

Part 3. "New Algebraic Studies of Magic Squares"
 Chapter 3. Study of Magic Squares of Order 5: Kanji Setsuda
 Section 1. Self-Complementary Magic Squares of Order 5

#1. Basic Conditions of Self-Complementary Magic Squares of Order 5

Let's study various types of Magic Squares of order 5. First of all we study the Self-Complementary one. It is defined with such equations as below. The last multi-equation (13) means Complementary Pairs of C are located symmetrically with respect to n13, the geometric center of this square.

[Figure 1: Basic Conditions for Self-Complementary Type]

1---2---3---4---5	$n_1 + n_2 + n_3 + n_4 + n_5 = K$ (1)
	$n_6 + n_7 + n_8 + n_9 + n_{10} = K$ (2)
6---7---8---9---10	$n_{11} + n_{12} + n_{13} + n_{14} + n_{15} = K$ (3)
	$n_{16} + n_{17} + n_{18} + n_{19} + n_{20} = K$ (4)
11---12---13---14---15	$n_{21} + n_{22} + n_{23} + n_{24} + n_{25} = K$ (5)
	$n_1 + n_6 + n_{11} + n_{16} + n_{21} = K$ (6)
16---17---18---19---20	$n_2 + n_7 + n_{12} + n_{17} + n_{22} = K$ (7)
	$n_3 + n_8 + n_{13} + n_{18} + n_{23} = K$ (8)
21---22---23---24---25	$n_4 + n_9 + n_{14} + n_{19} + n_{24} = K$ (9)
	$n_5 + n_{10} + n_{15} + n_{20} + n_{25} = K$... (10)
$n_1 + n_{25} = n_2 + n_{24} = n_3 + n_{23}$	$n_1 + n_7 + n_{13} + n_{19} + n_{25} = K$... (11)
$= n_4 + n_{22} = n_5 + n_{21} = n_6 + n_{20}$	$n_5 + n_9 + n_{13} + n_{17} + n_{21} = K$... (12)
$= n_7 + n_{19} + n_8 + n_{18} + n_9 + n_{17}$		
$= n_{10} + n_{16} = n_{11} + n_{15} = n_{12} + n_{14} = n_{13} + n_{13} = C$... (13)

The two magic constant K and C could be calculated beforehand.

(1)+(2)+(3)+(4)+(5)

$$n_1 + n_2 + n_3 + \dots + n_{13} + \dots + n_{24} + n_{25} = 5 * K$$

$$(25+1) \times 25 / 2 = 5 * K$$

Therefore K=65 ... (14)

(11)=(n1+n25)+(n7+n19)+n13=K

C + C + C/2 = K (n13=C/2; because n13+n13=C)

Multiply both sides by 2

$$5 * C = 2 * K$$

As a consequence C=130/5=26 ... (15)

The equations (13) take any value set out of those as follows:

(1,25),(2,24),(3,23),(4,22),(5,21),(6,20), ,(11,15),(12,14)

You may rewrite these equations into:

$$n_{25} = 26 - n_1; n_{24} = 26 - n_2; n_{23} = 26 - n_3; n_{22} = 26 - n_4; \dots; n_{15} = 26 - n_{11}; n_{14} = 26 - n_{12};$$

$$n_{13} = 26 - n_{13} = 13$$

Let me introduce you here a new notation of complementary pairs.

$$\underline{n}_1 = 26 - n_1; \underline{n}_2 = 26 - n_2; \underline{n}_3 = 26 - n_3; \underline{n}_4 = 26 - n_4; \dots; \underline{n}_{11} = 26 - n_{11}; \underline{n}_{12} = 26 - n_{12};$$

$$\underline{n}_{13} = 26 - n_{13}$$

You could put them back to the original style anytime you like:

$$\underline{n}_1 = n_{25}; \underline{n}_2 = n_{24}; \underline{n}_3 = n_{23}; \underline{n}_4 = n_{22}; \dots; \underline{n}_{12} = n_{14}$$

Or you could write these as:

$$n_1 + \underline{n}_1 = 26; n_2 + \underline{n}_2 = 26; n_3 + \underline{n}_3 = 26; n_4 + \underline{n}_4 = 26; \dots; n_{11} + \underline{n}_{11} = 26; n_{12} + \underline{n}_{12} = 26;$$

$$n_{13} + \underline{n}_{13} = 26$$

Now let's rewrite the basic diagram and conditions as follows.

[Figure 2: Basic Conditions for Self-Complementary Type]

1---2---3---4---5	$n1+ n2+ n3+ n4+ n5=K$ (1)
	$n6+ n7+ n8+ n9+n10=K$ (2)
6---7---8---9---10	$n11+n12+n13+n12+n11=K$ (3)
	$n10+ n9+ n8+ n7+ n6=K$ (4)
11--12--13--12--11	$n5+ n4+ n3+ n2+ n1=K$ (5)
	$n1+ n6+n11+n10+ n5=K$ (6)
10---9---8---7---6	$n2+ n7+n12+ n9+ n4=K$ (7)
	$n3+ n8+n13+ n8+ n3=K$ (8)
5---4---3---2---1	$n4+ n9+n12+ n7+ n2=K$ (9)
	$n5+n10+n11+ n6+ n1=K$... (10)
$n1+n1=n2+n2=n3+n3$	$n1+ n7+n13+ n7+ n1=K$... (11)
$=n4+n4=n5+n5=n6+n6$	$n5+ n9+n13+ n9+ n5=K$... (12)
$=n7+n7=n8+n8=n9+n9$	(Of course $K=65$)	
$=n10+n10=n11+n11=n12+n12=n13+n13=C=26$... (13)

You might easily find where any complementary pair is placed in the diagram.

Take primary diagonals for instance.

$$n1+ n7+n13+ n7+ n1=K \dots (11)$$

$$(n1+n1)+(n7+n7)+n13=C+C+13=26+26+13=65$$

Locations of complementary pairs make the sum of diagonal equal to the magic constant.

The other primary diagonal (12) also consists of two complementary pairs of 26 and the sum proves to be 65.

$$n5+ n9+n13+ n9+ n5=K \dots (12)$$

$$(n5+n5)+(n9+n9)+n13=C+C+13=26+26+13=65$$

If you find this characteristic structure in any line, you've just known its sum. You don't have to calculate the answer any more. You may well just see it.

#2. How Many Solutions could we find?

Under the basic conditions of Figure 1, I dictated some program codes for my personal computer and counted the solutions of our object.

```

/** Self-Complementary Magic Squares 5*5: */
/** File Name: 'MS55SC.c' Written & built by */
/** Kanji Setsuda on MacOS Xcode Feb.5, 2005 */
/**/
#include <stdio.h>
/**/
int cnt;
int cntr[5];
int carea[2];
short cnt2, cnt4;
short LSM, CC;
short nm[26], uflg[26], area[26];
short nt[4][26];
/**/
/* Main Program */
int main(){
short n;
printf("\n** Self-Complementary Magic Squares 5*5: Starting with n1=1 **\n");
for(n=0;n<26;n++){nm[n]=0; uflg[n]=0; area[n]=0;}
area[3]=1; area[8]=1; area[18]=1; area[23]=1;
area[11]=1; area[12]=1; area[14]=1; area[15]=1;

```

```

carea[0]=0; carea[1]=0;
LSM=65; CC=26; cnt=0; cnt4=0;
nm[13]=13; ufl g[13]=1;
stp01();
ufl g[13]=0; nm[13]=0;
if(cnt4<4){pr2ans(); }
printf("\n* Total Count = %d\n",cnt);
printf("  V1(A/B) = %d/%d\n", carea[0], carea[1]);
printf("  OK!\n");
return 0;
}
/* Begin The Search */
/**/
/* Set n1 & n25=CC-n1 & (n1<n25) */
void stp01(){
short a,b;
for(a=1; a<13; a++){b=CC-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[1]=a; nm[25]=b;
ufl g[a]=1; ufl g[b]=1;
stp02();
ufl g[b]=0; ufl g[a]=0; }
}
}
/**/
/* Set n5 & n21=CC-n5 & (n1<n5<n21) */
void stp02(){
short a,b;
for(a=nm[1]+1; a<13; a++){b=CC-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[5]=a; nm[21]=b;
ufl g[a]=1; ufl g[b]=1; cnt2=0;
stp03();
ufl g[b]=0; ufl g[a]=0; }
}
}
/**/
/* Set n2 & n24=CC-n2 */
void stp03(){
short a,b;
for(a=25; a>0; a--){b=CC-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[2]=a; nm[24]=b;
ufl g[a]=1; ufl g[b]=1;
stp04();
ufl g[b]=0; ufl g[a]=0; }
}
}
/**/
/* Set n4 & n22=CC-n4 */
void stp04(){
short a,b;
for(a=25; a>0; a--){b=CC-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[4]=a; nm[22]=b;
ufl g[a]=1; ufl g[b]=1;
stp05();
ufl g[b]=0; ufl g[a]=0; }
}
}

```

```

}
/**/
/* Set n3=LSM-n1-n2-n4-n5
   & n23=LSM-n25-n24-n22-n21 */
void stp05(){
short a, b;
a=LSM-nm[1]-nm[2]-nm[4]-nm[5];
b=LSM-nm[25]-nm[24]-nm[22]-nm[21];
if((0<a)&&(a<26)&&(a+b==CC)){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[3]=a; nm[23]=b;
uflg[a]=1; uflg[b]=1;
stp06();
uflg[b]=0; uflg[a]=0; }}
}
/**/
/* Set n6 & n20=CC-n6 */
void stp06(){
short a, b;
for(a=25; a>0; a--){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[6]=a; nm[20]=b;
uflg[a]=1; uflg[b]=1;
stp07();
uflg[b]=0; uflg[a]=0; }
}
}
/**/
/* Set n16 & n10=CC-n16 */
void stp07(){
short a, b;
for(a=25; a>0; a--){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[16]=a; nm[10]=b;
uflg[a]=1; uflg[b]=1;
stp08();
uflg[b]=0; uflg[a]=0; }
}
}
/**/
/* Set n11=LSM-n1-n6-n16-n21
   & n15=LSM-n25-n20-n10-n5 */
void stp08(){
short a, b;
a=LSM-nm[1]-nm[6]-nm[16]-nm[21];
b=LSM-nm[25]-nm[20]-nm[10]-nm[5];
if((0<a)&&(a<26)&&(a+b==CC)){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[11]=a; nm[15]=b;
uflg[a]=1; uflg[b]=1;
stp09();
uflg[b]=0; uflg[a]=0; }}
}
}
/* Search Level 2: */
/* Set n7 & n19=CC-n7 */
void stp09(){
short a, b;
for(a=1; a<26; a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[7]=a; nm[19]=b;

```

```

        ufl g[a]=1; ufl g[b]=1;
        stp10();
        ufl g[b]=0; ufl g[a]=0; }
    }
}
/**/
/* Set n9 & n17=CC-n9 */
void stp10(){
    short a, b;
    for(a=1; a<26; a++){b=CC-a;
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[9]=a; nm[17]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp11();
            ufl g[b]=0; ufl g[a]=0; }
        }
}
/**/
/* Set n8=LSM-n6-n7-n9-n10 & n18=LSM-n20-n19-n17-n16 */
void stp11(){
    short a, b;
    a=LSM-nm[6]-nm[7]-nm[9]-nm[10];
    b=LSM-nm[20]-nm[19]-nm[17]-nm[16];
    if((0<a)&&(a<26)&&(a+b==CC)){
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[8]=a; nm[18]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp12();
            ufl g[b]=0; ufl g[a]=0; }}
}
/**/
/* Check(n3+n8+n13+n18+n23=65) */
void stp12(void){
    if(nm[3]+nm[8]+nm[13]+nm[18]+nm[23]==LSM){stp13(); }
}
/**/
/* Set n12=LSM-n2-n7-n17-n22 & n14=LSM-n24-n19-n9-n4 */
void stp13(){
    short a, b;
    a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
    b=LSM-nm[24]-nm[19]-nm[9]-nm[4];
    if((0<a)&&(a<26)&&(a+b==CC)){
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[12]=a; nm[14]=b;
            ufl g[a]=1; ufl g[b]=1;
            stp14();
            ufl g[b]=0; ufl g[a]=0; }}
}
/**/
/* The Last Checks */
void stp14(void){
    short sm1, sm2, sm3;
    sm1=nm[11]+nm[12]+nm[13]+nm[14]+nm[15];
    sm2=nm[1]+nm[7]+nm[13]+nm[19]+nm[25];
    sm3=nm[5]+nm[9]+nm[13]+nm[17]+nm[21];
    if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){anspri nt(); }
}
/**/
/* Print the Answers */

```

```

void ansprint(){
    short n;
    cnt++;
    jarea();
    cnt2++;
    if(cnt2==1){
        for(n=1; n<26; n++){nt[cnt4][n]=nm[n];}
        cntr[cnt4]=cnt;
        cnt4++; if(cnt4==4){pr4ans(); cnt4=0;}}
}
/**/
/* Where does Value 1 appear? */
void jarea(void){
    short n;
    for(n=1; n<26; n++){
        if(nm[n]==1){carea[area[n]]++;}
    }
}
/**/
/* Print the 2 Answers */
void pr2ans(){
    short l, l5, m, n;
    printf("%16d/%16d/\n", cntr[0], cntr[1]);
    for(l=0; l<5; l++){l5=l*5;
        for(m=0; m<2; m++){
            printf(" %3d%3d%3d%3d%3d",
                nt[m][l5+1], nt[m][l5+2], nt[m][l5+3], nt[m][l5+4], nt[m][l5+5]);
        }
        printf("\n");
    }
}
/**/
/* Print the 4 Answers */
void pr4ans(){
    short l, l5, m, n;
    printf("%16d/%16d/%16d/%16d/\n", cntr[0], cntr[1], cntr[2], cntr[3]);
    for(l=0; l<5; l++){l5=l*5;
        for(m=0; m<4; m++){
            printf(" %3d%3d%3d%3d%3d",
                nt[m][l5+1], nt[m][l5+2], nt[m][l5+3], nt[m][l5+4], nt[m][l5+5]);
        }
        printf("\n");
    }
}
/**/

```

When I assumed the inequality condition $n_1 < n_5 < n_{21}$; $n_1 < n_{25}$, I could have got the solution set of 48544 self-complementary magic squares of order 5.
 When I started with $n_3=1$ and add $n_3 < n_{11} < n_{15}$; $n_3 < n_{23}$, I could have also got the solution set of 48544 SCMS55.

**** Self-Complementary Magic Squares 5*5: Starting with $n_1=1$ ****

1/					397/					1241/					1757/				
1	23	17	22	2	1	24	15	22	3	1	24	15	21	4	1	24	12	23	5
21	8	11	6	19	21	8	17	7	12	19	16	3	18	9	20	11	18	9	7
12	10	13	16	14	6	10	13	16	20	6	12	13	14	20	4	10	13	16	22
7	20	15	18	5	14	19	9	18	5	17	8	23	10	7	19	17	8	15	6
24	4	9	3	25	23	4	11	2	25	22	5	11	2	25	21	3	14	2	25

2585/ 1 24 11 23 6 21 8 17 12 7 4 16 13 10 22 19 14 9 18 5 20 3 15 2 25	3117/ 1 24 10 23 7 21 9 18 11 6 4 14 13 12 22 20 15 8 17 5 19 3 16 2 25	3977/ 1 24 9 23 8 22 7 5 15 16 14 20 13 6 12 10 11 21 19 4 18 3 17 2 25	4629/ 1 24 8 23 9 22 15 12 10 6 5 7 13 19 21 20 16 14 11 4 17 3 18 2 25
5689/ 1 24 7 23 10 21 4 11 12 17 18 20 13 6 8 9 14 15 22 5 16 3 19 2 25	6221/ 1 24 6 23 11 22 7 12 16 8 9 21 13 5 17 18 10 14 19 4 15 3 20 2 25	7157/ 1 24 5 23 12 22 17 7 11 8 10 6 13 20 16 18 15 19 9 4 14 3 21 2 25	7793/ 2 25 14 21 3 19 6 16 15 9 4 18 13 8 22 17 11 10 20 7 23 5 12 1 24
8201/ 2 25 11 23 4 21 8 17 7 12 6 10 13 16 20 14 19 9 18 5 22 3 15 1 24	8905/ 2 25 10 23 5 22 6 8 14 15 9 19 13 7 17 11 12 18 20 4 21 3 16 1 24	9597/ 2 25 9 23 6 21 15 7 14 8 4 10 13 16 22 18 12 19 11 5 20 3 17 1 24	10497/ 2 25 8 23 7 22 12 6 16 9 5 15 13 11 21 17 10 20 14 4 19 3 18 1 24
11065/ 2 25 7 23 8 22 15 5 14 9 6 10 13 16 20 17 12 21 11 4 18 3 19 1 24	11957/ 2 25 6 23 9 22 8 16 12 7 5 15 13 11 21 19 14 10 18 4 17 3 20 1 24	12509/ 2 25 5 23 10 22 8 17 11 7 6 14 13 12 20 19 15 9 18 4 16 3 21 1 24	13249/ 2 25 4 23 11 21 8 16 14 6 7 17 13 9 19 20 12 10 18 5 15 3 22 1 24
14009/ 2 23 6 22 12 25 9 16 8 7 5 11 13 15 21 19 18 10 17 1 14 4 20 3 24	14601/ 3 25 9 24 4 21 8 11 6 19 12 10 13 16 14 7 20 15 18 5 22 2 17 1 23	15173/ 3 25 8 24 5 22 7 14 6 16 9 11 13 15 17 10 20 12 19 4 21 2 18 1 23	15953/ 3 25 7 24 6 22 5 15 9 14 8 16 13 10 18 12 17 11 21 4 20 2 19 1 23
16669/ 3 25 6 24 7 22 11 14 8 10 5 9 13 17 21 16 18 12 15 4 19 2 20 1 23	17885/ 3 25 5 24 8 19 6 9 16 15 14 22 13 4 12 11 10 17 20 7 18 2 21 1 23	18613/ 3 25 4 24 9 21 7 12 15 10 8 20 13 6 18 16 11 14 19 5 17 2 22 1 23	19497/ 3 25 5 22 10 19 2 15 12 17 18 20 13 6 8 9 14 11 24 7 16 4 21 1 23
20297/ 3 24 5 22 11 25 7 9 14 10 6 18 13 8 20 16 12 17 19 1 15 4 21 2 23	21285/ 3 25 4 21 12 24 6 8 16 11 9 19 13 7 17 15 10 18 20 2 14 5 22 1 23	21805/ 4 25 7 24 5 23 8 9 10 15 6 14 13 12 20 11 16 17 18 3 21 2 19 1 22	22449/ 4 25 7 23 6 17 14 11 5 18 16 2 13 24 10 8 21 15 12 9 20 3 19 1 22
23241/ 4 25 5 24 7 23 6 11 8 17 10 14 13 12 16 9 18 15 20 3 19 2 21 1 22	23829/ 4 25 5 23 8 24 6 11 7 17 10 12 13 14 16 9 19 15 20 2 18 3 21 1 22	24793/ 4 25 3 24 9 21 6 11 8 19 16 14 13 12 10 7 18 15 20 5 17 2 23 1 22	25489/ 4 24 6 21 10 25 11 12 8 9 3 7 13 19 23 17 18 14 15 1 16 5 20 2 22
26217/ 4 25 2 23 11 21 9 18 10 7 6 12 13 14 20 19 16 8 17 5 15 3 24 1 22	26901/ 4 25 3 21 12 24 6 17 8 10 7 11 13 15 19 16 18 9 20 2 14 5 23 1 22	27597/ 5 25 7 22 6 23 8 9 14 11 2 16 13 10 24 15 12 17 18 3 20 4 19 1 21	28353/ 5 25 4 24 7 23 11 8 9 14 6 10 13 16 20 12 17 18 15 3 19 2 22 1 21

29565/				30265/				31081/				31761/							
5	25	3	24	8	5	24	4	23	9	5	25	2	23	10	5	25	2	22	11
22	9	15	7	12	25	7	8	11	14	20	14	4	18	9	20	16	12	9	8
6	10	13	16	20	6	16	13	10	20	7	15	13	11	19	7	3	13	23	19
14	19	11	17	4	12	15	18	19	1	17	8	22	12	6	18	17	14	10	6
18	2	23	1	21	17	3	22	2	21	16	3	24	1	21	15	4	24	1	21
32409/				33193/				33913/				34473/							
5	25	3	20	12	6	25	3	24	7	6	24	4	23	8	6	25	2	23	9
19	2	11	16	17	22	9	5	11	18	21	9	16	12	7	22	14	8	10	11
18	22	13	4	8	10	14	13	12	16	1	15	13	11	25	5	7	13	19	21
9	10	15	24	7	8	15	21	17	4	19	14	10	17	5	15	16	18	12	4
14	6	23	1	21	19	2	23	1	20	18	3	22	2	20	17	3	24	1	20
35033/				35733/				36333/				37313/							
6	25	2	22	10	6	25	2	21	11	6	25	3	19	12	7	25	2	23	8
23	5	8	14	15	23	10	4	9	19	24	5	11	16	9	21	14	10	9	11
9	19	13	7	17	14	8	13	18	12	4	18	13	8	22	4	6	13	20	22
11	12	18	21	3	7	17	22	16	3	17	10	15	21	2	15	17	16	12	5
16	4	24	1	20	15	5	24	1	20	14	7	23	1	20	18	3	24	1	19
37829/				38629/				39165/				40421/							
7	25	2	22	9	7	25	2	21	10	7	25	2	20	11	7	25	3	18	12
21	3	10	11	20	23	6	4	14	18	23	5	18	9	10	22	5	17	15	6
14	18	13	8	12	11	17	13	9	15	4	12	13	14	22	2	16	13	10	24
6	15	16	23	5	8	12	22	20	3	16	17	8	21	3	20	11	9	21	4
17	4	24	1	19	16	5	24	1	19	15	6	24	1	19	14	8	23	1	19
41081/				41829/				42641/				43365/							
8	25	2	21	9	8	25	2	20	10	8	25	2	19	11	8	25	3	17	12
23	4	16	7	15	23	4	14	5	19	21	3	20	12	9	22	15	5	16	7
6	12	13	14	20	11	9	13	17	15	4	16	13	10	22	2	6	13	20	24
11	19	10	22	3	7	21	12	22	3	17	14	6	23	5	19	10	21	11	4
17	5	24	1	18	16	6	24	1	18	15	7	24	1	18	14	9	23	1	18
44129/				44689/				45877/				46533/							
9	25	2	19	10	9	25	2	18	11	9	25	3	16	12	10	25	2	17	11
23	8	14	5	15	23	7	16	5	14	20	7	11	5	22	23	4	18	6	14
6	4	13	22	20	6	4	13	22	20	18	2	13	24	8	5	7	13	19	21
11	21	12	18	3	12	21	10	19	3	4	21	15	19	6	12	20	8	22	3
16	7	24	1	17	15	8	24	1	17	14	10	23	1	17	15	9	24	1	16
47205/				48065/															
10	25	3	15	12	11	24	1	17	12										
22	6	7	21	9	22	5	3	19	16										
2	18	13	8	24	8	20	13	6	18										
17	5	19	20	4	10	7	23	21	4										
14	11	23	1	16	14	9	25	2	15										

* Total Count = 48544; V1(A/B) = 31168/17376

** Self-Complementary Magic Squares 5*5:
Starting with n3=1 under n3<n11<n15; n3<n23 **

1/				1245/				1433/				2857/							
15	18	1	14	17	12	20	1	24	8	14	21	1	19	10	7	22	1	20	15
19	21	3	16	6	22	15	5	7	16	20	18	3	9	15	24	16	3	14	8
2	4	13	22	24	3	9	13	17	23	4	2	13	24	22	5	9	13	17	21
20	10	23	5	7	10	19	21	11	4	11	17	23	8	6	18	12	23	10	2
9	12	25	8	11	18	2	25	6	14	16	7	25	5	12	11	6	25	4	19

3297/ 12 17 1 16 19 22 21 3 11 8 6 2 13 24 20 18 15 23 5 4 7 10 25 9 14	4465/ 10 12 1 22 20 24 21 3 9 8 7 11 13 15 19 18 17 23 5 2 6 4 25 14 16	4645/ 10 19 1 21 14 20 22 3 9 11 8 2 13 24 18 15 17 23 4 6 12 5 25 7 16	5785/ 7 24 1 22 11 18 20 3 14 10 9 5 13 21 17 16 12 23 6 8 15 4 25 2 19
6125/ 8 21 1 20 15 19 22 3 12 9 10 2 13 24 16 17 14 23 4 7 11 6 25 5 18	7461/ 6 24 1 22 12 16 21 3 17 8 11 7 13 19 15 18 9 23 5 10 14 4 25 2 20	7649/ 8 19 1 21 16 20 22 3 9 11 12 2 13 24 14 15 17 23 4 6 10 5 25 7 18	8689/ 14 21 2 11 17 20 18 4 16 7 3 1 13 25 23 19 10 22 8 6 9 15 24 5 12
10013/ 16 15 2 7 25 23 14 6 17 5 4 8 13 18 22 21 9 20 12 3 1 19 24 11 10	10629/ 12 17 2 15 19 23 20 4 10 8 5 1 13 25 21 18 16 22 6 3 7 11 24 9 14	11769/ 9 15 2 23 16 19 25 4 12 5 6 8 13 18 20 21 14 22 1 7 10 3 24 11 17	12421/ 9 23 2 20 11 18 21 4 12 10 7 1 13 25 19 16 14 22 5 8 15 6 24 3 17
13393/ 5 23 2 19 16 25 15 4 12 9 8 6 13 20 18 17 14 22 11 1 10 7 24 3 21	13917/ 8 23 2 21 11 19 20 4 10 12 9 1 13 25 17 14 16 22 6 7 15 5 24 3 18	15053/ 5 25 2 15 18 23 14 4 17 7 10 6 13 20 16 19 9 22 12 3 8 11 24 1 21	15649/ 6 23 2 18 16 21 19 4 12 9 11 1 13 25 15 17 14 22 7 5 10 8 24 3 20
16681/ 8 19 2 11 25 23 16 4 17 5 12 6 13 20 14 21 9 22 10 3 1 15 24 7 18	17309/ 17 15 3 18 12 20 21 1 7 16 4 2 13 24 22 10 19 25 5 6 14 8 23 11 9	18353/ 8 20 3 24 10 22 19 1 11 12 5 9 13 17 21 14 15 25 7 4 16 2 23 6 18	18545/ 10 21 3 14 17 22 19 1 15 8 6 2 13 24 20 18 11 25 7 4 9 12 23 5 16
19489/ 12 22 3 8 20 24 9 1 21 10 7 11 13 15 19 16 5 25 17 2 6 18 23 4 14	19665/ 9 19 3 20 14 21 22 1 10 11 8 2 13 24 18 15 16 25 4 5 12 6 23 7 17	20753/ 11 24 3 8 19 22 12 1 20 10 9 5 13 21 17 16 6 25 14 4 7 18 23 2 15	20961/ 7 22 3 18 15 20 21 1 14 9 10 2 13 24 16 17 12 25 5 6 11 8 23 4 19
21933/ 4 24 3 20 14 21 18 1 16 9 11 7 13 19 15 17 10 25 8 5 12 6 23 2 22	22145/ 8 22 3 21 11 20 19 1 9 16 12 2 13 24 14 10 17 25 7 6 15 5 23 4 18	23157/ 10 19 4 15 17 23 20 2 12 8 5 1 13 25 21 18 14 24 6 3 9 11 22 7 16	24065/ 5 25 4 19 12 23 15 2 16 9 6 8 13 18 20 17 10 24 11 3 14 7 22 1 21
24573/ 9 21 4 20 11 18 23 2 12 10 7 1 13 25 19 16 14 24 3 8 15 6 22 5 17	25669/ 14 19 4 5 23 25 10 2 17 11 8 6 13 20 18 15 9 24 16 1 3 21 22 7 12	26169/ 8 20 4 21 12 19 23 2 10 11 9 1 13 25 17 15 16 24 3 7 14 5 22 6 18	27157/ 7 17 4 23 14 25 18 2 5 15 10 6 13 20 16 11 21 24 8 1 12 3 22 9 19
27701/ 6 21 4 16 18 23 19 2 12 9 11 1 13 25 15 17 14 24 7 3 8 10 22 5 20	28513/ 8 25 4 23 5 17 16 2 11 19 12 6 13 20 14 7 15 24 10 9 21 3 22 1 18	29177/ 17 12 5 16 15 23 19 1 4 18 6 2 13 24 20 8 22 25 7 3 11 10 21 14 9	30125/ 6 16 5 24 14 22 23 1 11 8 7 9 13 17 19 18 15 25 3 4 12 2 21 10 20

30329/ 9 16 5 15 20 23 22 1 12 7 8 2 13 24 18 19 14 25 4 3 6 11 21 10 17	31205/ 7 20 5 22 11 18 24 1 12 10 9 3 13 23 17 16 14 25 2 8 15 4 21 6 19	31577/ 6 23 5 19 12 18 22 1 15 9 10 2 13 24 16 17 11 25 4 8 14 7 21 3 20	32529/ 4 19 5 23 14 18 24 1 16 6 11 9 13 17 15 20 10 25 2 8 12 3 21 7 22
32697/ 6 23 5 16 15 17 22 1 18 7 12 2 13 24 14 19 8 25 4 9 11 10 21 3 20	33689/ 14 18 6 5 22 23 15 2 16 9 7 1 13 25 19 17 10 24 11 3 4 21 20 8 12	34493/ 14 15 6 5 25 23 16 2 17 7 8 4 13 22 18 19 9 24 10 3 1 21 20 11 12	35053/ 5 23 6 16 15 22 19 2 14 8 9 1 13 25 17 18 12 24 7 4 11 10 20 3 21
36029/ 3 25 6 19 12 21 18 2 15 9 10 4 13 22 16 17 11 24 8 5 14 7 20 1 23	36609/ 5 23 6 17 14 18 22 2 16 7 11 1 13 25 15 19 10 24 4 8 12 9 20 3 21	37553/ 3 25 6 15 16 21 17 2 18 7 12 4 13 22 14 19 8 24 9 5 10 11 20 1 23	38109/ 9 16 7 22 11 21 23 1 6 14 8 2 13 24 18 12 20 25 3 5 15 4 19 10 17
38829/ 3 24 7 20 11 22 18 1 14 10 9 5 13 21 17 16 12 25 8 4 15 6 19 2 23	39057/ 4 21 7 18 15 23 20 1 12 9 10 2 13 24 16 17 14 25 6 3 11 8 19 5 22	40097/ 9 22 7 6 21 24 12 1 18 10 11 3 13 23 15 16 8 25 14 2 5 20 19 4 17	40285/ 5 17 7 20 16 23 22 1 8 11 12 2 13 24 14 15 18 25 4 3 10 6 19 9 21
41085/ 4 23 8 16 14 21 20 2 15 7 9 1 13 25 17 19 11 24 6 5 12 10 18 3 22	41865/ 3 21 8 14 19 25 17 2 15 6 10 4 13 22 16 20 11 24 9 1 7 12 18 5 23	42313/ 4 19 8 20 14 21 23 2 10 9 11 1 13 25 15 17 16 24 3 5 12 6 18 7 22	43117/ 3 17 8 21 16 25 20 2 7 11 12 4 13 22 14 15 19 24 6 1 10 5 18 9 23
43829/ 4 19 9 15 18 23 21 1 14 6 10 2 13 24 16 20 12 25 5 3 8 11 17 7 22	44841/ 2 18 9 20 16 23 22 1 12 7 11 5 13 21 15 19 14 25 4 3 10 6 17 8 24	45013/ 4 19 9 15 18 20 23 1 16 5 12 2 13 24 14 21 10 25 3 6 8 11 17 7 22	46033/ 9 23 10 4 19 20 14 2 21 8 11 1 13 25 15 18 5 24 12 6 7 22 16 3 17
46953/ 5 19 10 8 23 25 15 2 17 6 12 4 13 22 14 20 9 24 11 1 3 18 16 7 21	47585/ 6 17 11 21 10 23 19 1 4 18 12 2 13 24 14 8 22 25 7 3 16 5 15 9 20		

[Count(n3==1) = 8688]; [Total Count = 48544]

Each solution of the two sets looks quite different, but it should have the one-to-one correspondence to each other. Difference comes from their own unique ways of making lists smart. We assumed different inequality conditions for the two types.

If you want to find the correspondence, or if you want to find your answer in the list, you will probably have to know how the list order is made at first, and then have to reflect your answer or rotate it to meet with those rules.

#3. What Parts of this Square can we Exchange?

How about the 'fundamental solutions'? Couldn't we get any smaller set of solutions?

Before making the fundamental solutions, we must seriously study what patterns might be identified with each other and how to prevent us from double counting of the same thing.

Give your careful look at the next three squares below.

[Figure 3: Different Faces of the Same Thing]

No. 1	No. 2 * * *	No. 3
1 23 17 22 2	1 22 17 23 2	1 23 17 22 2
21 8 11 6 19	21 6 11 8 19	* 7 20 15 18 5*
12 10 13 16 14	12 16 13 10 14	12 10 13 16 14
7 20 15 18 5	7 18 15 20 5	*21 8 11 6 19*
24 4 9 3 25	24 3 9 4 25	24 4 9 3 25
	* * *	

They are very similar, but you might find a little difference among them.

No.2 has got its 2nd and 4th columns exchanged with each other.

No.3 has got its 2nd and 4th rows exchanged with each other.

The members of any column or any row are not changed, and they always add up to the same constant, although the positions or orders of the members are partially changed.

Watch the two primary diagonals. Both members and positions are changed, though the sums of diagonals are just the same. They are always equal to magic constant 65.

Should we suppose these three squares as different and count three?

Or should we identify these with No.1 and suppose these three as the different faces of the same thing?

How do you like to do with it? It is the problem of your personal decision.

I decide to take them for the same thing and count once. I don't like to suppose them as different things any row or any column of which is the same with the original combination.

Look at the next diagrams below. They show another way how to exchange lines. Check every diagonal whether the sum of each is equal to 65.

For instance,

$$n5+n7+n13+n7+n5=(n5+n5)+(n7+n7)+n13=C+C+13=65; \text{ Very good!}$$

$$n5+n9+n13+n9+n5=(n5+n5)+(n9+n9)+n13=C+C+13=65; \text{ Very good!}$$

You can combine these three types of line-exchanges. You could almost make a new pattern by the combination of these exchanges. Whatever you may get, however, you should take them for the same thing and count once.

[Figure 4: Which Lines can we Exchange?]

Line(2-4)Exchange:

Original

1	2	3	4	5
6	7	8	9	10
11	12	13	12	11
10	9	8	7	6
5	4	3	2	1

Row(2-4)
Exchange

1	2	3	4	5
10	9	8	7	6
11	12	13	12	11
6	7	8	9	10
5	4	3	2	1

Column(2-4)
Exchange

1	4	3	2	5
6	9	8	7	10
11	12	13	12	11
10	7	8	9	6
5	2	3	4	1

Double Exchange:

Row & Column(2-4)
Exchange

1	4	3	2	5
10	7	8	9	6
11	12	13	12	11
6	9	8	7	10
5	2	3	4	1

Line(1-5)Exchange:

Original

1	2	3	4	5
6	7	8	9	10
11	12	13	12	11
10	9	8	7	6
5	4	3	2	1

Row(1-5) Exchange

5	4	3	2	1
6	7	8	9	10
11	12	13	12	11
10	9	8	7	6
1	2	3	4	5

Column(1-5) Exchange

5	2	3	4	1
10	7	8	9	6
11	12	13	12	11
6	9	8	7	10
1	4	3	2	5

Double Exchange: Row & Column(1-5) Exchange

1	4	3	2	5
10	7	8	9	6
11	12	13	12	11
6	9	8	7	10
5	2	3	4	1

Line(1-2)&(4-5)Exchange:

Original

1	2	3	4	5
6	7	8	9	10
11	12	13	12	11
10	9	8	7	6
5	4	3	2	1

Row(1-2)&(4-5) Exchange

6	7	8	9	10
1	2	3	4	5
11	12	13	12	11
5	4	3	2	1
10	9	8	7	6

Column(1-2)&(4-5) Exchange

2	1	3	5	4
7	6	8	10	9
12	11	13	11	12
9	10	8	6	7
4	5	3	1	2

Double Exchange: R&C(1-2)&(4-5) Exchange

7	6	8	10	9
2	1	3	5	4
12	11	13	11	12
4	5	3	1	2
9	10	8	6	7

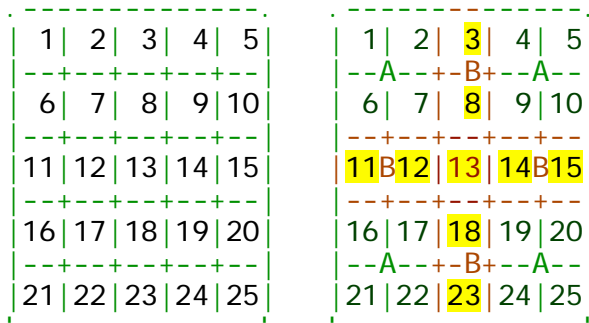
You could almost move n1 to anywhere you like, but you might soon notice that you could not move n1 onto the 3rd row or 3rd column from the other places by any combination of line-exchange.

There are three 'exclusive' areas in the square as follows:

- (1) n13: only takes the value 13.
- (2) Area A{n1,n2,n6,n7},{n4,n5,n9,n10},{n16,n17,n21,n22}, and {n19,n20,n24,n25}
- (3) Area B{n3,n8}, {n11,n12}, {n14,n15}, and {n18,n23}

Any number in any area could not move to the other areas. For instance, n3 could move to any positions in Area B {n3,n8,n11,n12, ..., n23} above, but it could not move into the Area A{n1,n2,n6,n7,n4,n5,n9, ..., n25}.

[Figure 5: Exclusive Areas]



As a result we are forced to build the two different 'Standard Forms' for representative of all self-complementary magic squares: The type with n1=1 and the type with n3=1.

#4. Fundamental Solutions found for each type

For type 1, I put $n_1=1$; $n_5<n_{21}$; $n_2<n_4$; $n_6<n_{16}$.

For type 2, I put $n_3=1$; $n_{11}<n_{15}$; $n_8<n_{18}$; $n_{11}<n_{12}<n_{14}$

**** Self-Complementary Magic Squares 5*5: Starting with $n_1=1$ ****

1/					5/					25/					35/				
1	22	17	23	2	1	21	18	23	2	1	20	19	23	2	1	19	20	23	2
8	19	11	21	6	17	11	20	10	7	12	9	21	15	8	15	16	12	17	5
12	16	13	10	14	4	14	13	12	22	10	22	13	4	16	4	18	13	8	22
20	5	15	7	18	19	16	6	15	9	18	11	5	17	14	21	9	14	10	11
24	3	9	4	25	24	3	8	5	25	24	3	7	6	25	24	3	6	7	25
73/					84/					100/					128/				
1	18	21	23	2	1	17	22	23	2	1	22	15	24	3	1	21	16	24	3
17	14	11	16	7	15	21	12	10	7	17	12	21	7	8	18	11	20	9	7
4	20	13	6	22	6	8	13	18	20	6	10	13	16	20	4	14	13	12	22
19	10	15	12	9	19	16	14	5	11	18	19	5	14	9	19	17	6	15	8
24	3	5	8	25	24	3	4	9	25	23	2	11	4	25	23	2	10	5	25
155/					163/					197/					240/				
1	20	17	24	3	1	19	18	24	3	1	18	19	24	3	1	17	20	24	3
11	18	19	5	12	16	15	20	9	5	16	20	9	15	5	16	14	22	5	8
16	4	13	22	10	4	12	13	14	22	4	14	13	12	22	7	11	13	15	19
14	21	7	8	15	21	17	6	11	10	21	11	17	6	10	18	21	4	12	10
23	2	9	6	25	23	2	8	7	25	23	2	7	8	25	23	2	6	9	25
260/					278/					311/					320/				
1	16	21	24	3	1	15	22	24	3	1	21	15	24	4	1	20	16	24	4
18	12	17	11	7	14	18	10	17	6	17	10	23	8	7	18	19	12	11	5
4	20	13	6	22	7	21	13	5	19	6	14	13	12	20	3	9	13	17	23
19	15	9	14	8	20	9	16	8	12	19	18	3	16	9	21	15	14	7	8
23	2	5	10	25	23	2	4	11	25	22	2	11	5	25	22	2	10	6	25
353/					372/					379/					405/				
1	19	17	24	4	1	18	19	23	4	1	17	19	24	4	1	16	20	24	4
11	14	21	16	3	15	24	5	12	9	15	12	23	10	5	18	19	14	9	5
8	20	13	6	18	10	6	13	20	16	6	18	13	8	20	3	11	13	15	23
23	10	5	12	15	17	14	21	2	11	21	16	3	14	11	21	17	12	7	8
22	2	9	7	25	22	3	7	8	25	22	2	7	9	25	22	2	6	10	25
432/					440/					447/					465/				
1	15	21	24	4	1	23	12	24	5	1	22	14	23	5	1	20	15	24	5
10	23	17	7	8	19	15	16	9	6	16	8	24	11	6	16	22	3	17	7
14	6	13	20	12	4	8	13	18	22	7	17	13	9	19	8	12	13	14	18
18	19	9	3	16	20	17	10	11	7	20	15	2	18	10	19	9	23	4	10
22	2	5	11	25	21	2	14	3	25	21	3	12	4	25	21	2	11	6	25
485/					505/					535/					570/				
1	19	16	24	5	1	18	17	24	5	1	17	18	24	5	1	16	19	24	5
18	12	20	11	4	15	23	7	16	4	12	19	20	3	11	18	20	9	14	4
3	17	13	9	23	6	12	13	14	20	16	4	13	22	10	3	15	13	11	23
22	15	6	14	8	22	10	19	3	11	15	23	6	7	14	22	12	17	6	8
21	2	10	7	25	21	2	9	8	25	21	2	8	9	25	21	2	7	10	25
603/					612/					635/					647/				
1	15	20	24	5	1	14	22	23	5	1	12	23	24	5	1	23	11	24	6
18	19	10	14	4	16	24	8	11	6	19	16	15	9	6	19	10	17	14	5
3	17	13	9	23	7	9	13	17	19	4	18	13	8	22	4	18	13	8	22
22	12	16	7	8	20	15	18	2	10	20	17	11	10	7	21	12	9	16	7
21	2	6	11	25	21	3	4	12	25	21	2	3	14	25	20	2	15	3	25

651/ 1 22 12 24 6 10 17 8 23 7 15 21 13 5 11 19 3 18 9 16 20 2 14 4 25	653/ 1 21 14 23 6 16 22 2 18 7 9 11 13 15 17 19 8 24 4 10 20 3 12 5 25	659/ 1 19 15 24 6 8 23 9 21 4 14 16 13 10 12 22 5 17 3 18 20 2 11 7 25	682/ 1 18 16 24 6 15 23 14 9 4 7 5 13 21 19 22 17 12 3 11 20 2 10 8 25
689/ 1 17 18 23 6 19 11 14 16 5 4 24 13 2 22 21 10 12 15 7 20 3 8 9 25	711/ 1 16 18 24 6 12 19 22 3 9 15 5 13 21 11 17 23 4 7 14 20 2 8 10 25	734/ 1 15 19 24 6 17 14 21 10 3 4 18 13 8 22 23 16 5 12 9 20 2 7 11 25	760/ 1 14 21 23 6 4 19 17 15 10 24 18 13 8 2 16 11 9 7 22 20 3 5 12 25
765/ 1 12 22 24 6 16 17 18 11 3 5 19 13 7 21 23 15 8 9 10 20 2 4 14 25	770/ 1 11 23 24 6 17 16 21 4 7 8 14 13 12 18 19 22 5 10 9 20 2 3 15 25	780/ 1 23 10 24 7 20 17 12 11 5 4 8 13 18 22 21 15 14 9 6 19 2 16 3 25	789/ 1 22 11 24 7 18 10 23 9 5 6 14 13 12 20 21 17 3 16 8 19 2 15 4 25
808/ 1 21 12 24 7 20 17 8 16 4 3 15 13 11 23 22 10 18 9 6 19 2 14 5 25	824/ 1 20 14 23 7 18 10 24 9 4 5 15 13 11 21 22 17 2 16 8 19 3 12 6 25	841/ 1 18 15 24 7 17 23 5 16 4 6 12 13 14 20 22 10 21 3 9 19 2 11 8 25	859/ 1 17 16 24 7 20 11 18 12 4 3 21 13 5 23 22 14 8 15 6 19 2 10 9 25
880/ 1 16 17 24 7 18 12 21 11 3 4 20 13 6 22 23 15 5 14 8 19 2 9 10 25	910/ 1 15 18 24 7 12 21 20 3 9 16 4 13 22 10 17 23 6 5 14 19 2 8 11 25	931/ 1 14 20 23 7 15 17 24 5 4 8 10 13 16 18 22 21 2 9 11 19 3 6 12 25	959/ 1 12 21 24 7 18 16 17 11 3 4 20 13 6 22 23 15 9 10 8 19 2 5 14 25
973/ 1 11 22 24 7 18 23 10 9 5 6 12 13 14 20 21 17 16 3 8 19 2 4 15 25	983/ 1 10 23 24 7 18 22 11 9 5 6 14 13 12 20 21 17 15 4 8 19 2 3 16 25	995/ 1 23 9 24 8 15 16 21 6 7 12 4 13 22 14 19 20 5 10 11 18 2 17 3 25	1003/ 1 22 10 24 8 12 17 6 23 7 15 21 13 5 11 19 3 20 9 14 18 2 16 4 25
1008/ 1 21 11 24 8 19 20 9 14 3 4 10 13 16 22 23 12 17 6 7 18 2 15 5 25	1023/ 1 20 12 24 8 21 19 10 11 4 3 9 13 17 23 22 15 16 7 5 18 2 14 6 25	1045/ 1 19 14 23 8 16 22 2 20 5 9 15 13 11 17 21 6 24 4 10 18 3 12 7 25	1060/ 1 17 15 24 8 19 14 23 4 5 6 10 13 16 20 21 22 3 12 7 18 2 11 9 25
1070/ 1 16 23 17 8 7 22 5 20 11 24 12 13 14 2 15 6 21 4 19 18 9 3 10 25	1071/ 1 15 17 24 8 19 12 21 10 3 4 20 13 6 22 23 16 5 14 7 18 2 9 11 25	1107/ 1 14 20 22 8 10 17 24 3 11 21 7 13 19 5 15 23 2 9 16 18 4 6 12 25	1117/ 1 12 20 24 8 21 15 16 9 4 3 19 13 7 23 22 17 10 11 5 18 2 6 14 25
1132/ 1 11 21 24 8 14 23 7 17 4 10 20 13 6 16 22 9 19 3 12 18 2 5 15 25	1145/ 1 10 22 24 8 12 23 6 17 7 15 21 13 5 11 19 9 20 3 14 18 2 4 16 25	1153/ 1 9 23 24 8 10 19 21 11 4 14 20 13 6 12 22 15 5 7 16 18 2 3 17 25	1158/ 1 23 8 24 9 20 11 14 16 4 5 19 13 7 21 22 10 12 15 6 17 2 18 3 25

1166/ 1 22 10 23 9 8 21 20 14 2 15 7 13 19 11 24 12 6 5 18 17 3 16 4 25	1169/ 1 21 10 24 9 20 19 12 11 3 4 8 13 18 22 23 15 14 7 6 17 2 16 5 25	1188/ 1 20 11 24 9 16 22 5 19 3 8 14 13 12 18 23 7 21 4 10 17 2 15 6 25	1212/ 1 19 12 24 9 20 21 10 11 3 4 8 13 18 22 23 15 16 5 6 17 2 14 7 25
1223/ 1 18 14 23 9 20 10 24 7 4 5 15 13 11 21 22 19 2 16 6 17 3 12 8 25	1244/ 1 16 18 21 9 22 19 6 15 3 2 14 13 12 24 23 11 20 7 4 17 5 8 10 25	1271/ 1 15 16 24 9 20 21 14 7 3 4 8 13 18 22 23 19 12 5 6 17 2 10 11 25	1292/ 1 14 18 23 9 16 19 22 2 6 11 5 13 21 15 20 24 4 7 10 17 3 8 12 25
1335/ 1 12 19 24 9 16 22 3 18 6 11 21 13 5 15 20 8 23 4 10 17 2 7 14 25	1366/ 1 11 20 24 9 16 19 22 5 3 8 12 13 14 18 23 21 4 7 10 17 2 6 15 25	1373/ 1 10 21 24 9 19 23 11 8 4 6 12 13 14 20 22 18 15 3 7 17 2 5 16 25	1411/ 1 8 23 24 9 16 22 7 14 6 11 21 13 5 15 20 12 19 4 10 17 2 3 18 25
1423/ 1 23 7 24 10 9 22 15 14 5 18 6 13 20 8 21 12 11 4 17 16 2 19 3 25	1556/ 1 23 6 24 11 18 19 14 10 4 9 5 13 21 17 22 16 12 7 8 15 2 20 3 25	1790/ 1 23 5 24 12 18 9 19 15 4 10 20 13 6 16 22 11 7 17 8 14 2 21 3 25	

* Total Count = 1948

** Self-Complementary Magic Squares 5*5: Starting with n3=1 **

1/ 15 19 1 14 16 18 21 3 17 6 2 4 13 22 24 20 9 23 5 8 10 12 25 7 11	78/ 10 24 1 21 9 19 18 3 14 11 4 6 13 20 22 15 12 23 8 7 17 5 25 2 16	134/ 7 22 1 20 15 24 16 3 14 8 5 9 13 17 21 18 12 23 10 2 11 6 25 4 19	150/ 7 24 1 22 11 21 17 3 14 10 6 8 13 18 20 16 12 23 9 5 15 4 25 2 19
181/ 10 12 1 22 20 24 21 3 9 8 7 11 13 15 19 18 17 23 5 2 6 4 25 14 16	191/ 21 15 1 22 6 14 17 3 7 24 8 10 13 16 18 2 19 23 9 12 20 4 25 11 5	209/ 6 24 1 19 15 17 18 3 22 5 10 12 13 14 16 21 4 23 8 9 11 7 25 2 20	217/ 11 20 1 19 14 23 18 5 10 9 2 4 13 22 24 17 16 21 8 3 12 7 25 6 15
279/ 12 20 1 24 8 22 15 5 7 16 3 9 13 17 23 10 19 21 11 4 18 2 25 6 14	293/ 8 23 1 17 16 24 15 5 14 7 4 6 13 20 22 19 12 21 11 2 10 9 25 3 18	341/ 7 22 1 24 11 23 16 5 9 12 6 8 13 18 20 14 17 21 10 3 15 2 25 4 19	370/ 10 14 1 18 22 20 23 5 15 2 7 9 13 17 19 24 11 21 3 6 4 8 25 12 16
378/ 9 14 1 19 22 20 23 5 15 2 8 10 13 16 18 24 11 21 3 6 4 7 25 12 17	390/ 4 23 1 19 18 24 14 5 16 6 9 11 13 15 17 20 10 21 12 2 8 7 25 3 22	402/ 7 22 1 15 20 18 17 5 23 2 10 12 13 14 16 24 3 21 9 8 6 11 25 4 19	426/ 10 23 1 17 14 20 18 7 15 5 2 4 13 22 24 21 11 19 8 6 12 9 25 3 16

	493/	499/	584/	606/
10 22 1 24 8	8 24 1 15 17	8 20 1 24 12	11 12 1 24 17	
14 21 7 17 6	23 14 7 16 5	16 23 7 15 4	23 21 7 4 10	
3 11 13 15 23	4 6 13 20 22	5 9 13 17 21	6 8 13 18 20	
20 9 19 5 12	21 10 19 12 3	22 11 19 3 10	16 22 19 5 3	
18 2 25 4 16	9 11 25 2 18	14 2 25 6 18	9 2 25 14 15	
	644/	660/	676/	727/
6 21 1 15 22	3 24 1 20 17	10 21 1 18 15	10 19 1 15 20	
24 14 7 17 3	21 15 7 18 4	19 20 9 14 3	22 18 9 14 2	
8 10 13 16 18	10 12 13 14 16	2 4 13 22 24	3 5 13 21 23	
23 9 19 12 2	22 8 19 11 5	23 12 17 6 7	24 12 17 8 4	
4 11 25 5 20	9 6 25 2 23	11 8 25 5 16	6 11 25 7 16	
	747/	813/	852/	891/
11 14 1 23 16	8 20 1 22 14	10 23 1 19 12	4 21 1 19 20	
19 24 9 8 5	16 23 9 15 2	11 22 9 21 2	24 15 9 14 3	
4 6 13 20 22	5 7 13 19 21	6 8 13 18 20	8 10 13 16 18	
21 18 17 2 7	24 11 17 3 10	24 5 17 4 15	23 12 17 11 2	
10 3 25 12 15	12 4 25 6 18	14 7 25 3 16	6 7 25 5 22	
	915/	926/	980/	987/
3 20 1 22 19	9 21 1 20 14	8 22 1 24 10	8 19 1 23 14	
21 18 9 15 2	23 16 11 8 7	17 20 11 12 5	24 16 11 5 9	
10 12 13 14 16	2 4 13 22 24	3 7 13 19 23	4 6 13 20 22	
24 11 17 8 5	19 18 15 10 3	21 14 15 6 9	17 21 15 10 2	
7 4 25 6 23	12 6 25 5 17	16 2 25 4 18	12 3 25 7 18	
	1021/	1034/	1065/	1083/
10 12 1 24 18	10 12 1 19 23	4 19 1 21 20	9 23 1 24 8	
23 20 11 4 7	22 21 11 9 2	23 17 11 12 2	22 7 11 5 20	
5 9 13 17 21	6 8 13 18 20	8 10 13 16 18	10 12 13 14 16	
19 22 15 6 3	24 17 15 5 4	24 14 15 9 3	6 21 15 19 4	
8 2 25 14 16	3 7 25 14 16	6 5 25 7 22	18 2 25 3 17	
	1087/	1234/	1399/	1599/
10 25 2 20 8	8 22 2 19 14	9 25 2 22 7	9 19 2 15 20	
19 17 4 14 11	25 15 6 10 9	20 15 8 10 12	22 18 10 14 1	
3 5 13 21 23	3 5 13 21 23	3 5 13 21 23	3 5 13 21 23	
15 12 22 9 7	17 16 20 11 1	14 16 18 11 6	25 12 16 8 4	
18 6 24 1 16	12 7 24 4 18	19 4 24 1 17	6 11 24 7 17	
	1757/	2009/	2108/	2202/
7 25 2 22 9	12 17 3 15 18	9 18 3 24 11	8 19 3 21 14	
20 15 12 10 8	16 24 5 19 1	16 25 7 12 5	16 24 9 15 1	
3 5 13 21 23	4 6 13 20 22	4 6 13 20 22	4 6 13 20 22	
18 16 14 11 6	25 7 21 2 10	21 14 19 1 10	25 11 17 2 10	
17 4 24 1 19	8 11 23 9 14	15 2 23 8 17	12 5 23 7 18	
	2303/	2412/	2503/	2587/
7 18 3 16 21	9 24 4 25 3	9 25 4 24 3	6 18 4 25 12	
24 17 11 12 1	16 18 6 11 14	12 20 8 15 10	17 24 10 11 3	
4 6 13 20 22	5 7 13 19 21	5 7 13 19 21	5 7 13 19 21	
25 14 15 9 2	12 15 20 8 10	16 11 18 6 14	23 15 16 2 9	
5 10 23 8 19	23 1 22 2 17	23 2 22 1 17	14 1 22 8 20	
	2687/	2747/	2776/	2817/
6 16 4 24 15	9 23 5 16 12	2 25 5 23 10	2 22 5 19 17	
25 17 12 3 8	11 22 7 24 1	22 15 9 12 7	23 16 11 14 1	
5 7 13 19 21	6 8 13 18 20	6 8 13 18 20	6 8 13 18 20	
18 23 14 9 1	25 2 19 4 15	19 14 17 11 4	25 12 15 10 3	
11 2 22 10 20	14 10 21 3 17	16 3 21 1 24	9 7 21 4 24	

2881/	2901/	2920/	2966/
21 24 6 10 4	22 15 6 8 14	3 15 6 25 16	14 23 7 1 20
3 15 8 25 14	23 2 10 5 25	21 22 12 8 2	22 2 9 21 11
7 9 13 17 19	7 9 13 17 19	7 9 13 17 19	8 10 13 16 18
12 1 18 11 23	1 21 16 24 3	24 18 14 4 5	15 5 17 24 4
22 16 20 2 5	12 18 20 11 4	10 1 20 11 23	6 25 19 3 12
2977/	2997/	3001/	3003/
3 24 7 25 6	12 23 8 1 21	3 25 8 24 5	3 25 9 22 6
12 21 11 17 4	20 4 10 24 7	10 20 12 19 4	24 5 11 7 18
8 10 13 16 18	9 11 13 15 17	9 11 13 15 17	10 12 13 14 16
22 9 15 5 14	19 2 16 22 6	22 7 14 6 16	8 19 15 21 2
20 1 19 2 23	5 25 18 3 14	21 2 18 1 23	20 4 17 1 23

* Total Count = 3034; V1(A/B) = 1948/1086

I have got 1948+1086=3034 'fundamental' solutions in all.
This total sum is quite reasonable. It is because 48544 = 3034 x 16.

#5. How to Transform and Identify it with the Fundamental?

When you happen to get something like examples shown below, you could transform it into the 'standard form,' and identify it with a certain fundamental in the list.
Try to exchange some available lines, and transform it into the form with n1=1 or n3=1.

You could move n1 to the top on the left hand side.

[Figure 6: How to Identify it with the Fundamental]

Ex. 1 [Start]	Col umn(1-2)&(4-5) Exchange	180 Degree Rotati on	Row & Col umn(2-4) Exchange
2 25 11 23 4	25 2 11 4 23	1 24 15 22 3	1 22 15 24 3
21 9 7 16 12	9 21 7 12 16	17 5 19 14 10	16 21 7 12 9
6 18 13 8 20	18 6 13 20 8	8 20 13 6 18	8 6 13 20 18
14 10 19 17 5	10 14 19 5 17	16 12 7 21 9	17 14 19 5 10
22 3 15 1 24	3 22 15 24 1	23 4 11 2 25	23 2 11 4 25 [Goal]

Ex. 2 [Start]	Col umn(1-2)&(4-5) Exchange	Row(1-2)&(4-5) Exchange
12 20 8 22 3	20 12 8 3 22	25 2 10 7 21
2 25 10 21 7	25 2 10 7 21	20 12 8 3 22
9 11 13 15 17	11 9 13 17 15	11 9 13 17 15
19 5 16 1 24	5 19 16 24 1	4 23 18 14 6
23 4 18 6 14	4 23 18 14 6	5 19 16 24 1

[180 Deg. Rotati on]	[Col umn(2-4)Exchange]
1 24 16 19 5	1 19 16 24 5
6 14 18 23 4	6 23 18 14 4
15 17 13 9 11	15 9 13 17 11
22 3 8 12 20	22 12 8 3 20
21 7 10 2 25	21 2 10 7 25 [Goal]

(Written on August 23, 2001; Revised on May 25, 2005 by Kanji Setsuda)

#6. Solution Sets and Transformations

Let's study about the relationship between 48544 solutions and 3034 ones. We often call the latter 'Fundamental'. But what reason can we say that for? Is each of 3034-solution set really the 'representative' solution of 48544 set? Or can we make 3034 into 48544 by any transformation system?

I say "yes". We can really make 3034 into 48544 only by several transformation rules. Take your careful look at these two types of transformations as below:
 00/ means 'No Operation'. The result is the original itself kept unchanged. 10/, 20/, 30/, 01/, 02/ and 03/ are the results of actual transformations. Every operation is quite simple, since you may only exchange some whole lines.

We have to combine {00,10,20,30} with {00,01,02,03} one by one and make 16 transformations in all. I am afraid it might look rather complicated.

[Figure 7-1: Transformation: Type 1]

** Line-Exchanges: (i i <-> i v), (i <-> i i) & (v <-> i v),
 and (i i <-> i v) & ((i <-> i i) & (v <-> i v)) **

00/No Operation	10/ *	20/	30/
1 2 3 4 5	1 4 3 2 5	2 1 3 5 4	4 1 3 5 2
6 7 8 9 10	6 9 8 7 10	7 6 8 10 9	9 6 8 10 7
11 12 13-12-11	11-12 13 12-11	12 11 13-11-12	-12 11 13-11 12
-10 -9 -8 -7 -6	-10 -7 -8 -9 -6	-9-10 -8 -6 -7	-7-10 -8 -6 -9
-5 -4 -3 -2 -1	-5 -2 -3 -4 -1	-4 -5 -3 -1 -2	-2 -5 -3 -1 -4
i i i i v	i v i i	i i i v i v	i v i v i i

[Figure 7-2: Transformation: Type 2]

** Line-Exchanges: (I I <-> I V), (I <-> I I) & (V <-> I V),
 and (I I <-> I V) & ((I <-> I I) & (V <-> I V)) **

00/No Operation	01/	02/	03/
I 1 2 3 4 5	1 2 3 4 5	* 6 7 8 9 10 I I	-10 -9 -8 -7 -6 I V
II 6 7 8 9 10 *	-10 -9 -8 -7 -6 I V	* 1 2 3 4 5 I	1 2 3 4 5 I
III 11 12 13-12-11	11 12 13-12-11	11 12 13-12-11	11 12 13-12-11
IV -10 -9 -8 -7 -6 *	6 7 8 9 10 I I	* -5 -4 -3 -2 -1 V	-5 -4 -3 -2 -1 V
V -5 -4 -3 -2 -1	-5 -4 -3 -2 -1	* -10 -9 -8 -7 -6 I V	6 7 8 9 10 I I

** Concept of 16 Transformations for our Object: **

00/	01/	02/	03/
1 2 3 4 5	1 2 3 4 5	6 7 8 9 10	-10 -9 -8 -7 -6
6 7 8 9 10	-10 -9 -8 -7 -6	1 2 3 4 5	1 2 3 4 5
11 12 13-12-11	11 12 13-12-11	11 12 13-12-11	11 12 13-12-11
-10 -9 -8 -7 -6	6 7 8 9 10	-5 -4 -3 -2 -1	-5 -4 -3 -2 -1
-5 -4 -3 -2 -1	-5 -4 -3 -2 -1	-10 -9 -8 -7 -6	6 7 8 9 10
10/	11/	12/	13/
1 4 3 2 5	1 4 3 2 5	6 9 8 7 10	-10 -7 -8 -9 -6
6 9 8 7 10	-10 -7 -8 -9 -6	1 4 3 2 5	1 4 3 2 5
11-12 13 12-11	11-12 13 12-11	11-12 13 12-11	11-12 13 12-11
-10 -7 -8 -9 -6	6 9 8 7 10	-5 -2 -3 -4 -1	-5 -2 -3 -4 -1
-5 -2 -3 -4 -1	-5 -2 -3 -4 -1	-10 -7 -8 -9 -6	6 9 8 7 10
20/	21/	22/	23/
2 1 3 5 4	2 1 3 5 4	7 6 8 10 9	-9-10 -8 -6 -7
7 6 8 10 9	-9-10 -8 -6 -7	2 1 3 5 4	2 1 3 5 4
12 11 13-11-12	12 11 13-11-12	12 11 13-11-12	12 11 13-11-12
-9-10 -8 -6 -7	7 6 8 10 9	-4 -5 -3 -1 -2	-4 -5 -3 -1 -2
-4 -5 -3 -1 -2	-4 -5 -3 -1 -2	-9-10 -8 -6 -7	7 6 8 10 9

30/	31/	32/	33/
4 1 3 5 2	4 1 3 5 2	9 6 8 10 7	-7-10 -8 -6 -9
9 6 8 10 7	-7-10 -8 -6 -9	4 1 3 5 2	4 1 3 5 2
-12 11 13-11 12	-12 11 13-11 12	-12 11 13-11 12	-12 11 13-11 12
-7-10 -8 -6 -9	9 6 8 10 7	-2 -5 -3 -1 -4	-2 -5 -3 -1 -4
-2 -5 -3 -1 -4	-2 -5 -3 -1 -4	-7-10 -8 -6 -9	9 6 8 10 7

Next list shows the first result of application of those rules above to the first fundamental:

[List: First Fundamental with 15 Transformations]

[1]

	1/	7825/	0/	0/
15 18 1 14 17	15 18 1 14 17	19 21 3 16 6	20 10 23 5 7	
19 21 3 16 6	20 10 23 5 7	15 18 1 14 17	15 18 1 14 17	
2 4 13 22 24	2 4 13 22 24	2 4 13 22 24	2 4 13 22 24	
20 10 23 5 7	19 21 3 16 6	9 12 25 8 11	9 12 25 8 11	
9 12 25 8 11	9 12 25 8 11	20 10 23 5 7	19 21 3 16 6	
	145/	7969/	0/	0/
15 14 1 18 17	15 14 1 18 17	19 16 3 21 6	20 5 23 10 7	
19 16 3 21 6	20 5 23 10 7	15 14 1 18 17	15 14 1 18 17	
2 22 13 4 24	2 22 13 4 24	2 22 13 4 24	2 22 13 4 24	
20 5 23 10 7	19 16 3 21 6	9 8 25 12 11	9 8 25 12 11	
9 8 25 12 11	9 8 25 12 11	20 5 23 10 7	19 16 3 21 6	
	159/	7983/	17316/	0/
18 15 1 17 14	18 15 1 17 14	21 19 3 6 16	10 20 23 7 5	
21 19 3 6 16	10 20 23 7 5	18 15 1 17 14	18 15 1 17 14	
4 2 13 24 22	4 2 13 24 22	4 2 13 24 22	4 2 13 24 22	
10 20 23 7 5	21 19 3 6 16	12 9 25 11 8	12 9 25 11 8	
12 9 25 11 8	12 9 25 11 8	10 20 23 7 5	21 19 3 6 16	
	0/	0/	0/	0/
14 15 1 17 18	14 15 1 17 18	16 19 3 6 21	5 20 23 7 10	
16 19 3 6 21	5 20 23 7 10	14 15 1 17 18	14 15 1 17 18	
22 2 13 24 4	22 2 13 24 4	22 2 13 24 4	22 2 13 24 4	
5 20 23 7 10	16 19 3 6 21	8 9 25 11 12	8 9 25 11 12	
8 9 25 11 12	8 9 25 11 12	5 20 23 7 10	16 19 3 6 21	

Can we find all results from 1/ to 15/ exist in the old 48544-solution list? You can only find some of them, but not all. As you see above, some answers appear in reverse with any reflection or rotations. You have to rewrite all results and make them smart under the list forming inequality condition such as (n3<n23)&&(n3<n11<n15).

** Concept of Reflection and Rotations by 90 Degrees **

00/	01/	02/	03/
1 2 3 4 5	21 16 11 6 1	25 24 23 22 21	5 10 15 20 25
6 7 8 9 10	22 17 12 7 2	20 19 18 17 16	4 9 14 19 24
11 12 13 14 15	23 18 13 8 3	15 14 13 12 11	3 8 13 18 23
16 17 18 19 20	24 19 14 9 4	10 9 8 7 6	2 7 12 17 22
21 22 23 24 25	25 20 15 10 5	5 4 3 2 1	1 6 11 16 21
10/	11/	12/	13/
5 4 3 2 1	25 20 15 10 5	21 22 23 24 25	1 6 11 16 21
10 9 8 7 6	24 19 14 9 4	16 17 18 19 20	2 7 12 17 22
15 14 13 12 11	23 18 13 8 3	11 12 13 14 15	3 8 13 18 23
20 19 18 17 16	22 17 12 7 2	6 7 8 9 10	4 9 14 19 24
25 24 23 22 21	21 16 11 6 1	1 2 3 4 5	5 10 15 20 25

The next list contains the extract results of those elaborate works.

** Self-Complementary Magic Squares 5*5: **
 ** Fundamentals with 15 Transformations **

[1]

1/ 15 18 1 14 17 19 21 3 16 6 2 4 13 22 24 20 10 23 5 7 9 12 25 8 11	7825/ 15 18 1 14 17 20 10 23 5 7 2 4 13 22 24 19 21 3 16 6 9 12 25 8 11	8695/ 19 15 2 9 20 21 18 4 12 10 3 1 13 25 23 16 14 22 8 5 6 17 24 11 7	8803/ 19 9 2 15 20 21 12 4 18 10 3 25 13 1 23 16 8 22 14 5 6 11 24 17 7
145/ 15 14 1 18 17 19 16 3 21 6 2 22 13 4 24 20 5 23 10 7 9 8 25 12 11	7969/ 15 14 1 18 17 20 5 23 10 7 2 22 13 4 24 19 16 3 21 6 9 8 25 12 11	16593/ 19 15 2 9 20 16 14 22 8 5 3 1 13 25 23 21 18 4 12 10 6 17 24 11 7	16701/ 19 9 2 15 20 16 8 22 14 5 3 25 13 1 23 21 12 4 18 10 6 11 24 17 7
159/ 18 15 1 17 14 21 19 3 6 16 4 2 13 24 22 10 20 23 7 5 12 9 25 11 8	7983/ 18 15 1 17 14 10 20 23 7 5 4 2 13 24 22 21 19 3 6 16 12 9 25 11 8	17316/ 21 19 3 6 16 18 15 1 17 14 4 2 13 24 22 12 9 25 11 8 10 20 23 7 5	22454/ 21 19 3 6 16 12 9 25 11 8 4 2 13 24 22 18 15 1 17 14 10 20 23 7 5
281/ 18 17 1 15 14 21 6 3 19 16 4 24 13 2 22 10 7 23 20 5 12 11 25 9 8	8105/ 18 17 1 15 14 10 7 23 20 5 4 24 13 2 22 21 6 3 19 16 12 11 25 9 8	17438/ 21 6 3 19 16 18 17 1 15 14 4 24 13 2 22 12 11 25 9 8 10 7 23 20 5	22576/ 21 6 3 19 16 12 11 25 9 8 4 24 13 2 22 18 17 1 15 14 10 7 23 20 5

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430/ 12 21 1 9 22 24 10 3 20 8 7 11 13 15 19 18 6 23 16 2 4 17 25 5 14	8254/ 12 21 1 9 22 18 6 23 16 2 7 11 13 15 19 24 10 3 20 8 4 17 25 5 14	17592/ 24 10 3 20 8 12 21 1 9 22 7 11 13 15 19 4 17 25 5 14 18 6 23 16 2	22730/ 24 10 3 20 8 4 17 25 5 14 7 11 13 15 19 12 21 1 9 22 18 6 23 16 2
441/ 12 9 1 21 22 24 20 3 10 8 7 15 13 11 19 18 16 23 6 2 4 5 25 17 14	8265/ 12 9 1 21 22 18 16 23 6 2 7 15 13 11 19 24 20 3 10 8 4 5 25 17 14	17600/ 24 20 3 10 8 12 9 1 21 22 7 15 13 11 19 4 5 25 17 14 18 16 23 6 2	22738/ 24 20 3 10 8 4 5 25 17 14 7 15 13 11 19 12 9 1 21 22 18 16 23 6 2
736/ 21 12 1 22 9 10 24 3 8 20 11 7 13 19 15 6 18 23 2 16 17 4 25 14 5	8560/ 21 12 1 22 9 6 18 23 2 16 11 7 13 19 15 10 24 3 8 20 17 4 25 14 5	17886/ 10 24 3 8 20 21 12 1 22 9 11 7 13 19 15 17 4 25 14 5 6 18 23 2 16	23024/ 10 24 3 8 20 17 4 25 14 5 11 7 13 19 15 21 12 1 22 9 6 18 23 2 16

	745/		8569/		17896/		23034/
21 22 1 12 9		21 22 1 12 9		10 8 3 24 20		10 8 3 24 20	
10 8 3 24 20		6 2 23 18 16		21 22 1 12 9		17 14 25 4 5	
11 19 13 7 15		11 19 13 7 15		11 19 13 7 15		11 19 13 7 15	
6 2 23 18 16		10 8 3 24 20		17 14 25 4 5		21 22 1 12 9	
17 14 25 4 5		17 14 25 4 5		6 2 23 18 16		6 2 23 18 16	

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	1191/		7315/		29218/		33178/
24 15 1 3 22		24 15 1 3 22		19 9 5 18 14		19 9 5 18 14	
19 9 5 18 14		12 8 21 17 7		24 15 1 3 22		4 23 25 11 2	
6 10 13 16 20		6 10 13 16 20		6 10 13 16 20		6 10 13 16 20	
12 8 21 17 7		19 9 5 18 14		4 23 25 11 2		24 15 1 3 22	
4 23 25 11 2		4 23 25 11 2		12 8 21 17 7		12 8 21 17 7	
	1211/		7335/		29237/		33197/
24 3 1 15 22		24 3 1 15 22		19 18 5 9 14		19 18 5 9 14	
19 18 5 9 14		12 17 21 8 7		24 3 1 15 22		4 11 25 23 2	
6 16 13 10 20		6 16 13 10 20		6 16 13 10 20		6 16 13 10 20	
12 17 21 8 7		19 18 5 9 14		4 11 25 23 2		24 3 1 15 22	
4 11 25 23 2		4 11 25 23 2		12 17 21 8 7		12 17 21 8 7	
	1469/		7593/		29493/		33453/
15 24 1 22 3		15 24 1 22 3		9 19 5 14 18		9 19 5 14 18	
9 19 5 14 18		8 12 21 7 17		15 24 1 22 3		23 4 25 2 11	
10 6 13 20 16		10 6 13 20 16		10 6 13 20 16		10 6 13 20 16	
8 12 21 7 17		9 19 5 14 18		23 4 25 2 11		15 24 1 22 3	
23 4 25 2 11		23 4 25 2 11		8 12 21 7 17		8 12 21 7 17	
	1543/		7667/		29567/		33527/
15 22 1 24 3		15 22 1 24 3		9 14 5 19 18		9 14 5 19 18	
9 14 5 19 18		8 7 21 12 17		15 22 1 24 3		23 2 25 4 11	
10 20 13 6 16		10 20 13 6 16		10 20 13 6 16		10 20 13 6 16	
8 7 21 12 17		9 14 5 19 18		23 2 25 4 11		15 22 1 24 3	
23 2 25 4 11		23 2 25 4 11		8 7 21 12 17		8 7 21 12 17	

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	1906/		6194/		25158/		25285/
9 23 1 14 18		9 23 1 14 18		20 9 4 8 24		20 8 4 9 24	
20 15 7 21 2		24 5 19 11 6		15 23 10 12 5		15 12 10 23 5	
4 10 13 16 22		4 10 13 16 22		7 1 13 25 19		7 25 13 1 19	
24 5 19 11 6		20 15 7 21 2		21 14 16 3 11		21 3 16 14 11	
8 12 25 3 17		8 12 25 3 17		2 18 22 17 6		2 17 22 18 6	
	1960/		6248/		26836/		26963/
9 14 1 23 18		9 14 1 23 18		20 9 4 8 24		20 8 4 9 24	
20 21 7 15 2		24 11 19 5 6		21 14 16 3 11		21 3 16 14 11	
4 16 13 10 22		4 16 13 10 22		7 1 13 25 19		7 25 13 1 19	
24 11 19 5 6		20 21 7 15 2		15 23 10 12 5		15 12 10 23 5	
8 3 25 12 17		8 3 25 12 17		2 18 22 17 6		2 17 22 18 6	
	2435/		6723/		38311/		40812/
23 9 1 18 14		23 9 1 18 14		15 20 7 2 21		15 20 7 2 21	
15 20 7 2 21		5 24 19 6 11		23 9 1 18 14		12 8 25 17 3	
10 4 13 22 16		10 4 13 22 16		10 4 13 22 16		10 4 13 22 16	
5 24 19 6 11		15 20 7 2 21		12 8 25 17 3		23 9 1 18 14	
12 8 25 17 3		12 8 25 17 3		5 24 19 6 11		5 24 19 6 11	

2557/	6845/	38433/	40934/
23 18 1 9 14	23 18 1 9 14	15 2 7 20 21	15 2 7 20 21
15 2 7 20 21	5 6 19 24 11	23 18 1 9 14	12 17 25 8 3
10 22 13 4 16	10 22 13 4 16	10 22 13 4 16	10 22 13 4 16
5 6 19 24 11	15 2 7 20 21	12 17 25 8 3	23 18 1 9 14
12 17 25 8 3	12 17 25 8 3	5 6 19 24 11	5 6 19 24 11

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2750/	5038/	12613/	12750/
23 20 1 5 16	23 20 1 5 16	19 23 2 10 11	19 10 2 23 11
19 4 9 18 15	11 8 17 22 7	4 20 12 21 8	4 21 12 20 8
2 12 13 14 24	2 12 13 14 24	9 1 13 25 17	9 25 13 1 17
11 8 17 22 7	19 4 9 18 15	18 5 14 6 22	18 6 14 5 22
10 21 25 6 3	10 21 25 6 3	15 16 24 3 7	15 3 24 16 7
2757/	5045/	13737/	13874/
23 5 1 20 16	23 5 1 20 16	19 23 2 10 11	19 10 2 23 11
19 18 9 4 15	11 22 17 8 7	18 5 14 6 22	18 6 14 5 22
2 14 13 12 24	2 14 13 12 24	9 1 13 25 17	9 25 13 1 17
11 22 17 8 7	19 18 9 4 15	4 20 12 21 8	4 21 12 20 8
10 6 25 21 3	10 6 25 21 3	15 16 24 3 7	15 3 24 16 7
3595/	5883/	44011/	45921/
20 23 1 16 5	20 23 1 16 5	4 19 9 15 18	4 19 9 15 18
4 19 9 15 18	8 11 17 7 22	20 23 1 16 5	21 10 25 3 6
12 2 13 24 14	12 2 13 24 14	12 2 13 24 14	12 2 13 24 14
8 11 17 7 22	4 19 9 15 18	21 10 25 3 6	20 23 1 16 5
21 10 25 3 6	21 10 25 3 6	8 11 17 7 22	8 11 17 7 22
3700/	5988/	44116/	46026/
20 16 1 23 5	20 16 1 23 5	4 15 9 19 18	4 15 9 19 18
4 15 9 19 18	8 7 17 11 22	20 16 1 23 5	21 3 25 10 6
12 24 13 2 14	12 24 13 2 14	12 24 13 2 14	12 24 13 2 14
8 7 17 11 22	4 15 9 19 18	21 3 25 10 6	20 16 1 23 5
21 3 25 10 6	21 3 25 10 6	8 7 17 11 22	8 7 17 11 22

[906]

3344/	5632/	42122/	42203/
22 3 1 19 20	22 3 1 19 20	15 22 8 6 14	15 6 8 22 14
15 24 9 5 12	14 21 17 2 11	24 3 10 7 21	24 7 10 3 21
8 10 13 16 18	8 10 13 16 18	9 1 13 25 17	9 25 13 1 17
14 21 17 2 11	15 24 9 5 12	5 19 16 23 2	5 23 16 19 2
6 7 25 23 4	6 7 25 23 4	12 20 18 4 11	12 4 18 20 11
3374/	5662/	42602/	42683/
22 19 1 3 20	22 19 1 3 20	15 22 8 6 14	15 6 8 22 14
15 5 9 24 12	14 2 17 21 11	5 19 16 23 2	5 23 16 19 2
8 16 13 10 18	8 16 13 10 18	9 1 13 25 17	9 25 13 1 17
14 2 17 21 11	15 5 9 24 12	24 3 10 7 21	24 7 10 3 21
6 23 25 7 4	6 23 25 7 4	12 20 18 4 11	12 4 18 20 11
3450/	5738/	43889/	45799/
3 22 1 20 19	3 22 1 20 19	24 15 9 12 5	24 15 9 12 5
24 15 9 12 5	21 14 17 11 2	3 22 1 20 19	7 6 25 4 23
10 8 13 18 16	10 8 13 18 16	10 8 13 18 16	10 8 13 18 16
21 14 17 11 2	24 15 9 12 5	7 6 25 4 23	3 22 1 20 19
7 6 25 4 23	7 6 25 4 23	21 14 17 11 2	21 14 17 11 2

3490/ 3 20 1 22 19 24 12 9 15 5 10 18 13 8 16 21 11 17 14 2 7 4 25 6 23	5778/ 3 20 1 22 19 21 11 17 14 2 10 18 13 8 16 24 12 9 15 5 7 4 25 6 23	43929/ 24 12 9 15 5 3 20 1 22 19 10 18 13 8 16 7 4 25 6 23 21 11 17 14 2	45839/ 24 12 9 15 5 7 4 25 6 23 10 18 13 8 16 3 20 1 22 19 21 11 17 14 2
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