

Part 4. "New Advanced Study of Magic Squares and Cubes"

Chapter 7. New Method of Composing High-Dimensional Extra-Cubic Objects and their Developed Forms: **Kanji Setsuda**

Section 2. Composition of Four-Dimensional Extra-Cubic Objects of Order 2 and their Developed Forms

#1. Our Purpose

I have found we can draw many pictures of the same ECO. We can use these pictures for some 'basic diagrams' or 'prototype forms' or, say, anything else.

This time let's use these pictures for the Basic Diagrams of our object. Let's design and compose ECO² again more strictly than before, and count and know how many primitive solutions it has in its 'Mother Set' in reality.

First of all we have to know how many simultaneous equations are really necessary for our definition stage.

#2. How do you define ECO²?

We now know we can draw so many different pictures as 24 for our object by the 4-time loops of for(...){ ... } sentences and with a little change in the order of variables {d0, d1, d2, d3}. Permutation taking 4 out of 4 is $4P_4=4 \times 3 \times 2 \times 1=24$.

Let me list out all 24 pictures here before all.

** All Possible View-Forms with 'Composite Conditions' **

1/d0 1---- 2 3--- 4 5-- - 6 7---- 8	/d1 9----10 11---12 13-- -14 15----16	n1+n2+n3+n4=C n1+n2+n5+n6=C n1+n3+n5+n7=C n2+n4+n6+n8=C n3+n4+n7+n8=C n5+n6+n7+n8=C	n9+n10+n11+n12=C n9+n10+n13+n14=C n9+n11+n13+n15=C n10+n12+n14+n16=C n11+n12+n15+n16=C n13+n14+n15+n16=C
2/d0 1---- 3 2--- 4 5-- - 7 6---- 8	/d1 9----11 10---12 13-- -15 14----16	n1+n2+n3+n4=C n1+n3+n5+n7=C n1+n2+n5+n6=C n3+n4+n7+n8=C n2+n4+n6+n8=C n5+n6+n7+n8=C	n9+n10+n11+n12=C n9+n11+n13+n15=C n9+n10+n13+n14=C n11+n12+n15+n16=C n10+n12+n14+n16=C n13+n14+n15+n16=C
3/d0 1---- 2 5--- 6 3-- - 4 7---- 8	/d1 9----10 13---14 11-- -12 15----16	n1+n2+n5+n6=C n1+n2+n3+n4=C n1+n3+n5+n7=C n2+n4+n6+n8=C n5+n6+n7+n8=C n3+n4+n7+n8=C	n9+n10+n13+n14=C n9+n10+n11+n12=C n9+n11+n13+n15=C n10+n12+n14+n16=C n13+n14+n15+n16=C n11+n12+n15+n16=C
4/d0 1---- 3 5--- 7 2-- - 4 6---- 8	/d1 9----11 13---15 10-- -12 14----16	n1+n3+n5+n7=C n1+n2+n3+n4=C n1+n2+n5+n6=C n3+n4+n7+n8=C n5+n6+n7+n8=C n2+n4+n6+n8=C	n9+n11+n13+n15=C n9+n10+n11+n12=C n9+n10+n13+n14=C n11+n12+n15+n16=C n13+n14+n15+n16=C n10+n12+n14+n16=C
5/d0 1---- 5 2--- 6 3-- - 7 4---- 8	/d1 9----13 10---14 11-- -15 12----16	n1+n2+n5+n6=C n1+n3+n5+n7=C n1+n2+n3+n4=C n5+n6+n7+n8=C n2+n4+n6+n8=C n3+n4+n7+n8=C	n9+n10+n13+n14=C n9+n11+n13+n15=C n9+n10+n11+n12=C n13+n14+n15+n16=C n10+n12+n14+n16=C n11+n12+n15+n16=C

6/d0

1	---	5	
	3	---	7
2	---	6	
4	---	8	

/d1

9	---	13	
	11	---	15
10	---	14	
12	---	16	

n1+n3+n5+n7=C
n1+n2+n5+n6=C
n1+n2+n3+n4=C
n5+n6+n7+n8=C
n3+n4+n7+n8=C
n2+n4+n6+n8=C

n9+n11+n13+n15=C
n9+n10+n13+n14=C
n9+n10+n11+n12=C
n13+n14+n15+n16=C
n11+n12+n15+n16=C
n10+n12+n14+n16=C

7/d0

1	---	2	
	3	---	4
9	---	10	
11	---	12	

/d1

5	---	6	
	7	---	8
13	---	14	
15	---	16	

n1+n2+n3+n4=C
n1+n2+n9+n10=C
n1+n3+n9+n11=C
n2+n4+n10+n12=C
n3+n4+n11+n12=C
n9+n10+n11+n12=C

n5+n6+n7+n8=C
n5+n6+n13+n14=C
n5+n7+n13+n15=C
n6+n8+n14+n16=C
n7+n8+n15+n16=C
n13+n14+n15+n16=C

8/d0

1	---	3	
	2	---	4
9	---	11	
10	---	12	

/d1

5	---	7	
	6	---	8
13	---	15	
14	---	16	

n1+n2+n3+n4=C
n1+n3+n9+n11=C
n1+n2+n9+n10=C
n3+n4+n11+n12=C
n2+n4+n10+n12=C
n9+n10+n11+n12=C

n5+n6+n7+n8=C
n5+n7+n13+n15=C
n5+n6+n13+n14=C
n7+n8+n15+n16=C
n6+n8+n14+n16=C
n13+n14+n15+n16=C

9/d0

1	---	2	
	5	---	6
9	---	10	
13	---	14	

/d1

3	---	4	
	7	---	8
11	---	12	
15	---	16	

n1+n2+n5+n6=C
n1+n2+n9+n10=C
n1+n5+n9+n13=C
n2+n6+n10+n14=C
n5+n6+n13+n14=C
n9+n10+n13+n14=C

n3+n4+n7+n8=C
n3+n4+n11+n12=C
n3+n7+n11+n15=C
n4+n8+n12+n16=C
n7+n8+n15+n16=C
n11+n12+n15+n16=C

10/d0

1	---	3	
	5	---	7
9	---	11	
13	---	15	

/d1

2	---	4	
	6	---	8
10	---	12	
14	---	16	

n1+n3+n5+n7=C
n1+n3+n9+n11=C
n1+n5+n9+n13=C
n3+n7+n11+n15=C
n5+n7+n13+n15=C
n9+n11+n13+n15=C

n2+n4+n6+n8=C
n2+n4+n10+n12=C
n2+n6+n10+n14=C
n4+n8+n12+n16=C
n6+n8+n14+n16=C
n10+n12+n14+n16=C

11/d0

1	---	5	
	2	---	6
9	---	13	
10	---	14	

/d1

3	---	7	
	4	---	8
11	---	15	
12	---	16	

n1+n2+n5+n6=C
n1+n5+n9+n13=C
n1+n2+n9+n10=C
n5+n6+n13+n14=C
n2+n6+n10+n14=C
n9+n10+n13+n14=C

n3+n4+n7+n8=C
n3+n7+n11+n15=C
n3+n4+n11+n12=C
n7+n8+n15+n16=C
n4+n8+n12+n16=C
n11+n12+n15+n16=C

12/d0

1	---	5	
	3	---	7
9	---	13	
11	---	15	

/d1

2	---	6	
	4	---	8
10	---	14	
12	---	16	

n1+n3+n5+n7=C
n1+n5+n9+n13=C
n1+n3+n9+n11=C
n5+n7+n13+n15=C
n3+n7+n11+n15=C
n9+n11+n13+n15=C

n2+n4+n6+n8=C
n2+n6+n10+n14=C
n2+n4+n10+n12=C
n6+n8+n14+n16=C
n4+n8+n12+n16=C
n10+n12+n14+n16=C

13/d0

1	---	2	
	9	---	10
3	---	4	
11	---	12	

/d1

5	---	6	
	13	---	14
7	---	8	
15	---	16	

n1+n2+n9+n10=C
n1+n2+n3+n4=C
n1+n3+n9+n11=C
n2+n4+n10+n12=C
n9+n10+n11+n12=C
n3+n4+n11+n12=C

n5+n6+n13+n14=C
n5+n6+n7+n8=C
n5+n7+n13+n15=C
n6+n8+n14+n16=C
n13+n14+n15+n16=C
n7+n8+n15+n16=C

14/d0

1	---	3	
	9	---	11
2	---	4	
10	---	12	

/d1

5	---	7	
	13	---	15
6	---	8	
14	---	16	

n1+n3+n9+n11=C
n1+n2+n3+n4=C
n1+n2+n9+n10=C
n3+n4+n11+n12=C
n9+n10+n11+n12=C
n2+n4+n10+n12=C

n5+n7+n13+n15=C
n5+n6+n7+n8=C
n5+n6+n13+n14=C
n7+n8+n15+n16=C
n13+n14+n15+n16=C
n6+n8+n14+n16=C

15/d0

1	---	2	
	9	---	10
5	---	6	
13	---	14	

/d1

3	---	4	
	11	---	12
7	---	8	
15	---	16	

	n1+n2+n9+n10=C	
	n1+n2+n5+n6=C	
	n1+n5+n9+n13=C	
	n2+n6+n10+n14=C	
	n9+n10+n13+n14=C	
	n5+n6+n13+n14=C	

	n3+n4+n11+n12=C	
	n3+n4+n7+n8=C	
	n3+n7+n11+n15=C	
	n4+n8+n12+n16=C	
	n11+n12+n15+n16=C	
	n7+n8+n15+n16=C	

16/d0

1	---	3	
	9	---	11
5	---	7	
13	---	15	

/d1

2	---	4	
	10	---	12
6	---	8	
14	---	16	

	n1+n3+n9+n11=C	
	n1+n3+n5+n7=C	
	n1+n5+n9+n13=C	
	n3+n7+n11+n15=C	
	n9+n11+n13+n15=C	
	n5+n7+n13+n15=C	

	n2+n4+n10+n12=C	
	n2+n4+n6+n8=C	
	n2+n6+n10+n14=C	
	n4+n8+n12+n16=C	
	n10+n12+n14+n16=C	
	n6+n8+n14+n16=C	

17/d0

1	---	5	
	9	---	13
2	---	6	
10	---	14	

/d1

3	---	7	
	11	---	15
4	---	8	
12	---	16	

	n1+n5+n9+n13=C	
	n1+n2+n5+n6=C	
	n1+n2+n9+n10=C	
	n5+n6+n13+n14=C	
	n9+n10+n13+n14=C	
	n2+n6+n10+n14=C	

	n3+n7+n11+n15=C	
	n3+n4+n7+n8=C	
	n3+n4+n11+n12=C	
	n7+n8+n15+n16=C	
	n11+n12+n15+n16=C	
	n4+n8+n12+n16=C	

18/d0

1	---	5	
	9	---	13
3	---	7	
11	---	15	

/d1

2	---	6	
	10	---	14
4	---	8	
12	---	16	

	n1+n5+n9+n13=C	
	n1+n3+n5+n7=C	
	n1+n3+n9+n11=C	
	n5+n7+n13+n15=C	
	n9+n11+n13+n15=C	
	n3+n7+n11+n15=C	

	n2+n6+n10+n14=C	
	n2+n4+n6+n8=C	
	n2+n4+n10+n12=C	
	n6+n8+n14+n16=C	
	n10+n12+n14+n16=C	
	n4+n8+n12+n16=C	

19/d0

1	---	9	
	2	---	10
3	---	11	
4	---	12	

/d1

5	---	13	
	6	---	14
7	---	15	
8	---	16	

	n1+n2+n9+n10=C	
	n1+n3+n9+n11=C	
	n1+n2+n3+n4=C	
	n9+n10+n11+n12=C	
	n2+n4+n10+n12=C	
	n3+n4+n11+n12=C	

	n5+n6+n13+n14=C	
	n5+n7+n13+n15=C	
	n5+n6+n7+n8=C	
	n13+n14+n15+n16=C	
	n6+n8+n14+n16=C	
	n7+n8+n15+n16=C	

20/d0

1	---	9	
	3	---	11
2	---	10	
4	---	12	

/d1

5	---	13	
	7	---	15
6	---	14	
8	---	16	

	n1+n3+n9+n11=C	
	n1+n2+n9+n10=C	
	n1+n2+n3+n4=C	
	n9+n10+n11+n12=C	
	n3+n4+n11+n12=C	
	n2+n4+n10+n12=C	

	n5+n7+n13+n15=C	
	n5+n6+n13+n14=C	
	n5+n6+n7+n8=C	
	n13+n14+n15+n16=C	
	n7+n8+n15+n16=C	
	n6+n8+n14+n16=C	

21/d0

1	---	9	
	2	---	10
5	---	13	
6	---	14	

/d1

3	---	11	
	4	---	12
7	---	15	
8	---	16	

	n1+n2+n9+n10=C	
	n1+n5+n9+n13=C	
	n1+n2+n5+n6=C	
	n9+n10+n13+n14=C	
	n2+n6+n10+n14=C	
	n5+n6+n13+n14=C	

	n3+n4+n11+n12=C	
	n3+n7+n11+n15=C	
	n3+n4+n7+n8=C	
	n11+n12+n15+n16=C	
	n4+n8+n12+n16=C	
	n7+n8+n15+n16=C	

22/d0

1	---	9	
	3	---	11
5	---	13	
7	---	15	

/d1

2	---	10	
	4	---	12
6	---	14	
8	---	16	

	n1+n3+n9+n11=C	
	n1+n5+n9+n13=C	
	n1+n3+n5+n7=C	
	n9+n11+n13+n15=C	
	n3+n7+n11+n15=C	
	n5+n7+n13+n15=C	

	n2+n4+n10+n12=C	
	n2+n6+n10+n14=C	
	n2+n4+n6+n8=C	
	n10+n12+n14+n16=C	
	n4+n8+n12+n16=C	
	n6+n8+n14+n16=C	

23/d0

1	---	9	
	5	---	13
2	---	10	
6	---	14	

/d1

3	---	11	
	7	---	15
4	---	12	
8	---	16	

	n1+n5+n9+n13=C	
	n1+n2+n9+n10=C	
	n1+n2+n5+n6=C	
	n9+n10+n13+n14=C	
	n5+n6+n13+n14=C	
	n2+n6+n10+n14=C	

	n3+n7+n11+n15=C	
	n3+n4+n11+n12=C	
	n3+n4+n7+n8=C	
	n11+n12+n15+n16=C	
	n7+n8+n15+n16=C	
	n4+n8+n12+n16=C	

$24/d0$ $1 \text{---} 9$ $ \quad 5 \text{---} 13$ $3 \text{---} -11$ $7 \text{---} 15$	$/d1$ $2 \text{---} 10$ $ \quad 6 \text{---} 14$ $4 \text{---} -12$ $8 \text{---} 16$	$n1+n5+n9+n13=C$ $n1+n3+n9+n11=C$ $n1+n3+n5+n7=C$ $n9+n11+n13+n15=C$ $n5+n7+n13+n15=C$ $n3+n7+n11+n15=C$	$n2+n6+n10+n14=C$ $n2+n4+n10+n12=C$ $n2+n4+n6+n8=C$ $n10+n12+n14+n16=C$ $n6+n8+n14+n16=C$ $n4+n8+n12+n16=C$
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We want to put those equations all to every little cube of order 2 just as above. Every 4 numbers on each surface of those little ones must add up to the magic C(onstant=34). But $24 \times 12 = 288$ equations are too many for us to deal with.

What do we have to do, then? Can we cut any equations?

If you watch the list above carefully, you will find many duplications of any kind. As you see, every simple reflection or rotation of a representative pattern has the same set of equations. You can remove those duplications for our purpose.

The next 4 view-forms below are supposed to be really necessary for us to define our ECO with.

**** The Selected 4 View-Forms with 'Composite Conditions' ****

$1/d0$ $1 \text{---} 2$ $ \quad 3 \text{---} 4$ $5 \text{---} -6$ $7 \text{---} 8$	$/d1$ $9 \text{---} 10$ $ \quad 11 \text{---} 12$ $13 \text{---} -14$ $15 \text{---} 16$	$n1+n2+n3+n4=C$ $n1+n2+n5+n6=C$ $n1+n3+n5+n7=C$ $n2+n4+n6+n8=C$ $n3+n4+n7+n8=C$ $n5+n6+n7+n8=C$	$n9+n10+n11+n12=C$ $n9+n10+n13+n14=C$ $n9+n11+n13+n15=C$ $n10+n12+n14+n16=C$ $n11+n12+n15+n16=C$ $n13+n14+n15+n16=C$
---	---	--	---

$2/d0$ $1 \text{---} 2$ $ \quad 3 \text{---} 4$ $9 \text{---} -10$ $11 \text{---} 12$	$/d1$ $5 \text{---} 6$ $ \quad 7 \text{---} 8$ $13 \text{---} -14$ $15 \text{---} 16$	$n1+n2+n3+n4=C$ $n1+n2+n9+n10=C$ $n1+n3+n9+n11=C$ $n2+n4+n10+n12=C$ $n3+n4+n11+n12=C$ $n9+n10+n11+n12=C$	$n5+n6+n7+n8=C$ $n5+n6+n13+n14=C$ $n5+n7+n13+n15=C$ $n6+n8+n14+n16=C$ $n7+n8+n15+n16=C$ $n13+n14+n15+n16=C$
--	--	---	--

$3/d0$ $1 \text{---} 2$ $ \quad 5 \text{---} 6$ $9 \text{---} -10$ $13 \text{---} 14$	$/d1$ $3 \text{---} 4$ $ \quad 7 \text{---} 8$ $11 \text{---} -12$ $15 \text{---} 16$	$n1+n2+n5+n6=C$ $n1+n2+n9+n10=C$ $n1+n5+n9+n13=C$ $n2+n6+n10+n14=C$ $n5+n6+n13+n14=C$ $n9+n10+n13+n14=C$	$n3+n4+n7+n8=C$ $n3+n4+n11+n12=C$ $n3+n7+n11+n15=C$ $n4+n8+n12+n16=C$ $n7+n8+n15+n16=C$ $n11+n12+n15+n16=C$
--	--	---	--

$4/d0$ $1 \text{---} 3$ $ \quad 5 \text{---} 7$ $9 \text{---} -11$ $13 \text{---} 15$	$/d1$ $2 \text{---} 4$ $ \quad 6 \text{---} 8$ $10 \text{---} -12$ $14 \text{---} 16$	$n1+n3+n5+n7=C$ $n1+n3+n9+n11=C$ $n1+n5+n9+n13=C$ $n3+n7+n11+n15=C$ $n5+n7+n13+n15=C$ $n9+n11+n13+n15=C$	$n2+n4+n6+n8=C$ $n2+n4+n10+n12=C$ $n2+n6+n10+n14=C$ $n4+n8+n12+n16=C$ $n6+n8+n14+n16=C$ $n10+n12+n14+n16=C$
--	--	---	--

But you may still find some duplications of the same equation. Of course, you can remove them all right and get the smart list of simultaneous equations.

On the other hand, we also want to add the next equations to our definition.

$$n1+n16=n2+n15=n3+n14=n4+n13=n5+n12=n6+n11=n7+n10=n8+n9=CC=C/2=17$$

These equations are so called 'Self-Complementary Conditions'. They mean:

$$n16=CC-n1; \quad n15=CC-n2; \quad n14=CC-n3; \quad \dots \quad n10=CC-n7; \quad n9=CC-n8;$$

Therefore $n13+n14+n15+n16$

$$=(CC-n4)+(CC-n3)+(CC-n2)+(CC-n1)=4*CC-(n1+n2+n3+n4)=C$$

$$\qquad \qquad \qquad 68 \qquad \qquad 34 \qquad \qquad 34$$

In the same way $n9+n10+n11+n12=4*CC-(n5+n6+n7+n8)=C$

$$n7+n8+n15+n16=4*CC-(n1+n2+n9+n10)=C$$

.....

As a consequence, when you assume $(n1+n2+n3+n4=C)$ at first, you don't have to define $(n13+n14+n15+n16=C)$ any longer. If you assume $(n1+n2+n9+n10=C)$ beforehand, you don't have to define $(n7+n8+n15+n16=C)$ any longer. ...

You can remove half of the equations above by these reasons.

**** Collection of 'Composite Conditions' ****

$$\begin{array}{lll} n1+n2+n3+n4=C \dots (1); & n1+n2+n5+n6=C \dots (2); & n1+n2+n9+n10=C \dots (3); \\ n1+n3+n5+n7=C \dots (4); & n1+n3+n9+n11=C \dots (5); & n1+n5+n9+n13=C \dots (6); \\ n2+n4+n6+n8=C \dots (7); & n2+n4+n10+n12=C \dots (8); & n2+n6+n10+n14=C \dots (9); \\ n3+n4+n7+n8=C \dots (10); & n3+n4+n11+n12=C \dots (11); & n5+n6+n7+n8=C \dots (12); \end{array}$$

By the way, do we really have to define 6 equations for each little cube? See the next algebraic calculations. They show if you assume any 4 equations on each little cube beforehand, you don't have to define the rest 2 any longer.

$$\begin{array}{lll} 1 \text{-----} 5 & n1+n2+n3+n4=C & \dots (1) \\ \backslash \quad \quad \backslash & n1+n2+n5+n6=C & \dots (2) \\ | \quad 2 \text{-----} 6 & n1+n3+n5+n7=C & \dots (3) \\ 3 \text{-----} 7 \quad | & n2+n4+n6+n8=C & \dots (4) \\ \backslash \quad \quad \backslash & & \\ 4 \text{-----} 8 & n1+n3+n5+n7=C & \dots (3) \\ & +) n2+n4+n6+n8=C & \dots (4) \\ \hline & n1+n2+n3+n4+n5+n6+n7+n8=2*C & \\ & (n1+n2+n3+n4)+(n5+n6+n7+n8)=2*C & \\ \text{But } & n1+n2+n3+n4=C & \dots (1) \\ \text{Therefore } & n5+n6+n7+n8=C & \dots (5) \\ & & \\ & n1+n3+n5+n7=C & \dots (3) \\ & +) n2+n4+n6+n8=C & \dots (4) \\ \hline & n1+n2+n3+n4+n5+n6+n7+n8=2*C & \\ & (n1+n2+n5+n6)+(n3+n4+n7+n8)=2*C & \\ \text{But } & n1+n2+n5+n6=C & \dots (2) \\ \text{Therefore } & n3+n4+n7+n8=C & \dots (6) \end{array}$$

You can remove more by this reason and get such the smartest list as follows.

**** Essential 9 Equations for 'Composite Conditions' ****

$$\begin{array}{lll} n1+n2+n3+n4=C \dots (1); & n1+n2+n5+n6=C \dots (2); & n1+n2+n9+n10=C \dots (3); \\ n1+n3+n5+n7=C \dots (4); & n1+n3+n9+n11=C \dots (5); & n1+n5+n9+n13=C \dots (6); \\ n2+n4+n6+n8=C \dots (7); & n2+n4+n10+n12=C \dots (8); & n2+n6+n10+n14=C \dots (9); \end{array}$$

These 9 equations are logically equivalent even to the 288 equations of the first diagrams. It means if you assume 9 beforehand, then you don't have to define anything more. But it doesn't mean that you must not define anything else.

You may well add any equations you have already removed from the lists above to our first definition. These additions will make no difference at all. You will have the same result constantly, even if you revive many other equations.

#3. Composition of ECO2⁴ Again

Let's design and compose our ECO2⁴ again strictly under these simultaneous equations with the Basic Diagrams below. It is almost all the same as Chapter 6.

The two diagrams on the right hand side show how we can convert it down into 2-dimensions and get the two types of magic squares of order 4. We really want to have these objects: 'Self-Complementary' and 'Composite & Complete' MS44.

*** Defi ni ti on ***

BD/d0	/d1	SC/	CC/
1---- 2	3---- 4	1 2 3 4	1 2 4 3
9--+10	11--+12	5 6 7 8	5 6 8 7
5-- - 6	7-- - 8	9 10 11 12	13 14 16 15
13----14	15----16	13 14 15 16	9 10 12 11

$n1+n2+n3+n4=C \dots (1);$ $n1+n2+n5+n6=C \dots (2);$ $n1+n2+n9+n10=C \dots (3);$
 $n1+n3+n5+n7=C \dots (4);$ $n1+n3+n9+n11=C \dots (5);$ $n1+n5+n9+n13=C \dots (6);$
 $n2+n4+n6+n8=C \dots (7);$ $n2+n4+n10+n12=C \dots (8);$ $n2+n6+n10+n14=C \dots (9);$
 $n1+n16=n2+n15=n3+n14=n4+n13=n5+n12=n6+n11=n7+n10=n8+n9=PSM=C/2 \dots (10);$

Let me show you a sample list of computer program of mine.
It is written in the smallest size so that you might understand it easily.

```

/** Make the Extra-Cubic Magic Objects of Order 2^4 **/
/** and Print All the 384 Solutions of Various Types **/
/** 'EC024.c' Built by Kanji Setsuda **/
/** on MacOS Xcode 1.5; Mar.20, 2005 **/
**/
#include <stdio.h>
**/
/* Variables */
short int cnt;
short CC, LSM, cnt2;
short nm[17], flg[17];
**/
/* Sub-Routines */
void stp01(void), stp02(void), stp03(void);
void stp04(void), stp05(void), stp06(void);
void stp07(void), stp08(void), stp09(void);
void stp10(void), stp11(void), stp12(void);
void ansprint(void);
**/
/* Main Program */
main(){
    short n;
    printf("\n*** Make the 4-Dimensional Magic Objects of Order 2 ***\n");
    printf("*** and Transform them into SC & CC Magic Squares 4x4 ***\n\n");
    for(n=0;n<17;n++){nm[n]=0; flg[n]=0;}
    CC=17; LSM=34; cnt=0;
    stp01(); /* Begin the Calculations */
    printf("\n [Count = %d]\n",cnt);
    printf(" OK!\n");
    return 0;
}
**/
/* Calculations */
/* Set n1 & n16 */
void stp01(){
    short a,b;
    for(a=1;a<17;a++){b=CC-a;
        if((flg[a]==0)&&(flg[b]==0)){
            nm[1]=a; nm[16]=b;

```

```

        fl g[a]=1; fl g[b]=1; cnt2=0;
        stp02();
        fl g[b]=0; fl g[a]=0; }
    }
}
/* Set n2 & n15 */
void stp02(){
    short a,b;
    for(a=16; a>0; a--){b=CC-a;
        if((fl g[a]==0)&&(fl g[b]==0)){
            nm[2]=a; nm[15]=b;
            fl g[a]=1; fl g[b]=1;
            stp03();
            fl g[b]=0; fl g[a]=0; }
        }
}
/* Set n5 & n12 */
void stp03(){
    short a,b;
    for(a=16; a>0; a--){b=CC-a;
        if((fl g[a]==0)&&(fl g[b]==0)){
            nm[5]=a; nm[12]=b;
            fl g[a]=1; fl g[b]=1;
            stp04();
            fl g[b]=0; fl g[a]=0; }
        }
}
/* Set n6=LSM-n1-n2-n5 & n11 */
void stp04(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[5];
    if((0<a)&&(a<17)){b=CC-a;
        if((fl g[a]==0)&&(fl g[b]==0)){
            nm[6]=a; nm[11]=b;
            fl g[a]=1; fl g[b]=1;
            stp05();
            fl g[b]=0; fl g[a]=0; }}
}
/* Set n3 & n14 */
void stp05(){
    short a,b;
    for(a=16; a>0; a--){b=CC-a;
        if((fl g[a]==0)&&(fl g[b]==0)){
            nm[3]=a; nm[14]=b;
            fl g[a]=1; fl g[b]=1;
            stp06();
            fl g[b]=0; fl g[a]=0; }
        }
}
/* Set n4=LSM-n1-n2-n3 & n13 */
void stp06(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[3];
    if((0<a)&&(a<17)){b=CC-a;
        if((0<b)&&(fl g[a]==0)&&(fl g[b]==0)){
            nm[4]=a; nm[13]=b;
            fl g[a]=1; fl g[b]=1;
            stp07();
            fl g[b]=0; fl g[a]=0; }}
}

```

```

}
/* Set n7=LSM-n1-n3-n5 & n10 */
void stp07(){
short a, b;
a=LSM-nm[1]-nm[3]-nm[5];
if((0<a)&&(a<17)){b=CC-a;
if((0<b)&&(flg[a]==0)&&(flg[b]==0)){
nm[7]=a; nm[10]=b;
flg[a]=1; flg[b]=1;
stp08();
flg[b]=0; flg[a]=0; }}
}
/* Set n8=LSM-n2-n4-n6 & n9 */
void stp08(){
short a, b, c;
a=LSM-nm[2]-nm[4]-nm[6];
b=LSM-nm[3]-nm[4]-nm[7];
c=LSM-nm[5]-nm[6]-nm[7];
if((0<a)&&(a<17)&&(a==b)){b=CC-a;
if((0<b)&&(b<17)&&(a==c)){
if((flg[a]==0)&&(flg[b]==0)){
nm[8]=a; nm[9]=b;
flg[a]=1; flg[b]=1;
stp09();
flg[b]=0; flg[a]=0; }}}
}
/**/
/* Check some Line-sums */
void stp09(){
short sm1, sm2, sm3;
sm1=nm[1]+nm[2]+nm[9]+nm[10];
sm2=nm[1]+nm[3]+nm[9]+nm[11];
sm3=nm[1]+nm[5]+nm[9]+nm[13];
if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){stp10(); }
}
/* Check some Line-sums */
void stp10(){
short sm1, sm2, sm3, sm4;
sm1=nm[2]+nm[4]+nm[10]+nm[12];
sm2=nm[2]+nm[6]+nm[10]+nm[14];
if((sm1==LSM)&&(sm2==LSM)){
anspri nt(); }
}
/**/
/* Print All Forms of Solutions of Extra-Cubic Object of Order 2^4 */
/**/
void anspri nt(void){
cnt++; cnt2++;
if(cnt2==1){
printf("%3d/d0 /d1 SC/ CC/\n", cnt);
printf("%4d----%2d%7d----%2d ", nm[1], nm[2], nm[3], nm[4]);
printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
nm[1], nm[2], nm[3], nm[4], nm[1], nm[2], nm[4], nm[3]);
printf(" | %2d---%2d | %2d---%2d", nm[9], nm[10], nm[11], nm[12]);
printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
nm[5], nm[6], nm[7], nm[8], nm[5], nm[6], nm[8], nm[7]);
printf("%4d--|-%2d |%4d--|-%2d |", nm[5], nm[6], nm[7], nm[8]);
printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
nm[9], nm[10], nm[11], nm[12], nm[13], nm[14], nm[16], nm[15]);
}
}

```

```

printf("%7d----%2d%7d----%2d", nm[13], nm[14], nm[15], nm[16]);
printf("   %3d%3d%3d%3d   %3d%3d%3d%3d\n",
nm[13], nm[14], nm[15], nm[16], nm[9], nm[10], nm[12], nm[11]);
}
}
/**/

```

Let me show you part of the execution result of this program as follows.

```

*** Make the 4-Dimensional Magic Objects of Order 2 ***
** and Transform them into SC & CC Magic Squares 4x4 **

```

```

1/d0      /d1      SC/      CC/
1----15   12---- 6     1 15 12 6   1 15 6 12
| 8---+--10 | 13---+-- 3   14 4 7 9   14 4 9 7
14--|- 4 | 7--|- 9 |   8 10 13 3   11 5 16 2
  11---- 5     2----16   11 5 2 16   8 10 3 13

2/d0      /d1      SC/      CC/
1----15   8----10     1 15 8 10   1 15 10 8
| 12---+-- 6 | 13---+-- 3   14 4 11 5   14 4 5 11
14--|- 4 | 11--|- 5 |   12 6 13 3   7 9 16 2
  7---- 9     2----16   7 9 2 16   12 6 3 13

3/d0      /d1      SC/      CC/
1----15   14---- 4     1 15 14 4   1 15 4 14
| 8---+--10 | 11---+-- 5   12 6 7 9   12 6 9 7
12--|- 6 | 7--|- 9 |   8 10 11 5   13 3 16 2
  13---- 3     2----16   13 3 2 16   8 10 5 11

4/d0      /d1      SC/      CC/
1----15   8----10     1 15 8 10   1 15 10 8
| 14---+-- 4 | 11---+-- 5   12 6 13 3   12 6 3 13
12--|- 6 | 13--|- 3 |   14 4 11 5   7 9 16 2
  7---- 9     2----16   7 9 2 16   14 4 5 11

5/d0      /d1      SC/      CC/
1----15   14---- 4     1 15 14 4   1 15 4 14
| 12---+-- 6 | 7---+-- 9   8 10 11 5   8 10 5 11
8--|-10 | 11--|- 5 |   12 6 7 9   13 3 16 2
  13---- 3     2----16   13 3 2 16   12 6 9 7

6/d0      /d1      SC/      CC/
1----15   12---- 6     1 15 12 6   1 15 6 12
| 14---+-- 4 | 7---+-- 9   8 10 13 3   8 10 3 13
8--|-10 | 13--|- 3 |   14 4 7 9   11 5 16 2
  11---- 5     2----16   11 5 2 16   14 4 9 7

7/d0      /d1      SC/      CC/
1----14   12---- 7     1 14 12 7   1 14 7 12
| 8---+--11 | 13---+-- 2   15 4 6 9   15 4 9 6
15--|- 4 | 6--|- 9 |   8 11 13 2   10 5 16 3
  10---- 5     3----16   10 5 3 16   8 11 2 13

8/d0      /d1      SC/      CC/
1----14   8----11     1 14 8 11   1 14 11 8
| 12---+-- 7 | 13---+-- 2   15 4 10 5   15 4 5 10
15--|- 4 | 10--|- 5 |   12 7 13 2   6 9 16 3
  6---- 9     3----16   6 9 3 16   12 7 2 13

9/d0      /d1      SC/      CC/
1----14   15---- 4     1 14 15 4   1 14 4 15
| 8---+--11 | 10---+-- 5   12 7 6 9   12 7 9 6
12--|- 7 | 6--|- 9 |   8 11 10 5   13 2 16 3
  13---- 2     3----16   13 2 3 16   8 11 5 10

```

10/d0	/d1	SC/	CC/
1----14	8----11	1 14 8 11	1 14 11 8
15---+ 4	10---+ 5	12 7 13 2	12 7 2 13
12-- - 7	13-- - 2	15 4 10 5	6 9 16 3
6-----9	3-----16	6 9 3 16	15 4 5 10

11/d0	/d1	SC/	CC/
1----14	15-----4	1 14 15 4	1 14 4 15
12---+ 7	6---+ 9	8 11 10 5	8 11 5 10
8-- -11	10-- - 5	12 7 6 9	13 2 16 3
13-----2	3-----16	13 2 3 16	12 7 9 6

12/d0	/d1	SC/	CC/
1----14	12-----7	1 14 12 7	1 14 7 12
15---+ 4	6---+ 9	8 11 13 2	8 11 2 13
8-- -11	13-- - 2	15 4 6 9	10 5 16 3
10-----5	3-----16	10 5 3 16	15 4 9 6

13/d0	/d1	SC/	CC/
1----12	14-----7	1 12 14 7	1 12 7 14
8---+13	11---+ 2	15 6 4 9	15 6 9 4
15-- - 6	4-- - 9	8 13 11 2	10 3 16 5
10-----3	5-----16	10 3 5 16	8 13 2 11

14/d0	/d1	SC/	CC/
1----12	8-----13	1 12 8 13	1 12 13 8
14---+ 7	11---+ 2	15 6 10 3	15 6 3 10
15-- - 6	10-- - 3	14 7 11 2	4 9 16 5
4-----9	5-----16	4 9 5 16	14 7 2 11

15/d0	/d1	SC/	CC/
1----12	15-----6	1 12 15 6	1 12 6 15
8---+13	10---+ 3	14 7 4 9	14 7 9 4
14-- - 7	4-- - 9	8 13 10 3	11 2 16 5
11-----2	5-----16	11 2 5 16	8 13 3 10

16/d0	/d1	SC/	CC/
1----12	8-----13	1 12 8 13	1 12 13 8
15---+ 6	10---+ 3	14 7 11 2	14 7 2 11
14-- - 7	11-- - 2	15 6 10 3	4 9 16 5
4-----9	5-----16	4 9 5 16	15 6 3 10

17/d0	/d1	SC/	CC/
1----12	15-----6	1 12 15 6	1 12 6 15
14---+ 7	4---+ 9	8 13 10 3	8 13 3 10
8-- -13	10-- - 3	14 7 4 9	11 2 16 5
11-----2	5-----16	11 2 5 16	14 7 9 4

18/d0	/d1	SC/	CC/
1----12	14-----7	1 12 14 7	1 12 7 14
15---+ 6	4---+ 9	8 13 11 2	8 13 2 11
8-- -13	11-- - 2	15 6 4 9	10 3 16 5
10-----3	5-----16	10 3 5 16	15 6 9 4

19/d0	/d1	SC/	CC/
1---- 8	14-----11	1 8 14 11	1 8 11 14
12---+13	7---+ 2	15 10 4 5	15 10 5 4
15-- -10	4-- - 5	12 13 7 2	6 3 16 9
6-----3	9-----16	6 3 9 16	12 13 2 7

20/d0	/d1	SC/	CC/
1---- 8	12-----13	1 8 12 13	1 8 13 12
14---+11	7---+ 2	15 10 6 3	15 10 3 6
15-- -10	6-- - 3	14 11 7 2	4 5 16 9
4-----5	9-----16	4 5 9 16	14 11 2 7

21/d0	/d1	SC/	CC/
1---- 8	15----10	1 8 15 10	1 8 10 15
12---+13	6---+ 3	14 11 4 5	14 11 5 4
14-- -11	4-- - 5	12 13 6 3	7 2 16 9
7---- 2	9----16	7 2 9 16	12 13 3 6

22/d0	/d1	SC/	CC/
1---- 8	12----13	1 8 12 13	1 8 13 12
15---+10	6---+ 3	14 11 7 2	14 11 2 7
14-- -11	7-- - 2	15 10 6 3	4 5 16 9
4---- 5	9----16	4 5 9 16	15 10 3 6

23/d0	/d1	SC/	CC/
1---- 8	15----10	1 8 15 10	1 8 10 15
14---+11	4---+ 5	12 13 6 3	12 13 3 6
12-- -13	6-- - 3	14 11 4 5	7 2 16 9
7---- 2	9----16	7 2 9 16	14 11 5 4

24/d0	/d1	SC/	CC/
1---- 8	14----11	1 8 14 11	1 8 11 14
15---+10	4---+ 5	12 13 7 2	12 13 2 7
12-- -13	7-- - 2	15 10 4 5	6 3 16 9
6---- 3	9----16	6 3 9 16	15 10 5 4

25/d0	/d1	SC/	CC/
2-----16	11----- 5	2 16 11 5	2 16 5 11
7---+ 9	14---+ 4	13 3 8 10	13 3 10 8
13-- - 3	8-- -10	7 9 14 4	12 6 15 1
12----- 6	1-----15	12 6 1 15	7 9 4 14

49/d0	/d1	SC/	CC/
3-----16	10----- 5	3 16 10 5	3 16 5 10
6---+ 9	15---+ 4	13 2 8 11	13 2 11 8
13-- - 2	8-- -11	6 9 15 4	12 7 14 1
12----- 7	1-----14	12 7 1 14	6 9 4 15

73/d0	/d1	SC/	CC/
4-----15	9----- 6	4 15 9 6	4 15 6 9
5---+10	16---+ 3	14 1 7 12	14 1 12 7
14-- - 1	7-- -12	5 10 16 3	11 8 13 2
11----- 8	2-----13	11 8 2 13	5 10 3 16

97/d0	/d1	SC/	CC/
5-----16	10----- 3	5 16 10 3	5 16 3 10
4---+ 9	15---+ 6	11 2 8 13	11 2 13 8
11-- - 2	8-- -13	4 9 15 6	14 7 12 1
14----- 7	1-----12	14 7 1 12	4 9 6 15

121/d0	/d1	SC/	CC/
6-----15	9----- 4	6 15 9 4	6 15 4 9
3---+10	16---+ 5	12 1 7 14	12 1 14 7
12-- - 1	7-- -14	3 10 16 5	13 8 11 2
13----- 8	2-----11	13 8 2 11	3 10 5 16

145/d0	/d1	SC/	CC/
7-----14	9----- 4	7 14 9 4	7 14 4 9
2---+11	16---+ 5	12 1 6 15	12 1 15 6
12-- - 1	6-- -15	2 11 16 5	13 8 10 3
13----- 8	3-----10	13 8 3 10	2 11 5 16

169/d0	/d1	SC/	CC/
8----13	10---- 3	8 13 10 3	8 13 3 10
1---+12	15---- 6	11 2 5 16	11 2 16 5
11-- - 2	5-- -16	1 12 15 6	14 7 9 4
14---- 7	4---- 9	14 7 4 9	1 12 6 15

193/d0	/d1	SC/	CC/
9----16	6---- 3	9 16 6 3	9 16 3 6
4---+ 5	15----10	7 2 12 13	7 2 13 12
7-- - 2	12-- -13	4 5 15 10	14 11 8 1
14----11	1---- 8	14 11 1 8	4 5 10 15

217/d0	/d1	SC/	CC/
10----15	5---- 4	10 15 5 4	10 15 4 5
3---+ 6	16---- 9	8 1 11 14	8 1 14 11
8-- - 1	11-- -14	3 6 16 9	13 12 7 2
13----12	2---- 7	13 12 2 7	3 6 9 16

241/d0	/d1	SC/	CC/
11----14	5---- 4	11 14 5 4	11 14 4 5
2---+ 7	16---- 9	8 1 10 15	8 1 15 10
8-- - 1	10-- -15	2 7 16 9	13 12 6 3
13----12	3---- 6	13 12 3 6	2 7 9 16

265/d0	/d1	SC/	CC/
12----13	6---- 3	12 13 6 3	12 13 3 6
1---+ 8	15----10	7 2 9 16	7 2 16 9
7-- - 2	9-- -16	1 8 15 10	14 11 5 4
14----11	4---- 5	14 11 4 5	1 8 10 15

289/d0	/d1	SC/	CC/
13----12	3---- 6	13 12 3 6	13 12 6 3
2---+ 7	16---- 9	8 1 10 15	8 1 15 10
8-- - 1	10-- -15	2 7 16 9	11 14 4 5
11----14	5---- 4	11 14 5 4	2 7 9 16

313/d0	/d1	SC/	CC/
14----11	4---- 5	14 11 4 5	14 11 5 4
1---+ 8	15----10	7 2 9 16	7 2 16 9
7-- - 2	9-- -16	1 8 15 10	12 13 3 6
12----13	6---- 3	12 13 6 3	1 8 10 15

337/d0	/d1	SC/	CC/
15----10	4---- 5	15 10 4 5	15 10 5 4
1---+ 8	14----11	6 3 9 16	6 3 16 9
6-- - 3	9-- -16	1 8 14 11	12 13 2 7
12----13	7---- 2	12 13 7 2	1 8 11 14

361/d0	/d1	SC/	CC/
16---- 9	3---- 6	16 9 3 6	16 9 6 3
2---+ 7	13----12	5 4 10 15	5 4 15 10
5-- - 4	10-- -15	2 7 13 12	11 14 1 8
11----14	8---- 1	11 14 8 1	2 7 12 13

[Count = 384]
OK!

We have got the 384 primitive solutions of developed ECO_2^4 in various forms:
(1) Developed forms of order 2×2^3 , (2) Self-complementary magic squares of order 4^2 ,
and (3) Composite and complete magic squares of order 4^2 .

