Part 4. "New Advanced Study of Magic Squares and Cubes"

Chapter 6. Fundamental Study of Multi-Dimensional Extra-Cubic Forms and their Decomposed Ones: **Kanji Setsuda**

Section 1. Study of Four-Dimensional Extra-Cubic Forms of Order 3 and their Decomposed Ones

#1. Dimensions of Magic Squares and Cubes

Let's try to make some experiments and enhance our insight of looking through the whole fundamental structures of magic figures.

Say. I would like to build some extra-cubic magic objects by increasing the dimensions up to 4.

Isn't it visible? No, it isn't. Is it possible? Yes, it is.

Every magic square and cube is built in the regular array N^m (N=3, 4, 5, 6, . . .; m=2 or 3). We know that we can make any memory array, for instance, of 3^4 , or 2^6 in our computer space. The array of 3^4 is denoted by the four coordinates as: n(0,0,0,0), n(0,0,0,1), ... , n(1,1,1,1), ... , n(2,2,2,2). We can store any data into any position n(i,j,k,l). Of course i,j,k,l=0,1,2.

If you could not make such a memory array as of 3^4 in your computer, you should have given up your wish to make such a crazy thing.

But you can really make such a memory array 3^4 in your computer space. Then you are able to make such a magic form as 4-dimensional extra-cubic object of order 3.

How does it look like? No one can draw a single picture of the object.

But we do not deal with the problem of appearance. The essential problem is how we should have the whole consecutive integer 1, 2, 3, 4, 5, ..., 80, 81 stored into the array 3^4. Instead of drawing pictures we must think how we can rearrange them and what conditions we must give to it first of all.

Dimension itself is difficult to explain, but I tried to make some figures for you to understand it easily. Take a look at the next figures full of axes.

Suppose n1 as the origin n(0,0,0,0), and you will see four axes: x-, y-, z- and w-axis meet on the same origin. Each axis directly stands for each dimension of the object. Four-dimensional objects must have four axes meeting on the same origin.

[Figure 1: 2-Dimensional Square
Built in the Regular Array 3^2]

```
              The Origin
n1--n2--n3    n(0,0)--n(0,1)--n(0,2)...X-axis
 |   |   |       |       |       |
n4--n5--n6    n(1,0)--n(1,1)--n(1,2)
 |   |   |       |       |       |
n7--n8--n9    n(2,0)--n(2,1)--n(2,2)
                 :
                Y-axis
```

```
       [3-dimensional Cube
 -X axis    Built in the Regular Array 3*3*3]
The Origin
  n(0,0,0)-----n(1,0,0)-----n(2,0,0)--- Z axis
   |n(0,0,1)     n(1,0,1)    |n(2,0,1)
   |  n(0,0,2)-----n(1,0,2)-----n(2,0,2)
   |      |\                 |      |
  n(0,1,0)| X  n(1,1,0)     n(2,1,0)|
   |n(0,1,1)axis n(1,1,1)    |n(2,1,1)
   |  n(0,1,2)     n(1,1,2)  |  n(2,1,2)
   |      |                  |      |
  n(0,2,0)|----n(1,2,0)-----n(2,2,0)|
   |n(0,2,1)     n(1,2,1)     n(2,2,1)
   |  n(0,2,2)-----n(1,2,2)-----n(2,2,2)
  Y axis
```

```
 [The Concept of 4-Dimensional Extra-Cube
          Built in the Regular Array 3*3*3*3]
            W axis: the 4th axis
             /
           n(2,0,0,0)
           /
          /
        n(1,0,0,0)
-X axis  /
       \/The Origin
      n(0,0,0,0)---n(0,1,0,0)---n(0,2,0,0)...Z axis
       |n(0,0,0,1)
       | n(0,0,0,2)
      n(0,0,1,0)\
       |        X axis
       |
      n(0,0,2,0)
       |
      Y axis
```

#2. Preparation for Making our Extra-Cubic Object 3^4

We are going to make our object in such a structural array as shown above.

I tried even to invent a new complete 'Decomposed Patterns.'

The three cubes below indicate a single 4-dimensional extra-cubic object all together.

```
[Figure 2: Four-Dimensional Extra Cubic Form of Order 3]
   *** Decomposition by 3 x(3 x 3 x 3) ***

  1------10------19       28------37------46       55------64------73
  | 2       11      |20      |29      38      |47      |56      65      |74
  4   3--13--12--22--21     31  30--40--39--49--48     58  57--67--66--76--75
  | 5 |     14      |23 |    |32 |     41      |50 |    |59 |     68      |77 |
  7---6--16--15--25  24     34--33--43--42--52  51     61--60--70--69--79  78
    8 |     17       26 |     35 |     44       53 |     62 |     71       80 |
      9------18------27        36------45------54        63------72------81
          [No.1]                   [No.2]                   [No.3]
```

[Notation by 3rd Increment]

```
0000----0100----0200      1000----1100----1200      2000----2100----2200
 |0001    0101   |0201     |1001    1101   |1201     |2001    2101   |2201
 |  0002----0102--+-0202   |  1002----1102--+-1202   |  2002----2102--+-2202
0010  | 0110    0210  |   1010  | 1110    1210  |   2010  | 2110    2210  |
 |0011|   0111   |0211|    |1011|   1111   |1211|    |2011|   2111   |2211|
 |  0012    0112 |  0212   |  1012    1112 |  1212   |  2012    2112 |  2212
0020--|-0120----0220  |   1020--|-1120----1220  |   2020--|-2120----2220  |
  0021|   0121    0221|     1021|   1121    1221|     2021|   2121    2221|
    0022----0122----0222      1022----1122----1222      2022----2122----2222
         [No.1]                    [No.2]                    [No.3]
```

You must combine them into a whole thing in your brain. Although you cannot see it, you can imagine such a structural data-storage in your memory array 3^4.

Though you may take the position names n(0,0,0,0), n(0,0,0,1), n(0,0,0,2), . . . , n(2,2,2,1), n(2,2,2,2), I call them as n1(the origin), n2, n3, n4, ... , n80, n81 in the old classical style. Just because I am used to it.

As a single magic cube the sum of every line, every row and every column must be equal to the same constant 123:

```
n1+n2+n3=K; n4+n5+n6=K; n7+n8+n9=K; n1+n4+n7=K; n2+n5+n8=K; n3+n6+n9=K;
n10+n11+n11=K; n13+n14+n15=K; n16+n17+n18=K; n10+n13+n16=K; n11+n14+n17=K;
n12+n15+n18=K; n19+n20+n21=K; n22+n23+n24=K; n25+n26+n27=K; n19+n22+n25=K;
n20+n23+n26=K; n21+n24+n27=K; n1+n10+n19=K; n2+n11+n20=K; n3+n12+n21=K;
n4+n13+n22=K; n5+n14+n23=K; n6+n15+n24=K; n7+n16+n25=K; n8+n17+n26=K;
n9+n18+n27=K;
n28+n29+n30=K; n31+n32+n33=K; n34+n35+n36=K; n28+n31+n34=K; n29+n32+n35=K;
n30+n33+n36=K; n37+n38+n39=K; n40+n41+n42=K; .....
n55+n56+n57=K; n58+n59+n60=K; n61+n62+n63=K; n55+n58+n61=K; ....
```

But these 81 simultaneous equations are not enough to define our object. The next 27 equations are also required. They are necessary for defining the relations in the 4th direction. Yes. Every position must be defined in the 4 directions.

```
n1+n28+n55=K; n2+n29+n56=K; n3+n30+n57=K; n4+n31+n58=K; n5+n32+n59=K;
n6+n33+n60=K; n7+n34+n61=K; n8+n35+n62=K; n9+n36+n63=K; n10+n37+n64=K;
n11+n38+n65=K; n12+n39+n66=K; n13+n40+n67=K; n14+n41+n68=K; n15+n42+n69=K; ....
```

You can easily understand this, if you take your careful look at the following figure. It shows another view-form of the same Extra-Cubic Object.

[Figure 3: Another View of Decomposition by 3 x(3 x 3 x 3)]

```
    1----- 2----- 3          4----- 5----- 6          7----- 8----- 9
    |10     11    |12        |13     14    |15        |16     17    |18
   28  19-29--20-30--21     31  22-32--23-33--24     34  25-35--26-36--27
    |37 |    38   |39 |      |40 |    41   |42 |      |43 |    44   |45 |
   55--46-56--47-57  48     58--49-59--50-60  51     61--52-62--53-63  54
     64 |    65     66 |      67 |    68     69 |      70 |    71     72 |
        73-----74-----75         76-----77-----78         79-----80-----81
          [No.1]                   [No.2]                   [No.3]
```

How about the complementary pairs of C? Are they located symmetrically? Should we take them for the necessary conditions?

```
n1+n81=n2+n80=n3+n79=n4+n78=n5+n77=.....=n39+n43=n40+n42=n41+n41=C
```

All complementary pairs of C(=82) are so reasonably placed that we might call them as 'symmetrically' located. The unique position n41(=C/2=41) exists on the geometric center of our object. We can add them to the basic conditions to make our object for a Self-Complementary type.

What do we think of the 4-agonals? As usual as in the case of self-complementary type, each primary 4-agonal consists of one complementary pair and n41.

```
n1+n41+n81=K; n3+n41+n79=K; n7+n41+n75=K; n9+n41+n73=K;
```

All these 4 equations are quite acceptable, because each equation is calculated as `C+n41=82+41=123=K`. If you accept the constant sum of complementary pairs of C, you must also accept them at the same time.

#3. What do we think about the Pan-4-agonals?

In this case I don't want to make it for a pan-4-agonal type. You may skip to the next step, but I think it isn't useless for us to study about the problem of pan-4-agonals. Where do they appear in the diagram of Decomposed patterns?

If you consider the figure above as a 'prototype' form, you could find many pan-4-agonals exist in it and add up to the same constant 123.

Where and how do we find them? Remember the pan4agonal with n2 runs 'parallel' to the primary 4-agonal with n1 in any usual cases.

Give your careful glance at the next diagrams below. They are invented for the extended forms of Figure 2 to find out pan4agonals easily.

For instance, the first primary 4-agonal 1+41+81=123 runs straight from No.1 cube through No.2 up to No.3 taking each representative. In reality 2+42+79=123 seems to run 'parallel' to the primary 4-agonal 1+41+81=123, and 3+40+80=123 runs also parallel to the same. We may accept them as the examples of real pan4agonals.

I found 8 pan4agonals with n1. There are as many pan4agonals as 8 x 9 in all. They are determined to be reused and rearranged for the object. Every line, every row and every column of the object consists of the same entries with the ones on

[Figure 4: Decomposition by 3 x(3 x 3 x 3)]
[Extended Form and Pan-4-agonals]

```
[No.1]                                            n1+n41+n81 = K;
  9          18          27           9           n1+n42+n80 = K;
    7          16          25           7         n1+n44+n78 = K;
  3   8      12  17      21  26       3   8       n1+n45+n77 = K;
    1---9------10--18------19--27-------1   9     n1+n50+n72 = K;
  6 | 2   7  15 |11  16  24 |20  25   6 | 2   7   n1+n51+n71 = K;
    4   3------13--12------22--21-------4---3     n1+n54+n68 = K;
  9 | 5 | 1  18 |14 |10  27 |23 |19   9 | 5 | 1   n1+n53+n69 = K
    7---6------16--15------25--24-------7   6           ....(PT1)
  3   8 | 4  12  17 |13  21  26 |22   3   8 | 4   n2+n42+n79 = K;
    1   9------10--18------19--27-------1---9     n2+n40+n81 = K;
      2   7      11  16      20  25       2   7   n2+n45+n76 = K;
        3          12          21           3     n2+n43+n78 = K;
          1          10          19           1   n2+n51+n70 = K;
                                                  n2+n49+n72 = K;
                                                  n2+n54+n67 = K;
[No.2]                                            n2+n52+n69 = K;
```

```
  36          45          54          36                ....(PT2)
    34          43          52          34        n3+n40+n80 = K;
  30  35      39  44      48  53      30  35      n3+n41+n79 = K;
    28--36------37--45------46--54------28  36    n3+n43+n77 = K;
  33 |29  34  42 |38  43  51 |47  52  33 |29  34  n3+n44+n76 = K;
    31  30------40--39------49--48------31--30    n3+n49+n71 = K;
  36 |32 |28  45 |41 |37  54 |50 |46  36 |32 |28  n3+n50+n70 = K;
    34--33------43--42------52--51------34  33    n3+n52+n68 = K;
  30  35 |31  39  44 |40  48  53 |49  30  35 |31  n3+n53+n67 = K;
    28  36------37--45------46--54------28--36          ....(PT3)
      29  34      38  43      47  52      29  34  n4+n44+n75 = K;
        30          39          48          30    n4+n45+n74 = K;
          28          37          46          28  n4+n38+n81 = K;
[No.3]                                            n4+n39+n80 = K;
  63          72          81          63          n4+n53+n66 = K;
    61          70          79          61        n4+n54+n65 = K;
  57  62      66  71      75  80      57  62      n4+n47+n72 = K;
    55--63------64--72------73--81------55  63    n4+n48+n71 = K;
  60 |56  61  69 |65  70  78 |74  79  60 |56  61        ....(PT4)
    58  57------67--66------76--75------58--57    n5+n45+n73 = K;
  63 |59 |55  72 |68 |64  81 |77 |73  63 |59 |55  n5+n44+n75 = K;
    61--60------70--69------79--78------61  60    n5+n37+n81 = K;
  57  62 |58  66  71 |67  75  80 |76  57  62 |58  n5+n39+n79 = K;
    55  63------64--72------73--81------55--63    n5+n54+n64 = K;
      56  61      65  70      74  79      56  61  n5+n52+n66 = K;
        57          66          75          57    n5+n46+n72 = K;
          55          64          73          55  n5+n48+n70 = K;
                                                        ....(PT5)
                                                    ...........
```

any pan4agonal of the 'prototype'.

#4. Making the 4-Dimensional Magic Object of Order 3

We use the basic names after the Figure 2. The conditions we give to the object are:

(1) Complementary pairs of C must be located 'symmetrically.'

n1+n81=n2+n80=n3+n79=n4+n78=n5+n77=.....=n39+n43=n40+n42=n41+n41=C

(2) About every line, row and column 81+27 simultaneous equations are taken for the necessary conditions.

n1+n2+n3=K; n4+n5+n6=K; n7+n8+n9=K; n1+n4+n7=K; n2+n5+n8=K; n3+n6+n9=K;
n10+n11+n11=K; n13+n14+n15=K; n16+n17+n18=K; n10+n13+n16=K; n11+n14+n17=K;
n12+n15+n18=K; n19+n20+n21=K; n22+n23+n24=K; n25+n26+n27=K; n19+n22+n25=K;
n20+n23+n26=K; n21+n24+n27=K; n1+n10+n19=K; n2+n11+n20=K; n3+n12+n21=K;
n4+n13+n22=K; n5+n14+n23=K; n6+n15+n24=K; n7+n16+n25=K; n8+n17+n26=K;
n9+n18+n27=K;
n28+n29+n30=K; n31+n32+n33=K; n34+n35+n36=K; n28+n31+n34=K; n29+n32+n35=K;
n30+n33+n36=K; n37+n38+n39=K; n40+n41+n42=K; .....
n55+n56+n57=K; n58+n59+n60=K; n61+n62+n63=K; n55+n58+n61=K; ....
n1+n28+n55=K; n2+n29+n56=K; n3+n30+n57=K; n4+n31+n58=K; n5+n32+n59=K;
n6+n33+n60=K; n7+n34+n61=K; n8+n35+n62=K; n9+n36+n63=K; n10+n37+n64=K;
n11+n38+n65=K; n12+n39+n66=K; n13+n40+n67=K; n14+n41+n68=K; n15+n42+n69=K;
.... n27+n54+n81=K

But if you take (1), you don't have to deal with all of (2) equations.

n81=C-n1; n80=C-n2; n79=C-n3; n78=C-n4; .... n42=C-n40;

Therefore you may well take care of half of (2) equations at least.

```
** Essential 56 Simultaneous Equations for our Object **

n1+n2+n3=S    ...(1);    n1+n4+n7=S    ...(2);    n1+n10+n19=S  ...(3);
n1+n28+n55=S  ...(4);    n2+n5+n8=S    ...(5);    n2+n11+n20=S  ...(6);
n2+n29+n56=S  ...(7);    n3+n6+n9=S    ...(8);    n3+n12+n21=S  ...(9);
n3+n30+n57=S  ...(10);   n4+n5+n6=S    ...(11);   n4+n13+n22=S  ...(12);
n4+n31+n58=S  ...(13);   n5+n14+n23=S  ...(14);   n5+n32+n59=S  ...(15);
n6+n15+n24=S  ...(16);   n6+n33+n60=S  ...(17);   n7+n8+n9=S    ...(18);
n7+n16+n25=S  ...(19);   n7+n34+n61=S  ...(20);   n8+n17+n26=S  ...(21);
n8+n35+n62=S  ...(22);   n9+n18+n27=S  ...(23);   n9+n36+n63=S  ...(24);
n10+n11+n12=S ...(25);   n10+n13+n16=S ...(26);   n10+n37+n64=S ...(27);
n11+n14+n17=S ...(28);   n11+n38+n65=S ...(29);   n12+n15+n18=S ...(30);
n12+n39+n66=S ...(31);   n13+n14+n15=S ...(32);   n13+n40+n67=S ...(33);
n14+n41+n68=S ...(34);   n16+n17+n18=S ...(35);   n19+n20+n21=S ...(36);
n19+n22+n25=S ...(37);   n20+n23+n26=S ...(38);   n21+n24+n27=S ...(39);
n22+n23+n24=S ...(40);   n25+n26+n27=S ...(41);   n28+n29+n30=S ...(42);
n28+n31+n34=S ...(43);   n28+n37+n46=S ...(44);   n29+n32+n35=S ...(45);
n29+n38+n47=S ...(46);   n30+n33+n36=S ...(47);   n30+n39+n48=S ...(48);
n31+n32+n33=S ...(49);   n31+n40+n49=S ...(50);   n32+n41+n50=S ...(51);
n34+n35+n36=S ...(52);   n37+n38+n39=S ...(53);   n37+n40+n43=S ...(54);
n38+n41+n44=S ...(55);   n40+n41+n42=S ...(56);
```

(3) We don't give any conditions about pan-4-agonals this time.

Under these conditions I tried to permute all entries and calculate. I have got something like the result object set.

But are they really the ‘4-dimensional extra-cubic form’? How do you recognize it? How do you examine them?

Step back to Figure 1-4 and find the next four axes:

n1+n2+n3=K; n1+n4+n7=K; n1+n10+n19=K; n1+n28+n55=K

What values do they take? Do they exchange their values with one another always in the equal manner? They might imply something four-dimensional.

I picked some examples out of the largest ‘Mother Set’ of solutions and listed the values of 4 axes:

[Figure 5: The Values of Four Axes]

```
      {n1,n2,n3},{n1,n4,n7},{n1,n10,n19},{n1,n28,n55}
[  1] (1,80,42), (1,78,44), (1,72,50), (1,54,68)
[  2] (1,80,42), (1,78,44), (1,54,68), (1,72,50)
[  3] (1,80,42), (1,77,45), (1,71,51), (1,53,69)
[  4] (1,80,42), (1,77,45), (1,53,69), (1,71,51)
[  5] (1,80,42), (1,72,50), (1,78,44), (1,54,68)
[  6] (1,80,42), (1,72,50), (1,54,68), (1,78,44)
[  7] (1,80,42), (1,71,51), (1,77,45), (1,53,69)
[  8] (1,80,42), (1,71,51), (1,53,69), (1,77,45)
[  9] (1,80,42), (1,54,68), (1,78,44), (1,72,50)
[ 10] (1,80,42), (1,54,68), (1,72,50), (1,78,44)
[ 11] (1,80,42), (1,53,69), (1,77,45), (1,71,51)
[ 12] (1,80,42), (1,53,69), (1,71,51), (1,77,45)
[ 13] (1,78,44), (1,80,42), (1,72,50), (1,54,68)
[ 14] (1,78,44), (1,80,42), (1,54,68), (1,72,50)
[ 15] (1,78,44), (1,77,45), (1,69,53), (1,51,71)
[ 16] (1,78,44), (1,77,45), (1,51,71), (1,69,53)
```

```
[ 17] (1,78,44), (1,72,50), (1,80,42), (1,54,68)
[ 18] (1,78,44), (1,72,50), (1,54,68), (1,80,42)
[ 19] (1,78,44), (1,69,53), (1,77,45), (1,51,71)
[ 20] (1,78,44), (1,69,53), (1,51,71), (1,77,45)
[ 21] (1,78,44), (1,54,68), (1,80,42), (1,72,50)
[ 22] (1,78,44), (1,54,68), (1,72,50), (1,80,42)
[ 23] (1,78,44), (1,51,71), (1,77,45), (1,69,53)
[ 24] (1,78,44), (1,51,71), (1,69,53), (1,77,45)
[ 25] (1,77,45), (1,80,42), (1,71,51), (1,53,69)
[ 26] (1,77,45), (1,80,42), (1,53,69), (1,71,51)
[ 27] (1,77,45), (1,78,44), (1,69,53), (1,51,71)
[ 28] (1,77,45), (1,78,44), (1,51,71), (1,69,53)
[ 29] (1,77,45), (1,71,51), (1,80,42), (1,53,69)
[ 30] (1,77,45), (1,71,51), (1,53,69), (1,80,42)
[ 31] (1,77,45), (1,69,53), (1,78,44), (1,51,71)
[ 32] (1,77,45), (1,69,53), (1,51,71), (1,78,44)
[ 33] (1,77,45), (1,53,69), (1,80,42), (1,71,51)
[ 34] (1,77,45), (1,53,69), (1,71,51), (1,80,42)
[ 35] (1,77,45), (1,51,71), (1,78,44), (1,69,53)
[ 36] (1,77,45), (1,51,71), (1,69,53), (1,78,44)
[ 37] (1,72,50), (1,80,42), (1,78,44), (1,54,68)
[ 38] (1,72,50), (1,80,42), (1,54,68), (1,78,44)
[ 39] (1,72,50), (1,78,44), (1,80,42), (1,54,68)
[ 40] (1,72,50), (1,78,44), (1,54,68), (1,80,42)
[ 41] (1,72,50), (1,71,51), (1,69,53), (1,45,77)
[ 42] (1,72,50), (1,71,51), (1,45,77), (1,69,53)
[ 43] (1,72,50), (1,69,53), (1,71,51), (1,45,77)
[ 44] (1,72,50), (1,69,53), (1,45,77), (1,71,51)
[ 45] (1,72,50), (1,54,68), (1,80,42), (1,78,44)
[ 46] (1,72,50), (1,54,68), (1,78,44), (1,80,42)
[ 47] (1,72,50), (1,45,77), (1,71,51), (1,69,53)
[ 48] (1,72,50), (1,45,77), (1,69,53), (1,71,51)
[ 49] (1,71,51), (1,80,42), (1,77,45), (1,53,69)
[ 50] (1,71,51), (1,80,42), (1,53,69), (1,77,45)
[ 51] (1,71,51), (1,77,45), (1,80,42), (1,53,69)
[ 52] (1,71,51), (1,77,45), (1,53,69), (1,80,42)
[ 53] (1,71,51), (1,72,50), (1,69,53), (1,45,77)
[ 54] (1,71,51), (1,72,50), (1,45,77), (1,69,53)
[ 55] (1,71,51), (1,69,53), (1,72,50), (1,45,77)
[ 56] (1,71,51), (1,69,53), (1,45,77), (1,72,50)
[ 57] (1,71,51), (1,53,69), (1,80,42), (1,77,45)
[ 58] (1,71,51), (1,53,69), (1,77,45), (1,80,42)
[ 59] (1,71,51), (1,45,77), (1,72,50), (1,69,53)
[ 60] (1,71,51), (1,45,77), (1,69,53), (1,72,50)
[ 61] (1,69,53), (1,78,44), (1,77,45), (1,51,71)
[ 62] (1,69,53), (1,78,44), (1,51,71), (1,77,45)
[ 63] (1,69,53), (1,77,45), (1,78,44), (1,51,71)
[ 64] (1,69,53), (1,77,45), (1,51,71), (1,78,44)
[ 65] (1,69,53), (1,72,50), (1,71,51), (1,45,77)
[ 66] (1,69,53), (1,72,50), (1,45,77), (1,71,51)
[ 67] (1,69,53), (1,71,51), (1,72,50), (1,45,77)
[ 68] (1,69,53), (1,71,51), (1,45,77), (1,72,50)
[ 69] (1,69,53), (1,51,71), (1,78,44), (1,77,45)
[ 70] (1,69,53), (1,51,71), (1,77,45), (1,78,44)
[ 71] (1,69,53), (1,45,77), (1,72,50), (1,71,51)
[ 72] (1,69,53), (1,45,77), (1,71,51), (1,72,50)
[ 73] (1,54,68), (1,80,42), (1,78,44), (1,72,50)
[ 74] (1,54,68), (1,80,42), (1,72,50), (1,78,44)
[ 75] (1,54,68), (1,78,44), (1,80,42), (1,72,50)
```

```
[ 76] (1,54,68), (1,78,44), (1,72,50), (1,80,42)
[ 77] (1,54,68), (1,72,50), (1,80,42), (1,78,44)
[ 78] (1,54,68), (1,72,50), (1,78,44), (1,80,42)
[ 79] (1,54,68), (1,53,69), (1,51,71), (1,45,77)
[ 80] (1,54,68), (1,53,69), (1,45,77), (1,51,71)
[ 81] (1,54,68), (1,51,71), (1,53,69), (1,45,77)
[ 82] (1,54,68), (1,51,71), (1,45,77), (1,53,69)
[ 83] (1,54,68), (1,45,77), (1,53,69), (1,51,71)
[ 84] (1,54,68), (1,45,77), (1,51,71), (1,53,69)
[ 85] (1,53,69), (1,80,42), (1,77,45), (1,71,51)
[ 86] (1,53,69), (1,80,42), (1,71,51), (1,77,45)
[ 87] (1,53,69), (1,77,45), (1,80,42), (1,71,51)
[ 88] (1,53,69), (1,77,45), (1,71,51), (1,80,42)
[ 89] (1,53,69), (1,71,51), (1,80,42), (1,77,45)
[ 90] (1,53,69), (1,71,51), (1,77,45), (1,80,42)
[ 91] (1,53,69), (1,54,68), (1,51,71), (1,45,77)
[ 92] (1,53,69), (1,54,68), (1,45,77), (1,51,71)
[ 93] (1,53,69), (1,51,71), (1,54,68), (1,45,77)
[ 94] (1,53,69), (1,51,71), (1,45,77), (1,54,68)
[ 95] (1,53,69), (1,45,77), (1,54,68), (1,51,71)
[ 96] (1,53,69), (1,45,77), (1,51,71), (1,54,68)
[ 97] (1,51,71), (1,78,44), (1,77,45), (1,69,53)
[ 98] (1,51,71), (1,78,44), (1,69,53), (1,77,45)
[ 99] (1,51,71), (1,77,45), (1,78,44), (1,69,53)
[100] (1,51,71), (1,77,45), (1,69,53), (1,78,44)
[101] (1,51,71), (1,69,53), (1,78,44), (1,77,45)
[102] (1,51,71), (1,69,53), (1,77,45), (1,78,44)
[103] (1,51,71), (1,54,68), (1,53,69), (1,45,77)
[104] (1,51,71), (1,54,68), (1,45,77), (1,53,69)
[105] (1,51,71), (1,53,69), (1,54,68), (1,45,77)
[106] (1,51,71), (1,53,69), (1,45,77), (1,54,68)
[107] (1,51,71), (1,45,77), (1,54,68), (1,53,69)
[108] (1,51,71), (1,45,77), (1,53,69), (1,54,68)
[109] (1,45,77), (1,72,50), (1,71,51), (1,69,53)
[110] (1,45,77), (1,72,50), (1,69,53), (1,71,51)
[111] (1,45,77), (1,71,51), (1,72,50), (1,69,53)
[112] (1,45,77), (1,71,51), (1,69,53), (1,72,50)
[113] (1,45,77), (1,69,53), (1,72,50), (1,71,51)
[114] (1,45,77), (1,69,53), (1,71,51), (1,72,50)
[115] (1,45,77), (1,54,68), (1,53,69), (1,51,71)
[116] (1,45,77), (1,54,68), (1,51,71), (1,53,69)
[117] (1,45,77), (1,53,69), (1,54,68), (1,51,71)
[118] (1,45,77), (1,53,69), (1,51,71), (1,54,68)
[119] (1,45,77), (1,51,71), (1,54,68), (1,53,69)
[120] (1,45,77), (1,51,71), (1,53,69), (1,54,68)
 [Count = 120]

 {1,42,80} 12 + {1,80,42} 36 = 48
 {1,44,78} 12 + {1,78,44} 36 = 48
 {1,45,77} 48 + {1,77,45} 48 = 96
 {1,50,72} 12 + {1,72,50} 36 = 48
 {1,51,71} 48 + {1,71,51} 48 = 96
 {1,53,69} 48 + {1,69,53} 48 = 96
 {1,54,68} 36 + {1,68,54} 12 = 48
-----------------------------------
                      96 x 5 = 480
```

In the 120 examples there are only 7 combinations of values on the axes. They appeared on the 4th axis as equally as on the other three axes.

This implies our success in making the 4-dimensional magic object, doesn't it?

You could get the reasonable solution set by giving some inequality conditions such as: n1<n81; n1<n9; n1<n73; and n2>n10.

I found 60 standard solutions with n1=1 and 2784 solutions in all. This count is a eighth of 22272 primitive solutions.

#5. Another Way of Decomposing: 3^4 = 3 x 3 x(3 x 3) = 9 x 9

I noticed there is another way of decomposing 3^4 = 3 x 3 x(3 x 3) = 9 x 9 beside our 3 x(3 x 3 x 3). Take a look at the Figure 6 below.

Each layer of the 3 cubes is placed flat in reasonable order on the two dimensional 9 x 9 sheet shown below. There are 3 x 3 blocks in it, each of which is already a semi-magic square of order 3, and on the whole you could suppose a single large 9 x 9 Self-Complementary magic square with 9 blocks of semi-squares 3 x 3.

```
[Figure 6: Decomposition
     by 3 x 3 x(3 x 3)]
  |    1    |    2    |    3    |
--+---------+---------+---------|
  | 1  2  3|28 29 30|55 56 57|
  |        |        |        |
1 | 4  5  6|31 32 33|58 59 60|
  |        |        |        |
  | 7  8  9|34 35 36|61 62 63|
--+---------+---------+---------|
  |10 11 12|37 38 39|64 65 66|
  |        |        |        |
2 |13 14 15|40 41 42|67 68 69|
  |        |        |        |
  |16 17 18|43 44 45|70 71 72|
--+---------+---------+---------|
  |19 20 21|46 47 48|73 74 75|
  |        |        |        |
3 |22 23 24|49 50 51|76 77 78|
  |        |        |        |
  |25 26 27|52 53 54|79 80 81|
------------------------------'
```

```
       [No.1]
 1------10------19
 | 2      11     |20
 4   3--13--12--22--21
 | 5 |    14     |23 |
 7---6--16--15--25  24
   8 |    17     26 |
     9------18------27
       [No.2]
28------37------46
 |29      38     |47
31  30--40--39--49--48
 |32 |    41     |50 |
34--33--43--42--52  51
  35 |    44     53 |
    36------45------54
       [No.3]
55------64------73
 |56      65     |74
58  57--67--66--76--75
 |59 |    68     |77 |
61--60--70--69--79  78
  62 |    71     80 |
    63------72------81
```

Could it be really a self-complementary magic square of order 9?

(n1+n2+n3)+(n28+n29+n30)+(n55+n56+n57)=3K=369
(n1+n4+n7)+(n10+n13+n16)+(n19+n22+n25)=3K=369
n1+n5+n9+n37+n41+n45+n73+n77+n81
=(n1+n81)+(n5+n77)+(n9+n73)+(n37+n45)+n41
=C+C+C+C+n41=82+82+82+82+41=369

......

All complementary pairs of C are located symmetrically.

Yes, it is a self-complementary magic square 9 x 9, and at the same time it is a special magic square that has 9 semi-squares of order 3 within.

In each block every 3 entries of each row or each column add up to the same constant 123, but the two primary diagonals do not always add up to 123.

In the central block the next two equations of diagonals always become true.

n37+n41+n45=C+n41=123; n39+n41+n43=C+n41=123

Whenever you make a 4-dimensional magic object of order 3, you could always get a Self-complementary magic square 9 x 9 automatically.
It is amazing, isn't it?

Does this imply the new hope for 'Dimension Converter'? I would say the double decompositions could make it possible. Everything depends on:

$3^4 = 3 \times (3 \times 3 \times 3) = 3 \times 3 \times (3 \times 3)$

(Written in English on July 26, 2001; Retyped on May 9, 2005: Kanji Setsuda)

#6. Decomposition by 9th and 3rd increment

Watch the next figures below. The first Square of Order 9 is the one we got through down-conversion from the 4-d extra-cubic form of order 3. It has 3x3 little blocks of semi-magic squares 3x3 within.
Why don't you analyze it by Euler's method?

```
[ 1]
   1  78  44     72  29  22     50  16  57
  80  37   6     31  27  65     12  59  52
  42   8  73     20  67  36     61  48  14

  54  11  58      5  79  39     64  33  26
  13  63  47     75  41   7     35  19  69
  56  49  18     43   3  77     24  71  28

  68  34  21     46  15  62      9  74  40
  30  23  70     17  55  51     76  45   2
  25  66  32     60  53  10     38   4  81
/Notation by 9th increment     /Decomposition
  00 85 47   78 31 23   54 16 62    084 732 516   057 813 462
  87 40 05   33 28 71   12 64 56    840 327 165   705 381 246
  45 07 80   21 73 38   66 52 14    408 273 651   570 138 624

  58 11 63   04 86 42   70 35 27    516 084 732   813 462 057
  13 68 51   82 44 06   37 20 75    165 840 327   381 246 705
  61 53 18   46 02 84   25 77 30    651 408 273   138 624 570

  74 36 22   50 15 67   08 81 43    732 516 084   462 057 813
  32 24 76   17 60 55   83 48 01    327 165 840   246 705 381
  26 72 34   65 57 10   41 03 88    273 651 408   624 570 138
                                      [High:9^1]      [Low:9^0]
```

The succeeding figures above show the new notation and decomposition by 9th increment.
Watch the two layers of decomposition [High:9^1], [Low:9^0] on the right hand.
Each 9 entries on every row and column in each layer always consists of {0, 1, 2, 3, 4, 5, 6, 7, 8}, and uses each of them strictly once. It has no repetition and no drop-off of 9 members.
Therefore the sum of each row or each column is calculated in the same way as:

$(0+1+2+3+4+5+6+7+8) \times 9^1 + (0+1+2+3+4+5+6+7+8) \times 9^0$

$36 \times 9 + 36 \times 1 = 360$(Decimal) (It is always true that n1 = 0)

360 here is logically equivalent to 369 in our classical notation.
Both high layer and low one consist of {0, 1, 2, 3, 4, 5, 6, 7, 8} by 9 times.

These beautiful properties are the ones of so called ‘Euler Square’.

But the two primary diagonals of that solution are made of:

```
1+37+73+5+41+77+9+45+81
= (0+4+8+0+4+8+0+4+8) x 9 +(0+0+0+4+4+4+8+8+8) x 1 = 360
57+59+61+39+41+43+21+23+25
= (6+6+6+4+4+4+2+2+2) x 9 +(2+4+6+2+4+6+2+4+6) x 1 = 360
```

Although their sums are all right, their combinations of members are against the Euler's Law.

I feel it is getting harder for us to build any ‘Euler Square’ of higher orders.

I could find only several ‘Euler Squares’ by 9th increment as in the list below.

```
*** Decomposed Forms of Magic Square 9x9 ***  [:Figure 7]
****  'Euler Squares' by 9th increment  ****

  1/Magic Square 9x9 Converted /D9i
   1 78 44 72 29 22 50 16 57   084732516 057813462
  80 37  6 31 27 65 12 59 52   840327165 705381246
  42  8 73 20 67 36 61 48 14   408273651 570138624
  54 11 58  5 79 39 64 33 26   516084732 813462057
  13 63 47 75 41  7 35 19 69   165840327 381246705
  56 49 18 43  3 77 24 71 28   651408273 138624570
  68 34 21 46 15 62  9 74 40   732516084 462057813
  30 23 70 17 55 51 76 45  2   327165840 246705381
  25 66 32 60 53 10 38  4 81   273651408 624570138

 13/Magic Square 9x9 Converted /D9i
   2 76 45 69 35 19 52 12 59   084732516 138570624
  78 44  1 34 21 68 11 58 54   840327165 570624138
  43  3 77 20 67 36 60 53 10   408273651 624138570
  49 18 56  8 73 42 66 32 25   516084732 381705246
  17 55 51 75 41  7 31 27 65   165840327 705246381
  57 50 16 40  9 74 26 64 33   651408273 246381705
  72 29 22 46 15 62  5 79 39   732516084 813057462
  28 24 71 14 61 48 81 38  4   327165840 057462813
  23 70 30 63 47 13 37  6 80   273651408 462813057

 19/Magic Square 9x9 Converted /D9i
   3 76 44 70 29 24 50 18 55   084732516 237615480
  80 39  4 33 25 65 10 59 54   840327165 723561048
  40  8 75 20 69 34 63 46 14   408273651 372156804
  52 11 60  5 81 37 66 31 26   516084732 615480237
  15 61 47 73 41  9 35 21 67   165840327 561048723
  56 51 16 45  1 77 22 71 30   651408273 156804372
  68 36 19 48 13 62  7 74 42   732516084 480237615
  28 23 72 17 57 49 78 43  2   327165840 048723561
  27 64 32 58 53 12 38  6 79   273651408 804372156

 28/Magic Square 9x9 Converted /D9i
   4 74 45 71 33 19 48 16 59   084732516 318750264
  80 42  1 30 25 68 13 56 54   840327165 750264318
  39  7 77 22 65 36 62 51 10   408273651 264318750
  47 18 58  6 73 44 70 32 21   516084732 183507642
  15 55 53 79 41  3 29 27 67   165840327 507642183
  61 50 12 38  9 76 24 64 35   651408273 642183507
  72 31 20 46 17 60  5 75 43   732516084 831075426
  28 26 69 14 57 52 81 40  2   327165840 075426831
  23 66 34 63 49 11 37  8 78   273651408 426831075
```

32/Magic Square 9x9 Converted /D9i

5	73	45	66	35	22	52	15	56	084732516	408273651
79	42	2	32	19	72	12	62	49	840327165	651408273
39	8	76	25	69	29	59	46	18	408273651	273651408
54	14	55	4	75	44	65	34	24	516084732	840327165
11	61	51	81	41	1	31	21	71	165840327	165840327
58	48	17	38	7	78	27	68	28	651408273	327165840
64	36	23	53	13	57	6	74	43	732516084	084732516
33	20	70	10	63	50	80	40	3	327165840	516084732
26	67	30	60	47	16	37	9	77	273651408	732516084

. . . . . .

The last figure below demonstrates our new decomposed layers by 3rd increment. You can get this list because it depends upon 81 = 9x9 = 3x3x3x3.

1/Magic Square 9x9 Converted /D9i

1	78	44	72	29	22	50	16	57	084732516	057813462
80	37	6	31	27	65	12	59	52	840327165	705381246
42	8	73	20	67	36	61	48	14	408273651	570138624
54	11	58	5	79	39	64	33	26	516084732	813462057
13	63	47	75	41	7	35	19	69	165840327	381246705
56	49	18	43	3	77	24	71	28	651408273	138624570
68	34	21	46	15	62	9	74	40	732516084	462057813
30	23	70	17	55	51	76	45	2	327165840	246705381
25	66	32	60	53	10	38	4	81	273651408	624570138

/Decomposition by 3rd increment

021	210	102	021	102	210	012	201	120	021	210	102
210	102	021	210	021	102	201	120	012	102	021	210
102	021	210	102	210	021	120	012	201	210	102	021
102	021	210	210	021	102	201	120	012	210	102	021
021	210	102	102	210	021	120	012	201	021	210	102
210	102	021	021	102	210	012	201	120	102	021	210
210	102	021	102	210	021	120	012	201	102	021	210
102	021	210	021	102	210	012	201	120	210	102	021
021	210	102	210	021	102	201	120	012	021	210	102
	$[3^3]$			$[3^2]$			$[3^1]$			$[3^0]$	

(1) Every 9 entries of each row or each column on each layer consist of {0, 1, 2} by 3 times. All sums are calculated in the same way as:

$$\{(0+1+2)x3\}x3^3 + \{(0+1+2)x3\}x3^2 + \{(0+1+2)x3\}x3^1 + \{(0+1+2)x3\}x3^0$$
$$= 9x27 + 9x9 + 9x3 + 9x1 = 243 + 81 + 27 + 9 = 360 \text{(Decimal)}$$

(2) Every 9 entries of each primary diagonal on each layer consists of {0, 1, 2} by 3 times. Their sums are also calculated exactly in the same way.

(3) Each layer consists of {0, 1, 2} by 27 times.

What should we call these perfect equalities?

I think it is beautiful enough to be called 'Complete Euler Square'.

The last long list below contains part of 2784 ‘standard’ solutions of special magic squares 9x9 and their decompositions by the 3rd increment.

Almost all solutions are no longer 'Euler Squares' by 9th increment, but they are still 'Complete Euler Squares' by the 3rd increment.

It is amazing, isn't it?

We have easily got this beautiful result unexpectedly. I must say we are lucky.

But, to tell the truth I have recently known we could only have got 48 solutions of ‘Complete Euler Squares’ of order 9 including multiple 3x3 little squares within.
The rest 2736 pieces in the list below always have the number pattern of {1, 1, 1, 1, 1, 1, 1, 1, 1} anywhere in some of the primary diagonals of any decomposed layer by the 3rd increment. You can only get those precious 48 jewels, just when you explicitly design them for and make them as ‘Complete Euler Squares’ of order 9.

(Written in English on July 31, 2002, Revised on May 6, 2005: Kanji Setsuda)

*** E-Mail Address:<jag12001@nifty.ne.jp>

*** Four Dimensional Extra-Cubic Object of Order 3 ***
** Developed into Multiple 3x3 SC Magic Squares 9^2 **

[1]

```
    1-----80-----42       78-----37----- 8       44----- 6-----73
     |72     31     |20      |29     27     |67      |22     65     |36
   54  50-13--12-56--61   11  16-63--59-49--48   58  57-47--52-18--14
     | 5 |    75     |43 |    |79 |    41     | 3 |   |39 |     7     |77 |
   68--64-30--35-25  24   34--33-23--19-66  71   21--26-70--69-32  28
      46 |    17      60 |     15 |    55      53 |    62 |    51      10 |
        9-----76-----38       74-----45----- 4       40----- 2-----81
                         1/   /D3i
   1 80 42 78 37  8 44  6 73  021210102 021210102 021102210 012201120
  72 31 20 29 27 67 22 65 36  210102021 102021210 210021102 201120012
  50 12 61 16 59 48 57 52 14  102021210 210102021 102210021 120012201
  54 13 56 11 63 49 58 47 18  102021210 210102021 210021102 201120012
   5 75 43 79 41  3 39  7 77  021210102 021210102 102210021 120012201
  64 35 24 33 19 71 26 69 28  210102021 102021210 021102210 012201120
  68 30 25 34 23 66 21 70 32  210102021 102021210 102210021 120012201
  46 17 60 15 55 53 62 51 10  102021210 210102021 021102210 012201120
   9 76 38 74 45  4 40  2 81  021210102 021210102 210021102 201120012
```

[2]

```
    1-----80-----42       78-----37----- 8       44----- 6-----73
     |54     13     |56      |11     63     |49      |58     47     |18
   72  68-31--30-20--25   29  34-27--23-67--66   22  21-65--70-36--32
     | 5 |    75     |43 |    |79 |    41     | 3 |   |39 |     7     |77 |
   50--46-12--17-61  60   16--15-59--55-48  53   57--62-52--51-14  10
      64 |    35      24 |     33 |    19      71 |    26 |    69      28 |
        9-----76-----38       74-----45----- 4       40----- 2-----81
                         2/   /D3i
   1 80 42 78 37  8 44  6 73  021210102 021210102 021102210 012201120
  54 13 56 11 63 49 58 47 18  102021210 210102021 210021102 201120012
  68 30 25 34 23 66 21 70 32  210102021 102021210 102210021 120012201
  72 31 20 29 27 67 22 65 36  210102021 102021210 210021102 201120012
   5 75 43 79 41  3 39  7 77  021210102 021210102 102210021 120012201
  46 17 60 15 55 53 62 51 10  102021210 210102021 021102210 012201120
  50 12 61 16 59 48 57 52 14  102021210 210102021 102210021 120012201
  64 35 24 33 19 71 26 69 28  210102021 102021210 021102210 012201120
   9 76 38 74 45  4 40  2 81  021210102 021210102 210021102 201120012
```

[3]

```
    1-----80-----42       77-----39----- 7       45----- 4-----74
    |71     33    |19     |30     25    |68      |22     65    |36
   53  51-15--10-55--62   12  16-61--59-50--48   58  56-47--54-18--13
    | 6 |    73   |44 |   |79 |    41   | 3 |    |38 |    9    |76 |
   69--64-28--35-26  24   34--32-23--21-66  70   20--27-72--67-31  29
     46 |    17     60 |    14 |    57     52 |    63 |    49     11 |
       8-----78-----37        75-----43----- 5       40----- 2-----81
                                 3/  /D3i
   1 80 42 77 39  7 45  4 74  021210102 021210102 021102210 012120201
  71 33 19 30 25 68 22 65 36  210102021 102021210 210021102 120201012
  51 10 62 16 59 48 56 54 13  102021210 210102021 102210021 201012120
  53 15 55 12 61 50 58 47 18  102021210 210102021 210021102 120201012
   6 73 44 79 41  3 38  9 76  021210102 021210102 102210021 201012120
  64 35 24 32 21 70 27 67 29  210102021 102021210 021102210 012120201
  69 28 26 34 23 66 20 72 31  210102021 102021210 102210021 201012120
  46 17 60 14 57 52 63 49 11  102021210 210102021 021102210 012120201
   8 78 37 75 43  5 40  2 81  021210102 021210102 210021102 120201012
```

[4]

```
    1-----80-----42       77-----39----- 7       45----- 4-----74
    |53     15    |55     |12     61    |50      |58     47    |18
   71  69-33--28-19--26   30  34-25--23-68--66   22  20-65--72-36--31
    | 6 |    73   |44 |   |79 |    41   | 3 |    |38 |    9    |76 |
   51--46-10--17-62  60   16--14-59--57-48  52   56--63-54--49-13  11
     64 |    35     24 |    32 |    21     70 |    27 |    67     29 |
       8-----78-----37        75-----43----- 5       40----- 2-----81
                                 4/  /D3i
   1 80 42 77 39  7 45  4 74  021210102 021210102 021102210 012120201
  53 15 55 12 61 50 58 47 18  102021210 210102021 210021102 120201012
  69 28 26 34 23 66 20 72 31  210102021 102021210 102210021 201012120
  71 33 19 30 25 68 22 65 36  210102021 102021210 210021102 120201012
   6 73 44 79 41  3 38  9 76  021210102 021210102 102210021 201012120
  46 17 60 14 57 52 63 49 11  102021210 210102021 021102210 012120201
  51 10 62 16 59 48 56 54 13  102021210 210102021 102210021 201012120
  64 35 24 32 21 70 27 67 29  210102021 102021210 021102210 012120201
   8 78 37 75 43  5 40  2 81  021210102 021210102 210021102 120201012
```

[5]

```
    1-----80-----42       72-----31-----20       50-----12-----61
    |78     37    | 8     |29     27    |67      |16     59    |48
   54  44-13-- 6-56--73    5  22-75--65-43--36   64  57-35--52-24--14
    |11 |    63   |49 |   |79 |    41   | 3 |    |33 |    19   |71 |
   68--58-30--47-25  18   46--39-17-- 7-60  77    9--26-76--69-38  28
     34 |    23     66 |    15 |    55     53 |    74 |    45      4 |
      21-----70-----32        62-----51-----10       40----- 2-----81
                                 5/  /D3i
   1 80 42 72 31 20 50 12 61  021210102 021102210 021210102 012201120
  78 37  8 29 27 67 16 59 48  210102021 210021102 102021210 201120012
  44  6 73 22 65 36 57 52 14  102021210 102210021 210102021 120012201
  54 13 56  5 75 43 64 35 24  102021210 210021102 210102021 201120012
  11 63 49 79 41  3 33 19 71  021210102 102210021 021210102 120012201
  58 47 18 39  7 77 26 69 28  210102021 021102210 102021210 012201120
  68 30 25 46 17 60  9 76 38  210102021 102210021 102021210 120012201
  34 23 66 15 55 53 74 45  4  102021210 021102210 210102021 012201120
  21 70 32 62 51 10 40  2 81  021210102 210021102 021210102 201120012
```

```
[6]
    1-----80-----42         72-----31-----20         50-----12-----61
     |54      13     |56       | 5      75     |43       |64      35     |24
   78   68-37--30- 8--25    29   46-27--17-67--60    16    9-59--76-48--38
     |11 |     63     |49 |    |79 |     41     | 3 |    |33 |     19     |71 |
   44--34- 6--23-73   66    22--15-65--55-36   53    57--74-52--45-14    4
      58 |     47      18 |     39 |     7      77 |     26 |     69      28 |
        21-----70-----32        62-----51-----10         40----- 2-----81
                                      6/  /D3i
  1 80 42 72 31 20 50 12 61  021210102 021102210 021210102 012201120
 54 13 56  5 75 43 64 35 24  102021210 210021102 210102021 201120012
 68 30 25 46 17 60  9 76 38  210102021 102210021 102021210 120012201
 78 37  8 29 27 67 16 59 48  210102021 210021102 102021210 201120012
 11 63 49 79 41  3 33 19 71  021210102 102210021 021210102 120012201
 34 23 66 15 55 53 74 45  4  102021210 021102210 210102021 012201120
 44  6 73 22 65 36 57 52 14  102021210 102210021 210102021 120012201
 58 47 18 39  7 77 26 69 28  210102021 021102210 102021210 012201120
 21 70 32 62 51 10 40  2 81  021210102 210021102 021210102 201120012
```

```
                        7/
 1 80 42 71 33 19 51 10 62
77 39  7 30 25 68 16 59 48
45  4 74 22 65 36 56 54 13
53 15 55  6 73 44 64 35 24
12 61 50 79 41  3 32 21 70
58 47 18 38  9 76 27 67 29
69 28 26 46 17 60  8 78 37
34 23 66 14 57 52 75 43  5
20 72 31 63 49 11 40  2 81

                        8/
 1 80 42 71 33 19 51 10 62
53 15 55  6 73 44 64 35 24
69 28 26 46 17 60  8 78 37
77 39  7 30 25 68 16 59 48
12 61 50 79 41  3 32 21 70
34 23 66 14 57 52 75 43  5
45  4 74 22 65 36 56 54 13
58 47 18 38  9 76 27 67 29
20 72 31 63 49 11 40  2 81

                        9/
 1 80 42 54 13 56 68 30 25
78 37  8 11 63 49 34 23 66
44  6 73 58 47 18 21 70 32
72 31 20  5 75 43 46 17 60
29 27 67 79 41  3 15 55 53
22 65 36 39  7 77 62 51 10
50 12 61 64 35 24  9 76 38
16 59 48 33 19 71 74 45  4
57 52 14 26 69 28 40  2 81

                       10/
 1 80 42 54 13 56 68 30 25
72 31 20  5 75 43 46 17 60
50 12 61 64 35 24  9 76 38
78 37  8 11 63 49 34 23 66
29 27 67 79 41  3 15 55 53
16 59 48 33 19 71 74 45  4
44  6 73 58 47 18 21 70 32
22 65 36 39  7 77 62 51 10
57 52 14 26 69 28 40  2 81

                       11/
 1 80 42 53 15 55 69 28 26
77 39  7 12 61 50 34 23 66
45  4 74 58 47 18 20 72 31
71 33 19  6 73 44 46 17 60
30 25 68 79 41  3 14 57 52
22 65 36 38  9 76 63 49 11
51 10 62 64 35 24  8 78 37
16 59 48 32 21 70 75 43  5
56 54 13 27 67 29 40  2 81

                       12/
 1 80 42 53 15 55 69 28 26
71 33 19  6 73 44 46 17 60
51 10 62 64 35 24  8 78 37
77 39  7 12 61 50 34 23 66
30 25 68 79 41  3 14 57 52
16 59 48 32 21 70 75 43  5
45  4 74 58 47 18 20 72 31
22 65 36 38  9 76 63 49 11
56 54 13 27 67 29 40  2 81

                       13/
 1 78 44 80 37  6 42  8 73
72 29 22 31 27 65 20 67 36
50 16 57 12 59 52 61 48 14
54 11 58 13 63 47 56 49 18
 5 79 39 75 41  7 43  3 77
64 33 26 35 19 69 24 71 28
68 34 21 30 23 70 25 66 32
46 15 62 17 55 51 60 53 10
 9 74 40 76 45  2 38  4 81

                       14/
 1 78 44 80 37  6 42  8 73
54 11 58 13 63 47 56 49 18
68 34 21 30 23 70 25 66 32
72 29 22 31 27 65 20 67 36
 5 79 39 75 41  7 43  3 77
46 15 62 17 55 51 60 53 10
50 16 57 12 59 52 61 48 14
64 33 26 35 19 69 24 71 28
 9 74 40 76 45  2 38  4 81

                       15/
 1 78 44 77 43  3 45  2 76
69 35 19 34 21 68 20 67 36
53 10 60 12 59 52 58 54 11
51 17 55 16 57 50 56 49 18
 8 73 42 75 41  7 40  9 74
64 33 26 32 25 66 27 65 31
71 28 24 30 23 70 22 72 29
46 15 62 14 61 48 63 47 13
 6 80 37 79 39  5 38  4 81

                       16/
 1 78 44 77 43  3 45  2 76
51 17 55 16 57 50 56 49 18
71 28 24 30 23 70 22 72 29
69 35 19 34 21 68 20 67 36
 8 73 42 75 41  7 40  9 74
46 15 62 14 61 48 63 47 13
53 10 60 12 59 52 58 54 11
64 33 26 32 25 66 27 65 31
 6 80 37 79 39  5 38  4 81

                       17/
 1 78 44 72 29 22 50 16 57
54 11 58  5 79 39 64 33 26
68 34 21 46 15 62  9 74 40
80 37  6 31 27 65 12 59 52
13 63 47 75 41  7 35 19 69
30 23 70 17 55 51 76 45  2
42  8 73 20 67 36 61 48 14
56 49 18 43  3 77 24 71 28
25 66 32 60 53 10 38  4 81

                       18/
 1 78 44 69 35 19 53 10 60
77 43  3 34 21 68 12 59 52
45  2 76 20 67 36 58 54 11
51 17 55  8 73 42 64 33 26
16 57 50 75 41  7 32 25 66
56 49 18 40  9 74 27 65 31
71 28 24 46 15 62  6 80 37
30 23 70 14 61 48 79 39  5
22 72 29 63 47 13 38  4 81
```

19/

1	78	44	69	35	19	53	10	60
51	17	55	8	73	42	64	33	26
71	28	24	46	15	62	6	80	37
77	43	3	34	21	68	12	59	52
16	57	50	75	41	7	32	25	66
30	23	70	14	61	48	79	39	5
45	2	76	20	67	36	58	54	11
56	49	18	40	9	74	27	65	31
22	72	29	63	47	13	38	4	81

20/

1	78	44	54	11	58	68	34	21
72	29	22	5	79	39	46	15	62
50	16	57	64	33	26	9	74	40
80	37	6	13	63	47	30	23	70
31	27	65	75	41	7	17	55	51
12	59	52	35	19	69	76	45	2
42	8	73	56	49	18	25	66	32
20	67	36	43	3	77	60	53	10
61	48	14	24	71	28	38	4	81

21/

1	78	44	51	17	55	71	28	24
77	43	3	16	57	50	30	23	70
45	2	76	56	49	18	22	72	29
69	35	19	8	73	42	46	15	62
34	21	68	75	41	7	14	61	48
20	67	36	40	9	74	63	47	13
53	10	60	64	33	26	6	80	37
12	59	52	32	25	66	79	39	5
58	54	11	27	65	31	38	4	81

22/

1	78	44	51	17	55	71	28	24
69	35	19	8	73	42	46	15	62
53	10	60	64	33	26	6	80	37
77	43	3	16	57	50	30	23	70
34	21	68	75	41	7	14	61	48
12	59	52	32	25	66	79	39	5
45	2	76	56	49	18	22	72	29
20	67	36	40	9	74	63	47	13
58	54	11	27	65	31	38	4	81

23/

1	77	45	80	39	4	42	7	74
71	30	22	33	25	65	19	68	36
51	16	56	10	59	54	62	48	13
53	12	58	15	61	47	55	50	18
6	79	38	73	41	9	44	3	76
64	32	27	35	21	67	24	70	29
69	34	20	28	23	72	26	66	31
46	14	63	17	57	49	60	52	11
8	75	40	78	43	2	37	5	81

24/

1	77	45	80	39	4	42	7	74
53	12	58	15	61	47	55	50	18
69	34	20	28	23	72	26	66	31
71	30	22	33	25	65	19	68	36
6	79	38	73	41	9	44	3	76
46	14	63	17	57	49	60	52	11
51	16	56	10	59	54	62	48	13
64	32	27	35	21	67	24	70	29
8	75	40	78	43	2	37	5	81

25/

1	77	45	78	43	2	44	3	76
69	34	20	35	21	67	19	68	36
53	12	58	10	59	54	60	52	11
51	16	56	17	57	49	55	50	18
8	75	40	73	41	9	42	7	74
64	32	27	33	25	65	26	66	31
71	30	22	28	23	72	24	70	29
46	14	63	15	61	47	62	48	13
6	79	38	80	39	4	37	5	81

26/

1	77	45	78	43	2	44	3	76
51	16	56	17	57	49	55	50	18
71	30	22	28	23	72	24	70	29
69	34	20	35	21	67	19	68	36
8	75	40	73	41	9	42	7	74
46	14	63	15	61	47	62	48	13
53	12	58	10	59	54	60	52	11
64	32	27	33	25	65	26	66	31
6	79	38	80	39	4	37	5	81

27/

1	77	45	71	30	22	51	16	56
53	12	58	6	79	38	64	32	27
69	34	20	46	14	63	8	75	40
80	39	4	33	25	65	10	59	54
15	61	47	73	41	9	35	21	67
28	23	72	17	57	49	78	43	2
42	7	74	19	68	36	62	48	13
55	50	18	44	3	76	24	70	29
26	66	31	60	52	11	37	5	81

28/

1	77	45	69	34	20	53	12	58
51	16	56	8	75	40	64	32	27
71	30	22	46	14	63	6	79	38
78	43	2	35	21	67	10	59	54
17	57	49	73	41	9	33	25	65
28	23	72	15	61	47	80	39	4
44	3	76	19	68	36	60	52	11
55	50	18	42	7	74	26	66	31
24	70	29	62	48	13	37	5	81

29/

1	77	45	53	12	58	69	34	20
71	30	22	6	79	38	46	14	63
51	16	56	64	32	27	8	75	40
80	39	4	15	61	47	28	23	72
33	25	65	73	41	9	17	57	49
10	59	54	35	21	67	78	43	2
42	7	74	55	50	18	26	66	31
19	68	36	44	3	76	60	52	11
62	48	13	24	70	29	37	5	81

30/

1	77	45	51	16	56	71	30	22
69	34	20	8	75	40	46	14	63
53	12	58	64	32	27	6	79	38
78	43	2	17	57	49	28	23	72
35	21	67	73	41	9	15	61	47
10	59	54	33	25	65	80	39	4
44	3	76	55	50	18	24	70	29
19	68	36	42	7	74	62	48	13
60	52	11	26	66	31	37	5	81

31/

1	72	50	80	31	12	42	20	61
54	5	64	13	75	35	56	43	24
68	46	9	30	17	76	25	60	38
78	29	16	37	27	59	8	67	48
11	79	33	63	41	19	49	3	71
34	15	74	23	55	45	66	53	4
44	22	57	6	65	52	73	36	14
58	39	26	47	7	69	18	77	28
21	62	40	70	51	2	32	10	81

32/

1	72	50	78	29	16	44	22	57
54	5	64	11	79	33	58	39	26
68	46	9	34	15	74	21	62	40
80	31	12	37	27	59	6	65	52
13	75	35	63	41	19	47	7	69
30	17	76	23	55	45	70	51	2
42	20	61	8	67	48	73	36	14
56	43	24	49	3	71	18	77	28
25	60	38	66	53	4	32	10	81

33/

1	72	50	71	49	3	51	2	70
69	47	7	46	9	68	8	67	48
53	4	66	6	65	52	64	54	5
45	23	55	22	57	44	56	43	24
20	61	42	63	41	19	40	21	62
58	39	26	38	25	60	27	59	37
77	28	18	30	17	76	16	78	29
34	15	74	14	73	36	75	35	13
12	80	31	79	33	11	32	10	81

34/

1	72	50	71	49	3	51	2	70
45	23	55	22	57	44	56	43	24
77	28	18	30	17	76	16	78	29
69	47	7	46	9	68	8	67	48
20	61	42	63	41	19	40	21	62
34	15	74	14	73	36	75	35	13
53	4	66	6	65	52	64	54	5
58	39	26	38	25	60	27	59	37
12	80	31	79	33	11	32	10	81

35/

1	72	50	69	47	7	53	4	66
71	49	3	46	9	68	6	65	52
51	2	70	8	67	48	64	54	5
45	23	55	20	61	42	58	39	26
22	57	44	63	41	19	38	25	60
56	43	24	40	21	62	27	59	37
77	28	18	34	15	74	12	80	31
30	17	76	14	73	36	79	33	11
16	78	29	75	35	13	32	10	81

36/

1	72	50	69	47	7	53	4	66
45	23	55	20	61	42	58	39	26
77	28	18	34	15	74	12	80	31
71	49	3	46	9	68	6	65	52
22	57	44	63	41	19	38	25	60
30	17	76	14	73	36	79	33	11
51	2	70	8	67	48	64	54	5
56	43	24	40	21	62	27	59	37
16	78	29	75	35	13	32	10	81

37/

1	72	50	45	23	55	77	28	18
71	49	3	22	57	44	30	17	76
51	2	70	56	43	24	16	78	29
69	47	7	20	61	42	34	15	74
46	9	68	63	41	19	14	73	36
8	67	48	40	21	62	75	35	13
53	4	66	58	39	26	12	80	31
6	65	52	38	25	60	79	33	11
64	54	5	27	59	37	32	10	81

38/

1	72	50	45	23	55	77	28	18
69	47	7	20	61	42	34	15	74
53	4	66	58	39	26	12	80	31
71	49	3	22	57	44	30	17	76
46	9	68	63	41	19	14	73	36
6	65	52	38	25	60	79	33	11
51	2	70	56	43	24	16	78	29
8	67	48	40	21	62	75	35	13
64	54	5	27	59	37	32	10	81

39/

1	71	51	80	33	10	42	19	62
53	6	64	15	73	35	55	44	24
69	46	8	28	17	78	26	60	37
77	30	16	39	25	59	7	68	48
12	79	32	61	41	21	50	3	70
34	14	75	23	57	43	66	52	5
45	22	56	4	65	54	74	36	13
58	38	27	47	9	67	18	76	29
20	63	40	72	49	2	31	11	81

40/

1	71	51	77	30	16	45	22	56
53	6	64	12	79	32	58	38	27
69	46	8	34	14	75	20	63	40
80	33	10	39	25	59	4	65	54
15	73	35	61	41	21	47	9	67
28	17	78	23	57	43	72	49	2
42	19	62	7	68	48	74	36	13
55	44	24	50	3	70	18	76	29
26	60	37	66	52	5	31	11	81

41/

1	71	51	72	49	2	50	3	70
69	46	8	47	9	67	7	68	48
53	6	64	4	65	54	66	52	5
45	22	56	23	57	43	55	44	24
20	63	40	61	41	21	42	19	62
58	38	27	39	25	59	26	60	37
77	30	16	28	17	78	18	76	29
34	14	75	15	73	35	74	36	13
12	79	32	80	33	10	31	11	81

42/

1	71	51	72	49	2	50	3	70
45	22	56	23	57	43	55	44	24
77	30	16	28	17	78	18	76	29
69	46	8	47	9	67	7	68	48
20	63	40	61	41	21	42	19	62
34	14	75	15	73	35	74	36	13
53	6	64	4	65	54	66	52	5
58	38	27	39	25	59	26	60	37
12	79	32	80	33	10	31	11	81

43/

1	71	51	69	46	8	53	6	64
45	22	56	20	63	40	58	38	27
77	30	16	34	14	75	12	79	32
72	49	2	47	9	67	4	65	54
23	57	43	61	41	21	39	25	59
28	17	78	15	73	35	80	33	10
50	3	70	7	68	48	66	52	5
55	44	24	42	19	62	26	60	37
18	76	29	74	36	13	31	11	81

44/

1	71	51	45	22	56	77	30	16
69	46	8	20	63	40	34	14	75
53	6	64	58	38	27	12	79	32
72	49	2	23	57	43	28	17	78
47	9	67	61	41	21	15	73	35
4	65	54	39	25	59	80	33	10
50	3	70	55	44	24	18	76	29
7	68	48	42	19	62	74	36	13
66	52	5	26	60	37	31	11	81

45/

1	69	53	78	35	10	44	19	60
51	8	64	17	73	33	55	42	26
71	46	6	28	15	80	24	62	37
77	34	12	43	21	59	3	68	52
16	75	32	57	41	25	50	7	66
30	14	79	23	61	39	70	48	5
45	20	58	2	67	54	76	36	11
56	40	27	49	9	65	18	74	31
22	63	38	72	47	4	29	13	81

46/

1	69	53	77	34	12	45	20	58
51	8	64	16	75	32	56	40	27
71	46	6	30	14	79	22	63	38
78	35	10	43	21	59	2	67	54
17	73	33	57	41	25	49	9	65
28	15	80	23	61	39	72	47	4
44	19	60	3	68	52	76	36	11
55	42	26	50	7	66	18	74	31
24	62	37	70	48	5	29	13	81

47/

1	69	53	72	47	4	50	7	66
45	20	58	23	61	39	55	42	26
77	34	12	28	15	80	18	74	31
71	46	6	49	9	65	3	68	52
22	63	38	57	41	25	44	19	60
30	14	79	17	73	33	76	36	11
51	8	64	2	67	54	70	48	5
56	40	27	43	21	59	24	62	37
16	75	32	78	35	10	29	13	81

48/

1	69	53	71	46	6	51	8	64
45	20	58	22	63	38	56	40	27
77	34	12	30	14	79	16	75	32
72	47	4	49	9	65	2	67	54
23	61	39	57	41	25	43	21	59
28	15	80	17	73	33	78	35	10
50	7	66	3	68	52	70	48	5
55	42	26	44	19	60	24	62	37
18	74	31	76	36	11	29	13	81

49/

1	54	68	53	67	3	69	2	52
51	65	7	64	9	50	8	49	66
71	4	48	6	47	70	46	72	5
45	59	19	58	21	44	20	43	60
56	25	42	27	41	55	40	57	26
22	39	62	38	61	24	63	23	37
77	10	36	12	35	76	34	78	11
16	33	74	32	73	18	75	17	31
30	80	13	79	15	29	14	28	81

50/

1	54	68	53	67	3	69	2	52
45	59	19	58	21	44	20	43	60
77	10	36	12	35	76	34	78	11
51	65	7	64	9	50	8	49	66
56	25	42	27	41	55	40	57	26
16	33	74	32	73	18	75	17	31
71	4	48	6	47	70	46	72	5
22	39	62	38	61	24	63	23	37
30	80	13	79	15	29	14	28	81

51/

1	54	68	51	65	7	71	4	48
53	67	3	64	9	50	6	47	70
69	2	52	8	49	66	46	72	5
45	59	19	56	25	42	22	39	62
58	21	44	27	41	55	38	61	24
20	43	60	40	57	26	63	23	37
77	10	36	16	33	74	30	80	13
12	35	76	32	73	18	79	15	29
34	78	11	75	17	31	14	28	81

52/

1	54	68	51	65	7	71	4	48
45	59	19	56	25	42	22	39	62
77	10	36	16	33	74	30	80	13
53	67	3	64	9	50	6	47	70
58	21	44	27	41	55	38	61	24
12	35	76	32	73	18	79	15	29
69	2	52	8	49	66	46	72	5
20	43	60	40	57	26	63	23	37
34	78	11	75	17	31	14	28	81

53/

1	54	68	45	59	19	77	10	36
53	67	3	58	21	44	12	35	76
69	2	52	20	43	60	34	78	11
51	65	7	56	25	42	16	33	74
64	9	50	27	41	55	32	73	18
8	49	66	40	57	26	75	17	31
71	4	48	22	39	62	30	80	13
6	47	70	38	61	24	79	15	29
46	72	5	63	23	37	14	28	81

54/

1	54	68	45	59	19	77	10	36
51	65	7	56	25	42	16	33	74
71	4	48	22	39	62	30	80	13
53	67	3	58	21	44	12	35	76
64	9	50	27	41	55	32	73	18
6	47	70	38	61	24	79	15	29
69	2	52	20	43	60	34	78	11
8	49	66	40	57	26	75	17	31
46	72	5	63	23	37	14	28	81

55/

1	53	69	54	67	2	68	3	52
51	64	8	65	9	49	7	50	66
71	6	46	4	47	72	48	70	5
45	58	20	59	21	43	19	44	60
56	27	40	25	41	57	42	55	26
22	38	63	39	61	23	62	24	37
77	12	34	10	35	78	36	76	11
16	32	75	33	73	17	74	18	31
30	79	14	80	15	28	13	29	81

56/

1	53	69	54	67	2	68	3	52
45	58	20	59	21	43	19	44	60
77	12	34	10	35	78	36	76	11
51	64	8	65	9	49	7	50	66
56	27	40	25	41	57	42	55	26
16	32	75	33	73	17	74	18	31
71	6	46	4	47	72	48	70	5
22	38	63	39	61	23	62	24	37
30	79	14	80	15	28	13	29	81

57/

1	53	69	51	64	8	71	6	46
45	58	20	56	27	40	22	38	63
77	12	34	16	32	75	30	79	14
54	67	2	65	9	49	4	47	72
59	21	43	25	41	57	39	61	23
10	35	78	33	73	17	80	15	28
68	3	52	7	50	66	48	70	5
19	44	60	42	55	26	62	24	37
36	76	11	74	18	31	13	29	81

58/

1	53	69	45	58	20	77	12	34
51	64	8	56	27	40	16	32	75
71	6	46	22	38	63	30	79	14
54	67	2	59	21	43	10	35	78
65	9	49	25	41	57	33	73	17
4	47	72	39	61	23	80	15	28
68	3	52	19	44	60	36	76	11
7	50	66	42	55	26	74	18	31
48	70	5	62	24	37	13	29	81

59/

1	51	71	54	65	4	68	7	48
45	56	22	59	25	39	19	42	62
77	16	30	10	33	80	36	74	13
53	64	6	67	9	47	3	50	70
58	27	38	21	41	61	44	55	24
12	32	79	35	73	15	76	18	29
69	8	46	2	49	72	52	66	5
20	40	63	43	57	23	60	26	37
34	75	14	78	17	28	11	31	81

60/

1	51	71	53	64	6	69	8	46
45	56	22	58	27	38	20	40	63
77	16	30	12	32	79	34	75	14
54	65	4	67	9	47	2	49	72
59	25	39	21	41	61	43	57	23
10	33	80	35	73	15	78	17	28
68	7	48	3	50	70	52	66	5
19	42	62	44	55	24	60	26	37
36	74	13	76	18	29	11	31	81

61/

2	81	40	78	37	8	43	5	75
70	32	21	29	27	67	24	64	35
51	10	62	16	59	48	56	54	13
52	14	57	11	63	49	60	46	17
6	73	44	79	41	3	38	9	76
65	36	22	33	19	71	25	68	30
69	28	26	34	23	66	20	72	31
47	18	58	15	55	53	61	50	12
7	77	39	74	45	4	42	1	80

62/

2	81	40	78	37	8	43	5	75
52	14	57	11	63	49	60	46	17
69	28	26	34	23	66	20	72	31
70	32	21	29	27	67	24	64	35
6	73	44	79	41	3	38	9	76
47	18	58	15	55	53	61	50	12
51	10	62	16	59	48	56	54	13
65	36	22	33	19	71	25	68	30
7	77	39	74	45	4	42	1	80

63/

2	81	40	76	38	9	45	4	74
72	31	20	29	27	67	22	65	36
49	11	63	18	58	47	56	54	13
52	14	57	12	61	50	59	48	16
5	75	43	79	41	3	39	7	77
66	34	23	32	21	70	25	68	30
69	28	26	35	24	64	19	71	33
46	17	60	15	55	53	62	51	10
8	78	37	73	44	6	42	1	80

. . . .

2776/

39	76	8	4	44	75	80	3	40
22	35	66	71	21	31	30	67	26
62	12	49	48	58	17	13	53	57
59	18	46	54	55	14	10	50	63
45	73	5	1	41	81	77	9	37
19	32	72	68	27	28	36	64	23
25	29	69	65	24	34	33	70	20
56	15	52	51	61	11	16	47	60
42	79	2	7	38	78	74	6	43

2777/

39	59	25	22	45	56	62	19	42
4	54	65	71	1	51	48	68	7
80	10	33	30	77	16	13	36	74
76	18	29	35	73	15	12	32	79
44	55	24	21	41	61	58	27	38
3	50	70	67	9	47	53	64	6
8	46	69	66	5	52	49	72	2
75	14	34	31	81	11	17	28	78
40	63	20	26	37	60	57	23	43

2778/

39	58	26	23	45	55	61	20	42
4	53	66	72	1	50	47	69	7
80	12	31	28	77	18	15	34	74
76	17	30	36	73	14	11	33	79
44	57	22	19	41	63	60	25	38
3	49	71	68	9	46	52	65	6
8	48	67	64	5	54	51	70	2
75	13	35	32	81	10	16	29	78
40	62	21	27	37	59	56	24	43

2779/

39	58	26	22	44	57	62	21	40
5	54	64	72	1	50	46	68	9
79	11	33	29	78	16	15	34	74
76	17	30	35	75	13	12	31	80
45	55	23	19	41	63	59	27	37
2	51	70	69	7	47	52	65	6
8	48	67	66	4	53	49	71	3
73	14	36	32	81	10	18	28	77
42	61	20	25	38	60	56	24	43

2780/

39	58	26	22	44	57	62	21	40
4	53	66	71	3	49	48	67	8
80	12	31	30	76	17	13	35	75
77	18	28	36	73	14	10	32	81
45	55	23	19	41	63	59	27	37
1	50	72	68	9	46	54	64	5
7	47	69	65	6	52	51	70	2
74	15	34	33	79	11	16	29	78
42	61	20	25	38	60	56	24	43

2781/

39	23	61	58	45	20	26	55	42
4	72	47	53	1	69	66	50	7
80	28	15	12	77	34	31	18	74
76	36	11	17	73	33	30	14	79
44	19	60	57	41	25	22	63	38
3	68	52	49	9	65	71	46	6
8	64	51	48	5	70	67	54	2
75	32	16	13	81	29	35	10	78
40	27	56	62	37	24	21	59	43

2782/

39	22	62	59	45	19	25	56	42
4	71	48	54	1	68	65	51	7
80	30	13	10	77	36	33	16	74
76	35	12	18	73	32	29	15	79
44	21	58	55	41	27	24	61	38
3	67	53	50	9	64	70	47	6
8	66	49	46	5	72	69	52	2
75	31	17	14	81	28	34	11	78
40	26	57	63	37	23	20	60	43

2783/

39	22	62	58	44	21	26	57	40
5	72	46	54	1	68	64	50	9
79	29	15	11	78	34	33	16	74
76	35	12	17	75	31	30	13	80
45	19	59	55	41	27	23	63	37
2	69	52	51	7	65	70	47	6
8	66	49	48	4	71	67	53	3
73	32	18	14	81	28	36	10	77
42	25	56	61	38	24	20	60	43

2784/

39	22	62	58	44	21	26	57	40
4	71	48	53	3	67	66	49	8
80	30	13	12	76	35	31	17	75
77	36	10	18	73	32	28	14	81
45	19	59	55	41	27	23	63	37
1	68	54	50	9	64	72	46	5
7	65	51	47	6	70	69	52	2
74	33	16	15	79	29	34	11	78
42	25	56	61	38	24	20	60	43