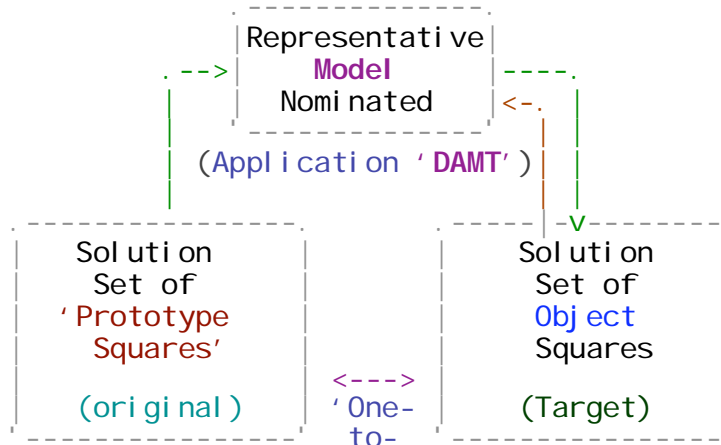


Chapter 1: Fundamental Study of 'Prototype' Squares and 'Do-it-After-the-Model' Transformation: **Kanji Setsuda**

Section 2: Prototype Squares of Order 4 and 'Do-it-After-the-Model Transformation'

Let's go on studying about the two 'parallel worlds' of squares and the beautiful 'bridge' between them.

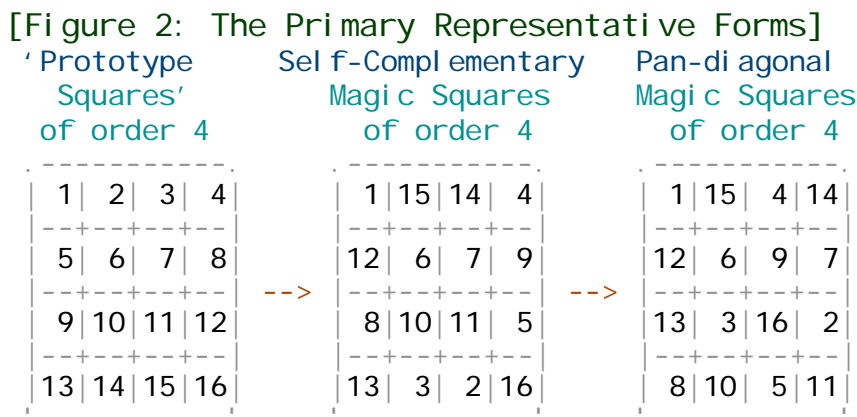
** 'Prototype Square' and 'Do-it-After-the-Model Transformation' **



[Figure 1] One' Correspondence

Now we are going to study those of order 4. One is the world of 'Prototype Squares', and the other is the world of magic squares: (1) Self-complementary type and (2) Pan-diagonal one.

Look at each example of these three squares shown below.



These three are all the primary representative squares of (1) 'Prototype Squares', (2) Self-complementary and (3) Pan-diagonal magic squares of Order 4.

We already know that each type of the latter two has the set of as many as 48 'standard' solutions (equivalent to 384 'primitive' solutions).

The primary prototype square is made after the regular array 4 x 4.

The value of each entry is always equal to the value of each position name itself.

n1=1, n2=2, n3=3, ... , n10=10, ... , n15=15 and n16=16

#1. 'Prototype Squares' for Self-Complementary Magic Squares of Order 4

15 16 13 14 15 16 13 14 [Figure 3: Extended Form]

3	4	1	2	3	4	1	2	Complementary pairs of 17: $1+16=2+15=3+14=4+13$ $=5+12=6+11=7+10=8+9=17$ Eight pairs are all located symmetrically. $1+2+3+4=10, 5+6+7+8=26,$ $9+10+11+12=42, \dots,$ $1+5+9+13=28, 2+6+10+14=32,$ $\dots, 4+8+12+16=40$
7	8	5	6	7	8	5	6	
11	12	9	10	11	12	9	10	
15	16	13	14	15	16	13	14	
3	4	1	2	3	4	1	2	

But $1+6+11+16=34, 1+8+11+14=34, 2+7+12+13=34, 2+5+12+15=34,$
 $3+8+9+14=34, 3+6+9+16=34, 4+5+10+15=34, 4+7+10+13=34$
 Four entries on every pan-diagonal add up to the same sum 34.

This 'Prototype Square' is, as you can see, already the 'simultaneous' type of both self-complementary and pan-diagonal, although no row and no column add up to the same constant.

On top of that it has some other interesting properties such as:

- $1+6=2+5, 2+7=3+6, 3+8=4+7, 5+10=6+9, 6+11=7+10, 7+12=8+11,$
- $9+14=10+13, 10+15=11+14, 11+16=12+15$ (Equality of Cross-sums)
- $1+7=3+5, 2+8=4+6, 1+8=4+5, 5+11=7+9, 5+12=8+9,$
- $6+12=8+10, 9+15=11+13, 10+16=12+14, 9+16=12+13,$
- $1+10=2+9, 5+14=6+13, 1+14=2+13, 2+11=3+10, 6+15=7+14,$
- $2+15=3+14, 3+12=4+11, 7+16=8+15, 3+16=4+15,$
- $1+11=3+9, 2+12=4+10, 5+15=7+13, 6+16=8+14, 1+16=4+13$
- (Equality of Cross-sums in broad sense)
- $1+4=2+3, 5+8=6+7, 9+12=10+11, 13+16=14+15,$
- $1+13=5+9, 2+14=6+10, 3+15=7+11, 4+16=8+12$
- (The sum of outer entries is equal to the sum of inner entries in a line.)

1-2. How many different patterns can you make for this set of prototype squares?
 Try to rearrange all entries and make new patterns under those strict conditions.
 Put it no inequality conditions like $n1 < n4, n1 < n13, n1 < n16$ or $n2 > n5$ before all.
 Of course you can use your personal computer to count them through.
 And you will get 384 different patterns in all like the list shown below.

*** List 1: 'Prototype Squares' of Order 4 (Part) ***

1/	2/	3/	4/	5/	6/
1 2 3 4	1 2 3 4	1 2 5 6	1 2 5 6	1 2 9 10	1 2 9 10
5 6 7 8	9 10 11 12	3 4 7 8	9 10 13 14	3 4 11 12	5 6 13 14
9 10 11 12	5 6 7 8	9 10 13 14	3 4 7 8	5 6 13 14	3 4 11 12
13 14 15 16	13 14 15 16	11 12 15 16	11 12 15 16	7 8 15 16	7 8 15 16
7/	8/	9/	10/	11/	12/
1 3 2 4	1 3 2 4	1 3 5 7	1 3 5 7	1 3 9 11	1 3 9 11
5 7 6 8	9 11 10 12	2 4 6 8	9 11 13 15	2 4 10 12	5 7 13 15
9 11 10 12	5 7 6 8	9 11 13 15	2 4 6 8	5 7 13 15	2 4 10 12
13 15 14 16	13 15 14 16	10 12 14 16	10 12 14 16	6 8 14 16	6 8 14 16
13/	14/	15/	16/	17/	18/
1 5 2 6	1 5 2 6	1 5 3 7	1 5 3 7	1 5 9 13	1 5 9 13
3 7 4 8	9 13 10 14	2 6 4 8	9 13 11 15	2 6 10 14	3 7 11 15
9 13 10 14	3 7 4 8	9 13 11 15	2 6 4 8	3 7 11 15	2 6 10 14
11 15 12 16	11 15 12 16	10 14 12 16	10 14 12 16	4 8 12 16	4 8 12 16
19/	20/	21/	22/	23/	24/
1 9 2 10	1 9 2 10	1 9 3 11	1 9 3 11	1 9 5 13	1 9 5 13
3 11 4 12	5 13 6 14	2 10 4 12	5 13 7 15	2 10 6 14	3 11 7 15
5 13 6 14	3 11 4 12	5 13 7 15	2 10 4 12	3 11 7 15	2 10 6 14
7 15 8 16	7 15 8 16	6 14 8 16	6 14 8 16	4 12 8 16	4 12 8 16

25/ 2 1 4 3 6 5 8 7 10 9 12 11 14 13 16 15	26/ 2 1 4 3 10 9 12 11 6 5 8 7 14 13 16 15	27/ 2 1 6 5 4 3 8 7 10 9 14 13 12 11 16 15	28/ 2 1 6 5 10 9 14 13 4 3 8 7 12 11 16 15	29/ 2 1 10 9 4 3 12 11 6 5 14 13 8 7 16 15	30/ 2 1 10 9 6 5 14 13 4 3 12 11 8 7 16 15
31/ 2 4 1 3 6 8 5 7 10 12 9 11 14 16 13 15	32/ 2 4 1 3 10 12 9 11 6 8 5 7 14 16 13 15	33/ 2 4 6 8 1 3 5 7 10 12 14 16 9 11 13 15	34/ 2 4 6 8 10 12 14 16 1 3 5 7 9 11 13 15	35/ 2 4 10 12 1 3 9 11 6 8 14 16 5 7 13 15	36/ 2 4 10 12 6 8 14 16 1 3 9 11 5 7 13 15
37/ 2 6 1 5 4 8 3 7 10 14 9 13 12 16 11 15	38/ 2 6 1 5 10 14 9 13 4 8 3 7 12 16 11 15	39/ 2 6 4 8 1 5 3 7 10 14 12 16 9 13 11 15	40/ 2 6 4 8 10 14 12 16 1 5 3 7 9 13 11 15	41/ 2 6 10 14 1 5 9 13 4 8 12 16 3 7 11 15	42/ 2 6 10 14 4 8 12 16 1 5 9 13 3 7 11 15
43/ 2 10 1 9 4 12 3 11 6 14 5 13 8 16 7 15	44/ 2 10 1 9 6 14 5 13 4 12 3 11 8 16 7 15	45/ 2 10 4 12 1 9 3 11 6 14 8 16 5 13 7 15	46/ 2 10 4 12 6 14 8 16 1 9 3 11 5 13 7 15	47/ 2 10 6 14 1 9 5 13 4 12 8 16 3 11 7 15	48/ 2 10 6 14 4 12 8 16 1 9 5 13 3 11 7 15
49/ 3 1 4 2 7 5 8 6 11 9 12 10 15 13 16 14	50/ 3 1 4 2 11 9 12 10 7 5 8 6 15 13 16 14	51/ 3 1 7 5 4 2 8 6 11 9 15 13 12 10 16 14	52/ 3 1 7 5 11 9 15 13 4 2 8 6 12 10 16 14	53/ 3 1 11 9 4 2 12 10 7 5 15 13 8 6 16 14	54/ 3 1 11 9 7 5 15 13 4 2 12 10 8 6 16 14
55/ 3 4 1 2 7 8 5 6 11 12 9 10 15 16 13 14	56/ 3 4 1 2 11 12 9 10 7 8 5 6 15 16 13 14	57/ 3 4 7 8 1 2 5 6 11 12 15 16 9 10 13 14	58/ 3 4 7 8 11 12 15 16 1 2 5 6 9 10 13 14	59/ 3 4 11 12 1 2 9 10 7 8 15 16 5 6 13 14	60/ 3 4 11 12 7 8 15 16 1 2 9 10 5 6 13 14
61/ 3 7 1 5 4 8 2 6 11 15 9 13 12 16 10 14	62/ 3 7 1 5 11 15 9 13 4 8 2 6 12 16 10 14	63/ 3 7 4 8 1 5 2 6 11 15 12 16 9 13 10 14	64/ 3 7 4 8 11 15 12 16 1 5 2 6 9 13 10 14	65/ 3 7 11 15 1 5 9 13 4 8 12 16 2 6 10 14	66/ 3 7 11 15 4 8 12 16 1 5 9 13 2 6 10 14
67/ 3 11 1 9 4 12 2 10 7 15 5 13 8 16 6 14	68/ 3 11 1 9 7 15 5 13 4 12 2 10 8 16 6 14	69/ 3 11 4 12 1 9 2 10 7 15 8 16 5 13 6 14	70/ 3 11 4 12 7 15 8 16 1 9 2 10 5 13 6 14	71/ 3 11 7 15 1 9 5 13 4 12 8 16 2 10 6 14	72/ 3 11 7 15 4 12 8 16 1 9 5 13 2 10 6 14
73/ 4 2 3 1 8 6 7 5 12 10 11 9 16 14 15 13	74/ 4 2 3 1 12 10 11 9 8 6 7 5 16 14 15 13	75/ 4 2 8 6 3 1 7 5 12 10 16 14 11 9 15 13	76/ 4 2 8 6 12 10 16 14 3 1 7 5 11 9 15 13	77/ 4 2 12 10 3 1 11 9 8 6 16 14 7 5 15 13	78/ 4 2 12 10 8 6 16 14 3 1 11 9 7 5 15 13

... (Skip) ...

355/ 15 16 7 8 11 12 3 4 13 14 5 6 9 10 1 2	356/ 15 16 7 8 13 14 5 6 11 12 3 4 9 10 1 2	357/ 15 16 11 12 7 8 3 4 13 14 9 10 5 6 1 2	358/ 15 16 11 12 13 14 9 10 7 8 3 4 5 6 1 2	359/ 15 16 13 14 7 8 5 6 11 12 9 10 3 4 1 2	360/ 15 16 13 14 11 12 9 10 7 8 5 6 3 4 1 2
361/ 16 8 12 4 14 6 10 2 15 7 11 3 13 5 9 1	362/ 16 8 12 4 15 7 11 3 14 6 10 2 13 5 9 1	363/ 16 8 14 6 12 4 10 2 15 7 13 5 11 3 9 1	364/ 16 8 14 6 15 7 13 5 12 4 10 2 11 3 9 1	365/ 16 8 15 7 12 4 11 3 14 6 13 5 10 2 9 1	366/ 16 8 15 7 14 6 13 5 12 4 11 3 10 2 9 1
367/ 16 12 8 4 14 10 6 2 15 11 7 3 13 9 5 1	368/ 16 12 8 4 15 11 7 3 14 10 6 2 13 9 5 1	369/ 16 12 14 10 8 4 6 2 15 11 13 9 7 3 5 1	370/ 16 12 14 10 15 11 13 9 8 4 6 2 7 3 5 1	371/ 16 12 15 11 8 4 7 3 14 10 13 9 6 2 5 1	372/ 16 12 15 11 14 10 13 9 8 4 7 3 6 2 5 1

16 14 8 6	16 14 8 6	16 14 12 10	16 14 12 10	16 14 15 13	16 14 15 13
12 10 4 2	15 13 7 5	8 6 4 2	15 13 11 9	8 6 7 5	12 10 11 9
15 13 7 5	12 10 4 2	15 13 11 9	8 6 4 2	12 10 11 9	8 6 7 5
11 9 3 1	11 9 3 1	7 5 3 1	7 5 3 1	4 2 3 1	4 2 3 1
16 15 8 7	16 15 8 7	16 15 12 11	16 15 12 11	16 15 14 13	16 15 14 13
12 11 4 3	14 13 6 5	8 7 4 3	14 13 10 9	8 7 6 5	12 11 10 9
14 13 6 5	12 11 4 3	14 13 10 9	8 7 4 3	12 11 10 9	8 7 6 5
10 9 2 1	10 9 2 1	6 5 2 1	6 5 2 1	4 3 2 1	4 3 2 1

[Count = 384]

1-3. How can you combine these 384 patterns of Prototype Squares above to those 384 primitive solutions of self-complementary magic squares below one by one?

** List 2: Self-Complementary Magic Squares 4*4: 384 Solutions **

1 15 14 4	1 15 14 4	1 15 12 6	1 15 12 6	1 15 8 10	1 15 8 10
12 6 7 9	8 10 11 5	14 4 7 9	8 10 13 3	14 4 11 5	12 6 13 3
8 10 11 5	12 6 7 9	8 10 13 3	14 4 7 9	12 6 13 3	14 4 11 5
13 3 2 16	13 3 2 16	11 5 2 16	11 5 2 16	7 9 2 16	7 9 2 16
1 14 15 4	1 14 15 4	1 14 12 7	1 14 12 7	1 14 8 11	1 14 8 11
12 7 6 9	8 11 10 5	15 4 6 9	8 11 13 2	15 4 10 5	12 7 13 2
8 11 10 5	12 7 6 9	8 11 13 2	15 4 6 9	12 7 13 2	15 4 10 5
13 2 3 16	13 2 3 16	10 5 3 16	10 5 3 16	6 9 3 16	6 9 3 16
1 12 15 6	1 12 15 6	1 12 14 7	1 12 14 7	1 12 8 13	1 12 8 13
14 7 4 9	8 13 10 3	15 6 4 9	8 13 11 2	15 6 10 3	14 7 11 2
8 13 10 3	14 7 4 9	8 13 11 2	15 6 4 9	14 7 11 2	15 6 10 3
11 2 5 16	11 2 5 16	10 3 5 16	10 3 5 16	4 9 5 16	4 9 5 16
1 8 15 10	1 8 15 10	1 8 14 11	1 8 14 11	1 8 12 13	1 8 12 13
14 11 4 5	12 13 6 3	15 10 4 5	12 13 7 2	15 10 6 3	14 11 7 2
12 13 6 3	14 11 4 5	12 13 7 2	15 10 4 5	14 11 7 2	15 10 6 3
7 2 9 16	7 2 9 16	6 3 9 16	6 3 9 16	4 5 9 16	4 5 9 16
2 16 13 3	2 16 13 3	2 16 11 5	2 16 11 5	2 16 7 9	2 16 7 9
11 5 8 10	7 9 12 6	13 3 8 10	7 9 14 4	13 3 12 6	11 5 14 4
7 9 12 6	11 5 8 10	7 9 14 4	13 3 8 10	11 5 14 4	13 3 12 6
14 4 1 15	14 4 1 15	12 6 1 15	12 6 1 15	8 10 1 15	8 10 1 15
2 13 16 3	2 13 16 3	2 13 11 8	2 13 11 8	2 13 7 12	2 13 7 12
11 8 5 10	7 12 9 6	16 3 5 10	7 12 14 1	16 3 9 6	11 8 14 1
7 12 9 6	11 8 5 10	7 12 14 1	16 3 5 10	11 8 14 1	16 3 9 6
14 1 4 15	14 1 4 15	9 6 4 15	9 6 4 15	5 10 4 15	5 10 4 15
2 11 16 5	2 11 16 5	2 11 13 8	2 11 13 8	2 11 7 14	2 11 7 14
13 8 3 10	7 14 9 4	16 5 3 10	7 14 12 1	16 5 9 4	13 8 12 1
7 14 9 4	13 8 3 10	7 14 12 1	16 5 3 10	13 8 12 1	16 5 9 4
12 1 6 15	12 1 6 15	9 4 6 15	9 4 6 15	3 10 6 15	3 10 6 15
2 7 16 9	2 7 16 9	2 7 13 12	2 7 13 12	2 7 11 14	2 7 11 14
13 12 3 6	11 14 5 4	16 9 3 6	11 14 8 1	16 9 5 4	13 12 8 1
11 14 5 4	13 12 3 6	11 14 8 1	16 9 3 6	13 12 8 1	16 9 5 4
8 1 10 15	8 1 10 15	5 4 10 15	5 4 10 15	3 6 10 15	3 6 10 15
3 16 13 2	3 16 13 2	3 16 10 5	3 16 10 5	3 16 6 9	3 16 6 9
10 5 8 11	6 9 12 7	13 2 8 11	6 9 15 4	13 2 12 7	10 5 15 4
6 9 12 7	10 5 8 11	6 9 15 4	13 2 8 11	10 5 15 4	13 2 12 7
15 4 1 14	15 4 1 14	12 7 1 14	12 7 1 14	8 11 1 14	8 11 1 14

55/ 3 13 16 2 10 8 5 11 6 12 9 7 15 1 4 14	56/ 3 13 16 2 6 12 9 7 10 8 5 11 15 1 4 14	57/ 3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14	58/ 3 13 10 8 6 12 15 1 16 2 5 11 9 7 4 14	59/ 3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14	60/ 3 13 6 12 10 8 15 1 16 2 9 7 5 11 4 14
61/ 3 10 16 5 13 8 2 11 6 15 9 4 12 1 7 14	62/ 3 10 16 5 6 15 9 4 13 8 2 11 12 1 7 14	63/ 3 10 13 8 16 5 2 11 6 15 12 1 9 4 7 14	64/ 3 10 13 8 6 15 12 1 16 5 2 11 9 4 7 14	65/ 3 10 6 15 16 5 9 4 13 8 12 1 2 11 7 14	66/ 3 10 6 15 13 8 12 1 16 5 9 4 2 11 7 14
67/ 3 6 16 9 13 12 2 7 10 15 5 4 8 1 11 14	68/ 3 6 16 9 10 15 5 4 13 12 2 7 8 1 11 14	69/ 3 6 13 12 16 9 2 7 10 15 8 1 5 4 11 14	70/ 3 6 13 12 10 15 8 1 16 9 2 7 5 4 11 14	71/ 3 6 10 15 16 9 5 4 13 12 8 1 2 7 11 14	72/ 3 6 10 15 13 12 8 1 16 9 5 4 2 7 11 14
73/ 4 15 14 1 9 6 7 12 5 10 11 8 16 3 2 13	74/ 4 15 14 1 5 10 11 8 9 6 7 12 16 3 2 13	75/ 4 15 9 6 14 1 7 12 5 10 16 3 11 8 2 13	76/ 4 15 9 6 5 10 16 3 14 1 7 12 11 8 2 13	77/ 4 15 5 10 14 1 11 8 9 6 16 3 7 12 2 13	78/ 4 15 5 10 9 6 16 3 14 1 11 8 7 12 2 13

. . . (Ski p). . .

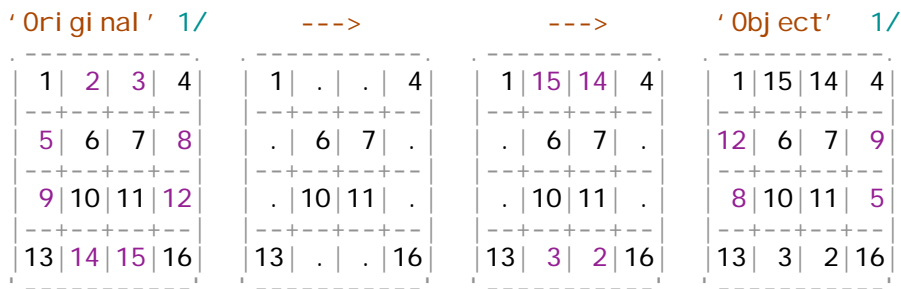
355/ 15 1 10 8 6 12 3 13 4 14 5 11 9 7 16 2	356/ 15 1 10 8 4 14 5 11 6 12 3 13 9 7 16 2	357/ 15 1 6 12 10 8 3 13 4 14 9 7 5 11 16 2	358/ 15 1 6 12 4 14 9 7 10 8 3 13 5 11 16 2	359/ 15 1 4 14 10 8 5 11 6 12 9 7 3 13 16 2	360/ 15 1 4 14 6 12 9 7 10 8 5 11 3 13 16 2
361/ 16 9 5 4 3 6 10 15 2 7 11 14 13 12 8 1	362/ 16 9 5 4 2 7 11 14 3 6 10 15 13 12 8 1	363/ 16 9 3 6 5 4 10 15 2 7 13 12 11 14 8 1	364/ 16 9 3 6 2 7 13 12 5 4 10 15 11 14 8 1	365/ 16 9 2 7 5 4 11 14 3 6 13 12 10 15 8 1	366/ 16 9 2 7 3 6 13 12 5 4 11 14 10 15 8 1
367/ 16 5 9 4 3 10 6 15 2 11 7 14 13 8 12 1	368/ 16 5 9 4 2 11 7 14 3 10 6 15 13 8 12 1	369/ 16 5 3 10 9 4 6 15 2 11 13 8 7 14 12 1	370/ 16 5 3 10 2 11 13 8 9 4 6 15 7 14 12 1	371/ 16 5 2 11 9 4 7 14 3 10 13 8 6 15 12 1	372/ 16 5 2 11 3 10 13 8 9 4 7 14 6 15 12 1
373/ 16 3 9 6 5 10 4 15 2 13 7 12 11 8 14 1	374/ 16 3 9 6 2 13 7 12 5 10 4 15 11 8 14 1	375/ 16 3 5 10 9 6 4 15 2 13 11 8 7 12 14 1	376/ 16 3 5 10 2 13 11 8 9 6 4 15 7 12 14 1	377/ 16 3 2 13 9 6 7 12 5 10 11 8 4 15 14 1	378/ 16 3 2 13 5 10 11 8 9 6 7 12 4 15 14 1
379/ 16 2 9 7 5 11 4 14 3 13 6 12 10 8 15 1	380/ 16 2 9 7 3 13 6 12 5 11 4 14 10 8 15 1	381/ 16 2 5 11 9 7 4 14 3 13 10 8 6 12 15 1	382/ 16 2 5 11 3 13 10 8 9 7 4 14 6 12 15 1	383/ 16 2 3 13 9 7 6 12 5 11 10 8 4 14 15 1	384/ 16 2 3 13 5 11 10 8 9 7 6 12 4 14 15 1

[Count = 384]

At first we have to find the way how to transform the primary representative form of Prototype Squares (Let's call it the "original") into the one of self-complementary magic squares (Let's call it the "object"). And then we have to examine if we can apply the very transformation method to the other 'original' squares one after another to make them all into our 'object' solutions without any fails.

Let me introduce you the concept of transformation I always assume recently. It could be shown in the next diagrams.

[Figure 4: The Transformation]



- (1) Hold 8 entries on the two primary diagonals still in the new 'object' form.
- (2) What do we have to do for the rest 8 ones?
Find four complementary pairs of 17 such as: (2, 15), (3, 14), (5, 12) and (8, 9); and exchange their locations with each partner as shown above.

1-4. Apply this way of transformation to each solution of the original, and you will get each other solution of the object. Examine if this transformation works all right.

The "Rules of Transformation" below should express this transformation method in another way. They are dictated literally after the object model.

- (1) Put the value of $n_1(\text{original})$ into $n_1(\text{object})$.
- (2) Put the value of $n_{15}(\text{original})$ into $n_2(\text{object})$.
- (3) Put the value of $n_{14}(\text{original})$ into $n_3(\text{object})$.
- (4) Put the value of $n_4(\text{original})$ into $n_4(\text{object})$.
-
- (9) Put the value of $n_8(\text{original})$ into $n_9(\text{object})$.
-
- (15) Put the value of $n_2(\text{original})$ into $n_{15}(\text{object})$.
- (16) Put the value of $n_{16}(\text{original})$ into $n_{16}(\text{object})$.

In your computer program these rules might be easy for you to write.

Once you select the representative model, make these rules after it and apply them to the other 'prototype' solutions one after another, you can automatically have got all the rest object squares.

Let's call this way of making objects as "Do-it-After-the-Model Transformation".

The next list shows the result of this transformation method.

Each number on the shoulder of object ('S/n') stands for the number of old list of Self-complementary solutions.

**** List 3: 'Prototype Squares' 4*4 and S-C MS44 Recomposed ****

<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>1/P</td><td>S/1</td></tr> <tr><td>1 2 3 4</td><td>1 15 14 4</td></tr> <tr><td>5 6 7 8</td><td>12 6 7 9</td></tr> <tr><td>9 10 11 12</td><td>8 10 11 5</td></tr> <tr><td>13 14 15 16</td><td>13 3 2 16</td></tr> </table>	1/P	S/1	1 2 3 4	1 15 14 4	5 6 7 8	12 6 7 9	9 10 11 12	8 10 11 5	13 14 15 16	13 3 2 16	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>2/P</td><td>S/2</td></tr> <tr><td>1 2 3 4</td><td>1 15 14 4</td></tr> <tr><td>9 10 11 12</td><td>8 10 11 5</td></tr> <tr><td>5 6 7 8</td><td>12 6 7 9</td></tr> <tr><td>13 14 15 16</td><td>13 3 2 16</td></tr> </table>	2/P	S/2	1 2 3 4	1 15 14 4	9 10 11 12	8 10 11 5	5 6 7 8	12 6 7 9	13 14 15 16	13 3 2 16	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>3/P</td><td>S/3</td></tr> <tr><td>1 2 5 6</td><td>1 15 12 6</td></tr> <tr><td>3 4 7 8</td><td>14 4 7 9</td></tr> <tr><td>9 10 13 14</td><td>8 10 13 3</td></tr> <tr><td>11 12 15 16</td><td>11 5 2 16</td></tr> </table>	3/P	S/3	1 2 5 6	1 15 12 6	3 4 7 8	14 4 7 9	9 10 13 14	8 10 13 3	11 12 15 16	11 5 2 16
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9 10 13 14	8 10 13 3																															
3 4 7 8	14 4 7 9																															
11 12 15 16	11 5 2 16																															
5/P	S/5																															
1 2 9 10	1 15 8 10																															
3 4 11 12	14 4 11 5																															
5 6 13 14	12 6 13 3																															
7 8 15 16	7 9 2 16																															
6/P	S/6																															
1 2 9 10	1 15 8 10																															
5 6 13 14	12 6 13 3																															
3 4 11 12	14 4 11 5																															
7 8 15 16	7 9 2 16																															
<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>7/P</td><td>S/7</td></tr> <tr><td>1 3 2 4</td><td>1 14 15 4</td></tr> <tr><td>5 7 6 8</td><td>12 7 6 9</td></tr> <tr><td>9 11 10 12</td><td>8 11 10 5</td></tr> <tr><td>13 15 14 16</td><td>13 2 3 16</td></tr> </table>	7/P	S/7	1 3 2 4	1 14 15 4	5 7 6 8	12 7 6 9	9 11 10 12	8 11 10 5	13 15 14 16	13 2 3 16	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>8/P</td><td>S/8</td></tr> <tr><td>1 3 2 4</td><td>1 14 15 4</td></tr> <tr><td>9 11 10 12</td><td>8 11 10 5</td></tr> <tr><td>5 7 6 8</td><td>12 7 6 9</td></tr> <tr><td>13 15 14 16</td><td>13 2 3 16</td></tr> </table>	8/P	S/8	1 3 2 4	1 14 15 4	9 11 10 12	8 11 10 5	5 7 6 8	12 7 6 9	13 15 14 16	13 2 3 16	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr><td>9/P</td><td>S/9</td></tr> <tr><td>1 3 5 7</td><td>1 14 12 7</td></tr> <tr><td>2 4 6 8</td><td>15 4 6 9</td></tr> <tr><td>9 11 13 15</td><td>8 11 13 2</td></tr> <tr><td>10 12 14 16</td><td>10 5 3 16</td></tr> </table>	9/P	S/9	1 3 5 7	1 14 12 7	2 4 6 8	15 4 6 9	9 11 13 15	8 11 13 2	10 12 14 16	10 5 3 16
7/P	S/7																															
1 3 2 4	1 14 15 4																															
5 7 6 8	12 7 6 9																															
9 11 10 12	8 11 10 5																															
13 15 14 16	13 2 3 16																															
8/P	S/8																															
1 3 2 4	1 14 15 4																															
9 11 10 12	8 11 10 5																															
5 7 6 8	12 7 6 9																															
13 15 14 16	13 2 3 16																															
9/P	S/9																															
1 3 5 7	1 14 12 7																															
2 4 6 8	15 4 6 9																															
9 11 13 15	8 11 13 2																															
10 12 14 16	10 5 3 16																															

10/P S/10	11/P S/11	12/P S/12
1 3 5 7 1 14 12 7	1 3 9 11 1 14 8 11	1 3 9 11 1 14 8 11
9 11 13 15 8 11 13 2	2 4 10 12 15 4 10 5	5 7 13 15 12 7 13 2
2 4 6 8 15 4 6 9	5 7 13 15 12 7 13 2	2 4 10 12 15 4 10 5
10 12 14 16 10 5 3 16	6 8 14 16 6 9 3 16	6 8 14 16 6 9 3 16
13/P S/13	14/P S/14	15/P S/15
1 5 2 6 1 12 15 6	1 5 2 6 1 12 15 6	1 5 3 7 1 12 14 7
3 7 4 8 14 7 4 9	9 13 10 14 8 13 10 3	2 6 4 8 15 6 4 9
9 13 10 14 8 13 10 3	3 7 4 8 14 7 4 9	9 13 11 15 8 13 11 2
11 15 12 16 11 2 5 16	11 15 12 16 11 2 5 16	10 14 12 16 10 3 5 16
16/P S/16	17/P S/17	18/P S/18
1 5 3 7 1 12 14 7	1 5 9 13 1 12 8 13	1 5 9 13 1 12 8 13
9 13 11 15 8 13 11 2	2 6 10 14 15 6 10 3	3 7 11 15 14 7 11 2
2 6 4 8 15 6 4 9	3 7 11 15 14 7 11 2	2 6 10 14 15 6 10 3
10 14 12 16 10 3 5 16	4 8 12 16 4 9 5 16	4 8 12 16 4 9 5 16
19/P S/19	20/P S/20	21/P S/21
1 9 2 10 1 8 15 10	1 9 2 10 1 8 15 10	1 9 3 11 1 8 14 11
3 11 4 12 14 11 4 5	5 13 6 14 12 13 6 3	2 10 4 12 15 10 4 5
5 13 6 14 12 13 6 3	3 11 4 12 14 11 4 5	5 13 7 15 12 13 7 2
7 15 8 16 7 2 9 16	7 15 8 16 7 2 9 16	6 14 8 16 6 3 9 16
22/P S/22	23/P S/23	24/P S/24
1 9 3 11 1 8 14 11	1 9 5 13 1 8 12 13	1 9 5 13 1 8 12 13
5 13 7 15 12 13 7 2	2 10 6 14 15 10 6 3	3 11 7 15 14 11 7 2
2 10 4 12 15 10 4 5	3 11 7 15 14 11 7 2	2 10 6 14 15 10 6 3
6 14 8 16 6 3 9 16	4 12 8 16 4 5 9 16	4 12 8 16 4 5 9 16
25/P S/25	26/P S/26	27/P S/27
2 1 4 3 2 16 13 3	2 1 4 3 2 16 13 3	2 1 6 5 2 16 11 5
6 5 8 7 11 5 8 10	10 9 12 11 7 9 12 6	4 3 8 7 13 3 8 10
10 9 12 11 7 9 12 6	6 5 8 7 11 5 8 10	10 9 14 13 7 9 14 4
14 13 16 15 14 4 1 15	14 13 16 15 14 4 1 15	12 11 16 15 12 6 1 15
28/P S/28	29/P S/29	30/P S/30
2 1 6 5 2 16 11 5	2 1 10 9 2 16 7 9	2 1 10 9 2 16 7 9
10 9 14 13 7 9 14 4	4 3 12 11 13 3 12 6	6 5 14 13 11 5 14 4
4 3 8 7 13 3 8 10	6 5 14 13 11 5 14 4	4 3 12 11 13 3 12 6
12 11 16 15 12 6 1 15	8 7 16 15 8 10 1 15	8 7 16 15 8 10 1 15
31/P S/31	32/P S/32	33/P S/33
2 4 1 3 2 13 16 3	2 4 1 3 2 13 16 3	2 4 6 8 2 13 11 8
6 8 5 7 11 8 5 10	10 12 9 11 7 12 9 6	1 3 5 7 16 3 5 10
10 12 9 11 7 12 9 6	6 8 5 7 11 8 5 10	10 12 14 16 7 12 14 1
14 16 13 15 14 1 4 15	14 16 13 15 14 1 4 15	9 11 13 15 9 6 4 15
34/P S/34	35/P S/35	36/P S/36
2 4 6 8 2 13 11 8	2 4 10 12 2 13 7 12	2 4 10 12 2 13 7 12
10 12 14 16 7 12 14 1	1 3 9 11 16 3 9 6	6 8 14 16 11 8 14 1
1 3 5 7 16 3 5 10	6 8 14 16 11 8 14 1	1 3 9 11 16 3 9 6
9 11 13 15 9 6 4 15	5 7 13 15 5 10 4 15	5 7 13 15 5 10 4 15
37/P S/37	38/P S/38	39/P S/39
2 6 1 5 2 11 16 5	2 6 1 5 2 11 16 5	2 6 4 8 2 11 13 8
4 8 3 7 13 8 3 10	10 14 9 13 7 14 9 4	1 5 3 7 16 5 3 10
10 14 9 13 7 14 9 4	4 8 3 7 13 8 3 10	10 14 12 16 7 14 12 1
12 16 11 15 12 1 6 15	12 16 11 15 12 1 6 15	9 13 11 15 9 4 6 15
40/P S/40	41/P S/41	42/P S/42
2 6 4 8 2 11 13 8	2 6 10 14 2 11 7 14	2 6 10 14 2 11 7 14
10 14 12 16 7 14 12 1	1 5 9 13 16 5 9 4	4 8 12 16 13 8 12 1
1 5 3 7 16 5 3 10	4 8 12 16 13 8 12 1	1 5 9 13 16 5 9 4
9 13 11 15 9 4 6 15	3 7 11 15 3 10 6 15	3 7 11 15 3 10 6 15

43/P S/43	44/P S/44	45/P S/45
2 10 1 9 2 7 16 9	2 10 1 9 2 7 16 9	2 10 4 12 2 7 13 12
4 12 3 11 13 12 3 6	6 14 5 13 11 14 5 4	1 9 3 11 16 9 3 6
6 14 5 13 11 14 5 4	4 12 3 11 13 12 3 6	6 14 8 16 11 14 8 1
8 16 7 15 8 1 10 15	8 16 7 15 8 1 10 15	5 13 7 15 5 4 10 15
46/P S/46	47/P S/47	48/P S/48
2 10 4 12 2 7 13 12	2 10 6 14 2 7 11 14	2 10 6 14 2 7 11 14
6 14 8 16 11 14 8 1	1 9 5 13 16 9 5 4	4 12 8 16 13 12 8 1
1 9 3 11 16 9 3 6	4 12 8 16 13 12 8 1	1 9 5 13 16 9 5 4
5 13 7 15 5 4 10 15	3 11 7 15 3 6 10 15	3 11 7 15 3 6 10 15
49/P S/49	50/P S/50	51/P S/51
3 1 4 2 3 16 13 2	3 1 4 2 3 16 13 2	3 1 7 5 3 16 10 5
7 5 8 6 10 5 8 11	11 9 12 10 6 9 12 7	4 2 8 6 13 2 8 11
11 9 12 10 6 9 12 7	7 5 8 6 10 5 8 11	11 9 15 13 6 9 15 4
15 13 16 14 15 4 1 14	15 13 16 14 15 4 1 14	12 10 16 14 12 7 1 14
52/P S/52	53/P S/53	54/P S/54
3 1 7 5 3 16 10 5	3 1 11 9 3 16 6 9	3 1 11 9 3 16 6 9
11 9 15 13 6 9 15 4	4 2 12 10 13 2 12 7	7 5 15 13 10 5 15 4
4 2 8 6 13 2 8 11	7 5 15 13 10 5 15 4	4 2 12 10 13 2 12 7
12 10 16 14 12 7 1 14	8 6 16 14 8 11 1 14	8 6 16 14 8 11 1 14
55/P S/55	56/P S/56	57/P S/57
3 4 1 2 3 13 16 2	3 4 1 2 3 13 16 2	3 4 7 8 3 13 10 8
7 8 5 6 10 8 5 11	11 12 9 10 6 12 9 7	1 2 5 6 16 2 5 11
11 12 9 10 6 12 9 7	7 8 5 6 10 8 5 11	11 12 15 16 6 12 15 1
15 16 13 14 15 1 4 14	15 16 13 14 15 1 4 14	9 10 13 14 9 7 4 14
58/P S/58	59/P S/59	60/P S/60
3 4 7 8 3 13 10 8	3 4 11 12 3 13 6 12	3 4 11 12 3 13 6 12
11 12 15 16 6 12 15 1	1 2 9 10 16 2 9 7	7 8 15 16 10 8 15 1
1 2 5 6 16 2 5 11	7 8 15 16 10 8 15 1	1 2 9 10 16 2 9 7
9 10 13 14 9 7 4 14	5 6 13 14 5 11 4 14	5 6 13 14 5 11 4 14
61/P S/61	62/P S/62	63/P S/63
3 7 1 5 3 10 16 5	3 7 1 5 3 10 16 5	3 7 4 8 3 10 13 8
4 8 2 6 13 8 2 11	11 15 9 13 6 15 9 4	1 5 2 6 16 5 2 11
11 15 9 13 6 15 9 4	4 8 2 6 13 8 2 11	11 15 12 16 6 15 12 1
12 16 10 14 12 1 7 14	12 16 10 14 12 1 7 14	9 13 10 14 9 4 7 14
64/P S/64	65/P S/65	66/P S/66
3 7 4 8 3 10 13 8	3 7 11 15 3 10 6 15	3 7 11 15 3 10 6 15
11 15 12 16 6 15 12 1	1 5 9 13 16 5 9 4	4 8 12 16 13 8 12 1
1 5 2 6 16 5 2 11	4 8 12 16 13 8 12 1	1 5 9 13 16 5 9 4
9 13 10 14 9 4 7 14	2 6 10 14 2 11 7 14	2 6 10 14 2 11 7 14
67/P S/67	68/P S/68	69/P S/69
3 11 1 9 3 6 16 9	3 11 1 9 3 6 16 9	3 11 4 12 3 6 13 12
4 12 2 10 13 12 2 7	7 15 5 13 10 15 5 4	1 9 2 10 16 9 2 7
7 15 5 13 10 15 5 4	4 12 2 10 13 12 2 7	7 15 8 16 10 15 8 1
8 16 6 14 8 1 11 14	8 16 6 14 8 1 11 14	5 13 6 14 5 4 11 14
70/P S/70	71/P S/71	72/P S/72
3 11 4 12 3 6 13 12	3 11 7 15 3 6 10 15	3 11 7 15 3 6 10 15
7 15 8 16 10 15 8 1	1 9 5 13 16 9 5 4	4 12 8 16 13 12 8 1
1 9 2 10 16 9 2 7	4 12 8 16 13 12 8 1	1 9 5 13 16 9 5 4
5 13 6 14 5 4 11 14	2 10 6 14 2 7 11 14	2 10 6 14 2 7 11 14
73/P S/73	74/P S/74	75/P S/75
4 2 3 1 4 15 14 1	4 2 3 1 4 15 14 1	4 2 8 6 4 15 9 6
8 6 7 5 9 6 7 12	12 10 11 9 5 10 11 8	3 1 7 5 14 1 7 12
12 10 11 9 5 10 11 8	8 6 7 5 9 6 7 12	12 10 16 14 5 10 16 3
16 14 15 13 16 3 2 13	16 14 15 13 16 3 2 13	11 9 15 13 11 8 2 13

. . . (Skip) . . .

376/P	S/376	377/P	S/377	378/P	S/378
16 14 12 10	16 3 5 10	16 14 15 13	16 3 2 13	16 14 15 13	16 3 2 13
15 13 11 9	2 13 11 8	8 6 7 5	9 6 7 12	12 10 11 9	5 10 11 8
8 6 4 2	9 6 4 15	12 10 11 9	5 10 11 8	8 6 7 5	9 6 7 12
7 5 3 1	7 12 14 1	4 2 3 1	4 15 14 1	4 2 3 1	4 15 14 1
379/P	S/379	380/P	S/380	381/P	S/381
16 15 8 7	16 2 9 7	16 15 8 7	16 2 9 7	16 15 12 11	16 2 5 11
12 11 4 3	5 11 4 14	14 13 6 5	3 13 6 12	8 7 4 3	9 7 4 14
14 13 6 5	3 13 6 12	12 11 4 3	5 11 4 14	14 13 10 9	3 13 10 8
10 9 2 1	10 8 15 1	10 9 2 1	10 8 15 1	6 5 2 1	6 12 15 1
382/P	S/382	383/P	S/383	384/P	S/384
16 15 12 11	16 2 5 11	16 15 14 13	16 2 3 13	16 15 14 13	16 2 3 13
14 13 10 9	3 13 10 8	8 7 6 5	9 7 6 12	12 11 10 9	5 11 10 8
8 7 4 3	9 7 4 14	12 11 10 9	5 11 10 8	8 7 6 5	9 7 6 12
6 5 2 1	6 12 15 1	4 3 2 1	4 14 15 1	4 3 2 1	4 14 15 1

*** Monitor List of Correspondence between Old List and New ***

```

??: 0
1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
25: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
49: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
73: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
97: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
121: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
145: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
169: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
193: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
217: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
241: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
265: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
289: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
313: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
337: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
361: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

[Count = 384/384]

I tried to check the solution number of each transformed answer by consulting with the old solution list of self-complementary magic squares 4x4.

These check results are shown in the monitor list above. I could find neither duplications nor drop-outs of any solution, but I could finally know there is the beautiful 'one-to-one correspondence' between the two sets of solutions.

As long as you have the solution set of 384 'Prototype Squares' and 'DAM Transformation', you can surely reconstruct the complete set of 384 solutions of self-complementary magic squares of order 4, whenever you want to. It is amazing!

What does this set of Prototype Squares really mean? What purpose does it exist for? I can only say now it seems to act like a 'variety generator' for the object. It also seems to be the set of 'Basic Forms' for our self-complementary magic squares 4x4.

#2. 'Prototype Squares' for Pan-diagonal Magic Squares of Order 4

How about for Pan-diagonal MS44? Do we have to get another set of 384 'Prototype Squares' of order 4 corresponding to them? Do we also have to have another 'Do-it-After-the-Model Transformation'? And with both of them can we really recompose those 384 solutions of Pan-diagonal magic squares of order 4?

2-1. First of all, we have to find the 'Best Model' out of the solution set of pan-diagonal magic squares of order 4. It is used for the 'DAM Transformation' and also used for making the solution set of 'Prototype Squares'.

I made some experiments to know which models were really suitable. I dictated the rules of 'DAMT' from the model at first and then applied them to the model itself to know if we can make it back into the prototype form.

The next list will tell you how we can take the best model out of the object solutions. I finally decided to take the solution No.4 for our new representative model.

**** List 4: Find the 'Best Model' for Pan-diagonal Magic Squares ****

*** Apply 'DAMT' to itself and Repeat it 4 times one after another ***

3/T0	/T1	/T2	/T3	/T4
1 15 4 14	1 5 14 10	1 12 10 3	1 2 3 4	1 15 4 14
12 6 9 7	2 6 13 9	15 6 8 13	5 6 7 8	12 6 9 7
13 3 16 2	8 4 11 15	7 14 16 5	9 10 11 12	13 3 16 2
8 10 5 11	7 3 12 16	9 4 2 11	13 14 15 16	8 10 5 11
4/T0	/T1	/T2	/T3	/T4
1 15 10 8	1 5 9 13	1 12 7 14	1 2 3 4	1 15 10 8
12 6 3 13	2 6 10 14	15 6 9 4	5 6 7 8	12 6 3 13
7 9 16 2	3 7 11 15	10 3 16 5	9 10 11 12	7 9 16 2
14 4 5 11	4 8 12 16	8 13 2 11	13 14 15 16	14 4 5 11
5/T0	/T1	/T2	/T3	/T4
1 15 4 14	1 9 14 6	1 13 6 10	1 12 10 3	1 2 3 4
8 10 5 11	11 3 8 16	16 4 11 7	7 14 16 5	5 6 7 8
13 3 16 2	12 4 7 15	2 14 5 9	15 6 8 13	9 10 11 12
12 6 9 7	2 10 13 5	15 3 12 8	9 4 2 11	13 14 15 16
8/T0	/T1	/T2	/T3	/T4
1 14 11 8	1 7 16 10	1 5 13 9	1 15 12 6	1 2 3 4
15 4 5 10	2 8 15 9	14 10 2 6	7 9 14 4	5 6 7 8
6 9 16 3	4 6 13 11	8 4 12 16	10 8 3 13	9 10 11 12
12 7 2 13	3 5 14 12	11 15 7 3	16 2 5 11	13 14 15 16
9/T0	/T1	/T2	/T3	/T4
1 14 4 15	1 11 15 5	1 16 5 12	1 10 12 3	1 2 3 4
12 7 9 6	3 9 13 7	4 13 8 9	15 8 6 13	5 6 7 8
13 2 16 3	8 14 10 4	6 11 2 15	7 16 14 5	9 10 11 12
8 11 5 10	6 16 12 2	7 10 3 14	9 2 4 11	13 14 15 16
11/T0	/T1	/T2	/T3	/T4
1 14 4 15	1 7 15 9	1 5 9 13	1 8 13 12	1 10 12 3
8 11 5 10	10 16 8 2	2 6 10 14	14 11 2 7	7 16 14 5
13 2 16 3	12 14 6 4	3 7 11 15	4 5 16 9	15 8 6 13
12 7 9 6	3 5 13 11	4 8 12 16	15 10 3 6	9 2 4 11
13/T0	/T1	/T2	/T3	/T4
1 12 7 14	1 5 9 13	1 15 10 8	1 2 3 4	1 12 7 14
15 6 9 4	2 6 10 14	12 6 3 13	5 6 7 8	15 6 9 4
10 3 16 5	3 7 11 15	7 9 16 2	9 10 11 12	10 3 16 5
8 13 2 11	4 8 12 16	14 4 5 11	13 14 15 16	8 13 2 11
14/T0	/T1	/T2	/T3	/T4
1 12 13 8	1 5 14 10	1 15 7 9	1 2 3 4	1 12 13 8
15 6 3 10	2 6 13 9	12 6 14 4	5 6 7 8	15 6 3 10
4 9 16 5	8 4 11 15	10 8 16 2	9 10 11 12	4 9 16 5
14 7 2 11	7 3 12 16	3 13 5 11	13 14 15 16	14 7 2 11

The next list shows the result of my newest experiment with the new set of Prototype Squares and the new DAM Transformation dictated as above.

This time I tried to make the standard set of 48 Prototype Squares under the next list-forming inequality conditions at first:

$n1 < n11$, $n1 < n14$, $n1 < n8$, and $n15 > n12$;

Because I should have finally got the 48 Standard Solutions of objects by 'DAMT'.

[' Prototype Squares' 4x4 and Pan-diagonal MS44 Recomposed by ' DAMT']

1/P 1/C		2/P 2/C		3/P 3/C	
1 2 3 4	1 15 4 14	1 2 3 4	1 15 4 14	1 2 5 6	1 15 6 12
5 6 7 8	12 6 9 7	9 10 11 12	8 10 5 11	3 4 7 8	14 4 9 7
9 10 11 12	13 3 16 2	5 6 7 8	13 3 16 2	9 10 13 14	11 5 16 2
13 14 15 16	8 10 5 11	13 14 15 16	12 6 9 7	11 12 15 16	8 10 3 13
4/P 4/C		5/P 5/C		6/P 6/C	
1 2 5 6	1 15 6 12	1 2 9 10	1 15 10 8	1 2 9 10	1 15 10 8
9 10 13 14	8 10 3 13	3 4 11 12	14 4 5 11	5 6 13 14	12 6 3 13
3 4 7 8	11 5 16 2	5 6 13 14	7 9 16 2	3 4 11 12	7 9 16 2
11 12 15 16	14 4 9 7	7 8 15 16	12 6 3 13	7 8 15 16	14 4 5 11
7/P 7/C		8/P 8/C		9/P 9/C	
1 3 2 4	1 14 4 15	1 3 2 4	1 14 4 15	1 3 5 7	1 14 7 12
5 7 6 8	12 7 9 6	9 11 10 12	8 11 5 10	9 11 13 15	8 11 2 13
9 11 10 12	13 2 16 3	5 7 6 8	13 2 16 3	2 4 6 8	10 5 16 3
13 15 14 16	8 11 5 10	13 15 14 16	12 7 9 6	10 12 14 16	15 4 9 6
10/P 10/C		11/P 11/C		12/P 12/C	
1 3 9 11	1 14 11 8	1 5 2 6	1 12 6 15	1 5 3 7	1 12 7 14
5 7 13 15	12 7 2 13	9 13 10 14	8 13 3 10	9 13 11 15	8 13 2 11
2 4 10 12	6 9 16 3	3 7 4 8	11 2 16 5	2 6 4 8	10 3 16 5
6 8 14 16	15 4 5 10	11 15 12 16	14 7 9 4	10 14 12 16	15 6 9 4
13/P 13/C		14/P 14/C		15/P 15/C	
2 1 4 3	2 16 3 13	2 1 4 3	2 16 3 13	2 1 6 5	2 16 5 11
6 5 8 7	11 5 10 8	10 9 12 11	7 9 6 12	4 3 8 7	13 3 10 8
10 9 12 11	14 4 15 1	6 5 8 7	14 4 15 1	10 9 14 13	12 6 15 1
14 13 16 15	7 9 6 12	14 13 16 15	11 5 10 8	12 11 16 15	7 9 4 14
16/P 16/C		17/P 17/C		18/P 18/C	
2 1 6 5	2 16 5 11	2 1 10 9	2 16 9 7	2 1 10 9	2 16 9 7
10 9 14 13	7 9 4 14	4 3 12 11	13 3 6 12	6 5 14 13	11 5 4 14
4 3 8 7	12 6 15 1	6 5 14 13	8 10 15 1	4 3 12 11	8 10 15 1
12 11 16 15	13 3 10 8	8 7 16 15	11 5 4 14	8 7 16 15	13 3 6 12
19/P 19/C		20/P 20/C		21/P 21/C	
2 4 1 3	2 13 3 16	2 4 1 3	2 13 3 16	2 4 6 8	2 13 8 11
6 8 5 7	11 8 10 5	10 12 9 11	7 12 6 9	10 12 14 16	7 12 1 14
10 12 9 11	14 1 15 4	6 8 5 7	14 1 15 4	1 3 5 7	9 6 15 4
14 16 13 15	7 12 6 9	14 16 13 15	11 8 10 5	9 11 13 15	16 3 10 5
22/P 22/C		23/P 23/C		24/P 24/C	
2 4 10 12	2 13 12 7	2 6 1 5	2 11 5 16	2 6 4 8	2 11 8 13
6 8 14 16	11 8 1 14	10 14 9 13	7 14 4 9	10 14 12 16	7 14 1 12
1 3 9 11	5 10 15 4	4 8 3 7	12 1 15 6	1 5 3 7	9 4 15 6
5 7 13 15	16 3 6 9	12 16 11 15	13 8 10 3	9 13 11 15	16 5 10 3
25/P 25/C		26/P 26/C		27/P 27/C	
3 1 4 2	3 16 2 13	3 1 4 2	3 16 2 13	3 1 7 5	3 16 5 10
7 5 8 6	10 5 11 8	11 9 12 10	6 9 7 12	4 2 8 6	13 2 11 8
11 9 12 10	15 4 14 1	7 5 8 6	15 4 14 1	11 9 15 13	12 7 14 1
15 13 16 14	6 9 7 12	15 13 16 14	10 5 11 8	12 10 16 14	6 9 4 15

28/P	28/C	29/P	29/C	30/P	30/C
3 1 7 5	3 16 5 10	3 1 11 9	3 16 9 6	3 1 11 9	3 16 9 6
11 9 15 13	6 9 4 15	4 2 12 10	13 2 7 12	7 5 15 13	10 5 4 15
4 2 8 6	12 7 14 1	7 5 15 13	8 11 14 1	4 2 12 10	8 11 14 1
12 10 16 14	13 2 11 8	8 6 16 14	10 5 4 15	8 6 16 14	13 2 7 12
31/P	31/C	32/P	32/C	33/P	33/C
3 4 1 2	3 13 2 16	3 4 1 2	3 13 2 16	3 4 7 8	3 13 8 10
7 8 5 6	10 8 11 5	11 12 9 10	6 12 7 9	11 12 15 16	6 12 1 15
11 12 9 10	15 1 14 4	7 8 5 6	15 1 14 4	1 2 5 6	9 7 14 4
15 16 13 14	6 12 7 9	15 16 13 14	10 8 11 5	9 10 13 14	16 2 11 5
34/P	34/C	35/P	35/C	36/P	36/C
3 4 11 12	3 13 12 6	4 2 3 1	4 15 1 14	4 2 3 1	4 15 1 14
7 8 15 16	10 8 1 15	8 6 7 5	9 6 12 7	12 10 11 9	5 10 8 11
1 2 9 10	5 11 14 4	12 10 11 9	16 3 13 2	8 6 7 5	16 3 13 2
5 6 13 14	16 2 7 9	16 14 15 13	5 10 8 11	16 14 15 13	9 6 12 7
37/P	37/C	38/P	38/C	39/P	39/C
4 2 8 6	4 15 6 9	4 2 8 6	4 15 6 9	4 2 12 10	4 15 10 5
3 1 7 5	14 1 12 7	12 10 16 14	5 10 3 16	3 1 11 9	14 1 8 11
12 10 16 14	11 8 13 2	3 1 7 5	11 8 13 2	8 6 16 14	7 12 13 2
11 9 15 13	5 10 3 16	11 9 15 13	14 1 12 7	7 5 15 13	9 6 3 16
40/P	40/C	41/P	41/C	42/P	42/C
4 2 12 10	4 15 10 5	4 3 2 1	4 14 1 15	4 3 2 1	4 14 1 15
8 6 16 14	9 6 3 16	8 7 6 5	9 7 12 6	12 11 10 9	5 11 8 10
3 1 11 9	7 12 13 2	12 11 10 9	16 2 13 3	8 7 6 5	16 2 13 3
7 5 15 13	14 1 8 11	16 15 14 13	5 11 8 10	16 15 14 13	9 7 12 6
43/P	43/C	44/P	44/C	45/P	45/C
4 3 8 7	4 14 7 9	4 3 12 11	4 14 11 5	5 1 6 2	5 16 2 11
12 11 16 15	5 11 2 16	8 7 16 15	9 7 2 16	13 9 14 10	4 9 7 14
2 1 6 5	10 8 13 3	2 1 10 9	6 12 13 3	7 3 8 4	15 6 12 1
10 9 14 13	15 1 12 6	6 5 14 13	15 1 8 10	15 11 16 12	10 3 13 8
46/P	46/C	47/P	47/C	48/P	48/C
5 1 7 3	5 16 3 10	6 2 5 1	6 15 1 12	6 2 8 4	6 15 4 9
13 9 15 11	4 9 6 15	14 10 13 9	3 10 8 13	14 10 16 12	3 10 5 16
6 2 8 4	14 7 12 1	8 4 7 3	16 5 11 2	5 1 7 3	13 8 11 2
14 10 16 12	11 2 13 8	16 12 15 11	9 4 14 7	13 9 15 11	12 1 14 7

[Count = 48/48]

[Monitor List of Correspondence between Old and New]

??: 0
 1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 17: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 33: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

OK!

We could successfully recompose the complete set of 48 standard solutions and find the one-to-one correspondence between the prototype and the pan-diagonal squares. It is amazing. But why? What do the Prototype Squares really mean?

The set of prototype squares looks like the complete set of 'Basic Forms'.

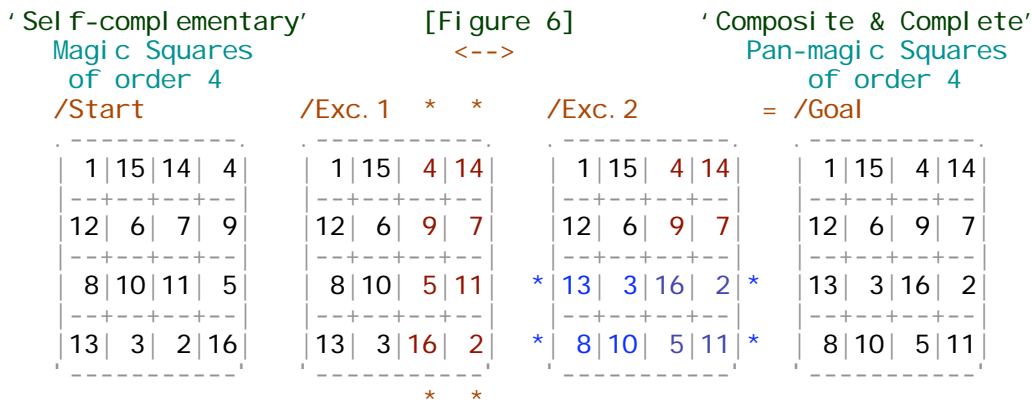
#3. Another Way of Making our Objects by Prototype Squares and 'DAMT'

We already know how to transform self-complementary squares into pan-diagonal ones or how to remake the latter into the former. The only things you have to do are:

- (1) Exchange the whole 4 entries between the 3rd and 4th columns.
- (2) Exchange the whole 4 entries between the 3rd and 4th rows.

As the result we can get a pan-diagonal magic square from a self-complementary

one. It happens to become a 'Composite & Complete' magic square at the same time.
 All complementary pairs of 17 are located on the pan-diagonals, nothing elsewhere.



Every 4 entries of any 2 x 2 block within the object add up to same total 34.

We can take this 'type converter' and add it to our job.

- (1) At first we make 384 solutions of self-complementary squares by the 384 Prototype Squares and 'DAM Transformation' in the same way as we just made.
- (2) And we make the 384 pan-diagonal squares from the 384 self-complementary ones by this type converter.

Why don't you do this experiment? Yes, let's try it now right here.
 The next long list shows the result of my recent experiment.

** Figure 7: 'Prototype Squares' 4x4 and the Goal Objects **

'Prototype'	Self-complementary	Complete Type
16 13 14 15 16 13	16 13 3 2 16 13	11 8 10 5 11 8
4 1 2 3 4 1	4 1 15 14 4 1	14 1 15 4 14 1
8 5 6 7 8 5	9 12 6 7 9 12	7 12 6 9 7 12
12 9 10 11 12 9	5 8 10 11 5 8	2 13 3 16 2 13
16 13 14 15 16 13	16 13 3 2 16 13	11 8 10 5 11 8
4 1 2 3 4 1	4 1 15 14 4 1	14 1 15 4 14 1

[Prototype Squares 4x4 and Re-compositions of Self-complementary(/S) and Complete(/C) Pan-magic Squares 4x4 Recomposed by serial Transformations]

[1]	1/P	1/S	1/C	[2]	2/P	2/S	2/C
1 2 3 4	1 15 14 4	1 15 4 14	1 2 3 4	1 15 14 4	1 15 4 14	1 15 4 14	1 15 4 14
5 6 7 8	12 6 7 9	12 6 9 7	9 10 11 12	8 10 11 5	8 10 5 11	8 10 5 11	8 10 5 11
9 10 11 12	8 10 11 5	13 3 16 2	5 6 7 8	12 6 7 9	13 3 16 2	13 3 16 2	13 3 16 2
13 14 15 16	13 3 2 16	8 10 5 11	13 14 15 16	13 3 2 16	12 6 9 7	12 6 9 7	12 6 9 7
[3]	3/P	3/S	3/C	[4]	4/P	4/S	4/C
1 2 5 6	1 15 12 6	1 15 6 12	1 2 5 6	1 15 12 6	1 15 6 12	1 15 6 12	1 15 6 12
3 4 7 8	14 4 7 9	14 4 9 7	9 10 13 14	8 10 13 3	8 10 3 13	8 10 3 13	8 10 3 13
9 10 13 14	8 10 13 3	11 5 16 2	3 4 7 8	14 4 7 9	11 5 16 2	11 5 16 2	11 5 16 2
11 12 15 16	11 5 2 16	8 10 3 13	11 12 15 16	11 5 2 16	14 4 9 7	14 4 9 7	14 4 9 7
[5]	5/P	5/S	5/C	[6]	6/P	6/S	6/C
1 2 9 10	1 15 8 10	1 15 10 8	1 2 9 10	1 15 8 10	1 15 10 8	1 15 10 8	1 15 10 8
3 4 11 12	14 4 11 5	14 4 5 11	5 6 13 14	12 6 13 3	12 6 3 13	12 6 3 13	12 6 3 13
5 6 13 14	12 6 13 3	7 9 16 2	3 4 11 12	14 4 11 5	7 9 16 2	7 9 16 2	7 9 16 2
7 8 15 16	7 9 2 16	12 6 3 13	7 8 15 16	7 9 2 16	14 4 5 11	14 4 5 11	14 4 5 11

[7]	7/P	7/S	7/C	[8]	8/P	8/S	8/C
1 3 2 4	1 14 15 4	1 14 4 15	1 3 2 4	1 14 15 4	1 14 4 15	1 14 4 15	1 14 4 15
5 7 6 8	12 7 6 9	12 7 9 6	9 11 10 12	8 11 10 5	8 11 5 10	8 11 5 10	8 11 5 10
9 11 10 12	8 11 10 5	13 2 16 3	5 7 6 8	12 7 6 9	13 2 16 3	13 2 16 3	13 2 16 3
13 15 14 16	13 2 3 16	8 11 5 10	13 15 14 16	13 2 3 16	12 7 9 6	12 7 9 6	12 7 9 6
[9]	9/P	9/S	9/C	[10]	10/P	10/S	10/C
1 3 5 7	1 14 12 7	1 14 7 12	1 3 5 7	1 14 12 7	1 14 7 12	1 14 7 12	1 14 7 12
2 4 6 8	15 4 6 9	15 4 9 6	9 11 13 15	8 11 13 2	8 11 2 13	8 11 2 13	8 11 2 13
9 11 13 15	8 11 13 2	10 5 16 3	2 4 6 8	15 4 6 9	10 5 16 3	10 5 16 3	10 5 16 3
10 12 14 16	10 5 3 16	8 11 2 13	10 12 14 16	10 5 3 16	15 4 9 6	15 4 9 6	15 4 9 6
[11]	11/P	11/S	11/C	[12]	12/P	12/S	12/C
1 3 9 11	1 14 8 11	1 14 11 8	1 3 9 11	1 14 8 11	1 14 11 8	1 14 11 8	1 14 11 8
2 4 10 12	15 4 10 5	15 4 5 10	5 7 13 15	12 7 13 2	12 7 2 13	12 7 2 13	12 7 2 13
5 7 13 15	12 7 13 2	6 9 16 3	2 4 10 12	15 4 10 5	6 9 16 3	6 9 16 3	6 9 16 3
6 8 14 16	6 9 3 16	12 7 2 13	6 8 14 16	6 9 3 16	15 4 5 10	15 4 5 10	15 4 5 10
[13]	13/P	13/S	13/C	[14]	14/P	14/S	14/C
1 5 2 6	1 12 15 6	1 12 6 15	1 5 2 6	1 12 15 6	1 12 6 15	1 12 6 15	1 12 6 15
3 7 4 8	14 7 4 9	14 7 9 4	9 13 10 14	8 13 10 3	8 13 3 10	8 13 3 10	8 13 3 10
9 13 10 14	8 13 10 3	11 2 16 5	3 7 4 8	14 7 4 9	11 2 16 5	11 2 16 5	11 2 16 5
11 15 12 16	11 2 5 16	8 13 3 10	11 15 12 16	11 2 5 16	14 7 9 4	14 7 9 4	14 7 9 4
[15]	15/P	15/S	15/C	[16]	16/P	16/S	16/C
1 5 3 7	1 12 14 7	1 12 7 14	1 5 3 7	1 12 14 7	1 12 7 14	1 12 7 14	1 12 7 14
2 6 4 8	15 6 4 9	15 6 9 4	9 13 11 15	8 13 11 2	8 13 2 11	8 13 2 11	8 13 2 11
9 13 11 15	8 13 11 2	10 3 16 5	2 6 4 8	15 6 4 9	10 3 16 5	10 3 16 5	10 3 16 5
10 14 12 16	10 3 5 16	8 13 2 11	10 14 12 16	10 3 5 16	15 6 9 4	15 6 9 4	15 6 9 4
[17]	17/P	17/S	17/C	[18]	18/P	18/S	18/C
1 5 9 13	1 12 8 13	1 12 13 8	1 5 9 13	1 12 8 13	1 12 13 8	1 12 13 8	1 12 13 8
2 6 10 14	15 6 10 3	15 6 3 10	3 7 11 15	14 7 11 2	14 7 2 11	14 7 2 11	14 7 2 11
3 7 11 15	14 7 11 2	4 9 16 5	2 6 10 14	15 6 10 3	4 9 16 5	4 9 16 5	4 9 16 5
4 8 12 16	4 9 5 16	14 7 2 11	4 8 12 16	4 9 5 16	15 6 3 10	15 6 3 10	15 6 3 10
[19]	19/P	19/S	19/C	[20]	20/P	20/S	20/C
1 9 2 10	1 8 15 10	1 8 10 15	1 9 2 10	1 8 15 10	1 8 10 15	1 8 10 15	1 8 10 15
3 11 4 12	14 11 4 5	14 11 5 4	5 13 6 14	12 13 6 3	12 13 3 6	12 13 3 6	12 13 3 6
5 13 6 14	12 13 6 3	7 2 16 9	3 11 4 12	14 11 4 5	7 2 16 9	7 2 16 9	7 2 16 9
7 15 8 16	7 2 9 16	12 13 3 6	7 15 8 16	7 2 9 16	14 11 5 4	14 11 5 4	14 11 5 4
[21]	21/P	21/S	21/C	[22]	22/P	22/S	22/C
1 9 3 11	1 8 14 11	1 8 11 14	1 9 3 11	1 8 14 11	1 8 11 14	1 8 11 14	1 8 11 14
2 10 4 12	15 10 4 5	15 10 5 4	5 13 7 15	12 13 7 2	12 13 2 7	12 13 2 7	12 13 2 7
5 13 7 15	12 13 7 2	6 3 16 9	2 10 4 12	15 10 4 5	6 3 16 9	6 3 16 9	6 3 16 9
6 14 8 16	6 3 9 16	12 13 2 7	6 14 8 16	6 3 9 16	15 10 5 4	15 10 5 4	15 10 5 4
[23]	23/P	23/S	23/C	[24]	24/P	24/S	24/C
1 9 5 13	1 8 12 13	1 8 13 12	1 9 5 13	1 8 12 13	1 8 13 12	1 8 13 12	1 8 13 12
2 10 6 14	15 10 6 3	15 10 3 6	3 11 7 15	14 11 7 2	14 11 2 7	14 11 2 7	14 11 2 7
3 11 7 15	14 11 7 2	4 5 16 9	2 10 6 14	15 10 6 3	4 5 16 9	4 5 16 9	4 5 16 9
4 12 8 16	4 5 9 16	14 11 2 7	4 12 8 16	4 5 9 16	15 10 3 6	15 10 3 6	15 10 3 6
[25]	25/P	25/S	25/C	[26]	26/P	26/S	26/C
2 1 4 3	2 16 13 3	2 16 3 13	2 1 4 3	2 16 13 3	2 16 3 13	2 16 3 13	2 16 3 13
6 5 8 7	11 5 8 10	11 5 10 8	10 9 12 11	7 9 12 6	7 9 6 12	7 9 6 12	7 9 6 12
10 9 12 11	7 9 12 6	14 4 15 1	6 5 8 7	11 5 8 10	14 4 15 1	14 4 15 1	14 4 15 1
14 13 16 15	14 4 1 15	7 9 6 12	14 13 16 15	14 4 1 15	11 5 10 8	11 5 10 8	11 5 10 8
...
[47]	47/P	47/S	47/C	[48]	48/P	48/S	48/C
2 10 6 14	2 7 11 14	2 7 14 11	2 10 6 14	2 7 11 14	2 7 14 11	2 7 14 11	2 7 14 11
1 9 5 13	16 9 5 4	16 9 4 5	4 12 8 16	13 12 8 1	13 12 1 8	13 12 1 8	13 12 1 8
4 12 8 16	13 12 8 1	3 6 15 10	1 9 5 13	16 9 5 4	3 6 15 10	3 6 15 10	3 6 15 10
3 11 7 15	3 6 10 15	13 12 1 8	3 11 7 15	3 6 10 15	16 9 4 5	16 9 4 5	16 9 4 5

[49]	49/P	49/S	49/C	[50]	50/P	50/S	50/C
3 1 4 2	3 16 13 2	3 16 2 13	3 16 2 13	3 1 4 2	3 16 13 2	3 16 13 2	3 16 2 13
7 5 8 6	10 5 8 11	10 5 11 8	10 5 11 8	11 9 12 10	6 9 12 7	6 9 12 7	6 9 7 12
11 9 12 10	6 9 12 7	15 4 14 1	15 4 14 1	7 5 8 6	10 5 8 11	15 4 14 1	15 4 14 1
15 13 16 14	15 4 1 14	6 9 7 12	6 9 7 12	15 13 16 14	15 4 1 14	10 5 11 8	10 5 11 8
[51]	51/P	51/S	51/C	[52]	52/P	52/S	52/C
3 1 7 5	3 16 10 5	3 16 5 10	3 16 5 10	3 1 7 5	3 16 10 5	3 16 5 10	3 16 5 10
4 2 8 6	13 2 8 11	13 2 11 8	13 2 11 8	11 9 15 13	6 9 15 4	6 9 4 15	6 9 4 15
11 9 15 13	6 9 15 4	12 7 14 1	12 7 14 1	4 2 8 6	13 2 8 11	12 7 14 1	12 7 14 1
12 10 16 14	12 7 1 14	6 9 4 15	6 9 4 15	12 10 16 14	12 7 1 14	13 2 11 8	13 2 11 8
[95]	95/P	95/S	95/C	[96]	96/P	96/S	96/C
4 12 8 16	4 5 9 16	4 5 16 9	4 5 16 9	4 12 8 16	4 5 9 16	4 5 16 9	4 5 16 9
2 10 6 14	15 10 6 3	15 10 3 6	15 10 3 6	3 11 7 15	14 11 7 2	14 11 2 7	14 11 2 7
3 11 7 15	14 11 7 2	1 8 13 12	1 8 13 12	2 10 6 14	15 10 6 3	1 8 13 12	1 8 13 12
1 9 5 13	1 8 12 13	14 11 2 7	14 11 2 7	1 9 5 13	1 8 12 13	15 10 3 6	15 10 3 6
[97]	97/P	97/S	97/C	[98]	98/P	98/S	98/C
5 1 6 2	5 16 11 2	5 16 2 11	5 16 2 11	5 1 6 2	5 16 11 2	5 16 2 11	5 16 2 11
7 3 8 4	10 3 8 13	10 3 13 8	10 3 13 8	13 9 14 10	4 9 14 7	4 9 7 14	4 9 7 14
13 9 14 10	4 9 14 7	15 6 12 1	15 6 12 1	7 3 8 4	10 3 8 13	15 6 12 1	15 6 12 1
15 11 16 12	15 6 1 12	4 9 7 14	4 9 7 14	15 11 16 12	15 6 1 12	10 3 13 8	10 3 13 8
[99]	99/P	99/S	99/C	[100]	100/P	100/S	100/C
5 1 7 3	5 16 10 3	5 16 3 10	5 16 3 10	5 1 7 3	5 16 10 3	5 16 3 10	5 16 3 10
6 2 8 4	11 2 8 13	11 2 13 8	11 2 13 8	13 9 15 11	4 9 15 6	4 9 6 15	4 9 6 15
13 9 15 11	4 9 15 6	14 7 12 1	14 7 12 1	6 2 8 4	11 2 8 13	14 7 12 1	14 7 12 1
14 10 16 12	14 7 1 12	4 9 6 15	4 9 6 15	14 10 16 12	14 7 1 12	11 2 13 8	11 2 13 8
[191]	191/P	191/S	191/C	[192]	192/P	192/S	192/C
8 16 7 15	8 1 10 15	8 1 15 10	8 1 15 10	8 16 7 15	8 1 10 15	8 1 15 10	8 1 15 10
4 12 3 11	13 12 3 6	13 12 6 3	13 12 6 3	6 14 5 13	11 14 5 4	11 14 4 5	11 14 4 5
6 14 5 13	11 14 5 4	2 7 9 16	2 7 9 16	4 12 3 11	13 12 3 6	2 7 9 16	2 7 9 16
2 10 1 9	2 7 16 9	11 14 4 5	11 14 4 5	2 10 1 9	2 7 16 9	13 12 6 3	13 12 6 3
[193]	193/P	193/S	193/C	[194]	194/P	194/S	194/C
9 1 10 2	9 16 7 2	9 16 2 7	9 16 2 7	9 1 10 2	9 16 7 2	9 16 2 7	9 16 2 7
11 3 12 4	6 3 12 13	6 3 13 12	6 3 13 12	13 5 14 6	4 5 14 11	4 5 11 14	4 5 11 14
13 5 14 6	4 5 14 11	15 10 8 1	15 10 8 1	11 3 12 4	6 3 12 13	15 10 8 1	15 10 8 1
15 7 16 8	15 10 1 8	4 5 11 14	4 5 11 14	15 7 16 8	15 10 1 8	6 3 13 12	6 3 13 12
[195]	195/P	195/S	195/C	[196]	196/P	196/S	196/C
9 1 11 3	9 16 6 3	9 16 3 6	9 16 3 6	9 1 11 3	9 16 6 3	9 16 3 6	9 16 3 6
10 2 12 4	7 2 12 13	7 2 13 12	7 2 13 12	13 5 15 7	4 5 15 10	4 5 10 15	4 5 10 15
13 5 15 7	4 5 15 10	14 11 8 1	14 11 8 1	10 2 12 4	7 2 12 13	14 11 8 1	14 11 8 1
14 6 16 8	14 11 1 8	4 5 10 15	4 5 10 15	14 6 16 8	14 11 1 8	7 2 13 12	7 2 13 12
...
[377]	377/P	377/S	377/C	[378]	378/P	378/S	378/C
16 14 15 13	16 3 2 13	16 3 13 2	16 3 13 2	16 14 15 13	16 3 2 13	16 3 13 2	16 3 13 2
8 6 7 5	9 6 7 12	9 6 12 7	9 6 12 7	12 10 11 9	5 10 11 8	5 10 8 11	5 10 8 11
12 10 11 9	5 10 11 8	4 15 1 14	4 15 1 14	8 6 7 5	9 6 7 12	4 15 1 14	4 15 1 14
4 2 3 1	4 15 14 1	5 10 8 11	5 10 8 11	4 2 3 1	4 15 14 1	9 6 12 7	9 6 12 7
[379]	379/P	379/S	379/C	[380]	380/P	380/S	380/C
16 15 8 7	16 2 9 7	16 2 7 9	16 2 7 9	16 15 8 7	16 2 9 7	16 2 7 9	16 2 7 9
12 11 4 3	5 11 4 14	5 11 14 4	5 11 14 4	14 13 6 5	3 13 6 12	3 13 12 6	3 13 12 6
14 13 6 5	3 13 6 12	10 8 1 15	10 8 1 15	12 11 4 3	5 11 4 14	10 8 1 15	10 8 1 15
10 9 2 1	10 8 15 1	3 13 12 6	3 13 12 6	10 9 2 1	10 8 15 1	5 11 14 4	5 11 14 4
[381]	381/P	381/S	381/C	[382]	382/P	382/S	382/C
16 15 12 11	16 2 5 11	16 2 11 5	16 2 11 5	16 15 12 11	16 2 5 11	16 2 11 5	16 2 11 5
8 7 4 3	9 7 4 14	9 7 14 4	9 7 14 4	14 13 10 9	3 13 10 8	3 13 8 10	3 13 8 10
14 13 10 9	3 13 10 8	6 12 1 15	6 12 1 15	8 7 4 3	9 7 4 14	6 12 1 15	6 12 1 15
6 5 2 1	6 12 15 1	3 13 8 10	3 13 8 10	6 5 2 1	6 12 15 1	9 7 14 4	9 7 14 4

14/P	11/S	11/C	16/P	12/S	12/C
1 5 2 6	1 12 15 6	1 12 6 15	1 5 3 7	1 12 14 7	1 12 7 14
9 13 10 14	8 13 10 3	8 13 3 10	9 13 11 15	8 13 11 2	8 13 2 11
3 7 4 8	14 7 4 9	11 2 16 5	2 6 4 8	15 6 4 9	10 3 16 5
11 15 12 16	11 2 5 16	14 7 9 4	10 14 12 16	10 3 5 16	15 6 9 4
25/P	13/S	13/C	26/P	14/S	14/C
2 1 4 3	2 16 13 3	2 16 3 13	2 1 4 3	2 16 13 3	2 16 3 13
6 5 8 7	11 5 8 10	11 5 10 8	10 9 12 11	7 9 12 6	7 9 6 12
10 9 12 11	7 9 12 6	14 4 15 1	6 5 8 7	11 5 8 10	14 4 15 1
14 13 16 15	14 4 1 15	7 9 6 12	14 13 16 15	14 4 1 15	11 5 10 8
27/P	15/S	15/C	28/P	16/S	16/C
2 1 6 5	2 16 11 5	2 16 5 11	2 1 6 5	2 16 11 5	2 16 5 11
4 3 8 7	13 3 8 10	13 3 10 8	10 9 14 13	7 9 14 4	7 9 4 14
10 9 14 13	7 9 14 4	12 6 15 1	4 3 8 7	13 3 8 10	12 6 15 1
12 11 16 15	12 6 1 15	7 9 4 14	12 11 16 15	12 6 1 15	13 3 10 8
29/P	17/S	17/C	30/P	18/S	18/C
2 1 10 9	2 16 7 9	2 16 9 7	2 1 10 9	2 16 7 9	2 16 9 7
4 3 12 11	13 3 12 6	13 3 6 12	6 5 14 13	11 5 14 4	11 5 4 14
6 5 14 13	11 5 14 4	8 10 15 1	4 3 12 11	13 3 12 6	8 10 15 1
8 7 16 15	8 10 1 15	11 5 4 14	8 7 16 15	8 10 1 15	13 3 6 12
31/P	19/S	19/C	32/P	20/S	20/C
2 4 1 3	2 13 16 3	2 13 3 16	2 4 1 3	2 13 16 3	2 13 3 16
6 8 5 7	11 8 5 10	11 8 10 5	10 12 9 11	7 12 9 6	7 12 6 9
10 12 9 11	7 12 9 6	14 1 15 4	6 8 5 7	11 8 5 10	14 1 15 4
14 16 13 15	14 1 4 15	7 12 6 9	14 16 13 15	14 1 4 15	11 8 10 5
34/P	21/S	21/C	36/P	22/S	22/C
2 4 6 8	2 13 11 8	2 13 8 11	2 4 10 12	2 13 7 12	2 13 12 7
10 12 14 16	7 12 14 1	7 12 1 14	6 8 14 16	11 8 14 1	11 8 1 14
1 3 5 7	16 3 5 10	9 6 15 4	1 3 9 11	16 3 9 6	5 10 15 4
9 11 13 15	9 6 4 15	16 3 10 5	5 7 13 15	5 10 4 15	16 3 6 9
38/P	23/S	23/C	40/P	24/S	24/C
2 6 1 5	2 11 16 5	2 11 5 16	2 6 4 8	2 11 13 8	2 11 8 13
10 14 9 13	7 14 9 4	7 14 4 9	10 14 12 16	7 14 12 1	7 14 1 12
4 8 3 7	13 8 3 10	12 1 15 6	1 5 3 7	16 5 3 10	9 4 15 6
12 16 11 15	12 1 6 15	13 8 10 3	9 13 11 15	9 4 6 15	16 5 10 3
49/P	0/S	25/C	50/P	0/S	26/C
3 1 4 2	0 0 0 0	3 16 2 13	3 1 4 2	0 0 0 0	3 16 2 13
7 5 8 6	0 0 0 0	10 5 11 8	11 9 12 10	0 0 0 0	6 9 7 12
11 9 12 10	0 0 0 0	15 4 14 1	7 5 8 6	0 0 0 0	15 4 14 1
15 13 16 14	0 0 0 0	6 9 7 12	15 13 16 14	0 0 0 0	10 5 11 8
51/P	25/S	27/C	52/P	26/S	28/C
3 1 7 5	3 16 10 5	3 16 5 10	3 1 7 5	3 16 10 5	3 16 5 10
4 2 8 6	13 2 8 11	13 2 11 8	11 9 15 13	6 9 15 4	6 9 4 15
11 9 15 13	6 9 15 4	12 7 14 1	4 2 8 6	13 2 8 11	12 7 14 1
12 10 16 14	12 7 1 14	6 9 4 15	12 10 16 14	12 7 1 14	13 2 11 8
53/P	27/S	29/C	54/P	28/S	30/C
3 1 11 9	3 16 6 9	3 16 9 6	3 1 11 9	3 16 6 9	3 16 9 6
4 2 12 10	13 2 12 7	13 2 7 12	7 5 15 13	10 5 15 4	10 5 4 15
7 5 15 13	10 5 15 4	8 11 14 1	4 2 12 10	13 2 12 7	8 11 14 1
8 6 16 14	8 11 1 14	10 5 4 15	8 6 16 14	8 11 1 14	13 2 7 12
55/P	0/S	31/C	56/P	0/S	32/C
3 4 1 2	0 0 0 0	3 13 2 16	3 4 1 2	0 0 0 0	3 13 2 16
7 8 5 6	0 0 0 0	10 8 11 5	11 12 9 10	0 0 0 0	6 12 7 9
11 12 9 10	0 0 0 0	15 1 14 4	7 8 5 6	0 0 0 0	15 1 14 4
15 16 13 14	0 0 0 0	6 12 7 9	15 16 13 14	0 0 0 0	10 8 11 5

58/P	29/S	33/C	60/P	30/S	34/C
3 4 7 8	3 13 10 8	3 13 8 10	3 4 11 12	3 13 6 12	3 13 12 6
11 12 15 16	6 12 15 1	6 12 1 15	7 8 15 16	10 8 15 1	10 8 1 15
1 2 5 6	16 2 5 11	9 7 14 4	1 2 9 10	16 2 9 7	5 11 14 4
9 10 13 14	9 7 4 14	16 2 11 5	5 6 13 14	5 11 4 14	16 2 7 9
62/P	31/S	0/C	64/P	32/S	0/C
3 7 1 5	3 10 16 5	0 0 0 0	3 7 4 8	3 10 13 8	0 0 0 0
11 15 9 13	6 15 9 4	0 0 0 0	11 15 12 16	6 15 12 1	0 0 0 0
4 8 2 6	13 8 2 11	0 0 0 0	1 5 2 6	16 5 2 11	0 0 0 0
12 16 10 14	12 1 7 14	0 0 0 0	9 13 10 14	9 4 7 14	0 0 0 0
73/P	0/S	35/C	74/P	0/S	36/C
4 2 3 1	0 0 0 0	4 15 1 14	4 2 3 1	0 0 0 0	4 15 1 14
8 6 7 5	0 0 0 0	9 6 12 7	12 10 11 9	0 0 0 0	5 10 8 11
12 10 11 9	0 0 0 0	16 3 13 2	8 6 7 5	0 0 0 0	16 3 13 2
16 14 15 13	0 0 0 0	5 10 8 11	16 14 15 13	0 0 0 0	9 6 12 7
75/P	33/S	37/C	76/P	34/S	38/C
4 2 8 6	4 15 9 6	4 15 6 9	4 2 8 6	4 15 9 6	4 15 6 9
3 1 7 5	14 1 7 12	14 1 12 7	12 10 16 14	5 10 16 3	5 10 3 16
12 10 16 14	5 10 16 3	11 8 13 2	3 1 7 5	14 1 7 12	11 8 13 2
11 9 15 13	11 8 2 13	5 10 3 16	11 9 15 13	11 8 2 13	14 1 12 7
77/P	35/S	39/C	78/P	36/S	40/C
4 2 12 10	4 15 5 10	4 15 10 5	4 2 12 10	4 15 5 10	4 15 10 5
3 1 11 9	14 1 11 8	14 1 8 11	8 6 16 14	9 6 16 3	9 6 3 16
8 6 16 14	9 6 16 3	7 12 13 2	3 1 11 9	14 1 11 8	7 12 13 2
7 5 15 13	7 12 2 13	9 6 3 16	7 5 15 13	7 12 2 13	14 1 8 11
79/P	0/S	41/C	80/P	0/S	42/C
4 3 2 1	0 0 0 0	4 14 1 15	4 3 2 1	0 0 0 0	4 14 1 15
8 7 6 5	0 0 0 0	9 7 12 6	12 11 10 9	0 0 0 0	5 11 8 10
12 11 10 9	0 0 0 0	16 2 13 3	8 7 6 5	0 0 0 0	16 2 13 3
16 15 14 13	0 0 0 0	5 11 8 10	16 15 14 13	0 0 0 0	9 7 12 6
82/P	37/S	43/C	84/P	38/S	44/C
4 3 8 7	4 14 9 7	4 14 7 9	4 3 12 11	4 14 5 11	4 14 11 5
12 11 16 15	5 11 16 2	5 11 2 16	8 7 16 15	9 7 16 2	9 7 2 16
2 1 6 5	15 1 6 12	10 8 13 3	2 1 10 9	15 1 10 8	6 12 13 3
10 9 14 13	10 8 3 13	15 1 12 6	6 5 14 13	6 12 3 13	15 1 8 10
86/P	39/S	0/C	88/P	40/S	0/C
4 8 2 6	4 9 15 6	0 0 0 0	4 8 3 7	4 9 14 7	0 0 0 0
12 16 10 14	5 16 10 3	0 0 0 0	12 16 11 15	5 16 11 2	0 0 0 0
3 7 1 5	14 7 1 12	0 0 0 0	2 6 1 5	15 6 1 12	0 0 0 0
11 15 9 13	11 2 8 13	0 0 0 0	10 14 9 13	10 3 8 13	0 0 0 0
98/P	0/S	45/C	100/P	0/S	46/C
5 1 6 2	0 0 0 0	5 16 2 11	5 1 7 3	0 0 0 0	5 16 3 10
13 9 14 10	0 0 0 0	4 9 7 14	13 9 15 11	0 0 0 0	4 9 6 15
7 3 8 4	0 0 0 0	15 6 12 1	6 2 8 4	0 0 0 0	14 7 12 1
15 11 16 12	0 0 0 0	10 3 13 8	14 10 16 12	0 0 0 0	11 2 13 8
101/P	41/S	0/C	102/P	42/S	0/C
5 1 13 9	5 16 4 9	0 0 0 0	5 1 13 9	5 16 4 9	0 0 0 0
6 2 14 10	11 2 14 7	0 0 0 0	7 3 15 11	10 3 15 6	0 0 0 0
7 3 15 11	10 3 15 6	0 0 0 0	6 2 14 10	11 2 14 7	0 0 0 0
8 4 16 12	8 13 1 12	0 0 0 0	8 4 16 12	8 13 1 12	0 0 0 0
106/P	43/S	0/C	112/P	44/S	0/C
5 6 7 8	5 11 10 8	0 0 0 0	5 7 6 8	5 10 11 8	0 0 0 0
13 14 15 16	4 14 15 1	0 0 0 0	13 15 14 16	4 15 14 1	0 0 0 0
1 2 3 4	16 2 3 13	0 0 0 0	1 3 2 4	16 3 2 13	0 0 0 0
9 10 11 12	9 7 6 12	0 0 0 0	9 11 10 12	9 6 7 12	0 0 0 0

122/P	0/S	47/C	124/P	0/S	48/C
6 2 5 1	0 0 0 0	6 15 1 12	6 2 8 4	0 0 0 0	6 15 4 9
14 10 13 9	0 0 0 0	3 10 8 13	14 10 16 12	0 0 0 0	3 10 5 16
8 4 7 3	0 0 0 0	16 5 11 2	5 1 7 3	0 0 0 0	13 8 11 2
16 12 15 11	0 0 0 0	9 4 14 7	13 9 15 11	0 0 0 0	12 1 14 7
125/P	45/S	0/C	126/P	46/S	0/C
6 2 14 10	6 15 3 10	0 0 0 0	6 2 14 10	6 15 3 10	0 0 0 0
5 1 13 9	12 1 13 8	0 0 0 0	8 4 16 12	9 4 16 5	0 0 0 0
8 4 16 12	9 4 16 5	0 0 0 0	5 1 13 9	12 1 13 8	0 0 0 0
7 3 15 11	7 14 2 11	0 0 0 0	7 3 15 11	7 14 2 11	0 0 0 0
130/P	47/S	0/C	136/P	48/S	0/C
6 5 8 7	6 12 9 7	0 0 0 0	6 8 5 7	6 9 12 7	0 0 0 0
14 13 16 15	3 13 16 2	0 0 0 0	14 16 13 15	3 16 13 2	0 0 0 0
2 1 4 3	15 1 4 14	0 0 0 0	2 4 1 3	15 4 1 14	0 0 0 0
10 9 12 11	10 8 5 11	0 0 0 0	10 12 9 11	10 5 8 11	0 0 0 0

[Count(P/S/C) = 384/48/48]

When you make this list, you must not make the solution level of Prototype Squares limited to the 48 standard ones. You should hold on making the 384 primitive ones.

Otherwise you would miss a part of one-to-one correspondences among the 3 sets of solutions. As you see in the last half of the list above, you could not always find all the complete correspondences among the three solutions. It is embarrassing.

#5. We could reconstruct all the 384 self-complementary squares and all the 384 pan-diagonal ones, whenever we have got the appropriate set of 'Prototype Squares' and the appropriate Model Solutions. But why? What makes the common set of 384 Prototype Squares work all right? Why is another set of them unnecessary?

Why can we build such a beautiful 'bridge' as the one-to-one correspondence between the parallel worlds of prototype squares and of object solutions?

[Figure 8: Various types of magic squares and their properties]

Types: /Properties/	Prototype squares	Self- Complementary	Pan- magi c	'C & C' pan-magi c
Compl ementary pai rs l oca ted symmetri cally	0	0	X	X
4 entri es on pan-di agonal s add up to 34	0	X	0	0
Every row & Col umn add up to the same 34	X	0	0	0
Any 2 x 2 bl ock wi thi n add up to 34	X	X	0	0
Equal i ty of Cross-sums	0	X	X	X
How many Standard Sol uti ons?	48	48	48	48

We can guess there exists anything deep, like 'inner structural relation' among them. We can also guess that the common set of prototype squares should mean the only one common origin of everything.

#6. I must confess here that the basic idea of 'Prototype squares' and 'Do-it-After-the-Model Transformation' is not my original invention. I borrowed it from Drs. Ollerenshaw and Brae, British researchers. But I examined it in every case of every order intensively and tried to extend their idea.

I say I could almost have got the new concept and skill reborn.

Drs. Ollerenshaw and Brae presented it in their page only about 'Most-Perfect Pan-diagonal' magic squares (that are equivalent to our 'C & C' pan-magic squares) of order 4, 8, 12, 16,

But this idea itself could be applied to broader objects, I thought, to any other typical magic squares in every order. I had to extend the concept of 'Prototype Squares' and the application 'DAMT' explicitly. I also had to improve my knowledge about the way how to make the set of Prototype Squares and how to select the representative model out of object squares.

In the case of 'Composite & Complete' magic squares of order 4, Drs. Ollerenshaw & Brae discovered the three 'Principal Squares' among the 'Reversible' ones.

They are equivalent to No.1, 3, and 5 in our lists above.

**** List 8: 'Principal Squares' ****

1/PT	/SC	/PD
1 2 3 4	1 15 14 4	1 15 4 14
5 6 7 8	12 6 7 9	12 6 9 7
9 10 11 12	8 10 11 5	13 3 16 2
13 14 15 16	13 3 2 16	8 10 5 11
3/PT	/SC	/PD
1 2 5 6	1 15 12 6	1 15 6 12
3 4 7 8	14 4 7 9	14 4 9 7
9 10 13 14	8 10 13 3	11 5 16 2
11 12 15 16	11 5 2 16	8 10 3 13
5/PT	/SC	/PD
1 2 9 10	1 15 8 10	1 15 10 8
3 4 11 12	14 4 11 5	14 4 5 11
5 6 13 14	12 6 13 3	7 9 16 2
7 8 15 16	7 9 2 16	12 6 3 13

These three solutions are the same with the ones classified before as 'Fundamental Three' in self-complementary and 'C & C' pan-magic squares of order 4.

Recent study revealed us that the 4-dimensional extra-cubic magic forms of order 2 is certainly the origin of these squares.

There are three different patterns of them. They are equivalent to those three below. The 'one-to-one correspondence' could be found among them.

**** List 9: 'Fundamental Three': EC02^4 and Developed Forms ****

1/n1	/n2	1/SC	/CC
1---12	8---13	1 15 8 10	1 15 10 8
15---6	10---3	14 4 11 5	14 4 5 11
14- -7	11- -2	12 6 13 3	7 9 16 2
4---9	5---16	7 9 2 16	12 6 3 13

2/n1	/n2	2/SC	/CC
1-----8	12----13	1 15 12 6	1 15 6 12
15----10	6-----3	14 4 7 9	14 4 9 7
14- -11	7- --2	8 10 13 3	11 5 16 2
4-----5	9----16	11 5 2 16	8 10 3 13
3/n1	/n2	3/SC	/CC
1-----8	14----11	1 15 14 4	1 15 4 14
15----10	4-----5	12 6 7 9	12 6 9 7
12- -13	7- --2	8 10 11 5	13 3 16 2
6-----3	9----16	13 3 2 16	8 10 5 11

Everything tells us about some deep relationships among them: the beautiful bridges built over the three different worlds.

#7. Why are there always only three Fundamentals?

Why are there only three 'fundamental' solutions in all and no more?

I think the 'C&C' pan-magic squares could tell us about the secret.

It depends upon the miraculous unification that three types of magic squares of order 4: (1) Pan-diagonal, (2) Complete, and (3) C&C pan-magic have the same solution set. Therefore you can find the same property as the most strict conditions of C&C pan-magic type equally among all types of pan-diagonal magic squares of order 4.

It is always very important how the 'adjacent numbers next to n1' could appear and take certain values, I have found. It is the problem of permutations.

In the 'Composite and Complete' magic squares, many kinds of complementary pairs are desperately needed.

[Figure 9: 'Composite and Complete' Squares]

15 16 13 14 15 16 13 14	
3 4 1 2 3 4 1 2	n1+ n2+ n5+ n6=S... (1)
7 8 5 6 7 8 5 6	n2+ n3+ n6+ n7=S... (2)
11 12 9 10 11 12 9 10	n3+ n4+ n7+ n8=S... (3)
15 16 13 14 15 16 13 14	n5+ n6+ n9+n10=S... (4)
3 4 1 2 3 4 1 2	n6+ n7+n10+n11=S... (5)
	n7+ n8+n11+n12=S... (6)
	n9+n10+n13+n14=S... (7)
	n10+n11+n14+n15=S... (8)
	n11+n12+n15+n16=S... (9)

$$n1+n2+n3+n4=K... (10); \quad n1+n5+n9+n13=K... (11);$$

$$n1+n6+n11+n16=K... (12) \quad (S=K=34)$$

- (1) $n1+n5=n3+n7=n10+n14=n12+n16=C1;$
 $n2+n6=n4+n8=n9+n13=n11+n15=D1; \quad C1+D1=K(=34)$
- (2) $n5+n9=n7+n11=n2+n14=n4+n16=C2;$
 $n1+n13=n6+n10=n3+n15=n8+n12=D2; \quad C2+D2=K$
- (3) $n1+n2=n9+n10=n7+n8=n15+n16=C3;$
 $n3+n4=n5+n6=n11+n12=n13+n14=D3; \quad C3+D3=K$
- (4) $n2+n3=n5+n8=n10+n11=n13+n16=C4;$
 $n1+n4=n6+n7=n9+n12=n14+n15=D4; \quad C4+D4=K$
- (5) $n1+n11=n6+n16=n2+n12=n7+n13=n3+n9=n8+n14=n4+n10=n5+n15=17$

The last 8 pairs of 17 are all located on its pan-diagonals.
 Each group of the others consists of 2 sets of 4 pairs of C1~4 and D1~4.
 [Notice that each group has (n1+nN) without any exceptions.]

Are there so many complementary pairs available as eight?
 Yes, there are. But not very many varieties.

**** Complementary Pairs For Magic Square 4x4 ** [Figure 10]**

- CC=6: C1(1, 5), C2(2, 4),
 DD=28: D1(16, 12), D2(15, 13),
- CC=7: C1(1, 6), C2(2, 5), C3(3, 4),
 DD=27: D1(16, 11), D2(15, 12), D3(14, 13),
- CC=8: C1(1, 7), C2(2, 6), C3(3, 5),
 DD=26: D1(16, 10), D2(15, 11), D3(14, 12),
- CC=9: C1(1, 8), C2(2, 7), C3(3, 6), C4(4, 5),
 DD=25: D1(16, 9), D2(15, 10), D3(14, 11), D4(13, 12), **OK!**
- CC=10: C1(1, 9), C2(3, 7), C3(4, 6),
 DD=24: D1(16, 8), D2(14, 10), D3(13, 11),
- CC=11: C1(1, 10), C2(2, 9), C3(5, 6),
 DD=23: D1(16, 7), D2(15, 8), D3(12, 11),
- CC=12: C1(1, 11), C2(2, 10), C3(3, 9),
 DD=22: D1(16, 6), D2(15, 7), D3(14, 8),
- CC=13: C1(1, 12), C2(2, 11), C3(3, 10), C4(4, 9),
 DD=21: D1(16, 5), D2(15, 6), D3(14, 7), D4(13, 8), **OK!**
- CC=14: C1(1, 13), C2(2, 12), C3(3, 11),
 DD=20: D1(16, 4), D2(15, 5), D3(14, 6),
- CC=15: C1(1, 14), C2(2, 13), C3(5, 10), C4(6, 9),
 DD=19: D1(16, 3), D2(15, 4), D3(12, 7), D4(11, 8), **OK!**
- CC=16: C1(1, 15), C2(3, 13), C3(5, 11), C4(7, 9),
 DD=18: D1(16, 2), D2(14, 4), D3(12, 6), D4(10, 8), **OK!**
- CC=DD=17: 1(1, 16), 2(2, 15), 3(3, 14), 4(4, 13),
 5(5, 12), 6(6, 11), 7(7, 10), 8(8, 9) **OK!**

There are only 5 groups available. It is the fate we should accept when we want to make pan-magic squares of order 4 in classical style using the integers 1~16.

The last group of CC=DD=17 is used only for making pan-diagonals.

How and where do you place the other 4 groups?

Notice that each group has (1, N) at the top position. Take it and suppose the 'representative pair' of its group. If you assume the 'standard form' with n1=1, the adjacent number next to n1 must take any value of N: {15, 14, 12, 8}.

That means each of {n2, n4, n5, n13} must take any value of {15, 14, 12, 8} but nothing else.

Once they are determined, all the other entries should be determined accordingly, say, in one way. It is amazing.

After all 'N next to 1' is always acting a very important role.

The variety of appearances of {15, 14, 12, 8} on {n2, n4, n5, n13} is calculated as

follows: ${}^4C_2 / {}^2P_2 = 6 / 2 = 3$

[Figure 11: The Numbers Next to N1]

n16	n13	(n1+n2+n5+n6)	+	(n1+n4+n13+n16)	=	34+34=68;
						But n1=1; n6+n16=17;
n4	--n1	--n2	--n3	--n4		2+(n2+n5+n4+n13)+17=68
						Therefore n2+n4+n5+n13=49
n5	n6	n7	n8			{15, 14, 12, 8} are reasonable.

(The original written on July 4, 2001; 2nd version on February 26, 2003;
Newest revision on October 17 by Kanji Setsuda on MacOS X and Xcode 2.2)

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```

/** 'Prototype Squares' of Order 4 and 2 Types of MS44: **/
/** 'Self-Complementary' and 'Complete Pan-magic' Types **/
/** 'ProtoT4S&C48.c' built by Kanji Setsuda **/
/** on Mar. 8, 2003; Oct. 14, 2006 **/
/** Working on MacOSX & Xcode 2.2 **/
/**/
#include <stdio.h>
/**/
short cntc, cnt;
short cnt1, cnt2, cnt3;
short LSM, PSM;
short nm[17], uflg[17];
short lnm[49];
short snm[17], cnm[17];
short ct[49][17];
short at[7][17];
/**/
/* Sub-Routines #1 */
void deep01(void), deep02(void), deep03(void), deep04(void);
void deep05(void), deep06(void), deep07(void), deep08(void);
void deep09(void);
void ansrecord(void);
void printsol(short x);
/**/
/* Sub-Routines #2 */
void stp01(void), stp02(void), stp03(void);
void stp04(void), stp05(void), stp06(void);
void stp07(void), stp08(void), stp09(void);
void recrdans(void);
void pr32ans();
void damts(void);
void convsc(void);
short fi ndnmb(void);
void prlnm(void);
/**/
/* Main Program */
int main(){
short n;
printf("\n [The 48 Standard Solutions of Complete Pan-magic Squares of Order 4]\n");
for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
LSM=34; PSM=17; cntc=0; cnt2=0;
deep01(); /* Make the Solution List of Complete MS44 */
printf(" [Count = %d]\n", cntc);
/**/
printf("\n [Prototype Squares of Order 4 and Recompositions of Self-complementary(/S)\n");
printf(" and Complete(/C) Pan-magic Squares 4x4 Recomposed by 'DAM Transformations']\n");

```

```

for(n=0;n<17;n++){nm[n]=0; ufl g[n]=0;}
cnt=0; cnt1=0; cnt2=0; cnt3=0;
stp01(); /* Make 'Prototype Squares' and Apply 'DAMT' */
printf(" [Count(P/S/C) = %d/%d/%d]\n", cnt, cnt1, cnt2);
printf("\n [Monitor List of Correspondence between Old and New of Complete Type]\n");
prl nm();
printf(" [Count(Ol d/New) = %d/%d]\n", cntc, cnt2);
printf(" OK!\n");
return 0;
}
/**/
/* Make the Solution List of Complete MS44 */
/** Basic Forms: **
14 15 16 13 14 15 16 13 14 15

  2 3 4 | 1 | 2 | 3 | 4 | 1 2 3      | 1 |15| 4|14|
  6 7 8 | 5 | 6 | 7 | 8 | 5 6 7      |12| 6| 9| 7|
 10 11 12| 9 |10|11|12| 9 10 11      |13| 3|16| 2|
 14 15 16|13|14|15|16|13 14 15      | 8|10| 5|11|
 2 3 4 1 2 3 4 1 2 3

** Basic Condi ti ons: **
n1+n2+n3+n4=C ... rw1; | n1+n5+n9+n13=C ... cl 1;
n5+n6+n7+n8=C ... rw2; | n2+n6+n10+n14=C ... cl 2;
n9+n10+n11+n12=C ... rw3; | n3+n7+n11+n15=C ... cl 3;
n13+n14+n15+n16=C ... rw4; | n4+n8+n12+n16=C ... cl 4;

** Complete Condi ti ons: **
n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC...cc

** Li st-forming Inequal i ty Condi ti ons: **
n1<n4, n1<n13, n1<n16 and n5<n2
*/
/* Set N1 & n11=PSM-n1 */
void deep01(){
short a, b;
for(a=1; a<17; a++){b=PSM-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[1]=a; nm[11]=b;
ufl g[a]=1; ufl g[b]=1;
deep02();
ufl g[b]=0; ufl g[a]=0; }
}
}
/* Set N2 & n12=PSM-n2 */
void deep02(){
short a, b;
for(a=16; a>0; a--){b=PSM-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[2]=a; nm[12]=b;
ufl g[a]=1; ufl g[b]=1;
deep03();
ufl g[b]=0; ufl g[a]=0; }
}
}
/* Set N4(>n1) & n10=PSM-n4 */
void deep03(){
short a, b;
for(a=16; a>nm[1]; a--){b=PSM-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[4]=a; nm[10]=b;
ufl g[a]=1; ufl g[b]=1;
deep04();
ufl g[b]=0; ufl g[a]=0; }
}
}

```

```

}
}
/* Set n3=LSM-n1-n2-n4 & n9=LSM-n10-n11-n12 */
/* Check n3+n9==PSM */
void deep04(){
short a, b;
a=LSM-nm[1]-nm[2]-nm[4];
if((0<a)&&(a<17)){
b=LSM-nm[10]-nm[11]-nm[12];
if(a+b==PSM){
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[3]=a; nm[9]=b;
ufl g[a]=1; ufl g[b]=1;
deep05();
ufl g[b]=0; ufl g[a]=0; }}}
}
/* Set n5(<n2) & n15=PSM-n5 */
void deep05(){
short a, b;
for(a=nm[2]-1; a>0; a--){b=PSM-a;
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[5]=a; nm[15]=b;
ufl g[a]=1; ufl g[b]=1;
deep06();
ufl g[b]=0; ufl g[a]=0; }
}
}
/* Set n13=LSM-n1-n5-n9 & n7=LSM-n3-n11-n15 */
/* Check n13+n7==PSM & n13>n1 */
void deep06(){
short a, b;
a=LSM-nm[1]-nm[5]-nm[9];
if((nm[1]<a)&&(a<17)){
b=LSM-nm[3]-nm[11]-nm[15];
if(a+b==PSM){
if((ufl g[a]==0)&&(ufl g[b]==0)){
nm[13]=a; nm[7]=b;
ufl g[a]=1; ufl g[b]=1;
deep07();
ufl g[b]=0; ufl g[a]=0; }}}
}
}
/* Set N6 & n16=PSM-n6 & n16>n1 */
void deep07(){
short a, b;
for(a=1; a<17; a++){b=PSM-a;
if((b>nm[1])&&(ufl g[a]==0)&&(ufl g[b]==0)){
nm[6]=a; nm[16]=b;
ufl g[a]=1; ufl g[b]=1;
deep08();
ufl g[b]=0; ufl g[a]=0; }
}
}
/* Set n8=LSM-n5-n6-n7 & n14=LSM-n13-n15-n16 */
/* Set n8=LSM-n4-n12-n16 & n14=LSM-n2-n6-n10 */
/* Check n8+n14==PSM */
void deep08(){
short a, b, c, d;
a=LSM-nm[5]-nm[6]-nm[7];
if((0<a)&&(a<17)){
b=LSM-nm[13]-nm[15]-nm[16];
if(a+b==PSM){
c=LSM-nm[4]-nm[12]-nm[16];
d=LSM-nm[2]-nm[6]-nm[10];
if((a==c)&&(b==d)){

```

```

        i f((ufl g[a]==0)&&(ufl g[b]==0)){
            nm[8]=a; nm[14]=b;
            ufl g[a]=1; ufl g[b]=1;
            deep09();
            ufl g[b]=0; ufl g[a]=0; }}}}
    }
/**/
/* Check Sums of 2 Primary Diagonals */
void deep09(){
    short sm1, sm2;
    sm1=nm[1]+nm[6]+nm[11]+nm[16];
    sm2=nm[4]+nm[7]+nm[10]+nm[13];
    i f((sm1==LSM)&&(sm2==LSM)){ansrecord();}
}
/**/
/* Record the Answers */
void ansrecord(){
    short n;
    ct[cntc][0]=cntc+1;
    for(n=1; n<17; n++){ct[cntc][n]=nm[n];}
    cntc++;
    at[cnt2][0]=cntc;
    for(n=1; n<17; n++){at[cnt2][n]=nm[n];}
    cnt2++;
    i f(cnt2==6){printsol (cnt2); cnt2=0;}
}
/**/
/* Print the CMS44 */
void printsol (short x){
    short l, l4, m, n;
    for(m=0; m<x; m++){
        pri ntf("%12d/", at[m][0]);
    }
    pri ntf("\n");
    for(l=0; l<4; l++){l4=l*4;
        for(m=0; m<x; m++){
            pri ntf(" ");
            for(n=1; n<5; n++){pri ntf("%3d", at[m][l4+n]);}
        }
        pri ntf("\n");
    }
}
/**/
/* Make 'Prototype Squares' and Apply 'DAMT' */
/** Basic Forms for Prototype Squares 4x4: **
    3 2 16 13 3 2 16 13 3 2

15 14 4 | 1|15|14| 4| 1 15 14 | 1 | 2| 3| 4|
-----+-----+-----+-----+-----
6 7 9 |12| 6| 7| 9|12 6 7 | 5 | 6| 7| 8|
-----+-----+-----+-----+-----
10 11 5 | 8|10|11| 5| 8 10 11 | 9 |10|11|12|
-----+-----+-----+-----+-----
3 2 16 |13| 3| 2|16|13 3 2 |13|14|15|16|
-----+-----+-----+-----+-----
15 14 4 1 15 14 4 1 15 14

** Basic Condi ti ons for Prototype Squares 4x4: **
n1+n15+n14+n4=C ... rw1; | n1+n12+n8+n13=C ... cl 1;
n12+n6+n7+n9=C ... rw2; | n15+n6+n10+n3=C ... cl 2;
n8+n10+n11+n5=C ... rw3; | n14+n7+n11+n2=C ... cl 3;
n13+n3+n2+n16=C ... rw4; | n4+n9+n5+n16=C ... cl 4;

** Self-complementary Condi ti ons for Prototype Squares 4x4: **
n1+n16=n15+n2=n14+n3=n4+n13=n12+n5=n6+n11=n7+n10=n9+n8=CC... scc
*/
/* Set N1 & n16=PSM-n1 */

```

```

void stp01(){
  short a, b;
  for(a=1; a<17; a++){b=PSM-a;
    if((ufl g[a]==0)&&(ufl g[b]==0)){
      nm[1]=a; nm[16]=b;
      ufl g[a]=1; ufl g[b]=1;
      stp02();
      ufl g[b]=0; ufl g[a]=0; }
  }
}
/* Set N15 & n2=PSM-n15 */
void stp02(){
  short a, b;
  for(a=16; a>0; a--){b=PSM-a;
    if((ufl g[a]==0)&&(ufl g[b]==0)){
      nm[15]=a; nm[2]=b;
      ufl g[a]=1; ufl g[b]=1;
      stp03();
      ufl g[b]=0; ufl g[a]=0; }
  }
}
/* Set N14 & n3 */
void stp03(){
  short a, b;
  for(a=16; a>0; a--){b=PSM-a;
    if((ufl g[a]==0)&&(ufl g[b]==0)){
      ufl g[a]=1; ufl g[b]=1;
      nm[14]=a; nm[3]=b;
      stp04();
      ufl g[b]=0; ufl g[a]=0; }
  }
}
/* Set n4=LSM-n1-n15-n14 & n13 */
void stp04(){
  short a, b;
  a=LSM-nm[1]-nm[15]-nm[14];
  if((0<a)&&(a<17)){
    b=LSM-nm[3]-nm[2]-nm[16];
    if(a+b==PSM){
      if((ufl g[a]==0)&&(ufl g[b]==0)){
        ufl g[a]=1; ufl g[b]=1;
        nm[4]=a; nm[13]=b;
        stp05();
        ufl g[b]=0; ufl g[a]=0; }}}
  }
}
/* Set N12 & n5=PSM-n12 */
void stp05(){
  short a, b;
  for(a=16; a>0; a--){b=PSM-a;
    if((ufl g[a]==0)&&(ufl g[b]==0)){
      nm[12]=a; nm[5]=b;
      ufl g[a]=1; ufl g[b]=1;
      stp06();
      ufl g[b]=0; ufl g[a]=0; }
  }
}
/* Set n8=LSM-n1-n12-n13 & n9 */
void stp06(){
  short a, b;
  a=LSM-nm[1]-nm[12]-nm[13];
  if((0<a)&&(a<17)){
    b=LSM-nm[4]-nm[5]-nm[16];
    if(a+b==PSM){
      if((ufl g[a]==0)&&(ufl g[b]==0)){

```

```

        ufl g[a]=1; ufl g[b]=1;
        nm[8]=a; nm[9]=b;
        stp07();
        ufl g[b]=0; ufl g[a]=0; }}}
}
/* Set N6 & n11 */
void stp07(){
    short a, b;
    for(a=1; a<17; a++){b=PSM-a;
        if((ufl g[a]==0)&&(ufl g[b]==0)){
            ufl g[a]=1; ufl g[b]=1;
            nm[6]=a; nm[11]=b;
            stp08();
            ufl g[b]=0; ufl g[a]=0; }
        }
}
/* Set n7=LSM-n12-n6-n9 & n10 */
void stp08(){
    short a, b, c, d;
    a=LSM-nm[12]-nm[6]-nm[9];
    if((0<a)&&(a<17)){
        b=LSM-nm[8]-nm[11]-nm[5];
        if(a+b==PSM){
            c=LSM-nm[14]-nm[11]-nm[2];
            d=LSM-nm[15]-nm[6]-nm[3];
            if((a==c)&&(b==d)){
                if((ufl g[a]==0)&&(ufl g[b]==0)){
                    nm[7]=a; nm[10]=b;
                    ufl g[a]=1; ufl g[b]=1;
                    recrdans();
                    ufl g[b]=0; ufl g[a]=0; }}}
        }
}
/**/
/* Record the Solutions and Print them out */
void recrdans(){
    short c3, m, n, f;
    c3=cnt3*3;
    at[c3][0]=cnt+1;
    for(n=1; n<17; n++){at[c3][n]=nm[n]; }
    cnt++; f=0;
    for(m=1; m<3; m++){for(n=0; n<17; n++){at[c3+m][n]=0; }}
    damts();
    if((snm[1]<snm[4])&&(snm[1]<snm[13])&&(snm[1]<snm[16])&&(snm[5]<snm[2])){
        for(n=1; n<17; n++){at[c3+1][n]=snm[n]; }
        at[c3+1][0]=cnt1+1; cnt1++; f++; }
    convsc();
    if((cnm[1]<cnm[4])&&(cnm[1]<cnm[13])&&(cnm[1]<cnm[16])&&(cnm[5]<cnm[2])){
        for(n=1; n<17; n++){at[c3+2][n]=cnm[n]; }
        n=fi ndnmb(); l nm[n]++;
        at[c3+2][0]=n; cnt2++; f++; }
    if(f>0){cnt3++; }
    if(cnt3==2){pr32ans(); cnt3=0; }
}
/**/
/* Print 2 Groups of Solutions */
void pr32ans(){
    short l, l4, m, n;
    printf("%11d/P%11d/S%11d/C%13d/P%11d/S%11d/C\n",
        at[0][0], at[1][0], at[2][0], at[3][0], at[4][0], at[5][0]);
    for(l=0; l<4; l++){l4=l*4;
        for(m=0; m<6; m=m+3){
            printf(" ");
            for(n=1; n<5; n++){printf("%3d", at[m][l4+n]); }
            printf(" ");
        }
    }
}

```

```

        for(n=1;n<5;n++){printf("%3d",at[m+1][l4+n]);}
        printf(" ");
        for(n=1;n<5;n++){printf("%3d",at[m+2][l4+n]);}
        if(m<3){printf(" ");}
    }
    printf("\n");
}
}
/**/
/*
16 13 14 15 16 13   16 13 3 2 16 13   11 8 10 5 11 8
4 | 1| 2| 3| 4| 1   4 | 1|15|14| 4| 1   14 | 1|15| 4|14| 1
---+---+---+---+
8 | 5| 6| 7| 8| 5   9 |12| 6| 7| 9|12   7 |12| 6| 9| 7|12
---+---+---+---+
12| 9|10|11|12| 9   5 | 8|10|11| 5| 8   2 |13| 3|16| 2|13
---+---+---+---+
16|13|14|15|16|13   16|13| 3| 2|16|13   11| 8|10| 5|11| 8
---+---+---+---+
4  1  2  3  4  1   4  1 15 14  4  1   14  1 15  4 14  1
*/
/* Transform Prototype into Self-Complementary Object */
void damts(void){
    snm[1]=nm[1];   snm[2]=nm[15];   snm[3]=nm[14];   snm[4]=nm[4];
    snm[5]=nm[12];  snm[6]=nm[6];    snm[7]=nm[7];    snm[8]=nm[9];
    snm[9]=nm[8];   snm[10]=nm[10];  snm[11]=nm[11];  snm[12]=nm[5];
    snm[13]=nm[13]; snm[14]=nm[3];   snm[15]=nm[2];   snm[16]=nm[16];
}
/**/
/* Transform Self-Complementary into Complete Pan-magic */
void convsc(void){
    cnm[1]=snm[1];  cnm[2]=snm[2];    cnm[3]=snm[4];    cnm[4]=snm[3];
    cnm[5]=snm[5];  cnm[6]=snm[6];    cnm[7]=snm[8];    cnm[8]=snm[7];
    cnm[9]=snm[13]; cnm[10]=snm[14];   cnm[11]=snm[16];  cnm[12]=snm[15];
    cnm[13]=snm[9];  cnm[14]=snm[10];   cnm[15]=snm[12];  cnm[16]=snm[11];
}
/**/
/* Find the Original Number */
short findnmb(){
    short mflg,cn,m,n;
    cn=0;
    for(m=0;m<cntc;m++){
        mflg=0;
        for(n=1;n<17;n++){if(ct[m][n]==cnm[n]){mflg++;}}
        if(mflg==16){cn=ct[m][0]; break;}
    }
    return cn;
}
/**/
/* Print the Correspondence Monitor */
void prlnm(){
    short n;
    printf(" ??:%3d",lnm[0]);
    for(n=1;n<=cntc;n++){
        if(n%16==1){printf("\n%4d:",n);}
        printf("%3d",lnm[n]);
    }
    printf("\n");
}
}
/**/

```