self-Complementary Magic Squares of Order 4: ****
**** List of the 48 'Standard Solutions' ****

1/		2/		3/		4/		5/		6/	
1 8 15 10	1 8 14 11	1 8 15 10	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13	1 8 14 11	1 8 12 13
12 13 6 3	12 13 7 2	14 11 4 5	14 11 7 2	15 10 4 5	15 10 6 3	12 13 7 2	14 11 7 2	15 10 4 5	15 10 6 3	12 13 7 2	14 11 7 2
14 11 4 5	15 10 4 5	12 13 6 3	15 10 6 3	12 13 7 2	14 11 7 2	15 10 4 5	15 10 6 3	12 13 7 2	14 11 7 2	15 10 4 5	15 10 6 3
7 2 9 16	6 3 9 16	7 2 9 16	4 5 9 16	6 3 9 16	4 5 9 16	6 3 9 16	4 5 9 16	6 3 9 16	4 5 9 16	6 3 9 16	4 5 9 16
7/		8/		9/		10/		11/		12/	
1 12 15 6	1 12 8 13	1 12 14 7	1 12 8 13	1 14 12 7	1 14 8 11	1 12 15 6	1 12 8 13	1 14 12 7	1 14 8 11	1 12 15 6	1 12 8 13
14 7 4 9	14 7 11 2	15 6 4 9	15 6 10 3	15 4 6 9	15 4 10 5	14 7 4 9	14 7 11 2	15 6 4 9	15 6 10 3	15 4 6 9	15 4 10 5
8 13 10 3	15 6 10 3	8 13 11 2	14 7 11 2	8 11 13 2	12 7 13 2	8 13 10 3	15 6 10 3	8 13 11 2	14 7 11 2	8 11 13 2	12 7 13 2
11 2 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 5 3 16	6 9 3 16	11 2 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 5 3 16	6 9 3 16

13/ 2 7 16 9 11 14 5 4 13 12 3 6 8 1 10 15	14/ 2 7 13 12 11 14 8 1 16 9 3 6 5 4 10 15	15/ 2 7 16 9 13 12 3 6 11 14 5 4 8 1 10 15	16/ 2 7 11 14 13 12 8 1 16 9 5 4 3 6 10 15	17/ 2 7 13 12 16 9 3 6 11 14 8 1 5 4 10 15	18/ 2 7 11 14 16 9 5 4 13 12 8 1 3 6 10 15
19/ 2 11 16 5 13 8 3 10 7 14 9 4 12 1 6 15	20/ 2 11 7 14 13 8 12 1 16 5 9 4 3 10 6 15	21/ 2 11 13 8 16 5 3 10 7 14 12 1 9 4 6 15	22/ 2 11 7 14 16 5 9 4 13 8 12 1 3 10 6 15	23/ 2 13 11 8 16 3 5 10 7 12 14 1 9 6 4 15	24/ 2 13 7 12 16 3 9 6 11 8 14 1 5 10 4 15
25/ 3 6 16 9 10 15 5 4 13 12 2 7 8 1 11 14	26/ 3 6 13 12 10 15 8 1 16 9 2 7 5 4 11 14	27/ 3 6 16 9 13 12 2 7 10 15 5 4 8 1 11 14	28/ 3 6 13 12 16 9 2 7 10 15 8 1 5 4 11 14	29/ 3 10 16 5 13 8 2 11 6 15 9 4 12 1 7 14	30/ 3 10 13 8 16 5 2 11 6 15 12 1 9 4 7 14
31/ 3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14	32/ 3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14	33/ 4 5 15 10 9 16 6 3 14 11 1 8 7 2 12 13	34/ 4 5 14 11 9 16 7 2 15 10 1 8 6 3 12 13	35/ 4 5 15 10 14 11 1 8 9 16 6 3 7 2 12 13	36/ 4 5 14 11 15 10 1 8 9 16 7 2 6 3 12 13
37/ 4 9 15 6 14 7 1 12 5 16 10 3 11 2 8 13	38/ 4 9 14 7 15 6 1 12 5 16 11 2 10 3 8 13	39/ 4 14 9 7 15 1 6 12 5 11 16 2 10 8 3 13	40/ 4 14 5 11 15 1 10 8 9 7 16 2 6 12 3 13	41/ 5 4 16 9 10 15 3 6 11 14 2 7 8 1 13 12	42/ 5 4 16 9 11 14 2 7 10 15 3 6 8 1 13 12
43/ 5 10 11 8 16 3 2 13 4 15 14 1 9 6 7 12	44/ 5 11 10 8 16 2 3 13 4 14 15 1 9 7 6 12	45/ 6 3 15 10 9 16 4 5 12 13 1 8 7 2 14 11	46/ 6 3 15 10 12 13 1 8 9 16 4 5 7 2 14 11	47/ 6 9 12 7 15 4 1 14 3 16 13 2 10 5 8 11	48/ 6 12 9 7 15 1 4 14 3 13 16 2 10 8 5 11

[Count = 48] OK!

If you want to have the 'fundamental solutions' of our object, you have to put such conditions as {n1=1; n16=16; n2<n3; n5<n9; and n2<n5;}. You will finally get the three fundamental solutions as follows.

** Self-Complementary Magic Squares of Order 4: **
 ** List of the 'Fundamental' Three Solutions **

1/ 1 8 15 10 12 13 6 3 14 11 4 5 7 2 9 16	2/ 1 8 14 11 12 13 7 2 15 10 4 5 6 3 9 16	3/ 1 8 12 13 14 11 7 2 15 10 6 3 4 5 9 16
---	---	---

What kinds of relations are there among those three levels of solutions? What is the meaning of the next factors of divisions? $384 = 48 \times 8 = 3 \times 16 \times 8$
 Can we assume any transformation systems to make them meaningful?

#6. What Transformation Systems can we Assume?

Take your careful look at the next concept diagrams below I prepared.

** Transformation System for Self-Complementary Magic Squares 4x4 **/

[2x4 Transformation making 1 into 8: Rotate and Reflect]

00/				01/				02/				03/			
1	2	3	4	13	9	5	1	16	15	14	13	4	8	12	16
5	6	7	8	14	10	6	2	12	11	10	9	3	7	11	15
9	10	11	12	15	11	7	3	8	7	6	5	2	6	10	14
13	14	15	16	16	12	8	4	4	3	2	1	1	5	9	13
10/				11/				12/				13/			
1	5	9	13	4	3	2	1	16	12	8	4	13	14	15	16
2	6	10	14	8	7	6	5	15	11	7	3	9	10	11	12
3	7	11	15	12	11	10	9	14	10	6	2	5	6	7	8
4	8	12	16	16	15	14	13	13	9	5	1	1	2	3	4

[4x4 Transformation making 1 into 16: Exchange some Lines]

00/				01/				02/				03/			
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
10/				11/				12/				13/			
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
20/				21/				22/				23/			
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
30/				31/				32/				33/			
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11

The upper set illustrates the combination of rotation by 90 degrees and the mirror reflection. These 8 patterns are always possible in every type and in every order.

The lower set of 16 patterns illustrates the combination of row exchangeability and column exchangeability.

You can really make different forms by those two sets of transformation and can even combine them to make one fundamental into 8x16 different patterns.

The next long list shows the result of those transformations.

** Reconstruction of All 'Self-Complementary' Magic Squares 4x4: **

** Start: The 'Fundamental' Three Solutions **

1/				2/				3/			
1	8	15	10	1	8	14	11	1	8	12	13
12	13	6	3	12	13	7	2	14	11	7	2
14	11	4	5	15	10	4	5	15	10	6	3
7	2	9	16	6	3	9	16	4	5	9	16

** Transform them into 'Primitive' Solutions and Compare them with the Old List **

[Fundamental #1]

1/	163/	384/	222/	8/	235/	377/	150/
1 8 15 10	7 14 12 1	16 9 2 7	10 3 5 16	1 12 14 7	10 15 8 1	16 5 3 10	7 2 9 16
12 13 6 3	2 11 13 8	5 4 11 14	15 6 4 9	8 13 11 2	3 6 13 12	9 4 6 15	14 11 4 5
14 11 4 5	9 4 6 15	3 6 13 12	8 13 11 2	15 6 4 9	5 4 11 14	2 11 13 8	12 13 6 3
7 2 9 16	16 5 3 10	10 15 8 1	1 12 14 7	10 3 5 16	16 9 2 7	7 14 12 1	1 8 15 10
22/	165/	363/	220/	11/	229/	374/	156/
1 15 8 10	7 14 12 1	16 2 9 7	10 3 5 16	1 12 14 7	10 8 15 1	16 5 3 10	7 9 2 16
12 6 13 3	9 4 6 15	5 11 4 14	8 13 11 2	15 6 4 9	3 13 6 12	2 11 13 8	14 4 11 5
14 4 11 5	2 11 13 8	3 13 6 12	15 6 4 9	8 13 11 2	5 11 4 14	9 4 6 15	12 6 13 3
7 9 2 16	16 5 3 10	10 8 15 1	1 12 14 7	10 3 5 16	16 2 9 7	7 14 12 1	1 15 8 10
174/	32/	211/	353/	187/	360/	198/	25/
8 1 10 15	2 11 13 8	9 16 7 2	15 6 4 9	8 13 11 2	15 10 1 8	9 4 6 15	2 7 16 9
13 12 3 6	7 14 12 1	4 5 14 11	10 3 5 16	1 12 14 7	6 3 12 13	16 5 3 10	11 14 5 4
11 14 5 4	16 5 3 10	6 3 12 13	1 12 14 7	10 3 5 16	4 5 14 11	7 14 12 1	13 12 3 6
2 7 16 9	9 4 6 15	15 10 1 8	8 13 11 2	15 6 4 9	9 16 7 2	2 11 13 8	8 1 10 15
180/	35/	205/	350/	189/	339/	196/	46/
8 10 1 15	2 11 13 8	9 7 16 2	15 6 4 9	8 13 11 2	15 1 10 8	9 4 6 15	2 16 7 9
13 3 12 6	16 5 3 10	4 14 5 11	1 12 14 7	10 3 5 16	6 12 3 13	7 14 12 1	11 5 14 4
11 5 14 4	7 14 12 1	6 12 3 13	10 3 5 16	1 12 14 7	4 14 5 11	16 5 3 10	13 3 12 6
2 16 7 9	9 4 6 15	15 1 10 8	8 13 11 2	15 6 4 9	9 7 16 2	2 11 13 8	8 10 1 15
3/	157/	382/	228/	14/	237/	371/	148/
1 8 15 10	7 12 14 1	16 9 2 7	10 5 3 16	1 14 12 7	10 15 8 1	16 3 5 10	7 2 9 16
14 11 4 5	2 13 11 8	3 6 13 12	15 4 6 9	8 11 13 2	5 4 11 14	9 6 4 15	12 13 6 3
12 13 6 3	9 6 4 15	5 4 11 14	8 11 13 2	15 4 6 9	3 6 13 12	2 13 11 8	14 11 4 5
7 2 9 16	16 3 5 10	10 15 8 1	1 14 12 7	10 5 3 16	16 9 2 7	7 12 14 1	1 8 15 10
24/	159/	361/	226/	17/	231/	368/	154/
1 15 8 10	7 12 14 1	16 2 9 7	10 5 3 16	1 14 12 7	10 8 15 1	16 3 5 10	7 9 2 16
14 4 11 5	9 6 4 15	3 13 6 12	8 11 13 2	15 4 6 9	5 11 4 14	2 13 11 8	12 6 13 3
12 6 13 3	2 13 11 8	5 11 4 14	15 4 6 9	8 11 13 2	3 13 6 12	9 6 4 15	14 4 11 5
7 9 2 16	16 3 5 10	10 8 15 1	1 14 12 7	10 5 3 16	16 2 9 7	7 12 14 1	1 15 8 10
172/	38/	213/	347/	181/	358/	204/	27/
8 1 10 15	2 13 11 8	9 16 7 2	15 4 6 9	8 11 13 2	15 10 1 8	9 6 4 15	2 7 16 9
11 14 5 4	7 12 14 1	6 3 12 13	10 5 3 16	1 14 12 7	4 5 14 11	16 3 5 10	13 12 3 6
13 12 3 6	16 3 5 10	4 5 14 11	1 14 12 7	10 5 3 16	6 3 12 13	7 12 14 1	11 14 5 4
2 7 16 9	9 6 4 15	15 10 1 8	8 11 13 2	15 4 6 9	9 16 7 2	2 13 11 8	8 1 10 15
178/	41/	207/	344/	183/	337/	202/	48/
8 10 1 15	2 13 11 8	9 7 16 2	15 4 6 9	8 11 13 2	15 1 10 8	9 6 4 15	2 16 7 9
11 5 14 4	16 3 5 10	6 12 3 13	1 14 12 7	10 5 3 16	4 14 5 11	7 12 14 1	13 3 12 6
13 3 12 6	7 12 14 1	4 14 5 11	10 5 3 16	1 14 12 7	6 12 3 13	16 3 5 10	11 5 14 4
2 16 7 9	9 6 4 15	15 1 10 8	8 11 13 2	15 4 6 9	9 7 16 2	2 13 11 8	8 10 1 15
284/	330/	101/	55/	269/	50/	116/	335/
12 13 6 3	14 7 1 12	5 4 11 14	3 10 16 5	12 1 7 14	3 6 13 12	5 16 10 3	14 11 4 5
1 8 15 10	11 2 8 13	16 9 2 7	6 15 9 4	13 8 2 11	10 15 8 1	4 9 15 6	7 2 9 16
7 2 9 16	4 9 15 6	10 15 8 1	13 8 2 11	6 15 9 4	16 9 2 7	11 2 8 13	1 8 15 10
14 11 4 5	5 16 10 3	3 6 13 12	12 1 7 14	3 10 16 5	5 4 11 14	14 7 1 12	12 13 6 3
271/	328/	114/	57/	266/	64/	119/	321/
12 6 13 3	14 7 1 12	5 11 4 14	3 10 16 5	12 1 7 14	3 13 6 12	5 16 10 3	14 4 11 5
1 15 8 10	4 9 15 6	16 2 9 7	13 8 2 11	6 15 9 4	10 8 15 1	11 2 8 13	7 9 2 16
7 9 2 16	11 2 8 13	10 8 15 1	6 15 9 4	13 8 2 11	16 2 9 7	4 9 15 6	1 15 8 10
14 4 11 5	5 16 10 3	3 13 6 12	12 1 7 14	3 10 16 5	5 11 4 14	14 7 1 12	12 6 13 3
311/	245/	74/	140/	306/	125/	79/	260/
13 12 3 6	11 2 8 13	4 5 14 11	6 15 9 4	13 8 2 11	6 3 12 13	4 9 15 6	11 14 5 4
8 1 10 15	14 7 1 12	9 16 7 2	3 10 16 5	12 1 7 14	15 10 1 8	5 16 10 3	2 7 16 9
2 7 16 9	5 16 10 3	15 10 1 8	12 1 7 14	3 10 16 5	9 16 7 2	14 7 1 12	8 1 10 15
11 14 5 4	4 9 15 6	6 3 12 13	13 8 2 11	6 15 9 4	4 5 14 11	11 2 8 13	13 12 3 6
297/	242/	88/	143/	304/	138/	81/	247/
13 3 12 6	11 2 8 13	4 14 5 11	6 15 9 4	13 8 2 11	6 12 3 13	4 9 15 6	11 5 14 4
8 10 1 15	5 16 10 3	9 7 16 2	12 1 7 14	3 10 16 5	15 1 10 8	14 7 1 12	2 16 7 9
2 16 7 9	14 7 1 12	15 1 10 8	3 10 16 5	12 1 7 14	9 7 16 2	5 16 10 3	8 10 1 15
11 5 14 4	4 9 15 6	6 12 3 13	13 8 2 11	6 15 9 4	4 14 5 11	11 2 8 13	13 3 12 6

287/	317/	98/	68/	282/	53/	103/	332/
12 13 6 3	14 1 7 12	5 4 11 14	3 16 10 5	12 7 1 14	3 6 13 12	5 10 16 3	14 11 4 5
7 2 9 16	11 8 2 13	10 15 8 1	6 9 15 4	13 2 8 11	16 9 2 7	4 15 9 6	1 8 15 10
1 8 15 10	4 15 9 6	16 9 2 7	13 2 8 11	6 9 15 4	10 15 8 1	11 8 2 13	7 2 9 16
14 11 4 5	5 10 16 3	3 6 13 12	12 7 1 14	3 16 10 5	5 4 11 14	14 1 7 12	12 13 6 3
273/	314/	112/	71/	280/	66/	105/	319/
12 6 13 3	14 1 7 12	5 11 4 14	3 16 10 5	12 7 1 14	3 13 6 12	5 10 16 3	14 4 11 5
7 9 2 16	4 15 9 6	10 8 15 1	13 2 8 11	6 9 15 4	16 2 9 7	11 8 2 13	1 15 8 10
1 15 8 10	11 8 2 13	16 2 9 7	6 9 15 4	13 2 8 11	10 8 15 1	4 15 9 6	7 9 2 16
14 4 11 5	5 10 16 3	3 13 6 12	12 7 1 14	3 16 10 5	5 11 4 14	14 1 7 12	12 6 13 3
308/	258/	77/	127/	293/	122/	92/	263/
13 12 3 6	11 8 2 13	4 5 14 11	6 9 15 4	13 2 8 11	6 3 12 13	4 15 9 6	11 14 5 4
2 7 16 9	14 1 7 12	15 10 1 8	3 16 10 5	12 7 1 14	9 16 7 2	5 10 16 3	8 1 10 15
8 1 10 15	5 10 16 3	9 16 7 2	12 7 1 14	3 16 10 5	15 10 1 8	14 1 7 12	2 7 16 9
11 14 5 4	4 15 9 6	6 3 12 13	13 2 8 11	6 9 15 4	4 5 14 11	11 8 2 13	13 12 3 6
295/	256/	90/	129/	290/	136/	95/	249/
13 3 12 6	11 8 2 13	4 14 5 11	6 9 15 4	13 2 8 11	6 12 3 13	4 15 9 6	11 5 14 4
2 16 7 9	5 10 16 3	15 1 10 8	12 7 1 14	3 16 10 5	9 7 16 2	14 1 7 12	8 10 1 15
8 10 1 15	14 1 7 12	9 7 16 2	3 16 10 5	12 7 1 14	15 1 10 8	5 10 16 3	2 16 7 9
11 5 14 4	4 15 9 6	6 12 3 13	13 2 8 11	6 9 15 4	4 14 5 11	11 8 2 13	13 3 12 6

[Fundamental #2]

2/	139/	383/	246/	7/	259/	378/	126/
1 8 14 11	6 15 12 1	16 9 3 6	11 2 5 16	1 12 15 6	11 14 8 1	16 5 2 11	6 3 9 16
12 13 7 2	3 10 13 8	5 4 10 15	14 7 4 9	8 13 10 3	2 7 13 12	9 4 7 14	15 10 4 5
15 10 4 5	9 4 7 14	2 7 13 12	8 13 10 3	14 7 4 9	5 4 10 15	3 10 13 8	12 13 7 2
6 3 9 16	16 5 2 11	11 14 8 1	1 12 15 6	11 2 5 16	16 9 3 6	6 15 12 1	1 8 14 11
16/	141/	369/	244/	9/	253/	376/	132/
1 14 8 11	6 15 12 1	16 3 9 6	11 2 5 16	1 12 15 6	11 8 14 1	16 5 2 11	6 9 3 16
12 7 13 2	9 4 7 14	5 10 4 15	8 13 10 3	14 7 4 9	2 13 7 12	3 10 13 8	15 4 10 5
15 4 10 5	3 10 13 8	2 13 7 12	14 7 4 9	8 13 10 3	5 10 4 15	9 4 7 14	12 7 13 2
6 9 3 16	16 5 2 11	11 8 14 1	1 12 15 6	11 2 5 16	16 3 9 6	6 15 12 1	1 14 8 11
173/	56/	212/	329/	188/	336/	197/	49/
8 1 11 14	3 10 13 8	9 16 6 3	14 7 4 9	8 13 10 3	14 11 1 8	9 4 7 14	3 6 16 9
13 12 2 7	6 15 12 1	4 5 15 10	11 2 5 16	1 12 15 6	7 2 12 13	16 5 2 11	10 15 5 4
10 15 5 4	16 5 2 11	7 2 12 13	1 12 15 6	11 2 5 16	4 5 15 10	6 15 12 1	13 12 2 7
3 6 16 9	9 4 7 14	14 11 1 8	8 13 10 3	14 7 4 9	9 16 6 3	3 10 13 8	8 1 11 14
186/	59/	199/	326/	191/	315/	194/	70/
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18/	135/	367/	250/	23/	255/	362/	130/
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170/	62/	215/	323/	175/	334/	210/	51/
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184/	65/	201/	320/	177/	313/	208/	72/
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283/	354/	102/	31/	270/	26/	115/	359/
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277/	352/	108/	33/	268/	40/	117/	345/
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312/	221/	73/	164/	305/	149/	80/	236/
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291/	218/	94/	167/	302/	162/	83/	223/
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285/	341/	100/	44/	276/	29/	109/	356/
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279/	338/	106/	47/	274/	42/	111/	343/
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310/	234/	75/	151/	299/	146/	86/	239/
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289/	232/	96/	153/	296/	160/	89/	225/
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[Fundamental #3]

4/	91/	381/	294/	13/	307/	372/	78/
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192/	107/	193/	278/	185/	267/	200/	118/
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	333/	342/	52/	43/	324/	30/	61/	355/	
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10	3 15 6	6 12 9 7	7 14 2 11	11 5 8 10	7 9 12 6	6 15 3 10	10 8 5 11	11 2 14 7	

* Monitor List of Correspondence *

