Chapter 2. New Study of Magic Squares of Order 4: Kanji Setsuda

Section 2. New Algebraic Study of Pan-Magic Squares 4x4

#1. Pan-Diagonal Magic Square of Order 4

Let's study about Pan-diagonal magic squares of order 4 here. I know there are various kinds of definition of this type, but I like to start simply with these diagrams and simultaneous equations as follows.

[Basic Form and Basic Definitions]

```
 .--.--.--.--.     * Rows and Columns *
 |n1|n2|n3|n4|      n1+ n2+ n3+ n4 = C ... rw1
 |--+--+--+--|      n5+ n6+ n7+ n8 = C ... rw2
 |n5|n6|n7|n8|      n9+n10+n11+n12 = C ... rw3
 |--+--+--+--|     n13+n14+n15+n16 = C ... rw4
 |n9|10|11|12|      n1+ n5+ n9+n13 = C ... cl1
 |--+--+--+--|      n2+ n6+n10+n14 = C ... cl2
 |13|14|15|16|      n3+ n7+n11+n15 = C ... cl3
 '--'--'--'--'      n4+ n8+n12+n16 = C ... cl4
```

[Extended Space, Moving Frame and Pan-diagonals]

```
      .--.--------.--.
 11 12|n9 10 11 12|n9|10 11
   .--.--.--------.--.--.
 15|16|13|14 15 16|13|14|15  * Pan-diagonal Equations *
   .--.-----------.--.--.     n1+ n6+n11+n16 = C  ... pd1
 n3|n4|n1|n2|n3|n4|n1|n2|n3   n1+ n8+n11+n14 = C  ... pd2
   .--|--+--+--+--|--. -|     n2+ n7+n12+n13 = C  ... pd3
 n7|n8|n5|n6|n7|n8|n5|n6|n7   n2+ n5+n12+n15 = C  ... pd4
   |- |--+--+--+--| -| -|     n3+ n8+ n9+n14 = C  ... pd5
 11|12|n9|10|11|12|n9|10|11   n3+ n6+ n9+n16 = C  ... pd6
   |- |--+--+--+--| -' -|     n4+ n5+n10+n15 = C  ... pd7
 15|16|13|14|15|16|13|14|15   n4+ n7+n10+n13 = C  ... pd8
   '--'-----------'--'--'     (C means the magic constant 34.)
 n3 n4|n1|n2 n3 n4|n1|n2 n3
      '--'--------'--'
```

The lower figure above shows the Extended Space and the moving frame of 4x4 square. This diagram is very useful for us to find any pan-diagonals, because we can always see them in 'straight lines' instead of those notorious 'broken, wrapping lines.'

Pan-diagonal Equations pd1~pd8 can be easily taken out of the Extended Space.

You can move any last row or column of the supposed square far to the opposite side and put it beyond the first, and you can make a new pan-magic square, as you know. In the Extended Space you can realize the same thing by moving the 'square frame' as illustrated above. New squares can be taken out only by picking up any 4x4 entries with the new frames moved to anywhere else.

```
* 4x4 New Squares Taken out of the Extended Space *
           00/            01/            02/            03/
   n1 n2 n3 n4    13 14 15 16     9 10 11 12     5  6  7  8
   n5 n6 n7 n8     1  2  3  4    13 14 15 16     9 10 11 12
   n9 10 11 12     5  6  7  8     1  2  3  4    13 14 15 16
   13 14 15 16     9 10 11 12     5  6  7  8     1  2  3  4
```

```
            10/          11/          12/          13/
   4  1  2  3   16 13 14 15   12  9 10 11    8  5  6  7
   8  5  6  7    4  1  2  3   16 13 14 15   12  9 10 11
  12  9 10 11    8  5  6  7    4  1  2  3   16 13 14 15
  16 13 14 15   12  9 10 11    8  5  6  7    4  1  2  3
            20/          21/          22/          23/
   3  4  1  2   15 16 13 14   11 12  9 10    7  8  5  6
   7  8  5  6    3  4  1  2   15 16 13 14   11 12  9 10
  11 12  9 10    7  8  5  6    3  4  1  2   15 16 13 14
  15 16 13 14   11 12  9 10    7  8  5  6    3  4  1  2
            30/          31/          32/          33/
   2  3  4  1   14 15 16 13   10 11 12  9    6  7  8  5
   6  7  8  5    2  3  4  1   14 15 16 13   10 11 12  9
  10 11 12  9    6  7  8  5    2  3  4  1   14 15 16 13
  14 15 16 13   10 11 12  9    6  7  8  5    2  3  4  1
```

You can finally take out those new pan-magic squares above as many as 4x4.
In any form two diagonals must add up to the magic constant C(=34), since it should just satisfy the original concept of 'pan-magic square' of order 4.
The constant sum of every pandiagonal makes it possible.

#2. How many Solutions can we find?

What does it actually look like? Now let's make our computer draw and count all pan-magic squares 4x4 at first, shall we?
We know there are three levels of solutions as follows: (1) 'Primitive', (2) 'Standard', and (3) 'Fundamental'. Let's have them all here.

```
** 384 'Primitive Solutions' of Pan-Diagonal Magic Squares 4x4 **
          1/          2/          3/          4/          5/          6/
   1  8 10 15   1  8 10 15   1  8 11 14   1  8 11 14   1  8 13 12   1  8 13 12
  12 13  3  6  14 11  5  4  12 13  2  7  15 10  5  4  14 11  2  7  15 10  3  6
   7  2 16  9   7  2 16  9   6  3 16  9   6  3 16  9   4  5 16  9   4  5 16  9
  14 11  5  4  12 13  3  6  15 10  5  4  12 13  2  7  15 10  3  6  14 11  2  7
          7/          8/          9/         10/         11/         12/
   1 12  6 15   1 12  6 15   1 12  7 14   1 12  7 14   1 12 13  8   1 12 13  8
   8 13  3 10  14  7  9  4   8 13  2 11  15  6  9  4  14  7  2 11  15  6  3 10
  11  2 16  5  11  2 16  5  10  3 16  5  10  3 16  5   4  9 16  5   4  9 16  5
  14  7  9  4   8 13  3 10  15  6  9  4   8 13  2 11  15  6  3 10  14  7  2 11
         13/         14/         15/         16/         17/         18/
   1 14  4 15   1 14  4 15   1 14  7 12   1 14  7 12   1 14 11  8   1 14 11  8
   8 11  5 10  12  7  9  6   8 11  2 13  15  4  9  6  12  7  2 13  15  4  5 10
  13  2 16  3  13  2 16  3  10  5 16  3  10  5 16  3   6  9 16  3   6  9 16  3
  12  7  9  6   8 11  5 10  15  4  9  6   8 11  2 13  15  4  5 10  12  7  2 13
         19/         20/         21/         22/         23/         24/
   1 15  4 14   1 15  4 14   1 15  6 12   1 15  6 12   1 15 10  8   1 15 10  8
   8 10  5 11  12  6  9  7   8 10  3 13  14  4  9  7  12  6  3 13  14  4  5 11
  13  3 16  2  13  3 16  2  11  5 16  2  11  5 16  2   7  9 16  2   7  9 16  2
  12  6  9  7   8 10  5 11  14  4  9  7   8 10  3 13  14  4  5 11  12  6  3 13
         25/         26/         27/         28/         29/         30/
   2  7  9 16   2  7  9 16   2  7 12 13   2  7 12 13   2  7 14 11   2  7 14 11
  11 14  4  5  13 12  6  3  11 14  1  8  16  9  6  3  13 12  1  8  16  9  4  5
   8  1 15 10   8  1 15 10   5  4 15 10   5  4 15 10   3  6 15 10   3  6 15 10
  13 12  6  3  11 14  4  5  16  9  6  3  11 14  1  8  16  9  4  5  13 12  1  8
```

			31/				32/				33/				34/				35/				36/
2	11	5	16	2	11	5	16	2	11	8	13	2	11	8	13	2	11	14	7	2	11	14	7
7	14	4	9	13	8	10	3	7	14	1	12	16	5	10	3	13	8	1	12	16	5	4	9
12	1	15	6	12	1	15	6	9	4	15	6	9	4	15	6	3	10	15	6	3	10	15	6
13	8	10	3	7	14	4	9	16	5	10	3	7	14	1	12	16	5	4	9	13	8	1	12

			37/				38/				39/				40/				41/				42/
2	13	3	16	2	13	3	16	2	13	8	11	2	13	8	11	2	13	12	7	2	13	12	7
7	12	6	9	11	8	10	5	7	12	1	14	16	3	10	5	11	8	1	14	16	3	6	9
14	1	15	4	14	1	15	4	9	6	15	4	9	6	15	4	5	10	15	4	5	10	15	4
11	8	10	5	7	12	6	9	16	3	10	5	7	12	1	14	16	3	6	9	11	8	1	14

			43/				44/				45/				46/				47/				48/
2	16	3	13	2	16	3	13	2	16	5	11	2	16	5	11	2	16	9	7	2	16	9	7
7	9	6	12	11	5	10	8	7	9	4	14	13	3	10	8	11	5	4	14	13	3	6	12
14	4	15	1	14	4	15	1	12	6	15	1	12	6	15	1	8	10	15	1	8	10	15	1
11	5	10	8	7	9	6	12	13	3	10	8	7	9	4	14	13	3	6	12	11	5	4	14

			49/				50/				51/				52/				53/				54/
3	6	9	16	3	6	9	16	3	6	12	13	3	6	12	13	3	6	15	10	3	6	15	10
10	15	4	5	13	12	7	2	10	15	1	8	16	9	7	2	13	12	1	8	16	9	4	5
8	1	14	11	8	1	14	11	5	4	14	11	5	4	14	11	2	7	14	11	2	7	14	11
13	12	7	2	10	15	4	5	16	9	7	2	10	15	1	8	16	9	4	5	13	12	1	8

			55/				56/				57/				58/				59/				60/
3	10	5	16	3	10	5	16	3	10	8	13	3	10	8	13	3	10	15	6	3	10	15	6
6	15	4	9	13	8	11	2	6	15	1	12	16	5	11	2	13	8	1	12	16	5	4	9
12	1	14	7	12	1	14	7	9	4	14	7	9	4	14	7	2	11	14	7	2	11	14	7
13	8	11	2	6	15	4	9	16	5	11	2	6	15	1	12	16	5	4	9	13	8	1	12

			61/				62/				63/				64/				65/				66/
3	13	2	16	3	13	2	16	3	13	8	10	3	13	8	10	3	13	12	6	3	13	12	6
6	12	7	9	10	8	11	5	6	12	1	15	16	2	11	5	10	8	1	15	16	2	7	9
15	1	14	4	15	1	14	4	9	7	14	4	9	7	14	4	5	11	14	4	5	11	14	4
10	8	11	5	6	12	7	9	16	2	11	5	6	12	1	15	16	2	7	9	10	8	1	15

			67/				68/				69/				70/				71/				72/
3	16	2	13	3	16	2	13	3	16	5	10	3	16	5	10	3	16	9	6	3	16	9	6
6	9	7	12	10	5	11	8	6	9	4	15	13	2	11	8	10	5	4	15	13	2	7	12
15	4	14	1	15	4	14	1	12	7	14	1	12	7	14	1	8	11	14	1	8	11	14	1
10	5	11	8	6	9	7	12	13	2	11	8	6	9	4	15	13	2	7	12	10	5	4	15

			73/				74/				75/				76/				77/				78/
4	5	10	15	4	5	10	15	4	5	11	14	4	5	11	14	4	5	16	9	4	5	16	9
9	16	3	6	14	11	8	1	9	16	2	7	15	10	8	1	14	11	2	7	15	10	3	6
7	2	13	12	7	2	13	12	6	3	13	12	6	3	13	12	1	8	13	12	1	8	13	12
14	11	8	1	9	16	3	6	15	10	8	1	9	16	2	7	15	10	3	6	14	11	2	7

			79/				80/				81/				82/				83/				84/
4	9	6	15	4	9	6	15	4	9	7	14	4	9	7	14	4	9	16	5	4	9	16	5
5	16	3	10	14	7	12	1	5	16	2	11	15	6	12	1	14	7	2	11	15	6	3	10
11	2	13	8	11	2	13	8	10	3	13	8	10	3	13	8	1	12	13	8	1	12	13	8
14	7	12	1	5	16	3	10	15	6	12	1	5	16	2	11	15	6	3	10	14	7	2	11

			85/				86/				87/				88/				89/				90/
4	14	1	15	4	14	1	15	4	14	7	9	4	14	7	9	4	14	11	5	4	14	11	5
5	11	8	10	9	7	12	6	5	11	2	16	15	1	12	6	9	7	2	16	15	1	8	10
16	2	13	3	16	2	13	3	10	8	13	3	10	8	13	3	6	12	13	3	6	12	13	3
9	7	12	6	5	11	8	10	15	1	12	6	5	11	2	16	15	1	8	10	9	7	2	16

			91/				92/				93/				94/				95/				96/
4	15	1	14	4	15	1	14	4	15	6	9	4	15	6	9	4	15	10	5	4	15	10	5
5	10	8	11	9	6	12	7	5	10	3	16	14	1	12	7	9	6	3	16	14	1	8	11
16	3	13	2	16	3	13	2	11	8	13	2	11	8	13	2	7	12	13	2	7	12	13	2
9	6	12	7	5	10	8	11	14	1	12	7	5	10	3	16	14	1	8	11	9	6	3	16

```
         97/          98/          99/         100/         101/         102/
  5  4  9 16   5  4  9 16   5  4 14 11   5  4 14 11   5  4 15 10   5  4 15 10
 10 15  6  3  11 14  7  2  10 15  1  8  16  9  7  2  11 14  1  8  16  9  6  3
  8  1 12 13   8  1 12 13   3  6 12 13   3  6 12 13   2  7 12 13   2  7 12 13
 11 14  7  2  10 15  6  3  16  9  7  2  10 15  1  8  16  9  6  3  11 14  1  8

 . . . . . .

        355/         356/         357/         358/         359/         360/
 15 10  3  6  15 10  3  6  15 10  5  4  15 10  5  4  15 10  8  1  15 10  8  1
  1  8 13 12   4  5 16  9   1  8 11 14   6  3 16  9   4  5 11 14   6  3 13 12
 14 11  2  7  14 11  2  7  12 13  2  7  12 13  2  7   9 16  2  7   9 16  2  7
  4  5 16  9   1  8 13 12   6  3 16  9   1  8 11 14   6  3 13 12   4  5 11 14
        361/         362/         363/         364/         365/         366/
 16  2  7  9  16  2  7  9  16  2 11  5  16  2 11  5  16  2 13  3  16  2 13  3
  3 13 12  6   5 11 14  4   3 13  8 10   9  7 14  4   5 11  8 10   9  7 12  6
 10  8  1 15  10  8  1 15   6 12  1 15   6 12  1 15   4 14  1 15   4 14  1 15
  5 11 14  4   3 13 12  6   9  7 14  4   3 13  8 10   9  7 12  6   5 11  8 10
        367/         368/         369/         370/         371/         372/
 16  3  6  9  16  3  6  9  16  3 10  5  16  3 10  5  16  3 13  2  16  3 13  2
  2 13 12  7   5 10 15  4   2 13  8 11   9  6 15  4   5 10  8 11   9  6 12  7
 11  8  1 14  11  8  1 14   7 12  1 14   7 12  1 14   4 15  1 14   4 15  1 14
  5 10 15  4   2 13 12  7   9  6 15  4   2 13  8 11   9  6 12  7   5 10  8 11
        373/         374/         375/         376/         377/         378/
 16  5  4  9  16  5  4  9  16  5 10  3  16  5 10  3  16  5 11  2  16  5 11  2
  2 11 14  7   3 10 15  6   2 11  8 13   9  4 15  6   3 10  8 13   9  4 14  7
 13  8  1 12  13  8  1 12   7 14  1 12   7 14  1 12   6 15  1 12   6 15  1 12
  3 10 15  6   2 11 14  7   9  4 15  6   2 11  8 13   9  4 14  7   3 10  8 13
        379/         380/         381/         382/         383/         384/
 16  9  4  5  16  9  4  5  16  9  6  3  16  9  6  3  16  9  7  2  16  9  7  2
  2  7 14 11   3  6 15 10   2  7 12 13   5  4 15 10   3  6 12 13   5  4 14 11
 13 12  1  8  13 12  1  8  11 14  1  8  11 14  1  8  10 15  1  8  10 15  1  8
  3  6 15 10   2  7 14 11   5  4 15 10   2  7 12 13   5  4 14 11   3  6 12 13
* Counts according to the value of n1 *
 [ 1: 24] [ 2: 24] [ 3: 24] [ 4: 24] [ 5: 24] [ 6: 24] [ 7: 24] [ 8: 24]
 [ 9: 24] [10: 24] [11: 24] [12: 24] [13: 24] [14: 24] [15: 24] [16: 24]
* Total Count = 384 *
```

The next list is a sample program dictated for the purpose of making the 'Standard Solutions' of pan-diagonal magic squares of order 4. The list-forming inequality conditions I put for them are {n1<n4; n1<n13; n1<n16 and n2<n5}.

```c
/** Pandiagonal Magic Squares of Order 4 **/
/** 'MS44PD48.c' built by Kanji Setsuda  **/
/**  on Aug. 8, 2000; Dec.16, 2005; **/
/**  Worked on MacOSX and Xcode 1.5 **/
/**/
#include <stdio.h>
/**/
short cnt, cnt2;
short LSM;
short nm[17], uflg[17];
short anm[9][17];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
```

```
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void), stp16(void);
void ansprint(void);
void printans(short x);
/**/
/* Main Program */
int main(){
 short n;
  printf("\n** 48 Standard Solutions of Pan-diagonal Magic Squares 4x4 **\n");
  for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
  LSM=34; cnt=0; cnt2=0;
  stp01();/* Begin the Calculations */
  if(cnt2>0){printans(cnt2);}
  printf(" [Count = %d]\n",cnt);
  printf("  OK!\n");
 return 0;
}
/* Begin the Calculations */
/* Set n1 */
void stp01(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[1]=a; uflg[a]=1;
      stp02();
      uflg[a]=0;}
  }
}
/* Set n2 */
void stp02(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[2]=a; uflg[a]=1;
      stp03();
      uflg[a]=0;}
  }
}
/* Set n5 & n5>n2 */
void stp03(){
 short a;
  for(a=nm[2]+1;a<17;a++){
    if(uflg[a]==0){
      nm[5]=a; uflg[a]=1;
      stp04();
      uflg[a]=0;}
  }
}
/* Set n6 */
void stp04(){
 short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[6]=a; uflg[a]=1;
      stp05();
      uflg[a]=0;}
  }
}
```

```
/* Set n4 & n1<n4 */
void stp05(){
 short a;
  for(a=nm[1]+1;a<17;a++){
    if(uflg[a]==0){
      nm[4]=a; uflg[a]=1;
      stp06();
      uflg[a]=0;}
  }
}
/* Set n3=LSM-n1-n2-n4 */
void stp06(){
 short a;
  a=LSM-nm[1]-nm[2]-nm[4];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[3]=a; uflg[a]=1;
    stp07();
    uflg[a]=0;}
}
/* Set n13 & n1<n13 */
void stp07(){
 short a;
  for(a=nm[1]+1;a<17;a++){
    if(uflg[a]==0){
      nm[13]=a; uflg[a]=1;
      stp08();
      uflg[a]=0;}
  }
}
/* Set n9=LSM-n1-n5-n13 */
void stp08(){
 short a;
  a=LSM-nm[1]-nm[5]-nm[13];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[9]=a; uflg[a]=1;
    stp09();
    uflg[a]=0;}
}
/* Set n16=LSM-n3-n6-n9 & n1<n16 */
void stp09(){
 short a;
  a=LSM-nm[3]-nm[6]-nm[9];
  if((nm[1]<a)&&(a<17)&&(uflg[a]==0)){
    nm[16]=a; uflg[a]=1;
    stp10();
    uflg[a]=0;}
}
/* Set n11=LSM-n1-n6-n16 */
void stp10(){
 short a;
  a=LSM-nm[1]-nm[6]-nm[16];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[11]=a; uflg[a]=1;
    stp11();
    uflg[a]=0;}
}
/* Set n7 */
void stp11(){
 short a;
```

```
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[7]=a; uflg[a]=1;
      stp12();
      uflg[a]=0;}
  }
}
/* Set n8=LSM-n5-n6-n7 */
void stp12(){
 short a;
  a=LSM-nm[5]-nm[6]-nm[7];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[8]=a; uflg[a]=1;
    stp13();
    uflg[a]=0;}
}
/* Set n10=LSM-n4-n7-n13 */
void stp13(){
 short a;
  a=LSM-nm[4]-nm[7]-nm[13];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[10]=a; uflg[a]=1;
    stp14();
    uflg[a]=0;}
}
/* Set n12=LSM-n9-n10-n11 */
void stp14(){
 short a,b,c;
  a=LSM-nm[9]-nm[10]-nm[11];
  b=LSM-nm[4]-nm[8]-nm[16];
  c=LSM-nm[2]-nm[7]-nm[13];
  if((0<a)&&(a<17)){
    if((a==b)&&(a==c)&&(uflg[a]==0)){
      nm[12]=a; uflg[a]=1;
      stp15();
      uflg[a]=0;}}
}
/* Set n14=LSM-n2-n6-n10 */
void stp15(){
 short a,b,c;
  a=LSM-nm[2]-nm[6]-nm[10];
  b=LSM-nm[1]-nm[8]-nm[11];
  c=LSM-nm[3]-nm[8]-nm[9];
  if((0<a)&&(a<17)){
    if((a==b)&&(a==c)&&(uflg[a]==0)){
      nm[14]=a; uflg[a]=1;
      stp16();
      uflg[a]=0;}}
}
/* Set n15=LSM-n3-n7-n11 */
void stp16(){
 short a,b,c,d;
  a=LSM-nm[3]-nm[7]-nm[11];
  b=LSM-nm[2]-nm[5]-nm[12];
  c=LSM-nm[4]-nm[5]-nm[10];
  d=LSM-nm[13]-nm[14]-nm[16];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    if((a==b)&&(a==c)&&(a==d)){
      nm[15]=a; uflg[a]=1;
```

```c
       ansprint();
       uflg[a]=0;}}
}
/**/
/* Print the Answers */
void ansprint(){
 short n;
  cnt++;
  anm[cnt2][0]=cnt;
  for(n=1;n<17;n++){anm[cnt2][n]=nm[n];}
  cnt2++; if(cnt2==6){printans(6); cnt2=0;}
}
/* Print the Answers */
void printans(short x){
 short m,n;
  for(m=0;m<x;m++){printf("%13d/",anm[m][0]);}
  printf("\n");
  for(m=0;m<x;m++){printf(" .--.--.--.--.");}
  printf("\n");
  for(m=0;m<x;m++){
    printf(" |%2d|%2d|%2d|%2d|",anm[m][1],anm[m][2],anm[m][3],anm[m][4]);}
  printf("\n");
  for(m=0;m<x;m++){printf(" |--+--+--+--|");}
  printf("\n");
  for(m=0;m<x;m++){
    printf(" |%2d|%2d|%2d|%2d|",anm[m][5],anm[m][6],anm[m][7],anm[m][8]);}
  printf("\n");
  for(m=0;m<x;m++){printf(" |--+--+--+--|");}
  printf("\n");
  for(m=0;m<x;m++){
    printf(" |%2d|%2d|%2d|%2d|",anm[m][9],anm[m][10],anm[m][11],anm[m][12]);}
  printf("\n");
  for(m=0;m<x;m++){printf(" |--+--+--+--|");}
  printf("\n");
  for(m=0;m<x;m++){
    printf(" |%2d|%2d|%2d|%2d|",anm[m][13],anm[m][14],anm[m][15],anm[m][16]);}
  printf("\n");
  for(m=0;m<x;m++){printf(" '--'--'--'--'");}
  printf("\n");
}
/**/
```

```
*** 48 'Standard Solutions' of Pan-diagonal Magic Squares 4x4  ***
           1/           2/           3/           4/           5/           6/
 .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--.
 | 1| 8|11|14| | 1| 8|10|15| | 1| 8|13|12| | 1| 8|10|15| | 1| 8|13|12| | 1| 8|11|14|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |12|13| 2| 7| |12|13| 3| 6| |14|11| 2| 7| |14|11| 5| 4| |15|10| 3| 6| |15|10| 5| 4|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 | 6| 3|16| 9| | 7| 2|16| 9| | 4| 5|16| 9| | 7| 2|16| 9| | 4| 5|16| 9| | 6| 3|16| 9|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |15|10| 5| 4| |14|11| 5| 4| |15|10| 3| 6| |12|13| 3| 6| |14|11| 2| 7| |12|13| 2| 7|
 '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--'
           7/           8/           9/          10/          11/          12/
 .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--.
 | 1|12|13| 8| | 1|12| 6|15| | 1|12|13| 8| | 1|12| 7|14| | 1|14|11| 8| | 1|14| 7|12|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |14| 7| 2|11| |14| 7| 9| 4| |15| 6| 3|10| |15| 6| 9| 4| |15| 4| 5|10| |15| 4| 9| 6|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 | 4| 9|16| 5| |11| 2|16| 5| | 4| 9|16| 5| |10| 3|16| 5| | 6| 9|16| 3| |10| 5|16| 3|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |15| 6| 3|10| | 8|13| 3|10| |14| 7| 2|11| | 8|13| 2|11| |12| 7| 2|13| | 8|11| 2|13|
 '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--'
```

```
          13/             14/             15/             16/             17/             18/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 2| 7|12|13|   | 2| 7| 9|16|   | 2| 7|14|11|   | 2| 7| 9|16|   | 2| 7|14|11|   | 2| 7|12|13|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|11|14| 1| 8|   |11|14| 4| 5|   |13|12| 1| 8|   |13|12| 6| 3|   |16| 9| 4| 5|   |16| 9| 6| 3|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 5| 4|15|10|   | 8| 1|15|10|   | 3| 6|15|10|   | 8| 1|15|10|   | 3| 6|15|10|   | 5| 4|15|10|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|16| 9| 6| 3|   |13|12| 6| 3|   |16| 9| 4| 5|   |11|14| 4| 5|   |13|12| 1| 8|   |11|14| 1| 8|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

          19/             20/             21/             22/             23/             24/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 2|11|14| 7|   | 2|11| 5|16|   | 2|11|14| 7|   | 2|11| 8|13|   | 2|13|12| 7|   | 2|13| 8|11|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|13| 8| 1|12|   |13| 8|10| 3|   |16| 5| 4| 9|   |16| 5|10| 3|   |16| 3| 6| 9|   |16| 3|10| 5|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 3|10|15| 6|   |12| 1|15| 6|   | 3|10|15| 6|   | 9| 4|15| 6|   | 5|10|15| 4|   | 9| 6|15| 4|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|16| 5| 4| 9|   | 7|14| 4| 9|   |13| 8| 1|12|   | 7|14| 1|12|   |11| 8| 1|14|   | 7|12| 1|14|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

          25/             26/             27/             28/             29/             30/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 3| 6|15|10|   | 3| 6| 9|16|   | 3| 6|15|10|   | 3| 6|12|13|   | 3|10|15| 6|   | 3|10| 5|16|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|13|12| 1| 8|   |13|12| 7| 2|   |16| 9| 4| 5|   |16| 9| 7| 2|   |13| 8| 1|12|   |13| 8|11| 2|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 2| 7|14|11|   | 8| 1|14|11|   | 2| 7|14|11|   | 5| 4|14|11|   | 2|11|14| 7|   |12| 1|14| 7|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|16| 9| 4| 5|   |10|15| 4| 5|   |13|12| 1| 8|   |10|15| 1| 8|   |16| 5| 4| 9|   | 6|15| 4| 9|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

          31/             32/             33/             34/             35/             36/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 3|10|15| 6|   | 3|10| 8|13|   | 3|13|12| 6|   | 3|13| 8|10|   | 4| 5|16| 9|   | 4| 5|10|15|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|16| 5| 4| 9|   |16| 5|11| 2|   |16| 2| 7| 9|   |16| 2|11| 5|   |14|11| 2| 7|   |14|11| 8| 1|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 2|11|14| 7|   | 9| 4|14| 7|   | 5|11|14| 4|   | 9| 7|14| 4|   | 1| 8|13|12|   | 7| 2|13|12|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|13| 8| 1|12|   | 6|15| 1|12|   |10| 8| 1|15|   | 6|12| 1|15|   |15|10| 3| 6|   | 9|16| 3| 6|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

          37/             38/             39/             40/             41/             42/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 4| 5|16| 9|   | 4| 5|11|14|   | 4| 9|16| 5|   | 4| 9| 6|15|   | 4| 9|16| 5|   | 4| 9| 7|14|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|15|10| 3| 6|   |15|10| 8| 1|   |14| 7| 2|11|   |14| 7|12| 1|   |15| 6| 3|10|   |15| 6|12| 1|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 1| 8|13|12|   | 6| 3|13|12|   | 1|12|13| 8|   |11| 2|13| 8|   | 1|12|13| 8|   |10| 3|13| 8|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|14|11| 2| 7|   | 9|16| 2| 7|   |15| 6| 3|10|   | 5|16| 3|10|   |14| 7| 2|11|   | 5|16| 2|11|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

          43/             44/             45/             46/             47/             48/
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --
| 4|14|11| 5|   | 4|14| 7| 9|   | 5| 4|15|10|   | 5| 4|14|11|   | 6| 3|16| 9|   | 6| 3|13|12|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
|15| 1| 8|10|   |15| 1|12| 6|   |16| 9| 6| 3|   |16| 9| 7| 2|   |15|10| 5| 4|   |15|10| 8| 1|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 6|12|13| 3|   |10| 8|13| 3|   | 2| 7|12|13|   | 3| 6|12|13|   | 1| 8|11|14|   | 4| 5|11|14|
|--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|   |--+--+--+--|
| 9| 7| 2|16|   | 5|11| 2|16|   |11|14| 1| 8|   |10|15| 1| 8|   |12|13| 2| 7|   | 9|16| 2| 7|
 -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --     -- -- -- --

[Count = 48]  OK!
```

It may be of this type what you often see for the solution list of our object. But it really changes its 'faces' according to the list forming inequality conditions taken.

The next list is made up under the conditions: `n1==1; n2<n4; n5<n13 and n2<n5`

```
** 'Fundamental Three' of Pan-diagonal Magic Squares 4x4 **
[1]
  5  4 15 10  5  4 15 10
        .--.--.--.--.        * Sums of Rows and Columns *
 11 14| 1| 8|11|14| 1  8       1+ 8+11+14 = 34 ... rw1
        |--+--+--+--|          12+13+ 2+ 7 = 34 ... rw2
  2  7|12|13| 2| 7|12 13       6+ 3+16+ 9 = 34 ... rw3
        |--+--+--+--|          15+10+ 5+ 4 = 34 ... rw4
 16  9| 6| 3|16| 9| 6  3       1+12+ 6+15 = 34 ... cl1
        |--+--+--+--|           8+13+ 3+10 = 34 ... cl2
  5  4|15|10| 5| 4|15 10      11+ 2+16+ 5 = 34 ... cl3
        '--'--'--'--'          14+ 7+ 9+ 4 = 34 ... cl4
 11 14  1  8 11 14  1  8
* Sums of Pan-diagonals *
   1+13+16+ 4 = 34 ... pd1;  1+ 7+16+10 = 34 ... pb1;
   8+ 2+ 9+15 = 34 ... pd2;  8+12+ 9+ 5 = 34 ... pb2;
  11+ 7+ 6+10 = 34 ... pd3; 11+13+ 6+ 4 = 34 ... pb3;
  14+12+ 3+ 5 = 34 ... pd4; 14+ 2+ 3+15 = 34 ... pb4;
  ... This is certainly a Pan-diagonal Magic Type!
[2]
  5  4 14 11  5  4 14 11
        .--.--.--.--.         * Sums of Rows and Columns *
 10 15| 1| 8|10|15| 1  8       1+ 8+10+15 = 34 ... rw1
        |--+--+--+--|          12+13+ 3+ 6 = 34 ... rw2
  3  6|12|13| 3| 6|12 13       7+ 2+16+ 9 = 34 ... rw3
        |--+--+--+--|          14+11+ 5+ 4 = 34 ... rw4
 16  9| 7| 2|16| 9| 7  2       1+12+ 7+14 = 34 ... cl1
        |--+--+--+--|           8+13+ 2+11 = 34 ... cl2
  5  4|14|11| 5| 4|14 11      10+ 3+16+ 5 = 34 ... cl3
        '--'--'--'--'          15+ 6+ 9+ 4 = 34 ... cl4
 10 15  1  8 10 15  1  8
* Sums of Pan-diagonals *
   1+13+16+ 4 = 34 ... pd1;  1+ 6+16+11 = 34 ... pb1;
   8+ 3+ 9+14 = 34 ... pd2;  8+12+ 9+ 5 = 34 ... pb2;
  10+ 6+ 7+11 = 34 ... pd3; 10+13+ 7+ 4 = 34 ... pb3;
  15+12+ 2+ 5 = 34 ... pd4; 15+ 3+ 2+14 = 34 ... pb4;
  ... This is certainly a Pan-diagonal Magic Type!
[3]
  3  6 15 10  3  6 15 10
        .--.--.--.--.         * Sums of Rows and Columns *
 13 12| 1| 8|13|12| 1  8       1+ 8+13+12 = 34 ... rw1
        |--+--+--+--|          14+11+ 2+ 7 = 34 ... rw2
  2  7|14|11| 2| 7|14 11       4+ 5+16+ 9 = 34 ... rw3
        |--+--+--+--|          15+10+ 3+ 6 = 34 ... rw4
 16  9| 4| 5|16| 9| 4  5       1+14+ 4+15 = 34 ... cl1
        |--+--+--+--|           8+11+ 5+10 = 34 ... cl2
  3  6|15|10| 3| 6|15 10      13+ 2+16+ 3 = 34 ... cl3
        '--'--'--'--'          12+ 7+ 9+ 6 = 34 ... cl4
 13 12  1  8 13 12  1  8
* Sums of Pan-diagonals *
   1+11+16+ 6 = 34 ... pd1;  1+ 7+16+10 = 34 ... pb1;
   8+ 2+ 9+15 = 34 ... pd2;  8+14+ 9+ 3 = 34 ... pb2;
  13+ 7+ 4+10 = 34 ... pd3; 13+11+ 4+ 6 = 34 ... pb3;
  12+14+ 5+ 3 = 34 ... pd4; 12+ 2+ 5+15 = 34 ... pb4;
  ... This is certainly a Pan-diagonal Magic Type!
```

```
 [Count = 3]  OK!
```

#3. Various Types of Pan-magic Squares have the Same Solution Set.

Can you imagine what solution sets you might make for pan-magic squares 4x4 under such four different definitions as listed below?

No.1 is just the same with the previous definition of this section, as you see.

In the case of order 8, we know we could have four independent sets of solutions, different from each other, under similar style of definitions. How about order 4?

```
1. [Basic Form and Basic Definitions] for 'Pan-diagonal' Type:

 15 16 13 14 15 16 13 14
      .--.--.--.--.         * Rows and Columns *
  3  4|n1|n2|n3|n4| 1  2      n1+ n2+ n3+ n4 = C ... rw1
      |--+--+--+--|           n5+ n6+ n7+ n8 = C ... rw2
  7  8|n5|n6|n7|n8| 5  6      n9+n10+n11+n12 = C ... rw3
      |--+--+--+--|          n13+n14+n15+n16 = C ... rw4
 11 12|n9|10|11|12| 9 10      n1+ n5+ n9+n13 = C ... cl1
      |--+--+--+--|           n2+ n6+n10+n14 = C ... cl2
 15 16|13|14|15|16|13 14      n3+ n7+n11+n15 = C ... cl3
      '--'--'--'--'           n4+ n8+n12+n16 = C ... cl4
  3  4  1  2  3  4  1  2
* Pan-diagonal Conditions *
   n1+ n6+n11+n16 = C ... pd1;  n1+ n8+n11+n14 = C ... pb1;
   n2+ n7+n12+n13 = C ... pd2;  n2+ n5+n12+n15 = C ... pb2;
   n3+ n8+ n9+n14 = C ... pd3;  n3+ n6+ n9+n16 = C ... pb3;
   n4+ n5+n10+n15 = C ... pd4;  n4+ n7+n10+n13 = C ... pb4;

2. [Basic Form and Basic Definitions] for 'Complete' Type:

  .--.--.--.--.    * Rows and Columns *
  |n1|n2|n3|n4|     n1+ n2+ n3+ n4 = C ... rw1
  |--+--+--+--|     n5+ n6+ n7+ n8 = C ... rw2
  |n5|n6|n7|n8|     n9+n10+n11+n12 = C ... rw3
  |--+--+--+--|    n13+n14+n15+n16 = C ... rw4
  |n9|10|11|12|     n1+ n5+ n9+n13 = C ... cl1
  |--+--+--+--|     n2+ n6+n10+n14 = C ... cl2
  |13|14|15|16|     n3+ n7+n11+n15 = C ... cl3
  '--'--'--'--'     n4+ n8+n12+n16 = C ... cl4
* Complete Conditions *
  n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC=C/2 ... cmplt

3. [Basic Form and Basic Definitions] for 'Composite' Type:

  .--.--.--.--.    * Composite Conditions *
  |n1|n2|n3|n4|     n1+ n2+ n5+ n6 = C ... cst1
  |--+--+--+--|     n2+ n3+ n6+ n7 = C ... cst2
  |n5|n6|n7|n8|     n3+ n4+ n7+ n8 = C ... cst3
  |--+--+--+--|     n5+ n6+ n9+n10 = C ... cst4
  |n9|10|11|12|     n6+ n7+n10+n11 = C ... cst5
  |--+--+--+--|     n7+ n8+n11+n12 = C ... cst6
  |13|14|15|16|     n9+n10+n13+n14 = C ... cst7
  '--'--'--'--'    n10+n11+n14+n15 = C ... cst8
                   n11+n12+n15+n16 = C ... cst9
* Additional Conditions *
  n1+n2+n3+n4=C ...rw1;   n1+n5+n9+n13=C ...cl1;  n1+n6+n11+n16=C ...pd1;
```

4. [Basic Form and Basic Definitions] for 'Composite and Complete' Type:

```
 .--.--.--.--.    * Composite Conditions *      * Row and Column *
 |n1|n2|n3|n4|      n1+ n2+ n5+ n6 = C ...cst1    n1+ n2+ n3+ n4 = C...rw1
 |--+--+--+--|      n2+ n3+ n6+ n7 = C ...cst2    n1+ n5+ n9+n13 = C...cl1
 |n5|n6|n7|n8|      n3+ n4+ n7+ n8 = C ...cst3
 |--+--+--+--|      n5+ n6+ n9+n10 = C ...cst4
 |n9|10|11|12|      n6+ n7+n10+n11 = C ...cst5
 |--+--+--+--|      n7+ n8+n11+n12 = C ...cst6
 |13|14|15|16|      n9+n10+n13+n14 = C ...cst7
 '--'--'--'--'    n10+n11+n14+n15 = C ...cst8
                  n11+n12+n15+n16 = C ...cst9
```

* Complete Conditions *

```
n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC=C/2 ... cmplt
```

In the case of order 4, whenever we start making under any of those four different definitions above, we cannot but make the only one solution set, completely the same in three levels. We cannot make any other solution set at all.

It is truly amazing. Every time when I saw the same result of those four types, I used to be extremely surprised. I began to think about what it really means.

Let's have some algebraic study of our object here for a while.

#4. Important Properties of Pan-Diagonal Magic Squares

I want to start our argument with the Definition No.1 for 'Pan-diagonal' type.

```
15 16 13 14 15 16 13 14
      .--.--.--.--.         * Rows and Columns *
 3  4|n1|n2|n3|n4| 1  2       n1+ n2+ n3+ n4 = C ... rw1
      |--+--+--+--|           n5+ n6+ n7+ n8 = C ... rw2
 7  8|n5|n6|n7|n8| 5  6       n9+n10+n11+n12 = C ... rw3
      |--+--+--+--|          n13+n14+n15+n16 = C ... rw4
11 12|n9|10|11|12| 9 10       n1+ n5+ n9+n13 = C ... cl1
      |--+--+--+--|           n2+ n6+n10+n14 = C ... cl2
15 16|13|14|15|16|13 14       n3+ n7+n11+n15 = C ... cl3
      '--'--'--'--'           n4+ n8+n12+n16 = C ... cl4
 3  4  1  2  3  4  1  2

  n1+ n6+n11+n16 = C ... pd1;  n1+ n8+n11+n14 = C ... pb1;
  n2+ n7+n12+n13 = C ... pd2;  n2+ n5+n12+n15 = C ... pb2;
  n3+ n8+ n9+n14 = C ... pd3;  n3+ n6+ n9+n16 = C ... pb3;
  n4+ n5+n10+n15 = C ... pd4;  n4+ n7+n10+n13 = C ... pb4;

rw1+rw2+rw3+rw4
 n1+n2+n3+n4+n5+n6+n7+n8+n9+ ... +n15+n16 = 4*C
         1+2+3+4+5+6+7+8+9+ ... +15+16 = 4*C
                      (1+16)*16/2 = 4*C
 Therefore                        C = 34     ... r0
Compare and Calculate the Pandiagonal equations as follows:
   n1+ n6+n11+n16=C ... pd1 --> (n1+n11)+(n6+n16)=C
   n1+ n8+n11+n14=C ... pb1 --> (n1+n11)+(n8+n14)=C
 Therefore                      n6+n16=n8+n14  (=CC1)
   n2+ n7+n12+n13=C ... pd2 --> (n2+n12)+(n7+n13)=C
   n2+ n5+n12+n15=C ... pb2 --> (n2+n12)+(n5+n15)=C
 Therefore                      n5+n15=n7+n13  (=CC2)
   n3+ n8+ n9+n14=C ... pd3 --> (n3+n9)+(n8+n14)=C
   n3+ n6+ n9+n16=C ... pb3 --> (n3+n9)+(n6+n16)=C
```

```
  Therefore                      n6+n16=n8+n14  (=CC1)
    n4+ n5+n10+n15=C ... pd4 --> (n4+n10)+(n5+n15)=C
    n4+ n7+n10+n13=C ... pb4 --> (n4+n10)+(n7+n13)=C
  Therefore                      n5+n15=n7+n13  (=CC2)

  (n5+n15)+(n6+n16)+(n7+n13)+(n8+n14)=CC2+CC1+CC2+CC1
  =(n5+n6+n7+n8)+(n13+n14+n15+n16)=2*C
    2*(CC1+CC2)=2*C
  Therefore                               CC1+CC2=C  ... r1
In the same way
    (n1+n11)+(n6+n16)=C ... pd1
    (n3+n9)+(n6+n16)=C  ... pb3 --> n1+n11=n3+n9  (=DD1)
    (n2+n12)+(n7+n13)=C ... pd2
    (n4+n10)+(n7+n13)=C ... pb4 --> n2+n12=n4+n10 (=DD2)
  (n1+n11)+(n2+n12)+(n3+n9)+(n4+n10)
    =(n1+n2+n3+n4)+(n9+n10+n11+n12)=2*C
    2*(DD1+DD2)=2*C    Therefore          DD1+DD2=C  ... r2
On the other hand (n1+n6+n11+n16)=C  ... pd1
    (n1+n11)+(n6+n16)=C;  Therefore       DD1+CC1=C  ... r3
  (n4+n7+n10+n13)=C  ... pb4
    (n4+n10)+(n7+n13)=C   Therefore       DD2+CC2=C  ... r4
r1~r4 tell us as a consequence:
    CC1=CC2=DD1=DD2=CC2=C/2=17
That means: n1+n11=n2+n12=n3+n9=n4+n10
                =n5+n16=n6+n16=n7+n13=n8+n14=17      ... cmplt
```

8 Complementary Pairs of 17 are all located only on the pan-diagonals.
Each Pan-diagonal magic square 4x4 is always of 'Complete' type at the same time.

#5. Pan-Magic Square and Complementary Pairs

The next figure shows where we can easily find Complementary Pairs by the new notation.

n1+$\underline{\text{n1}}$=CC; n2+$\underline{\text{n2}}$=CC; n3+$\underline{\text{n3}}$=CC; ... ; n7+$\underline{\text{n7}}$=CC; n8+$\underline{\text{n8}}$=CC (=17)

We can also say in our pan-diagonal object:

$\underline{\text{n1}}$=CC-n1=n11; $\underline{\text{n2}}$=CC-n2=n12; $\underline{\text{n3}}$=CC-n3=n9; ...; $\underline{\text{n7}}$=CC-n7=n13; $\underline{\text{n8}}$=CC-n8=n14

[Complementary Pairs and Pan-Diagonals]

n3	n4	n1	n2	n3	n4	n1	n2	n3
n7	n8	n5	n6	n7	n8	n5	n6	n7
$\underline{\text{n1}}$	$\underline{\text{n2}}$	$\underline{\text{n3}}$	$\underline{\text{n4}}$	$\underline{\text{n1}}$	$\underline{\text{n2}}$	$\underline{\text{n3}}$	$\underline{\text{n4}}$	$\underline{\text{n1}}$
$\underline{\text{n5}}$	$\underline{\text{n6}}$	$\underline{\text{n7}}$	$\underline{\text{n8}}$	$\underline{\text{n5}}$	$\underline{\text{n6}}$	$\underline{\text{n7}}$	$\underline{\text{n8}}$	$\underline{\text{n5}}$

n1+ n6+ $\underline{\text{n1}}$+ $\underline{\text{n6}}$ = C ...pd1'
n1+ n8+ $\underline{\text{n1}}$+ $\underline{\text{n8}}$ = C ...pb1'
n2+ n7+ $\underline{\text{n2}}$+ $\underline{\text{n7}}$ = C ...pd2'
n2+ n5+ $\underline{\text{n2}}$+ $\underline{\text{n5}}$ = C ...pb2'
n3+ n8+ $\underline{\text{n3}}$+ $\underline{\text{n8}}$ = C ...pd3'
n3+ n6+ $\underline{\text{n3}}$+ $\underline{\text{n6}}$ = C ...pb3'
n4+ n5+ $\underline{\text{n4}}$+ $\underline{\text{n5}}$ = C ...pd4'
n4+ n7+ $\underline{\text{n4}}$+ $\underline{\text{n7}}$ = C ...pb4'

Each pandiagonal consists of 2 complementary pairs of 17.

Let's put their own names for the four blocks in the next diagram as shown below and calculate all sums of them.

{n1, n2, n5, n6}=block A; {n3, n4, n7, n8}=block B;
{$\underline{\text{n3}}$, $\underline{\text{n4}}$, $\underline{\text{n7}}$, $\underline{\text{n8}}$}=block C; {$\underline{\text{n1}}$, $\underline{\text{n2}}$, $\underline{\text{n5}}$, $\underline{\text{n6}}$}=block D;

```
.-----------.
|n1|n2|n3|n4|    bA+bB=(n1+n2+n3+n4)+(n5+n6+n7+n8)=2*C;
|-bA--|--bB-|    bA+bD=(n1+n1)+(n2+n2)+(n5+n5)+(n6+n6)=4x17=68=2*C;
|n5|n6|n7|n8|    bB+bD=(n3+n7+n11+n15)+(n4+n8+n12+n16)=2*C;
|-----+-----|    bB+bC=(n3+n3)+(n4+n4)+(n7+n7)+(n8+n8)=4x17=68=2*C;
|n3|n4|n1|n2|    The first two shows bB=bD=2*C/2=C (=34)
|-bC--|--bD-|    The second two shows bA=bB=2*C/2=C (=34)
|n7|n8|n5|n6|     .....
'-----------'    Consequently bA=bB=bC=bD=C (=34)
```

This means half of 'Composite Conditions' in Definition No.4 are always true.

What about {n2, n3, n6, n7}?

```
        n1+n2+n3+n4=C;
     +) n5+n6+n7+n8=C;
     --------------------
      (n2+n3+n6+n7)+(n1+n4+n5+n8)=2*C
But   (n2+n3+n6+n7)+(n1+n4+n5+n8)=2*C
Therefore   n1+n4+n5+n8 = n1+n4+n5+n8
But   (n1+n5+n4+n8)+(n4+n8+n1+n5)
  =(n1+n1)+(n4+n4)+(n5+n5)+(n8+n8)=4x17=68=2*C
As a consequence   n1+n4+n5+n8 = n1+n4+n5+n8 = C
That means   n2+n3+n6+n7 = C
```

In the same way all the other half of 'Composite Conditions' are always true.

After all we can say any Pan-diagonal magic square proves to be always the 'Composite' pan-magic type, and also the 'Composite and Complete' type at the same time, in the case of order 4. This makes it possible that every type has the same solution set. Only our algebraic study could reveal this inner relations among them.

#6. What Transformation System can we Assume?

Let's go on our algebraic study and step to the transformation system.

Which lines can we exchange in our object? Won't you see the next diagrams using the new notation for 'Complementary Pairs of 17'?

Let me skip all those boring explanations and jump over to the conclusion.

We know we can exchange the 2nd<->4th row/column with each other, and also exchange the 1st<->3rd row/column with each other.

We could even combine those two types of exchanges without destroying anything about the constant sums we always expect.

Therefore, we can take such a powerful transformation system with this possibility of line exchanges as follows.

```
[Possibility of Line Exchange]
    Basic Form        (ii-iv)Exchange    (II-IV)Exchange
   .-----------.      .-----------.      .-----------.
  I|n1|n2|n3|n4|     I|n1|n4|n3|n2|     I|n1|n2|n3|n4|
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
 II|n5|n6|n7|n8|    II|n5|n8|n7|n6|    IV|n7|n8|n5|n6|
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
III|n3|n4|n1|n2|   III|n3|n2|n1|n4|   III|n3|n4|n1|n2|
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
 IV|n7|n8|n5|n6|    IV|n7|n6|n5|n8|    II|n5|n6|n7|n8|
   '-----------'      '-----------'      '-----------'
     i ii iii iv        i  iv iii ii        i ii iii iv
```

```
   (ii-iv)&(II-IV)Ex    (i-iii)&(I-III)Ex
     .-----------.        .-----------.
    I|n1|n4|n3|n2|     III|n1|n4|n3|n2|
     |--+--+--+--|        |--+--+--+--|
   IV|n7|n6|n5|n8|      II|n7|n6|n5|n8|
     |--+--+--+--|        |--+--+--+--|
  III|n3|n2|n1|n4|       I|n3|n2|n1|n4|
     |--+--+--+--|        |--+--+--+--|
   II|n5|n8|n7|n6|      IV|n5|n8|n7|n6|
     '-----------'        '-----------'
       i iv iii ii          iii ii  i iv
```

Please see the next diagram I presented at the beginning of this section here again.

```
* 4x4 New Squares Taken out of the Extended Space *
             00/           01/           02/           03/
   n1 n2 n3 n4   13 14 15 16    9 10 11 12    5  6  7  8
   n5 n6 n7 n8    1  2  3  4   13 14 15 16    9 10 11 12
   n9 10 11 12    5  6  7  8    1  2  3  4   13 14 15 16
   13 14 15 16    9 10 11 12    5  6  7  8    1  2  3  4
             10/           11/           12/           13/
    4  1  2  3   16 13 14 15   12  9 10 11    8  5  6  7
    8  5  6  7    4  1  2  3   16 13 14 15   12  9 10 11
   12  9 10 11    8  5  6  7    4  1  2  3   16 13 14 15
   16 13 14 15   12  9 10 11    8  5  6  7    4  1  2  3
             20/           21/           22/           23/
    3  4  1  2   15 16 13 14   11 12  9 10    7  8  5  6
    7  8  5  6    3  4  1  2   15 16 13 14   11 12  9 10
   11 12  9 10    7  8  5  6    3  4  1  2   15 16 13 14
   15 16 13 14   11 12  9 10    7  8  5  6    3  4  1  2
             30/           31/           32/           33/
    2  3  4  1   14 15 16 13   10 11 12  9    6  7  8  5
    6  7  8  5    2  3  4  1   14 15 16 13   10 11 12  9
   10 11 12  9    6  7  8  5    2  3  4  1   14 15 16 13
   14 15 16 13   10 11 12  9    6  7  8  5    2  3  4  1
```

We can explain what it really means by the line exchangeability, can't we?

Take 4 examples 00/,03/,30/ and 33/ and get them smart with n1=1 always on the left top as shown below. What parts of 03/,30/ and 33/ are different from the ones of original 00/?

Yes. We can transform the original into others with those line exchangeabilities between the 2nd<->4th rows/columns and the combination of two types.

```
             00/           03/           30/           33/
 .--.--.--.--. .--.--.--.--. .--.--.--.--. .--.--.--.--.
 |n1|n2|n3|n4| |n1|n2|n3|n4| |n1|n4|n3|n2| |n1|n4|n3|n2|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |n5|n6|n7|n8| |13|14|15|16| |n5|n8|n7|n6| |13|16|15|14|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |n9|10|11|12| |n9|10|11|12| |n9|12|11|10| |n9|12|11|10|
 |--+--+--+--| |--+--+--+--| |--+--+--+--| |--+--+--+--|
 |13|14|15|16| |n5|n6|n7|n8| |13|16|15|14| |n5|n8|n7|n6|
 '--'--'--'--' '--'--'--'--' '--'--'--'--' '--'--'--'--'
```

#7. Reconstruction of All Pandiagonal MS44 from the Fundamental Three

Now let's have an experiment to reconstruct all 384 'primitive solutions' from the

'Fundamental Three' by such transformation system as listed below.

```
* Transformation System for Pan-diagonal MS44 *
1. [4x4 Transformations by the 'Row/Column Shift']
          00/          01/          02/          03/
    1  2  3  4  13 14 15 16   9 10 11 12   5  6  7  8
    5  6  7  8   1  2  3  4  13 14 15 16   9 10 11 12
    9 10 11 12   5  6  7  8   1  2  3  4  13 14 15 16
   13 14 15 16   9 10 11 12   5  6  7  8   1  2  3  4
          10/          11/          12/          13/
    4  1  2  3  16 13 14 15  12  9 10 11   8  5  6  7
    8  5  6  7   4  1  2  3  16 13 14 15  12  9 10 11
   12  9 10 11   8  5  6  7   4  1  2  3  16 13 14 15
   16 13 14 15  12  9 10 11   8  5  6  7   4  1  2  3
          20/          21/          22/          23/
    3  4  1  2  15 16 13 14  11 12  9 10   7  8  5  6
    7  8  5  6   3  4  1  2  15 16 13 14  11 12  9 10
   11 12  9 10   7  8  5  6   3  4  1  2  15 16 13 14
   15 16 13 14  11 12  9 10   7  8  5  6   3  4  1  2
          30/          31/          32/          33/
    2  3  4  1  14 15 16 13  10 11 12  9   6  7  8  5
    6  7  8  5   2  3  4  1  14 15 16 13  10 11 12  9
   10 11 12  9   6  7  8  5   2  3  4  1  14 15 16 13
   14 15 16 13  10 11 12  9   6  7  8  5   2  3  4  1

2. [Mirror Reflection and Rotation by 90 Degrees Clockwise]
          00/          01/          02/          03/
    1  2  3  4  13  9  5  1  16 15 14 13   4  8 12 16
    5  6  7  8  14 10  6  2  12 11 10  9   3  7 11 15
    9 10 11 12  15 11  7  3   8  7  6  5   2  6 10 14
   13 14 15 16  16 12  8  4   4  3  2  1   1  5  9 13
          10/          11/          12/          13/
    1  5  9 13   4  3  2  1  16 12  8  4  13 14 15 16
    2  6 10 14   8  7  6  5  15 11  7  3   9 10 11 12
    3  7 11 15  12 11 10  9  14 10  6  2   5  6  7  8
    4  8 12 16  16 15 14 13  13  9  5  1   1  2  3  4
```

We plan to make each Fundamental into 16x8 forms by the combination of these two types of transformation, and finally make 3x16x8=384 'primitive solutions'.

First of all, let me show you our starting points: the Fundamental Three.
And then you see the long list demonstrating the result of our reconstruction.

```
** Reconstruct 'Pan-diagonal' Magic Squares 4x4 **
** Start: The 'Fundamental' Three Solutions: **
          1/          2/          3/
   1  8 10 15   1  8 11 14   1  8 13 12
  12 13  3  6  12 13  2  7  14 11  2  7
   7  2 16  9   6  3 16  9   4  5 16  9
  14 11  5  4  15 10  5  4  15 10  3  6

** Transform them into 'Primitive' Solutions and Compare them with the Old List **
```

[Fundamental #1]

```
         1/          330/           74/          353/            8/          360/           79/          335/
 1  8 10 15   14  7 12  1    4  5 11 14   15  6  9  4    1 12  7 14   15 10  8  1    4  9  6 15   14 11  5  4
12 13  3  6   11  2 13  8    9 16  2  7   10  3 16  5    8 13  2 11    6  3 13 12    5 16  3 10    7  2 16  9
 7  2 16  9    5 16  3 10    6  3 13 12    8 13  2 11   10  3 16  5    9 16  2  7   11  2 13  8   12 13  3  6
14 11  5  4    4  9  6 15   15 10  8  1    1 12  7 14   15  6  9  4    4  5 11 14   14  7 12  1    1  8 10 15

       332/          157/          213/           92/          317/           77/          204/          148/
14 11  5  4    7 12  1 14    9 16  2  7    4 15  6  9   14  1 12  7    4  5 11 14    9  6 15  4    7  2 16  9
 1  8 10 15    2 13  8 11    6  3 13 12    5 10  3 16   11  8 13  2   15 10  8  1   16  3 10  5   12 13  3  6
12 13  3  6   16  3 10  5   15 10  8  1   11  8 13  2    5 10  3 16    6  3 13 12    2 13  8 11    1  8 10 15
 7  2 16  9    9  6 15  4    4  5 11 14   14  1 12  7    4 15  6  9    9 16  2  7    7 12  1 14   14 11  5  4

       150/          269/          125/          198/          163/          211/          140/          284/
 7  2 16  9   12  1 14  7    6  3 13 12    9  4 15  6    7 14  1 12    9 16  2  7    6 15  4  9   12 13  3  6
14 11  5  4   13  8 11  2   15 10  8  1   16  5 10  3    2 11  8 13    4  5 11 14    3 10  5 16    1  8 10 15
 1  8 10 15    3 10  5 16    4  5 11 14    2 11  8 13   16  5 10  3   15 10  8  1   13  8 11  2   14 11  5  4
12 13  3  6    6 15  4  9    9 16  2  7    7 14  1 12    9  4 15  6    6  3 13 12   12  1 14  7    7  2 16  9

       287/           14/          358/          127/          282/          122/          347/            3/
12 13  3  6    1 14  7 12   15 10  8  1    6  9  4 15   12  7 14  1    6  3 13 12   15  4  9  6    1  8 10 15
 7  2 16  9    8 11  2 13    4  5 11 14    3 16  5 10   13  2 11  8    9 16  2  7   10  5 16  3   14 11  5  4
14 11  5  4   10  5 16  3    9 16  2  7   13  2 11  8    3 16  5 10    4  5 11 14    8 11  2 13    7  2 16  9
 1  8 10 15   15  4  9  6    6  3 13 12   12  7 14  1    6  9  4 15   15 10  8  1    1 14  7 12   12 13  3  6

       339/           81/          114/          220/          350/          229/          119/           88/
15  1  8 10    4  9  6 15    5 11 14  4   10  3 16  5   15  6  9  4   10  8  1 15    5 16  3 10    4 14 11  5
 6 12 13  3   14  7 12  1   16  2  7  9    8 13  2 11    1 12  7 14    3 13 12  6   11  2 13  8    9  7  2 16
 9  7  2 16   11  2 13  8    3 13 12  6    1 12  7 14    8 13  2 11   16  2  7  9   14  7 12  1    6 12 13  3
 4 14 11  5    5 16  3 10   10  8  1 15   15  6  9  4   10  3 16  5    5 11 14  4    4  9  6 15   15  1  8 10

        90/          202/          361/          105/           95/          112/          368/          207/
 4 14 11  5    9  6 15  4   16  2  7  9    5 10  3 16    4 15  6  9    5 11 14  4   16  3 10  5    9  7  2 16
15  1  8 10    7 12  1 14    3 13 12  6   11  8 13  2   14  1 12  7   10  8  1 15    2 13  8 11    6 12 13  3
 6 12 13  3    2 13  8 11   10  8  1 15   14  1 12  7   11  8 13  2    3 13 12  6    7 12  1 14   15  1  8 10
 9  7  2 16   16  3 10  5    5 11 14  4    4 15  6  9    5 10  3 16   16  2  7  9    9  6 15  4    4 14 11  5

       205/          143/           64/          374/          196/          363/           57/          138/
 9  7  2 16    6 15  4  9    3 13 12  6   16  5 10  3    9  4 15  6   16  2  7  9    3 10  5 16    6 12 13  3
 4 14 11  5   12  1 14  7   10  8  1 15    2 11  8 13    7 14  1 12    5 11 14  4   13  8 11  2   15  1  8 10
15  1  8 10   13  8 11  2    5 11 14  4    7 14  1 12    2 11  8 13   10  8  1 15   12  1 14  7    4 14 11  5
 6 12 13  3    3 10  5 16   16  2  7  9    9  4 15  6   16  5 10  3    3 13 12  6    6 15  4  9    9  7  2 16

       136/          344/          231/           71/          129/           66/          226/          337/
 6 12 13  3   15  4  9  6   10  8  1 15    3 16  5 10    6  9  4 15    3 13 12  6   10  5 16  3   15  1  8 10
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 4 14 11  5    8 11  2 13   16  2  7  9   12  7 14  1   13  2 11  8    5 11 14  4    1 14  7 12    9  7  2 16
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       235/          116/          260/          187/          222/          174/          245/          101/
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        98/          371/           27/          258/          103/          263/           38/          382/
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       384/           55/          311/           32/          377/           25/          306/           50/
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        53/          228/          172/          293/           68/          308/          181/          237/
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 2 16  9  7   4  9  6 15  12  6  3 13  10  3 16  5  15  6  9  4   7  9 16  2   5 16  3 10  13  3  6 12
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        249/          41/         154/         314/         256/         319/         159/          48/
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         46/         304/         271/         165/          35/         156/         266/         297/
 2 16  9  7  13  8 11  2  12  6  3 13   7 14  1 12   2 11  8 13   7  9 16  2  12  1 14  7  13  3  6 12
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13  3  6 12  12  1 14  7   7  9 16  2   2 11  8 13   7 14  1 12  12  6  3 13  13  8 11  2   2 16  9  7
        295/         183/          24/         280/         290/         273/          17/         178/
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[Fundamental #2]

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          2/         354/          73/         329/           7/         336/          80/         359/
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        356/         133/         215/          86/         341/          75/         210/         124/
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 6  3 16  9   9  7 14  4   4  5 10 15  15  1 12  6   4 14  7  9   9 16  3  6   6 12  1 15  15 10  5  4
        126/         270/         149/         197/         139/         212/         164/         283/
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         96/         208/         367/         111/          89/         106/         362/         201/
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        199/         167/          40/         376/         194/         369/          33/         162/
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        160/         320/         255/          47/         153/          42/         250/         313/
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259/	115/	236/	188/	246/	173/	221/	102/
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2 7 12 13	4 9 7 14	3 6 9 16	1 12 6 15	14 7 9 4	13 12 7 2	15 6 12 1	16 9 6 3
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5 4 15 10	10 3 13 8	8 1 14 11	11 2 16 5	8 13 3 10	10 15 4 5	5 16 2 11	11 14 1 8

100/	365/	51/	234/	109/	239/	62/	380/
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16 9 6 3	3 13 8 10	10 15 4 5	5 11 2 16	10 8 13 3	3 6 9 16	16 2 11 5	5 4 15 10

383/	31/	312/	56/	378/	49/	305/	26/
16 9 6 3	2 11 5 16	13 12 7 2	3 10 8 13	16 5 11 2	3 6 9 16	13 8 10 3	2 7 12 13
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2 7 12 13	13 8 10 3	3 6 9 16	16 5 11 2	3 10 8 13	13 12 7 2	2 11 5 16	16 9 6 3

29/	252/	170/	299/	44/	310/	175/	261/
2 7 12 13	11 5 16 2	8 1 14 11	13 3 10 8	2 16 5 11	13 12 7 2	8 10 3 13	11 14 1 8
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186/	218/	345/	9/	191/	16/	352/	223/
8 11 14 1	10 3 13 8	15 4 5 10	1 12 6 15	8 13 3 10	1 14 11 8	15 6 12 1	10 5 4 15
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225/	65/	130/	338/	232/	343/	135/	72/
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13 2 7 12	9 7 14 4	1 14 11 8	5 11 2 16	4 14 7 9	12 7 2 13	16 2 11 5	8 11 14 1
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70/	302/	277/	141/	59/	132/	268/	291/
3 16 9 6	13 8 10 3	12 7 2 13	6 15 1 12	3 10 8 13	6 9 16 3	12 1 15 6	13 2 7 12
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289/	177/	18/	274/	296/	279/	23/	184/
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[Fundamental #3]

4/	348/	121/	281/	13/	288/	128/	357/
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4 5 16 9	3 16 2 13	7 2 11 14	8 11 5 10	13 2 16 3	9 16 5 4	10 5 11 8	14 11 2 7
15 10 3 6	6 9 7 12	12 13 8 1	1 14 4 15	12 7 9 6	6 3 10 15	15 4 14 1	1 8 13 12

355/	85/	216/	134/	342/	123/	209/	76/
15 10 3 6	4 14 1 15	9 16 5 4	6 12 7 9	15 1 14 4	6 3 10 15	9 7 12 6	4 5 16 9
1 8 13 12	5 11 8 10	7 2 11 14	3 13 2 16	10 8 11 5	12 13 8 1	16 2 13 3	14 11 2 7
14 11 2 7	16 2 13 3	12 13 8 1	10 8 11 5	3 13 2 16	7 2 11 14	5 11 8 10	1 8 13 12
4 5 16 9	9 7 12 6	6 3 10 15	15 1 14 4	6 12 7 9	9 16 5 4	4 14 1 15	15 10 3 6

78/	318/	147/	203/	91/	214/	158/	331/
4 5 16 9	14 1 15 4	7 2 11 14	9 6 12 7	4 15 1 14	9 16 5 4	7 12 6 9	14 11 2 7
15 10 3 6	11 8 10 5	12 13 8 1	16 3 13 2	5 10 8 11	6 3 10 15	2 13 3 16	1 8 13 12
1 8 13 12	2 13 3 16	6 3 10 15	5 10 8 11	16 3 13 2	12 13 8 1	11 8 10 5	15 10 3 6
14 11 2 7	7 12 6 9	9 16 5 4	4 15 1 14	9 6 12 7	7 2 11 14	14 1 15 4	4 5 16 9

333/	19/	286/	152/	324/	145/	275/	6/
14 11 2 7	1 15 4 14	12 13 8 1	7 9 6 12	14 4 15 1	7 2 11 14	12 6 9 7	1 8 13 12
4 5 16 9	8 10 5 11	6 3 10 15	2 16 3 13	11 5 10 8	9 16 5 4	13 3 16 2	15 10 3 6
15 10 3 6	13 3 16 2	9 16 5 4	11 5 10 8	2 16 3 13	6 3 10 15	8 10 5 11	4 5 16 9
1 8 13 12	12 6 9 7	7 2 11 14	14 4 15 1	7 9 6 12	12 13 8 1	1 15 4 14	14 11 2 7

267/	131/	60/	292/	278/	301/	69/	142/
12 1 8 13	6 9 7 12	3 10 15 6	13 2 16 3	12 7 9 6	13 8 1 12	3 16 2 13	6 15 10 3
7 14 11 2	15 4 14 1	16 5 4 9	8 11 5 10	1 14 4 15	2 11 14 7	10 5 11 8	9 4 5 16
9 4 5 16	10 5 11 8	2 11 14 7	1 14 4 15	8 11 5 10	16 5 4 9	15 4 14 1	7 14 11 2
6 15 10 3	3 16 2 13	13 8 1 12	12 7 9 6	13 2 16 3	3 10 15 6	6 9 7 12	12 1 8 13

144/	206/	373/	63/	137/	58/	364/	195/
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9 4 5 16	16 2 13 3	3 10 15 6	6 12 7 9	3 13 2 16	16 5 4 9	9 7 12 6	6 15 10 3

193/	161/	34/	370/	200/	375/	39/	168/
9 4 5 16	7 12 6 9	2 11 14 7	16 3 13 2	9 6 12 7	16 5 4 9	2 13 3 16	7 14 11 2
6 15 10 3	14 1 15 4	13 8 1 12	5 10 8 11	4 15 1 14	3 10 15 6	11 8 10 5	12 1 8 13
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7 14 11 2	2 13 3 16	16 5 4 9	9 6 12 7	16 3 13 2	2 11 14 7	7 12 6 9	9 4 5 16

166/	272/	303/	45/	155/	36/	298/	265/
7 14 11 2	12 6 9 7	13 8 1 12	2 16 3 13	7 9 6 12	2 11 14 7	13 3 16 2	12 1 8 13
9 4 5 16	1 15 4 14	3 10 15 6	11 5 10 8	14 4 15 1	16 5 4 9	8 10 5 11	6 15 10 3
6 15 10 3	8 10 5 11	16 5 4 9	14 4 15 1	11 5 10 8	3 10 15 6	1 15 4 14	9 4 5 16
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307/	67/	238/	182/	294/	171/	227/	54/
13 12 1 8	3 16 2 13	10 15 6 3	8 11 5 10	13 2 16 3	8 1 12 13	10 5 11 8	3 6 15 10
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3 6 15 10	10 5 11 8	8 1 12 13	13 2 16 3	8 11 5 10	10 15 6 3	3 16 2 13	13 12 1 8

52/	366/	99/	233/	61/	240/	110/	379/
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381/	37/	264/	104/	372/	97/	257/	28/
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2 7 14 11	11 8 10 5	5 4 9 16	16 3 13 2	5 10 8 11	11 14 7 2	2 13 3 16	16 9 4 5

30/	300/	169/	251/	43/	262/	176/	309/
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3 6 15 10	1 15 4 14	5 4 9 16	7 9 6 12	14 4 15 1	10 15 6 3	12 6 9 7	16 9 4 5
13 12 1 8	8 10 5 11	11 14 7 2	2 16 3 13	11 5 10 8	8 1 12 13	13 3 16 2	2 7 14 11

192/	224/	351/	15/	185/	10/	346/	217/
8 13 12 1	10 5 11 8	15 6 3 10	1 14 4 15	8 11 5 10	1 12 13 8	15 4 14 1	10 3 6 15
11 2 7 14	3 16 2 13	4 9 16 5	12 7 9 6	13 2 16 3	14 7 2 11	6 9 7 12	5 16 9 4
5 16 9 4	6 9 7 12	14 7 2 11	13 2 16 3	12 7 9 6	4 9 16 5	3 16 2 13	11 2 7 14
10 3 6 15	15 4 14 1	1 12 13 8	8 11 5 10	1 14 4 15	15 6 3 10	10 5 11 8	8 13 12 1

219/	113/	82/	340/	230/	349/	87/	120/
10 3 6 15	5 11 8 10	4 9 16 5	15 1 14 4	10 8 11 5	15 6 3 10	4 14 1 15	5 16 9 4
8 13 12 1	16 2 13 3	14 7 2 11	6 12 7 9	3 13 2 16	1 12 13 8	9 7 12 6	11 2 7 14
11 2 7 14	9 7 12 6	1 12 13 8	3 13 2 16	6 12 7 9	14 7 2 11	16 2 13 3	8 13 12 1
5 16 9 4	4 14 1 15	15 6 3 10	10 8 11 5	15 1 14 4	4 9 16 5	5 11 8 10	10 3 6 15

118/	254/	325/	93/	107/	84/	316/	243/
5 16 9 4	11 8 10 5	14 7 2 11	4 15 1 14	5 10 8 11	4 9 16 5	14 1 15 4	11 2 7 14
10 3 6 15	2 13 3 16	1 12 13 8	9 6 12 7	16 3 13 2	15 6 3 10	7 12 6 9	8 13 12 1
8 13 12 1	7 12 6 9	15 6 3 10	16 3 13 2	9 6 12 7	1 12 13 8	2 13 3 16	10 3 6 15
11 2 7 14	14 1 15 4	4 9 16 5	5 10 8 11	4 15 1 14	14 7 2 11	11 8 10 5	5 16 9 4

241/	179/	12/	322/	248/	327/	21/	190/
11 2 7 14	8 10 5 11	1 12 13 8	14 4 15 1	11 5 10 8	14 7 2 11	1 15 4 14	8 13 12 1
5 16 9 4	13 3 16 2	15 6 3 10	7 9 6 12	2 16 3 13	4 9 16 5	12 6 9 7	10 3 6 15
10 3 6 15	12 6 9 7	4 9 16 5	2 16 3 13	7 9 6 12	15 6 3 10	13 3 16 2	5 16 9 4
8 13 12 1	1 15 4 14	14 7 2 11	11 5 10 8	14 4 15 1	1 12 13 8	8 10 5 11	11 2 7 14

```
* Monitor List of Correspondence between Old and New *
  ??: 0
   1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  25: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  49: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  73: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  97: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 121: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 145: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 169: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 193: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 217: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 241: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 265: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 289: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 313: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 337: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 361: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 [OK!]
```

The last beautiful result of Monitor list tells us of our success in reconstructing all 384 primitive solutions from the Fundamental Three. We could make no other type of solutions, nor lose any one we expected at all.

The Fundamental Three prove to be really the representative solutions of all.

We know what the solution counts in three levels really mean: 3x16x8=48x8=384.

You may be surprised at the similarity between a self-complementary type and a pan-diagonal magic one. Yes. There are so many similar properties between them.

#8. Type Conversion between S-C and P-D

I must mention now about the possibility of type conversion here. You can indeed transform one type into another by some simple line exchanges. Take your careful look at the next figures.

```
 [Figure 5: How to Transform One into the Other Type]
                          (Step 1)           (Step 2) GOAL!
 Self-complementary   (iii-iv)Exchange    (III-IV)Exchange
     .-----------.       .-----------.       .-----------.
   I |n1|n2|n3|n4|     I |n1|n2|n4|n3|     I |n1|n2|n4|n3|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
  II |n5|n6|n7|n8|    II |n5|n6|n8|n7|    II |n5|n6|n8|n7|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
 III |n8|n7|n6|n5|   III |n8|n7|n5|n6|    IV |n4|n3|n1|n2|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
  IV |n4|n3|n2|n1|    IV |n4|n3|n1|n2|   III |n8|n7|n5|n6|
     '-----------'       '-----------'       '-----------'
       i ii iii iv         i  ii iv iii        i  ii  iv iii

                         (Step 1)           (Step 2) GOAL!
     Pan-diagonal     (iii-iv)Exchange    (III-IV)Exchange
     .-----------.       .-----------.       .-----------.
   I |n1|n2|n3|n4|     I |n1|n2|n4|n3|     I |n1|n2|n4|n3|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
  II |n5|n6|n7|n8|    II |n5|n6|n8|n7|    II |n5|n6|n8|n7|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
 III |n3|n4|n1|n2|   III |n3|n4|n2|n1|    IV |n7|n8|n6|n5|
     |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
  IV |n7|n8|n5|n6|    IV |n7|n8|n6|n5|   III |n3|n4|n2|n1|
     '-----------'       '-----------'       '-----------'
      i  ii iii iv         i  ii iv iii        i  ii  iv iii
```

The upper step-figures show how to transform a self-complementary type into a pan-magic one. Exchange the 3rd<->4th rows and columns at the same time. Watch the result and find all complementary pairs are located only on pan-diagonals.

The lower step-figures show how to transform a pan-diagonal magic type back into a self-complementary one. Exchange the 3rd<->4th rows and columns at the same time. Watch the result and find all complementary pairs are located symmetrically.

Though each result is not completely the same with the other start form, it is still a meaningful transformation. The solution counts of those two types are also the same. We may well accept the one-to-one correspondence should exist between them.

```
*** Comparison between 'Self-Complementary' and 'Pan-diagonal' Types: ***
** List of the Fundamental Three Solutions of Magic Squares of Order 4 **
            S/1/P                      S/2/P                      S/3/P
 .--.--.--.--.  .--.--.--.--.  .--.--.--.--.  .--.--.--.--.  .--.--.--.--.  .--.--.--.--.
 | 1| 8|15|10|  | 1| 8|10|15|  | 1| 8|14|11|  | 1| 8|11|14|  | 1| 8|12|13|  | 1| 8|13|12|
 |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|
 |12|13| 6| 3|  |12|13| 3| 6|  |12|13| 7| 2|  |12|13| 2| 7|  |14|11| 7| 2|  |14|11| 2| 7|
 |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|
 |14|11| 4| 5|  | 7| 2|16| 9|  |15|10| 4| 5|  | 6| 3|16| 9|  |15|10| 6| 3|  | 4| 5|16| 9|
 |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|  |--+--+--+--|
 | 7| 2| 9|16|  |14|11| 5| 4|  | 6| 3| 9|16|  |15|10| 5| 4|  | 4| 5| 9|16|  |15|10| 3| 6|
 '--'--'--'--'  '--'--'--'--'  '--'--'--'--'  '--'--'--'--'  '--'--'--'--'  '--'--'--'--'
 [Count = S/3:3/P]
  OK!
```

#9. Is One-to-one Correspondence broken between the Standard Solution Sets?

But, I noticed the one-to-one correspondence is broken between the two sets of 48 Standard Solutions afterward. Look carefully at the next list shown below.

```
** Transformation of MS44: Between 'Self-Complementary' and 'Complete' **
** Comparison between /Self-Complementary and /Complete Types **
          S/ 1/C                  S/ 2/C                  S/ 3/C
  1  8 15 10   1  8 10 15   1  8 14 11   1  8 11 14   1  8 15 10   1  8 10 15
 12 13  6  3  12 13  3  6  12 13  7  2  12 13  2  7  14 11  4  5  14 11  5  4
 14 11  4  5   7  2 16  9  15 10  4  5   6  3 16  9  12 13  6  3   7  2 16  9
  7  2  9 16  14 11  5  4   6  3  9 16  15 10  5  4   7  2  9 16  12 13  3  6
          S/ 4/C                  S/ 5/C                  S/ 6/C
  1  8 12 13   1  8 13 12   1  8 14 11   1  8 11 14   1  8 12 13   1  8 13 12
 14 11  7  2  14 11  2  7  15 10  4  5  15 10  5  4  15 10  6  3  15 10  3  6
 15 10  6  3   4  5 16  9  12 13  7  2   6  3 16  9  14 11  7  2   4  5 16  9
  4  5  9 16  15 10  3  6   6  3  9 16  12 13  2  7   4  5  9 16  14 11  2  7
          S/ 7/C                  S/ 8/C                  S/ 9/C
  1 12 15  6   1 12  6 15   1 12  8 13   1 12 13  8   1 12 14  7   1 12  7 14
 14  7  4  9  14  7  9  4  14  7 11  2  14  7  2 11  15  6  4  9  15  6  9  4
  8 13 10  3  11  2 16  5  15  6 10  3   4  9 16  5   8 13 11  2  10  3 16  5
 11  2  5 16   8 13  3 10   4  9  5 16  15  6  3 10  10  3  5 16   8 13  2 11
          S/10/C                  S/11/C                  S/12/C
  1 12  8 13   1 12 13  8   1 14 12  7   1 14  7 12   1 14  8 11   1 14 11  8
 15  6 10  3  15  6  3 10  15  4  6  9  15  4  9  6  15  4 10  5  15  4  5 10
 14  7 11  2   4  9 16  5   8 11 13  2  10  5 16  3  12  7 13  2   6  9 16  3
  4  9  5 16  14  7  2 11  10  5  3 16   8 11  2 13   6  9  3 16  12  7  2 13
          S/13/C                  S/14/C                  S/15/C
  2  7 16  9   2  7  9 16   2  7 13 12   2  7 12 13   2  7 16  9   2  7  9 16
 11 14  5  4  11 14  4  5  11 14  8  1  11 14  1  8  13 12  3  6  13 12  6  3
 13 12  3  6   8  1 15 10  16  9  3  6   5  4 15 10  11 14  5  4   8  1 15 10
  8  1 10 15  13 12  6  3   5  4 10 15  16  9  6  3   8  1 10 15  11 14  4  5
```

S/16/C								S/17/C								S/18/C							
2	7	11	14	2	7	14	11	2	7	13	12	2	7	12	13	2	7	11	14	2	7	14	11
13	12	8	1	13	12	1	8	16	9	3	6	16	9	6	3	16	9	5	4	16	9	4	5
16	9	5	4	3	6	15	10	11	14	8	1	5	4	15	10	13	12	8	1	3	6	15	10
3	6	10	15	16	9	4	5	5	4	10	15	11	14	1	8	3	6	10	15	13	12	1	8
S/19/C								S/20/C								S/21/C							
2	11	16	5	2	11	5	16	2	11	7	14	2	11	14	7	2	11	13	8	2	11	8	13
13	8	3	10	13	8	10	3	13	8	12	1	13	8	1	12	16	5	3	10	16	5	10	3
7	14	9	4	12	1	15	6	16	5	9	4	3	10	15	6	7	14	12	1	9	4	15	6
12	1	6	15	7	14	4	9	3	10	6	15	16	5	4	9	9	4	6	15	7	14	1	12
S/22/C								S/23/C								S/24/C							
2	11	7	14	2	11	14	7	2	13	11	8	2	13	8	11	2	13	7	12	2	13	12	7
16	5	9	4	16	5	4	9	16	3	5	10	16	3	10	5	16	3	9	6	16	3	6	9
13	8	12	1	3	10	15	6	7	12	14	1	9	6	15	4	11	8	14	1	5	10	15	4
3	10	6	15	13	8	1	12	9	6	4	15	7	12	1	14	5	10	4	15	11	8	1	14

25/S				0/T				25/C				26/S				0/T				26/C			
3	6	16	9	3	6	9	16	3	6	9	16	3	6	13	12	3	6	12	13	3	6	15	10
10	15	5	4	10	15	4	5	13	12	7	2	10	15	8	1	10	15	1	8	13	12	1	8
13	12	2	7	8	1	14	11	8	1	14	11	16	9	2	7	5	4	14	11	2	7	14	11
8	1	11	14	13	12	7	2	10	15	4	5	5	4	11	14	16	9	7	2	16	9	4	5
27/S				25/T				27/C				28/S				27/T				28/C			
3	6	16	9	3	6	9	16	3	6	12	13	3	6	13	12	3	6	12	13	3	6	15	10
13	12	2	7	13	12	7	2	16	9	7	2	16	9	2	7	16	9	7	2	16	9	4	5
10	15	5	4	8	1	14	11	5	4	14	11	10	15	8	1	5	4	14	11	2	7	14	11
8	1	11	14	10	15	4	5	10	15	1	8	5	4	11	14	10	15	1	8	13	12	1	8
29/S				29/T				29/C				30/S				31/T				30/C			
3	10	16	5	3	10	5	16	3	10	5	16	3	10	13	8	3	10	8	13	3	10	15	6
13	8	2	11	13	8	11	2	13	8	11	2	16	5	2	11	16	5	11	2	13	8	1	12
6	15	9	4	12	1	14	7	12	1	14	7	6	15	12	1	9	4	14	7	2	11	14	7
12	1	7	14	6	15	4	9	6	15	4	9	9	4	7	14	6	15	1	12	16	5	4	9
31/S				33/T				31/C				32/S				34/T				32/C			
3	13	10	8	3	13	8	10	3	10	8	13	3	13	6	12	3	13	12	6	3	10	15	6
16	2	5	11	16	2	11	5	16	5	11	2	16	2	9	7	16	2	7	9	16	5	4	9
6	12	15	1	9	7	14	4	9	4	14	7	10	8	15	1	5	11	14	4	2	11	14	7
9	7	4	14	6	12	1	15	6	15	1	12	5	11	4	14	10	8	1	15	13	8	1	12
33/S				0/T				33/C				34/S				0/T				34/C			
4	5	15	10	4	5	10	15	3	13	8	10	4	5	14	11	4	5	11	14	3	13	12	6
9	16	6	3	9	16	3	6	16	2	11	5	9	16	7	2	9	16	2	7	16	2	7	9
14	11	1	8	7	2	13	12	9	7	14	4	15	10	1	8	6	3	13	12	5	11	14	4
7	2	12	13	14	11	8	1	6	12	1	15	6	3	12	13	15	10	8	1	10	8	1	15
35/S				35/T				35/C				36/S				37/T				36/C			
4	5	15	10	4	5	10	15	4	5	10	15	4	5	14	11	4	5	11	14	4	5	16	9
14	11	1	8	14	11	8	1	14	11	8	1	15	10	1	8	15	10	8	1	14	11	2	7
9	16	6	3	7	2	13	12	7	2	13	12	9	16	7	2	6	3	13	12	1	8	13	12
7	2	12	13	9	16	3	6	9	16	3	6	6	3	12	13	9	16	2	7	15	10	3	6
37/S				39/T				37/C				38/S				41/T				38/C			
4	9	15	6	4	9	6	15	4	5	11	14	4	9	14	7	4	9	7	14	4	5	16	9
14	7	1	12	14	7	12	1	15	10	8	1	15	6	1	12	15	6	12	1	15	10	3	6
5	16	10	3	11	2	13	8	6	3	13	12	5	16	11	2	10	3	13	8	1	8	13	12
11	2	8	13	5	16	3	10	9	16	2	7	10	3	8	13	5	16	2	11	14	11	2	7
39/S				43/T				39/C				40/S				44/T				40/C			
4	14	9	7	4	14	7	9	4	9	6	15	4	14	5	11	4	14	11	5	4	9	16	5
15	1	6	12	15	1	12	6	14	7	12	1	15	1	10	8	15	1	8	10	14	7	2	11
5	11	16	2	10	8	13	3	11	2	13	8	9	7	16	2	6	12	13	3	1	12	13	8
10	8	3	13	5	11	2	16	5	16	3	10	6	12	3	13	9	7	2	16	15	6	3	10

```
        41/S          0/T         41/C         42/S          0/T         42/C
  5  4 16  9   5  4  9 16   4  9  7 14    5  4 16  9   5  4  9 16   4  9 16  5
 10 15  3  6  10 15  6  3  15  6 12  1   11 14  2  7  11 14  7  2  15  6  3 10
 11 14  2  7   8  1 12 13  10  3 13  8   10 15  3  6   8  1 12 13   1 12 13  8
  8  1 13 12  11 14  7  2   5 16  2 11    8  1 13 12  10 15  6  3  14  7  2 11
        43/S          0/T         43/C         44/S          0/T         44/C
  5 10 11  8   5 10  8 11   4 14  7  9    5 11 10  8   5 11  8 10   4 14 11  5
 16  3  2 13  16  3 13  2  15  1 12  6   16  2  3 13  16  2 13  3  15  1  8 10
  4 15 14  1   9  6 12  7  10  8 13  3    4 14 15  1   9  7 12  6   6 12 13  3
  9  6  7 12   4 15  1 14   5 11  2 16    9  7  6 12   4 14  1 15   9  7  2 16
        45/S          0/T         45/C         46/S          0/T         46/C
  6  3 15 10   6  3 10 15   5  4 14 11    6  3 15 10   6  3 10 15   5  4 15 10
  9 16  4  5   9 16  5  4  16  9  7  2   12 13  1  8  12 13  8  1  16  9  6  3
 12 13  1  8   7  2 11 14   3  6 12 13    9 16  4  5   7  2 11 14   2  7 12 13
  7  2 14 11  12 13  8  1  10 15  1  8    7  2 14 11   9 16  5  4  11 14  1  8
        47/S          0/T         47/C         48/S          0/T         48/C
  6  9 12  7   6  9  7 12   6  3 13 12    6 12  9  7   6 12  7  9   6  3 16  9
 15  4  1 14  15  4 14  1  15 10  8  1   15  1  4 14  15  1 14  4  15 10  5  4
  3 16 13  2  10  5 11  8   4  5 11 14    3 13 16  2  10  8 11  5   1  8 11 14
 10  5  8 11   3 16  2 13   9 16  2  7   10  8  5 11   3 13  2 16  12 13  2  7
```

```
* Monitor List of Solution Correspondence between /T and /C *
 ??: 12
  1:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 25:  1  0  1  0  1  0  1  0  1  1  1  0  1  0  1  0  1  0  1  1  0  0  0  0
 [Count = S/48:48/C]  OK!
```

I could find no problem in the first half of the two lists. But I found the one-to-one correspondence was partially broken in the last half. I could not find any 'perfect partners' for twelve solutions there. I was shocked. What was wrong with them?

The problem is the list-forming inequality conditions. When they are transformed, some results in the last half should get against the conditions: `n1<n4; n1<n13; n1<n16` and `n2<n5`, and cannot finally find any 'perfect partners' in our list.

When I examined between the two sets of 'Primitive solutions' (that are listed under no inequality conditions), I could find no problem there.

See the next list shown below. You can certainly see there exists the complete one-to-one correspondence between the 'self-complementary' magic squares 4x4 and the 'complete' ones (same with 'pan-diagonal' type).

```
** Transformation of MS44: Between 'Self-Complementary' and 'Complete' **
**  Comparison between /S-C and /C in the List of Primitive Solutions  **
          S/ 1/C                    S/ 2/C                    S/ 3/C
  1  8 15 10   1  8 10 15    1  8 14 11   1  8 11 14    1  8 15 10   1  8 10 15
 12 13  6  3  12 13  3  6   12 13  7  2  12 13  2  7   14 11  4  5  14 11  5  4
 14 11  4  5   7  2 16  9   15 10  4  5   6  3 16  9   12 13  6  3   7  2 16  9
  7  2  9 16  14 11  5  4    6  3  9 16  15 10  5  4    7  2  9 16  12 13  3  6
          S/ 4/C                    S/ 5/C                    S/ 6/C
  1  8 12 13   1  8 13 12    1  8 14 11   1  8 11 14    1  8 12 13   1  8 13 12
 14 11  7  2  14 11  2  7   15 10  4  5  15 10  5  4   15 10  6  3  15 10  3  6
 15 10  6  3   4  5 16  9   12 13  7  2   6  3 16  9   14 11  7  2   4  5 16  9
  4  5  9 16  15 10  3  6    6  3  9 16  12 13  2  7    4  5  9 16  14 11  2  7
          S/ 7/C                    S/ 8/C                    S/ 9/C
  1 12 15  6   1 12  6 15    1 12 14  7   1 12  7 14    1 12 15  6   1 12  6 15
  8 13 10  3   8 13  3 10    8 13 11  2   8 13  2 11   14  7  4  9  14  7  9  4
 14  7  4  9  11  2 16  5   15  6  4  9  10  3 16  5    8 13 10  3  11  2 16  5
 11  2  5 16  14  7  9  4   10  3  5 16  15  6  9  4   11  2  5 16   8 13  3 10
```

S/10/C

1	12	8	13	1	12	13	8
14	7	11	2	14	7	2	11
15	6	10	3	4	9	16	5
4	9	5	16	15	6	3	10

S/11/C

1	12	14	7	1	12	7	14
15	6	4	9	15	6	9	4
8	13	11	2	10	3	16	5
10	3	5	16	8	13	2	11

S/12/C

1	12	8	13	1	12	13	8
15	6	10	3	15	6	3	10
14	7	11	2	4	9	16	5
4	9	5	16	14	7	2	11

S/13/C

1	14	15	4	1	14	4	15
8	11	10	5	8	11	5	10
12	7	6	9	13	2	16	3
13	2	3	16	12	7	9	6

S/14/C

1	14	12	7	1	14	7	12
8	11	13	2	8	11	2	13
15	4	6	9	10	5	16	3
10	5	3	16	15	4	9	6

S/15/C

1	14	15	4	1	14	4	15
12	7	6	9	12	7	9	6
8	11	10	5	13	2	16	3
13	2	3	16	8	11	5	10

S/16/C

1	14	8	11	1	14	11	8
12	7	13	2	12	7	2	13
15	4	10	5	6	9	16	3
6	9	3	16	15	4	5	10

S/17/C

1	14	12	7	1	14	7	12
15	4	6	9	15	4	9	6
8	11	13	2	10	5	16	3
10	5	3	16	8	11	2	13

S/18/C

1	14	8	11	1	14	11	8
15	4	10	5	15	4	5	10
12	7	13	2	6	9	16	3
6	9	3	16	12	7	2	13

S/19/C

1	15	14	4	1	15	4	14
8	10	11	5	8	10	5	11
12	6	7	9	13	3	16	2
13	3	2	16	12	6	9	7

S/20/C

1	15	12	6	1	15	6	12
8	10	13	3	8	10	3	13
14	4	7	9	11	5	16	2
11	5	2	16	14	4	9	7

S/21/C

1	15	14	4	1	15	4	14
12	6	7	9	12	6	9	7
8	10	11	5	13	3	16	2
13	3	2	16	8	10	5	11

S/22/C

1	15	8	10	1	15	10	8
12	6	13	3	12	6	3	13
14	4	11	5	7	9	16	2
7	9	2	16	14	4	5	11

S/23/C

1	15	12	6	1	15	6	12
14	4	7	9	14	4	9	7
8	10	13	3	11	5	16	2
11	5	2	16	8	10	3	13

S/24/C

1	15	8	10	1	15	10	8
14	4	11	5	14	4	5	11
12	6	13	3	7	9	16	2
7	9	2	16	12	6	3	13

S/25/C

2	7	16	9	2	7	9	16
11	14	5	4	11	14	4	5
13	12	3	6	8	1	15	10
8	1	10	15	13	12	6	3

S/26/C

2	7	13	12	2	7	12	13
11	14	8	1	11	14	1	8
16	9	3	6	5	4	15	10
5	4	10	15	16	9	6	3

S/27/C

2	7	16	9	2	7	9	16
13	12	3	6	13	12	6	3
11	14	5	4	8	1	15	10
8	1	10	15	11	14	4	5

. . . . . .

S/193/C

9	4	16	5	9	4	5	16
6	15	3	10	6	15	10	3
7	14	2	11	12	1	8	13
12	1	13	8	7	14	11	2

S/194/C

9	4	7	14	9	4	14	7
6	15	12	1	6	15	1	12
16	5	2	11	3	10	8	13
3	10	13	8	16	5	11	2

S/195/C

9	4	16	5	9	4	5	16
7	14	2	11	7	14	11	2
6	15	3	10	12	1	8	13
12	1	13	8	6	15	10	3

S/196/C

9	4	6	15	9	4	15	6
7	14	12	1	7	14	1	12
16	5	3	10	2	11	8	13
2	11	13	8	16	5	10	3

S/197/C

9	4	7	14	9	4	14	7
16	5	2	11	16	5	11	2
6	15	12	1	3	10	8	13
3	10	13	8	6	15	1	12

S/198/C

9	4	6	15	9	4	15	6
16	5	3	10	16	5	10	3
7	14	12	1	2	11	8	13
2	11	13	8	7	14	1	12

S/199/C

9	6	16	3	9	6	3	16
4	15	5	10	4	15	10	5
7	12	2	13	14	1	8	11
14	1	11	8	7	12	13	2

S/200/C

9	6	7	12	9	6	12	7
4	15	14	1	4	15	1	14
16	3	2	13	5	10	8	11
5	10	11	8	16	3	13	2

S/201/C

9	6	16	3	9	6	3	16
7	12	2	13	7	12	13	2
4	15	5	10	14	1	8	11
14	1	11	8	4	15	10	5

S/202/C

9	6	4	15	9	6	15	4
7	12	14	1	7	12	1	14
16	3	5	10	2	13	8	11
2	13	11	8	16	3	10	5

S/203/C

9	6	7	12	9	6	12	7
16	3	2	13	16	3	13	2
4	15	14	1	5	10	8	11
5	10	11	8	4	15	1	14

S/204/C

9	6	4	15	9	6	15	4
16	3	5	10	16	3	10	5
7	12	14	1	2	13	8	11
2	13	11	8	7	12	1	14

S/205/C

9	7	16	2	9	7	2	16
4	14	5	11	4	14	11	5
6	12	3	13	15	1	8	10
15	1	10	8	6	12	13	3

S/206/C

9	7	6	12	9	7	12	6
4	14	15	1	4	14	1	15
16	2	3	13	5	11	8	10
5	11	10	8	16	2	13	3

S/207/C

9	7	16	2	9	7	2	16
6	12	3	13	6	12	13	3
4	14	5	11	15	1	8	10
15	1	10	8	4	14	11	5

```
          S/208/C                    S/209/C                    S/210/C
  9  7  4 14   9  7 14  4   9  7  6 12   9  7 12  6   9  7  4 14   9  7 14  4
  6 12 15  1   6 12  1 15  16  2  3 13  16  2 13  3  16  2  5 11  16  2 11  5
 16  2  5 11   3 13  8 10   4 14 15  1   5 11  8 10   6 12 15  1   3 13  8 10
  3 13 10  8  16  2 11  5   5 11 10  8   4 14  1 15   3 13 10  8   6 12  1 15
          S/211/C                    S/212/C                    S/213/C
  9 16  7  2   9 16  2  7   9 16  6  3   9 16  3  6   9 16  7  2   9 16  2  7
  4  5 14 11   4  5 11 14   4  5 15 10   4  5 10 15   6  3 12 13   6  3 13 12
  6  3 12 13  15 10  8  1   7  2 12 13  14 11  8  1   4  5 14 11  15 10  8  1
 15 10  1  8   6  3 13 12  14 11  1  8   7  2 13 12  15 10  1  8   4  5 11 14
          S/214/C                    S/215/C                    S/216/C
  9 16  4  5   9 16  5  4   9 16  6  3   9 16  3  6   9 16  4  5   9 16  5  4
  6  3 15 10   6  3 10 15   7  2 12 13   7  2 13 12   7  2 14 11   7  2 11 14
  7  2 14 11  12 13  8  1   4  5 15 10  14 11  8  1   6  3 15 10  12 13  8  1
 12 13  1  8   7  2 11 14  14 11  1  8   4  5 10 15  12 13  1  8   6  3 10 15
          S/217/C                    S/218/C                    S/219/C
 10  3 15  6  10  3  6 15  10  3  8 13  10  3 13  8  10  3 15  6  10  3  6 15
  5 16  4  9   5 16  9  4   5 16 11  2   5 16  2 11   8 13  1 12   8 13 12  1
  8 13  1 12  11  2  7 14  15  6  1 12   4  9  7 14   5 16  4  9  11  2  7 14
 11  2 14  7   8 13 12  1   4  9 14  7  15  6 12  1  11  2 14  7   5 16  9  4

 . . . . . .

          S/370/C                    S/371/C                    S/372/C
 16  3  2 13  16  3 13  2  16  3  5 10  16  3 10  5  16  3  2 13  16  3 13  2
  5 10 11  8   5 10  8 11   9  6  4 15   9  6 15  4   9  6  7 12   9  6 12  7
  9  6  7 12   4 15  1 14   2 13 11  8   7 12  1 14   5 10 11  8   4 15  1 14
  4 15 14  1   9  6 12  7   7 12 14  1   2 13  8 11   4 15 14  1   5 10  8 11
          S/373/C                    S/374/C                    S/375/C
 16  5  9  4  16  5  4  9  16  5  3 10  16  5 10  3  16  5  9  4  16  5  4  9
  2 11  7 14   2 11 14  7   2 11 13  8   2 11  8 13   3 10  6 15   3 10 15  6
  3 10  6 15  13  8  1 12   9  4  6 15   7 14  1 12   2 11  7 14  13  8  1 12
 13  8 12  1   3 10 15  6   7 14 12  1   9  4 15  6  13  8 12  1   2 11 14  7
          S/376/C                    S/377/C                    S/378/C
 16  5  2 11  16  5 11  2  16  5  3 10  16  5 10  3  16  5  2 11  16  5 11  2
  3 10 13  8   3 10  8 13   9  4  6 15   9  4 15  6   9  4  7 14   9  4 14  7
  9  4  7 14   6 15  1 12   2 11 13  8   7 14  1 12   3 10 13  8   6 15  1 12
  6 15 12  1   9  4 14  7   7 14 12  1   2 11  8 13   6 15 12  1   3 10  8 13
          S/379/C                    S/380/C                    S/381/C
 16  9  5  4  16  9  4  5  16  9  3  6  16  9  6  3  16  9  5  4  16  9  4  5
  2  7 11 14   2  7 14 11   2  7 13 12   2  7 12 13   3  6 10 15   3  6 15 10
  3  6 10 15  13 12  1  8   5  4 10 15  11 14  1  8   2  7 11 14  13 12  1  8
 13 12  8  1   3  6 15 10  11 14  8  1   5  4 15 10  13 12  8  1   2  7 14 11
          S/382/C                    S/383/C                    S/384/C
 16  9  2  7  16  9  7  2  16  9  3  6  16  9  6  3  16  9  2  7  16  9  7  2
  3  6 13 12   3  6 12 13   5  4 10 15   5  4 15 10   5  4 11 14   5  4 14 11
  5  4 11 14  10 15  1  8   2  7 13 12  11 14  1  8   3  6 13 12  10 15  1  8
 10 15  8  1   5  4 14 11  11 14  8  1   2  7 12 13  10 15  8  1   3  6 12 13
[Count = S/384:384/C]

* Monitor List of Solution Correspondence between /S and /C *
 ??:  0
  1:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 25:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 49:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 73:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 97:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
121:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
```

```
145:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
169:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
193:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
217:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
241:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
265:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
289:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
313:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
337:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
361:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 OK!
```

How impressive our Monitor list is!

It moved me very much, but puzzled me more. I could not help thinking about what the true reason is for such a beautiful concordance. I had to study for a long time.

At last I came up to the idea of true origin, 4 dimensional extra-cubic object of order 2. I could find one of the answers for that, I believe.

(Original written in English in 2002; Revised on Sep.3, 2003;
Newest revision on Dec.18, 2005 by Kanji Setsuda on Mac OS Xcode 1.5)

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