

## Chapter 6: Fundamental Studies of Multi-Dimensional Extra-Cubic Magic Forms and their Decompositions: Kanji Setsuda

### Section 3-2: 'Fundamental' Solutions and Transformation Method of 'Composite & Complete' Magic Squares 8x8

#1. In the former section we studied about 6-dimensional extra-cubic magic forms of order 2 and their decompositions. As a result we have got 15 fundamental solutions of 'C&C' magic Cubes of order 4 and 10 fundamental solutions of 'C&C' magic Squares of order 8. They are taken out of the higher dimensional magic object by down-converting into lower dimensional ones.

But could we say loudly that they are really the 'fundamental solutions' of all 'C&C' magic cubes and squares? What evidences could we demonstrate for them?

We once examined if the 'Prototype' solution set and 'DAM' transformation were applicable in this case. The result was quite satisfactory. We found we could re-construct all objects only by some representative models transforming the prototype solutions.

#2. But now we want to have something more direct and clearer to prove that it is true. Couldn't we get such things in the real world of magic cubes and squares?

We want to demonstrate that those 'fundamental' solutions are able to make all the objects we want, say, by transforming the fundamentals into all objects.

We want to find some simple rules of transformation, for instance, such as the ones I invented for 'C&C' magic cubes of order 4 (See #11 of this article).

Let's study and invent those transformation methods for our 'C&C' magic squares of order 8.

#3. The original source we have at first is the list of Fundamental Ten solutions:

```
*** 'Composite & Complete' Magic Square 8*8 ***
***   Print the Fundamental Ten Solutions   ***
```

1/ 8								2/10								3/12							
1	63	6	60	14	52	9	55	1	63	6	60	22	44	17	47	1	63	6	60	38	28	33	31
62	4	57	7	49	15	54	12	62	4	57	7	41	23	46	20	62	4	57	7	25	39	30	36
19	45	24	42	32	34	27	37	11	53	16	50	32	34	27	37	11	53	16	50	48	18	43	21
48	18	43	21	35	29	40	26	56	10	51	13	35	29	40	26	56	10	51	13	19	45	24	42
51	13	56	10	64	2	59	5	43	21	48	18	64	2	59	5	27	37	32	34	64	2	59	5
16	50	11	53	3	61	8	58	24	42	19	45	3	61	8	58	40	26	35	29	3	61	8	58
33	31	38	28	46	20	41	23	33	31	38	28	54	12	49	15	17	47	22	44	54	12	49	15
30	36	25	39	17	47	22	44	30	36	25	39	9	55	14	52	46	20	41	23	9	55	14	52
4/18								5/20								6/24							
1	63	10	56	26	40	17	47	1	63	10	56	42	24	33	31	1	63	18	48	50	16	33	31
62	4	53	11	37	27	46	20	62	4	53	11	21	43	30	36	62	4	45	19	13	51	30	36
7	57	16	50	32	34	23	41	7	57	16	50	48	18	39	25	7	57	24	42	56	10	39	25
60	6	51	13	35	29	44	22	60	6	51	13	19	45	28	38	60	6	43	21	11	53	28	38
39	25	48	18	64	2	55	9	23	41	32	34	64	2	55	9	15	49	32	34	64	2	47	17
28	38	19	45	3	61	12	54	44	22	35	29	3	61	12	54	52	14	35	29	3	61	20	46
33	31	42	24	58	8	49	15	17	47	26	40	58	8	49	15	9	55	26	40	58	8	41	23
30	36	21	43	5	59	14	52	46	20	37	27	5	59	14	52	54	12	37	27	5	59	22	44

```

              7/32                      8/34                      9/36
1 63  4 62 12 54  9 55   1 63  4 62 20 46 17 47   1 63  4 62 36 30 33 31
60  6 57  7 49 15 52 14  60  6 57  7 41 23 44 22  60  6 57  7 25 39 28 38
21 43 24 42 32 34 29 35  13 51 16 50 32 34 29 35  13 51 16 50 48 18 45 19
48 18 45 19 37 27 40 26  56 10 53 11 37 27 40 26  56 10 53 11 21 43 24 42
53 11 56 10 64  2 61  3  45 19 48 18 64  2 61  3  29 35 32 34 64  2 61  3
16 50 13 51  5 59  8 58  24 42 21 43  5 59  8 58  40 26 37 27  5 59  8 58
33 31 36 30 44 22 41 23  33 31 36 30 52 14 49 15  17 47 20 46 52 14 49 15
28 38 25 39 17 47 20 46  28 38 25 39  9 55 12 54  44 22 41 23  9 55 12 54

              10/48
1 63  4 62  8 58  5 59
56 10 53 11 49 15 52 14
25 39 28 38 32 34 29 35
48 18 45 19 41 23 44 22
57  7 60  6 64  2 61  3
16 50 13 51  9 55 12 54
33 31 36 30 40 26 37 27
24 42 21 43 17 47 20 46   [Count = 10]

```

They are taken out of the 6-dimensional extra-cubic object of order 2.  
They naturally inherited some impressive properties from the origin:

### \*\* Composite & Complete Magic Square 8x8 \*\*

```

n1 n2 n3 n4 n5 n6 n7 n8   n1+n2+n3+n4=130;
n9 10 11 12 13 14 15 16   n1+n9+n17+n25=130;
17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32   n1=1; n37=64;
33 34 35 36 37 38 39 40   {n2, n4, n9, n21, n25, n35}
41 42 43 44 45 46 47 48   = {32, 48, 56, 60, 62, 63}
49 50 51 52 53 54 55 56
57 58 59 60 61 62 63 64

```

Yes. They are naturally multiple type of 4 magic squares of order 4 included within. The numbers {63, 62, 60, 56, 48, 32} next to **n1** could appear only in {**n2**, **n4**, **n9**, **n21**, **n25**, **n35**} and couldn't come elsewhere. But **n21** and **n35** look like strange, don't they? Though they are not on the axes, they are waiting in their 'reserved seats' for appearing on the axes. It is because you can exchange **n21** and **n25** with each other, and exchange also **n4** and **n35** at any time you want to. You may know the next type of transformations is applicable to those positions.

### \*\* Inner Transformations by exchanging half-a-lines \*\*

```

[1]      * *           * *->[2]
n1 n2 n3 n4 n5 n6 n7 n8   n1 n2 n3 n4 n5 n6 n7 n8
n9 10 11 12 13 14 15 16   n9 10 11 12 13 14 15 16
*17 18 19 20 21 22 23 24*  29 30 31 32 25 26 27 28
*25 26 27 28 29 30 31 32*  21 22 23 24 17 18 19 20
 33 34 35 36 37 38 39 40   33 34 35 36 37 38 39 40
 41 42 43 44 45 46 47 48   41 42 43 44 45 46 47 48
*49 50 51 52 53 54 55 56*  61 62 63 64 57 58 59 60
*57 58 59 60 61 62 63 64*  53 54 55 56 49 50 51 52
      * * |           * * |

```

```

      |           |
[3]   v           ->[4]=[2]+[3]   v
n1 n2 36 35 n5 n6 40 39   n1 n2 36 35 n5 n6 40 39
n9 10 44 43 13 14 48 47   n9 10 44 43 13 14 48 47
17 18 52 51 21 22 56 55   29 30 64 63 25 26 60 59
25 26 60 59 29 30 64 63   21 22 56 55 17 18 52 51
33 34 n4 n3 37 38 n8 n7   33 34 n4 n3 37 38 n8 n7
41 42 12 11 45 46 16 15   41 42 12 11 45 46 16 15
49 50 20 19 53 54 24 23   61 62 32 31 57 58 28 20
57 58 28 20 61 62 32 31   53 54 24 23 49 50 20 19

```

#4. The object goal we have to reach is the list of 90 solutions of 'C&C' magic squares. Let me show you the list here.

```

*** 'Composite & Complete' Magic Square 8*8 ***
***   Print the Fundamental 90 Solutions   ***

```

```

      1/           2/           3/
  1 63  5 59 14 52 10 56   1 63  5 59 14 52 10 56   1 63  5 59 22 44 18 48
 62  4 58  8 49 15 53 11   62  4 58  8 49 15 53 11   62  4 58  8 41 23 45 19
 17 47 21 43 30 36 26 40   19 45 23 41 32 34 28 38   9 55 13 51 30 36 26 40
 46 20 42 24 33 31 37 27   48 18 44 22 35 29 39 25   54 12 50 16 33 31 37 27
 51 13 55  9 64  2 60  6   51 13 55  9 64  2 60  6   43 21 47 17 64  2 60  6
 16 50 12 54  3 61  7 57   16 50 12 54  3 61  7 57   24 42 20 46  3 61  7 57
 35 29 39 25 48 18 44 22   33 31 37 27 46 20 42 24   35 29 39 25 56 10 52 14
 32 34 28 38 19 45 23 41   30 36 26 40 17 47 21 43   32 34 28 38 11 53 15 49

      4/           5/           6/
  1 63  5 59 22 44 18 48   1 63  5 59 38 28 34 32   1 63  5 59 38 28 34 32
 62  4 58  8 41 23 45 19   62  4 58  8 25 39 29 35   62  4 58  8 25 39 29 35
 11 53 15 49 32 34 28 38   9 55 13 51 46 20 42 24   11 53 15 49 48 18 44 22
 56 10 52 14 35 29 39 25   54 12 50 16 17 47 21 43   56 10 52 14 19 45 23 41
 43 21 47 17 64  2 60  6   27 37 31 33 64  2 60  6   27 37 31 33 64  2 60  6
 24 42 20 46  3 61  7 57   40 26 36 30  3 61  7 57   40 26 36 30  3 61  7 57
 33 31 37 27 54 12 50 16   19 45 23 41 56 10 52 14   17 47 21 43 54 12 50 16
 30 36 26 40  9 55 13 51   48 18 44 22 11 53 15 49   46 20 42 24  9 55 13 51

      7/           [No.1]           8/           9/
  1 63  6 60 14 52  9 55   1 63  6 60 14 52  9 55   1 63  6 60 22 44 17 47
 62  4 57  7 49 15 54 12   62  4 57  7 49 15 54 12   62  4 57  7 41 23 46 20
 17 47 22 44 30 36 25 39   19 45 24 42 32 34 27 37   9 55 14 52 30 36 25 39
 46 20 41 23 33 31 38 28   48 18 43 21 35 29 40 26   54 12 49 15 33 31 38 28
 51 13 56 10 64  2 59  5   51 13 56 10 64  2 59  5   43 21 48 18 64  2 59  5
 16 50 11 53  3 61  8 58   16 50 11 53  3 61  8 58   24 42 19 45  3 61  8 58
 35 29 40 26 48 18 43 21   33 31 38 28 46 20 41 23   35 29 40 26 56 10 51 13
 32 34 27 37 19 45 24 42   30 36 25 39 17 47 22 44   32 34 27 37 11 53 16 50

[No.2]           10/           11/           [No.3]           12/
  1 63  6 60 22 44 17 47   1 63  6 60 38 28 33 31   1 63  6 60 38 28 33 31
 62  4 57  7 41 23 46 20   62  4 57  7 25 39 30 36   62  4 57  7 25 39 30 36
 11 53 16 50 32 34 27 37   9 55 14 52 46 20 41 23   11 53 16 50 48 18 43 21
 56 10 51 13 35 29 40 26   54 12 49 15 17 47 22 44   56 10 51 13 19 45 24 42
 43 21 48 18 64  2 59  5   27 37 32 34 64  2 59  5   27 37 32 34 64  2 59  5
 24 42 19 45  3 61  8 58   40 26 35 29  3 61  8 58   40 26 35 29  3 61  8 58
 33 31 38 28 54 12 49 15   19 45 24 42 56 10 51 13   17 47 22 44 54 12 49 15
 30 36 25 39  9 55 14 52   48 18 43 21 11 53 16 50   46 20 41 23  9 55 14 52

```

13/										14/										15/									
1	63	9	55	26	40	18	48	1	63	9	55	26	40	18	48	1	63	9	55	42	24	34	32						
62	4	54	12	37	27	45	19	62	4	54	12	37	27	45	19	62	4	54	12	21	43	29	35						
5	59	13	51	30	36	22	44	7	57	15	49	32	34	24	42	5	59	13	51	46	20	38	28						
58	8	50	16	33	31	41	23	60	6	52	14	35	29	43	21	58	8	50	16	17	47	25	39						
39	25	47	17	64	2	56	10	39	25	47	17	64	2	56	10	23	41	31	33	64	2	56	10						
28	38	20	46	3	61	11	53	28	38	20	46	3	61	11	53	44	22	36	30	3	61	11	53						
35	29	43	21	60	6	52	14	33	31	41	23	58	8	50	16	19	45	27	37	60	6	52	14						
32	34	24	42	7	57	15	49	30	36	22	44	5	59	13	51	48	18	40	26	7	57	15	49						
16/										17/										[No.4] 18/									
1	63	9	55	42	24	34	32	1	63	10	56	26	40	17	47	1	63	10	56	26	40	17	47						
62	4	54	12	21	43	29	35	62	4	53	11	37	27	46	20	62	4	53	11	37	27	46	20						
7	57	15	49	48	18	40	26	5	59	14	52	30	36	21	43	7	57	16	50	32	34	23	41						
60	6	52	14	19	45	27	37	58	8	49	15	33	31	42	24	60	6	51	13	35	29	44	22						
23	41	31	33	64	2	56	10	39	25	48	18	64	2	55	9	39	25	48	18	64	2	55	9						
44	22	36	30	3	61	11	53	28	38	19	45	3	61	12	54	28	38	19	45	3	61	12	54						
17	47	25	39	58	8	50	16	35	29	44	22	60	6	51	13	33	31	42	24	58	8	49	15						
46	20	38	28	5	59	13	51	32	34	23	41	7	57	16	50	30	36	21	43	5	59	14	52						

. . . . . (If you want the full list, [please click here.](#))

82/										83/										84/									
1	62	2	61	12	55	11	56	1	62	5	58	15	52	11	56	1	62	7	60	15	52	9	54						
48	19	47	20	37	26	38	25	48	19	44	23	34	29	38	25	48	19	42	21	34	29	40	27						
5	58	6	57	16	51	15	52	2	61	6	57	16	51	12	55	2	61	8	59	16	51	10	53						
44	23	43	24	33	30	34	29	47	20	43	24	33	30	37	26	47	20	41	22	33	30	39	28						
53	10	54	9	64	3	63	4	50	13	54	9	64	3	60	7	50	13	56	11	64	3	58	5						
28	39	27	40	17	46	18	45	31	36	27	40	17	46	21	42	31	36	25	38	17	46	23	44						
49	14	50	13	60	7	59	8	49	14	53	10	63	4	59	8	49	14	55	12	63	4	57	6						
32	35	31	36	21	42	22	41	32	35	28	39	18	45	22	41	32	35	26	37	18	45	24	43						
85/										86/										87/									
1	60	2	59	22	47	21	48	1	60	2	59	38	31	37	32	1	60	3	58	23	46	21	48						
56	13	55	14	35	26	36	25	56	13	55	14	19	42	20	41	56	13	54	15	34	27	36	25						
3	58	4	57	24	45	23	46	3	58	4	57	40	29	39	30	2	59	4	57	24	45	22	47						
54	15	53	16	33	28	34	27	54	15	53	16	17	44	18	43	55	14	53	16	33	28	35	26						
43	18	44	17	64	5	63	6	27	34	28	33	64	5	63	6	42	19	44	17	64	5	62	7						
30	39	29	40	9	52	10	51	46	23	45	24	9	52	10	51	31	38	29	40	9	52	11	50						
41	20	42	19	62	7	61	8	25	36	26	35	62	7	61	8	41	20	43	18	63	6	61	8						
32	37	31	38	11	50	12	49	48	21	47	22	11	50	12	49	32	37	30	39	10	51	12	49						
88/										89/										90/									
1	60	3	58	39	30	37	32	1	60	2	59	14	55	13	56	1	60	3	58	15	54	13	56						
56	13	54	15	18	43	20	41	48	21	47	22	35	26	36	25	48	21	46	23	34	27	36	25						
2	59	4	57	40	29	38	31	3	58	4	57	16	53	15	54	2	59	4	57	16	53	14	55						
55	14	53	16	17	44	19	42	46	23	45	24	33	28	34	27	47	22	45	24	33	28	35	26						
26	35	28	33	64	5	62	7	51	10	52	9	64	5	63	6	50	11	52	9	64	5	62	7						
47	22	45	24	9	52	11	50	30	39	29	40	17	44	18	43	31	38	29	40	17	44	19	42						
25	36	27	34	63	6	61	8	49	12	50	11	62	7	61	8	49	12	51	10	63	6	61	8						
48	21	46	23	10	51	12	49	32	37	31	38	19	42	20	41	32	37	30	39	18	43	20	41						

[Count = 90]

Could we make the 10 solutions of the former list above into the 90 solutions of the latter one? Could we transform each one of 10 into the other 9 object solutions?

#5. What we have to do at first is to classify 90 solutions into 10 groups. Each group must have only one of the Fundamental Ten solutions. It should be considered as the representative solution, or the flag standing for the same group.

Pick up any one of 90 solutions and examine if it is similar to each representative flag. If you find any common properties between them, you could take it into the same group. The more similarities, the better. Half of all entries must be the same at least.

```
*** Find Similar Squares among the Fundamental ***
**** 90 Solutions of 'C&C' Magic Squares 8x8 ****
***** To the Fundamental Ten Solutions *****
```

[2 : 1]

1 63 5 59 14 52 10 56	1 63 6 60 14 52 9 55
62 4 58 8 49 15 53 11	62 4 57 7 49 15 54 12
19 45 23 41 32 34 28 38	19 45 24 42 32 34 27 37
48 18 44 22 35 29 39 25	48 18 43 21 35 29 40 26
51 13 55 9 64 2 60 6	51 13 56 10 64 2 59 5
16 50 12 54 3 61 7 57	16 50 11 53 3 61 8 58
33 31 37 27 46 20 42 24	33 31 38 28 46 20 41 23
30 36 26 40 17 47 21 43	30 36 25 39 17 47 22 44

	2/O	<-- Unmatched -->	1/F
* * 5 59 *	* * 10 56	* * 6 60 *	* * 9 55
* * 58 8 *	* * 53 11	* * 57 7 *	* * 54 12
* * 23 41 *	* * 28 38	* * 24 42 *	* * 27 37
* * 44 22 *	* * 39 25	* * 43 21 *	* * 40 26
* * 55 9 *	* * 60 6	* * 56 10 *	* * 59 5
* * 12 54 *	* * 7 57	* * 11 53 *	* * 8 58
* * 37 27 *	* * 42 24	* * 38 28 *	* * 41 23
* * 26 40 *	* * 21 43	* * 25 39 *	* * 22 44

[7 : 1]

1 63 6 60 14 52 9 55	1 63 6 60 14 52 9 55
62 4 57 7 49 15 54 12	62 4 57 7 49 15 54 12
17 47 22 44 30 36 25 39	19 45 24 42 32 34 27 37
46 20 41 23 33 31 38 28	48 18 43 21 35 29 40 26
51 13 56 10 64 2 59 5	51 13 56 10 64 2 59 5
16 50 11 53 3 61 8 58	16 50 11 53 3 61 8 58
35 29 40 26 48 18 43 21	33 31 38 28 46 20 41 23
32 34 27 37 19 45 24 42	30 36 25 39 17 47 22 44

	7/O	<-- Unmatched -->	1/F
* * * * *	* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *	* * * * *
17 47 22 44 30 36 25 39	19 45 24 42 32 34 27 37	48 18 43 21 35 29 40 26	
46 20 41 23 33 31 38 28			
* * * * *	* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *	* * * * *
35 29 40 26 48 18 43 21	33 31 38 28 46 20 41 23		
32 34 27 37 19 45 24 42	30 36 25 39 17 47 22 44		

[8 : 1]

1 63 6 60 14 52 9 55	1 63 6 60 14 52 9 55
62 4 57 7 49 15 54 12	62 4 57 7 49 15 54 12
19 45 24 42 32 34 27 37	19 45 24 42 32 34 27 37
48 18 43 21 35 29 40 26	48 18 43 21 35 29 40 26
51 13 56 10 64 2 59 5	51 13 56 10 64 2 59 5
16 50 11 53 3 61 8 58	16 50 11 53 3 61 8 58
33 31 38 28 46 20 41 23	33 31 38 28 46 20 41 23
30 36 25 39 17 47 22 44	30 36 25 39 17 47 22 44

```

      8/O      <-- Unmatched -->      1/F
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *
* * * * * * * *

```

```

      10/O      <-- Unmatched -->      1/F
* * * * 22 44 17 47
* * * * 41 23 46 20
11 53 16 50 * * * *
56 10 51 13 * * * *
43 21 48 18 * * * *
24 42 19 45 * * * *
* * * * 54 12 49 15
* * * * 9 55 14 52

```

```

      32/O      <-- Unmatched -->      1/F
* * 4 62 12 54 * *
60 6 * * * * 52 14
21 43 * * * * 29 35
* * 45 19 37 27 * *
53 11 * * * * 61 3
* * 13 51 5 59 * *
* * 36 30 44 22 * *
28 38 * * * * 20 46

```

[4 : 2]

```

1 63 5 59 22 44 18 48
62 4 58 8 41 23 45 19
11 53 15 49 32 34 28 38
56 10 52 14 35 29 39 25
43 21 47 17 64 2 60 6
24 42 20 46 3 61 7 57
33 31 37 27 54 12 50 16
30 36 26 40 9 55 13 51

```

```

      4/O      <-- Unmatched -->      2/F
* * 5 59 * * 18 48
* * 58 8 * * 45 19
* * 15 49 * * 28 38
* * 52 14 * * 39 25
* * 47 17 * * 60 6
* * 20 46 * * 7 57
* * 37 27 * * 50 16
* * 26 40 * * 13 51

```

```

      8/O      <-- Unmatched -->      2/F
* * * * 14 52 9 55
* * * * 49 15 54 12
19 45 24 42 * * * *
48 18 43 21 * * * *
51 13 56 10 * * * *
16 50 11 53 * * * *
* * * * 46 20 41 23
* * * * 17 47 22 44

```

[9 : 2]

```

1 63 6 60 22 44 17 47
62 4 57 7 41 23 46 20
9 55 14 52 30 36 25 39
54 12 49 15 33 31 38 28
43 21 48 18 64 2 59 5
24 42 19 45 3 61 8 58
35 29 40 26 56 10 51 13
32 34 27 37 11 53 16 50

```

9/O	<-- Unmatched -->	2/F
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
9 55 14 52 30 36 25 39	11 53 16 50 32 34 27 37	
54 12 49 15 33 31 38 28	56 10 51 13 35 29 40 26	
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
35 29 40 26 56 10 51 13	33 31 38 28 54 12 49 15	
32 34 27 37 11 53 16 50	30 36 25 39 9 55 14 52	

10/O	<-- Unmatched -->	2/F
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *

12/O	<-- Unmatched -->	2/F
* * * * 38 28 33 31	* * * * 22 44 17 47	
* * * * 25 39 30 36	* * * * 41 23 46 20	
* * * * 48 18 43 21	* * * * 32 34 27 37	
* * * * 19 45 24 42	* * * * 35 29 40 26	
27 37 32 34 * * * *	43 21 48 18 * * * *	
40 26 35 29 * * * *	24 42 19 45 * * * *	
17 47 22 44 * * * *	33 31 38 28 * * * *	
46 20 41 23 * * * *	30 36 25 39 * * * *	

18/O	<-- Unmatched -->	2/F
* * 10 56 26 40 * *	* * 6 60 22 44 * *	
* * 53 11 37 27 * *	* * 57 7 41 23 * *	
7 57 * * * * 23 41	11 53 * * * * 27 37	
60 6 * * * * 44 22	56 10 * * * * 40 26	
39 25 * * * * 55 9	43 21 * * * * 59 5	
28 38 * * * * 12 54	24 42 * * * * 8 58	
* * 42 24 58 8 * *	* * 38 28 54 12 * *	
* * 21 43 5 59 * *	* * 25 39 9 55 * *	

. . . . .

First two pairs above are quite similar to each other. You could easily imagine how to transform each one into the other, couldn't you? The third pair shows No.8 in the 90 Solution List is just the same with No.1 in the list of Fundamental 10.

Let's take No.1, 2, 7 and 8 together into the same group of a 'family' led by F1.

In order to make such a large family as has 9 members in each, you should find anything more, even similar to No.1, 2, and 7 instead of No.8(=F1).

59/	<-- Unmatched -->	1/F3
* * * * *	* * * * *	* * * * *
48 18 44 22 35 29 39 25	62 4 58 8 49 15 53 11	
3 61 7 57 16 50 12 54	17 47 21 43 30 36 26 40	
* * * * *	* * * * *	* * * * *
* * * * *	* * * * *	* * * * *
30 36 26 40 17 47 21 43	16 50 12 54 3 61 7 57	
49 15 53 11 62 4 58 8	35 29 39 25 48 18 44 22	
* * * * *	* * * * *	* * * * *

```

60/ <-- Unmatched --> 1/F4
* 63 6 * * 52 9 * * 59 2 * * 56 13 *
* 18 43 * * 29 40 * * 22 47 * * 25 36 *
* 61 8 * * 50 11 * * 57 4 * * 54 15 *
* 20 41 * * 31 38 * * 24 45 * * 27 34 *
* 13 56 * * 2 59 * * 9 52 * * 6 63 *
* 36 25 * * 47 22 * * 40 29 * * 43 18 *
* 15 54 * * 4 57 * * 11 50 * * 8 61 *
* 34 27 * * 45 24 * * 38 31 * * 41 20 *

71/ <-- Unmatched --> 1/F4
* * 17 46 * * 35 32 * * 19 48 * * 33 30
* * 44 23 * * 26 37 * * 42 21 * * 28 39
* * 18 45 * * 36 31 * * 20 47 * * 34 29
* * 43 24 * * 25 38 * * 41 22 * * 27 40
* * 30 33 * * 48 19 * * 32 35 * * 46 17
* * 39 28 * * 21 42 * * 37 26 * * 23 44
* * 29 34 * * 47 20 * * 31 36 * * 45 18
* * 40 27 * * 22 41 * * 38 25 * * 24 43

72/ <-- Unmatched --> 1/F5
* * 19 48 * * 33 30 * * 17 46 * * 35 32
* * 42 21 * * 28 39 * * 44 23 * * 26 37
* * 20 47 * * 34 29 * * 18 45 * * 36 31
* * 41 22 * * 27 40 * * 43 24 * * 25 38
* * 32 35 * * 46 17 * * 30 33 * * 48 19
* * 37 26 * * 23 44 * * 39 28 * * 21 42
* * 31 36 * * 45 18 * * 29 34 * * 47 20
* * 38 25 * * 24 43 * * 40 27 * * 22 41

89/ <-- Unmatched --> 1/F6
* 60 2 * * 55 13 * * 63 5 * * 52 10 *
* 21 47 * * 26 36 * * 18 44 * * 29 39 *
* 58 4 * * 53 15 * * 61 7 * * 50 12 *
* 23 45 * * 28 34 * * 20 42 * * 31 37 *
* 10 52 * * 5 63 * * 13 55 * * 2 60 *
* 39 29 * * 44 18 * * 36 26 * * 47 21 *
* 12 50 * * 7 61 * * 15 53 * * 4 58 *
* 37 31 * * 42 20 * * 34 28 * * 45 23 *

```

Any pair of examples above is so similar that half of 64 entries are the same, and you could easily find how to transform each one into the other. You could take them together into the same family group of F1.

The next list below shows the result of my original classification:  
Each of 10 groups has 9 members led by the representative F10 at the top position.

```

*** 'Composite & Complete' Magic Square 8x8 ***
*** 10 Groups of Fundamental 90 Solutions ***

```

```

[No.1] 8/ 7/ 60/
1 63 6 60 14 52 9 55 1 63 6 60 14 52 9 55 1 63 6 60 14 52 9 55
62 4 57 7 49 15 54 12 62 4 57 7 49 15 54 12 48 18 43 21 35 29 40 26
19 45 24 42 32 34 27 37 17 47 22 44 30 36 25 39 3 61 8 58 16 50 11 53
48 18 43 21 35 29 40 26 46 20 41 23 33 31 38 28 46 20 41 23 33 31 38 28
51 13 56 10 64 2 59 5 51 13 56 10 64 2 59 5 51 13 56 10 64 2 59 5
16 50 11 53 3 61 8 58 16 50 11 53 3 61 8 58 30 36 25 39 17 47 22 44
33 31 38 28 46 20 41 23 35 29 40 26 48 18 43 21 49 15 54 12 62 4 57 7
30 36 25 39 17 47 22 44 32 34 27 37 19 45 24 42 32 34 27 37 19 45 24 42

```



44/								43/								55/							
1	63	3	61	8	58	6	60	1	63	3	61	8	58	6	60	1	63	3	61	8	58	6	60
56	10	54	12	49	15	51	13	56	10	54	12	49	15	51	13	48	18	46	20	41	23	43	21
25	39	27	37	32	34	30	36	17	47	19	45	24	42	22	44	9	55	11	53	16	50	14	52
48	18	46	20	41	23	43	21	40	26	38	28	33	31	35	29	40	26	38	28	33	31	35	29
57	7	59	5	64	2	62	4	57	7	59	5	64	2	62	4	57	7	59	5	64	2	62	4
16	50	14	52	9	55	11	53	16	50	14	52	9	55	11	53	24	42	22	44	17	47	19	45
33	31	35	29	40	26	38	28	41	23	43	21	48	18	46	20	49	15	51	13	56	10	54	12
24	42	22	44	17	47	19	45	32	34	30	36	25	39	27	37	32	34	30	36	25	39	27	37
74/								73/								81/							
1	62	2	61	8	59	7	60	1	62	2	61	8	59	7	60	1	62	2	61	8	59	7	60
56	11	55	12	49	14	50	13	56	11	55	12	49	14	50	13	48	19	47	20	41	22	42	21
25	38	26	37	32	35	31	36	17	46	18	45	24	43	23	44	9	54	10	53	16	51	15	52
48	19	47	20	41	22	42	21	40	27	39	28	33	30	34	29	40	27	39	28	33	30	34	29
57	6	58	5	64	3	63	4	57	6	58	5	64	3	63	4	57	6	58	5	64	3	63	4
16	51	15	52	9	54	10	53	16	51	15	52	9	54	10	53	24	43	23	44	17	46	18	45
33	30	34	29	40	27	39	28	41	22	42	21	48	19	47	20	49	14	50	13	56	11	55	12
24	43	23	44	17	46	18	45	32	35	31	36	25	38	26	37	32	35	31	36	25	38	26	37

[Count: 9 X 10]

This list has no repetitions, and no missing solutions, either.

As long as we could have got this list of 10 groups, we are almost close to our goal.

The next job we have to do is to find the way how to make one into the other 8 for each group. Make your job by some simple process of transformations, as common as possible in all groups, and you could really make all 90 from F10.

#6. How could you discover and invent the way how to transform the Fundamental into the other 8 solutions in the same group?

- (1) Pick up one and compare it precisely to the Fundamental representative.
- (2) Collect any pair of solutions whose 'unmatched pattern' is quite similar.

\*\*\* Collection of Similar Patterns of the 90 \*\*\*  
 \*\*\*\* Solutions of 'C&C' Magic Squares 8x8 \*\*\*\*  
 \*\*\*\* To the Most Fundamental Ten Solutions \*\*\*

[Trans. Type 1]

				2/O	<--	Unmatched	-->					1/F			
*	*	5	59	*	*	10	56	*	*	6	60	*	*	9	55
*	*	58	8	*	*	53	11	*	*	57	7	*	*	54	12
*	*	23	41	*	*	28	38	*	*	24	42	*	*	27	37
*	*	44	22	*	*	39	25	*	*	43	21	*	*	40	26
*	*	55	9	*	*	60	6	*	*	56	10	*	*	59	5
*	*	12	54	*	*	7	57	*	*	11	53	*	*	8	58
*	*	37	27	*	*	42	24	*	*	38	28	*	*	41	23
*	*	26	40	*	*	21	43	*	*	25	39	*	*	22	44
				4/O	<--	Unmatched	-->					2/F			
*	*	5	59	*	*	18	48	*	*	6	60	*	*	17	47
*	*	58	8	*	*	45	19	*	*	57	7	*	*	46	20
*	*	15	49	*	*	28	38	*	*	16	50	*	*	27	37
*	*	52	14	*	*	39	25	*	*	51	13	*	*	40	26
*	*	47	17	*	*	60	6	*	*	48	18	*	*	59	5
*	*	20	46	*	*	7	57	*	*	19	45	*	*	8	58
*	*	37	27	*	*	50	16	*	*	38	28	*	*	49	15
*	*	26	40	*	*	13	51	*	*	25	39	*	*	14	52

6/O <-- Unmatched --> 3/F  
 14/O <-- Unmatched --> 4/F  
 16/O <-- Unmatched --> 5/F  
 22/O <-- Unmatched --> 6/F  
 26/O <-- Unmatched --> 7/F  
 28/O <-- Unmatched --> 8/F  
 30/O <-- Unmatched --> 9/F  
 44/O <-- Unmatched --> 10/F

[Trans. Type 2]

7/O <-- Unmatched --> 1/F  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 17 47 22 44 30 36 25 39 19 45 24 42 32 34 27 37  
 46 20 41 23 33 31 38 28 48 18 43 21 35 29 40 26  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 35 29 40 26 48 18 43 21 33 31 38 28 46 20 41 23  
 32 34 27 37 19 45 24 42 30 36 25 39 17 47 22 44

9/O <-- Unmatched --> 2/F  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 9 55 14 52 30 36 25 39 11 53 16 50 32 34 27 37  
 54 12 49 15 33 31 38 28 56 10 51 13 35 29 40 26  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*  
 35 29 40 26 56 10 51 13 33 31 38 28 54 12 49 15  
 32 34 27 37 11 53 16 50 30 36 25 39 9 55 14 52

11/O <-- Unmatched --> 3/F  
 17/O <-- Unmatched --> 4/F  
 19/O <-- Unmatched --> 5/F  
 23/O <-- Unmatched --> 6/F  
 31/O <-- Unmatched --> 7/F  
 33/O <-- Unmatched --> 8/F  
 35/O <-- Unmatched --> 9/F  
 47/O <-- Unmatched --> 10/F

[Trans. Type 4]

32/O <-- Unmatched --> 1/F  
 \* \* 4 62 12 54 \* \* \* \* \* \* \* \* \* \*  
 60 6 \* \* \* \* 52 14 62 4 \* \* \* \* \* 54 12  
 21 43 \* \* \* \* 29 35 19 45 \* \* \* \* \* 27 37  
 \* \* 45 19 37 27 \* \* \* \* \* \* \* \* \* \*  
 53 11 \* \* \* \* 61 3 51 13 \* \* \* \* \* 59 5  
 \* \* 13 51 5 59 \* \* \* \* \* 11 53 3 61 \* \*  
 \* \* 36 30 44 22 \* \* \* \* \* 38 28 46 20 \* \*  
 28 38 \* \* \* \* 20 46 30 36 \* \* \* \* \* 22 44

34/O <-- Unmatched --> 2/F  
 36/O <-- Unmatched --> 3/F  
 8/O <-- Unmatched --> 7/F  
 10/O <-- Unmatched --> 8/F  
 12/O <-- Unmatched --> 9/F

[Trans. Type 5]

12/O <-- Unmatched --> 2/F  
 \* \* \* \* 38 28 33 31 \* \* \* \* 22 44 17 47  
 \* \* \* \* 25 39 30 36 \* \* \* \* 41 23 46 20  
 \* \* \* \* 48 18 43 21 \* \* \* \* 32 34 27 37  
 \* \* \* \* 19 45 24 42 \* \* \* \* 35 29 40 26  
 27 37 32 34 \* \* \* \* 43 21 48 18 \* \* \* \*  
 40 26 35 29 \* \* \* \* 24 42 19 45 \* \* \* \*  
 17 47 22 44 \* \* \* \* 33 31 38 28 \* \* \* \*  
 46 20 41 23 \* \* \* \* 30 36 25 39 \* \* \* \*

10/O <-- Unmatched --> 3/F  
 20/O <-- Unmatched --> 4/F  
 18/O <-- Unmatched --> 5/F  
 36/O <-- Unmatched --> 8/F  
 34/O <-- Unmatched --> 9/F

[Trans. **Type 7**]

18/O <-- Unmatched --> 2/F  
 \* \* 10 56 26 40 \* \* \* \* 6 60 22 44 \* \*  
 \* \* 53 11 37 27 \* \* \* \* 57 7 41 23 \* \*  
 7 57 \* \* \* \* 23 41 11 53 \* \* \* \* 27 37  
 60 6 \* \* \* \* 44 22 56 10 \* \* \* \* 40 26  
 39 25 \* \* \* \* 55 9 43 21 \* \* \* \* 59 5  
 28 38 \* \* \* \* 12 54 24 42 \* \* \* \* 8 58  
 \* \* 42 24 58 8 \* \* \* \* 38 28 54 12 \* \*  
 \* \* 21 43 5 59 \* \* \* \* 25 39 9 55 \* \*  
 20/O <-- Unmatched --> 3/F  
 10/O <-- Unmatched --> 4/F  
 12/O <-- Unmatched --> 5/F

[Trans. **Type 8**]

10/O <-- Unmatched --> 1/F  
 \* \* \* \* 22 44 17 47 \* \* \* \* 14 52 9 55  
 \* \* \* \* 41 23 46 20 \* \* \* \* 49 15 54 12  
 11 53 16 50 \* \* \* \* 19 45 24 42 \* \* \* \*  
 56 10 51 13 \* \* \* \* 48 18 43 21 \* \* \* \*  
 43 21 48 18 \* \* \* \* 51 13 56 10 \* \* \* \*  
 24 42 19 45 \* \* \* \* 16 50 11 53 \* \* \* \*  
 \* \* \* \* 54 12 49 15 \* \* \* \* 46 20 41 23  
 \* \* \* \* 9 55 14 52 \* \* \* \* 17 47 22 44  
 8/O <-- Unmatched --> 2/F  
 34/O <-- Unmatched --> 7/F  
 32/O <-- Unmatched --> 8/F

[Trans. **Type 9**]

24/O <-- Unmatched --> 5/F  
 \* \* 18 48 50 16 \* \* \* \* 10 56 42 24 \* \*  
 \* \* 45 19 13 51 \* \* \* \* 53 11 21 43 \* \*  
 \* \* 24 42 56 10 \* \* \* \* 16 50 48 18 \* \*  
 \* \* 43 21 11 53 \* \* \* \* 51 13 19 45 \* \*  
 15 49 \* \* \* \* 47 17 23 41 \* \* \* \* 55 9  
 52 14 \* \* \* \* 20 46 44 22 \* \* \* \* 12 54  
 9 55 \* \* \* \* 41 23 17 47 \* \* \* \* 49 15  
 54 12 \* \* \* \* 22 44 46 20 \* \* \* \* 14 52  
 20/O <-- Unmatched --> 6/F

[Trans. **Type 10**]

48/O <-- Unmatched --> 7/F  
 \* \* \* \* 8 58 5 59 \* \* \* \* 12 54 9 55  
 56 10 53 11 \* \* \* \* 60 6 57 7 \* \* \* \*  
 25 39 28 38 \* \* \* \* 21 43 24 42 \* \* \* \*  
 \* \* \* \* 41 23 44 22 \* \* \* \* 37 27 40 26  
 57 7 60 6 \* \* \* \* 53 11 56 10 \* \* \* \*  
 \* \* \* \* 9 55 12 54 \* \* \* \* 5 59 8 58  
 \* \* \* \* 40 26 37 27 \* \* \* \* 44 22 41 23  
 24 42 21 43 \* \* \* \* 28 38 25 39 \* \* \* \*  
 32/O <-- Unmatched --> 10/F

## [Other Types]

```

                60/ <-- Unmatched --> F1/7
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
48 18 43 21 35 29 40 26   62  4 57  7 49 15 54 12
 3 61  8 58 16 50 11 53   17 47 22 44 30 36 25 39
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *
30 36 25 39 17 47 22 44   16 50 11 53  3 61  8 58
49 15 54 12 62  4 57  7   35 29 40 26 48 18 43 21
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *

```

```

                89/ <-- Unmatched --> 1F/4
* 60  2  *  * 55 13  *   * 63  5  *  * 52 10  *
* 21 47  *  * 26 36  *   * 18 44  *  * 29 39  *
* 58  4  *  * 53 15  *   * 61  7  *  * 50 12  *
* 23 45  *  * 28 34  *   * 20 42  *  * 31 37  *
* 10 52  *  *  5 63  *   * 13 55  *  *  2 60  *
* 39 29  *  * 44 18  *   * 36 26  *  * 47 21  *
* 12 50  *  *  7 61  *   * 15 53  *  *  4 58  *
* 37 31  *  * 42 20  *   * 34 28  *  * 45 23  *

```

The first two transformation types **1** and **2** are found common in all groups.

Both Type 1 and Type 2 are doing the same thing in two different directions: left-right and up-downward. You could combine these two types just as: Type 2 after Type 1, or Type 1 after Type 2. You may call it the new Type 3.

Let's take Type 1, 2 and 3 for our transformation process.

In the group of F1, for instance, you could make No. 2, 7 and 1 from No.8(=F1) by these three types of transformations.

But you should look for any other type of transformations for the next step, I suggest, from such children as No.1, 2 and 7 instead of the parent F1=No.8.

After a lot of my 'try-and-err' trials, I found we can always make our job successfully when you use only 6 types of transformations. Let me skip to my conclusion.

Take the last two types above and call them Type 4 and 5, and Type 6 for the combination of 4 & 5.

```

/** Transformation Programs for Making the Most Fundamental Ten ***/
/** of 'C&C' Magic Square 8*8 into the Fundamental 90 Solutions **/
/**/
short cn[65], dn[65];
/**/
/* Sub-Routines */
void trans1(void), trans2(void), trans3(void);
void trans4(void), trans5(void), trans6(void), reflect(void);
/**/
/* Transformation: Type 1 */
void trans1(){
  dn[1]=cn[1]; dn[2]=cn[2]; dn[3]=cn[40]; dn[4]=cn[39];
  dn[5]=cn[5]; dn[6]=cn[6]; dn[7]=cn[36]; dn[8]=cn[35];
  dn[9]=cn[9]; dn[10]=cn[10]; dn[11]=cn[48]; dn[12]=cn[47];
  dn[13]=cn[13]; dn[14]=cn[14]; dn[15]=cn[44]; dn[16]=cn[43];

```

```

dn[17]=cn[17]; dn[18]=cn[18]; dn[19]=cn[56]; dn[20]=cn[55];
  dn[21]=cn[21]; dn[22]=cn[22]; dn[23]=cn[52]; dn[24]=cn[51];
dn[25]=cn[25]; dn[26]=cn[26]; dn[27]=cn[64]; dn[28]=cn[63];
  dn[29]=cn[29]; dn[30]=cn[30]; dn[31]=cn[60]; dn[32]=cn[59];
dn[33]=cn[33]; dn[34]=cn[34]; dn[35]=cn[8]; dn[36]=cn[7];
  dn[37]=cn[37]; dn[38]=cn[38]; dn[39]=cn[4]; dn[40]=cn[3];
dn[41]=cn[41]; dn[42]=cn[42]; dn[43]=cn[16]; dn[44]=cn[15];
  dn[45]=cn[45]; dn[46]=cn[46]; dn[47]=cn[12]; dn[48]=cn[11];
dn[49]=cn[49]; dn[50]=cn[50]; dn[51]=cn[24]; dn[52]=cn[23];
  dn[53]=cn[53]; dn[54]=cn[54]; dn[55]=cn[20]; dn[56]=cn[19];
dn[57]=cn[57]; dn[58]=cn[58]; dn[59]=cn[32]; dn[60]=cn[31];
  dn[61]=cn[61]; dn[62]=cn[62]; dn[63]=cn[28]; dn[64]=cn[27];
}
/**/
/* Transformation: Type 2 */
void trans2(){
  dn[1]=cn[1]; dn[2]=cn[2]; dn[3]=cn[3]; dn[4]=cn[4];
  dn[5]=cn[5]; dn[6]=cn[6]; dn[7]=cn[7]; dn[8]=cn[8];
dn[9]=cn[9]; dn[10]=cn[10]; dn[11]=cn[11]; dn[12]=cn[12];
  dn[13]=cn[13]; dn[14]=cn[14]; dn[15]=cn[15]; dn[16]=cn[16];
dn[17]=cn[61]; dn[18]=cn[62]; dn[19]=cn[63]; dn[20]=cn[64];
  dn[21]=cn[57]; dn[22]=cn[58]; dn[23]=cn[59]; dn[24]=cn[60];
dn[25]=cn[53]; dn[26]=cn[54]; dn[27]=cn[55]; dn[28]=cn[56];
  dn[29]=cn[49]; dn[30]=cn[50]; dn[31]=cn[51]; dn[32]=cn[52];
dn[33]=cn[33]; dn[34]=cn[34]; dn[35]=cn[35]; dn[36]=cn[36];
  dn[37]=cn[37]; dn[38]=cn[38]; dn[39]=cn[39]; dn[40]=cn[40];
dn[41]=cn[41]; dn[42]=cn[42]; dn[43]=cn[43]; dn[44]=cn[44];
  dn[45]=cn[45]; dn[46]=cn[46]; dn[47]=cn[47]; dn[48]=cn[48];
dn[49]=cn[29]; dn[50]=cn[30]; dn[51]=cn[31]; dn[52]=cn[32];
  dn[53]=cn[25]; dn[54]=cn[26]; dn[55]=cn[27]; dn[56]=cn[28];
dn[57]=cn[21]; dn[58]=cn[22]; dn[59]=cn[23]; dn[60]=cn[24];
  dn[61]=cn[17]; dn[62]=cn[18]; dn[63]=cn[19]; dn[64]=cn[20];
}
/**/
/* Transformation: Type 3 = 1 & 2 */
void trans3(){
  short n;
  trans1();
  for(n=1; n<65; n++){ cn[n]=dn[n]; }
  trans2();
}
/**/
/* Transformation: Type 4 */
void trans4(){
  dn[1]=cn[1]; dn[2]=cn[2]; dn[3]=cn[3]; dn[4]=cn[4];
  dn[5]=cn[5]; dn[6]=cn[6]; dn[7]=cn[7]; dn[8]=cn[8];
dn[9]=cn[53]; dn[10]=cn[54]; dn[11]=cn[55]; dn[12]=cn[56];
  dn[13]=cn[49]; dn[14]=cn[50]; dn[15]=cn[51]; dn[16]=cn[52];
dn[17]=cn[45]; dn[18]=cn[46]; dn[19]=cn[47]; dn[20]=cn[48];
  dn[21]=cn[41]; dn[22]=cn[42]; dn[23]=cn[43]; dn[24]=cn[44];
dn[25]=cn[25]; dn[26]=cn[26]; dn[27]=cn[27]; dn[28]=cn[28];
  dn[29]=cn[29]; dn[30]=cn[30]; dn[31]=cn[31]; dn[32]=cn[32];
dn[33]=cn[33]; dn[34]=cn[34]; dn[35]=cn[35]; dn[36]=cn[36];

```

```

    dn[37]=cn[37]; dn[38]=cn[38]; dn[39]=cn[39]; dn[40]=cn[40];
    dn[41]=cn[21]; dn[42]=cn[22]; dn[43]=cn[23]; dn[44]=cn[24];
    dn[45]=cn[17]; dn[46]=cn[18]; dn[47]=cn[19]; dn[48]=cn[20];
    dn[49]=cn[13]; dn[50]=cn[14]; dn[51]=cn[15]; dn[52]=cn[16];
    dn[53]=cn[9]; dn[54]=cn[10]; dn[55]=cn[11]; dn[56]=cn[12];
    dn[57]=cn[57]; dn[58]=cn[58]; dn[59]=cn[59]; dn[60]=cn[60];
    dn[61]=cn[61]; dn[62]=cn[62]; dn[63]=cn[63]; dn[64]=cn[64];
}
/**/
/* Transformation: Type 5 */
void trans5() {
    dn[1]=cn[1]; dn[2]=cn[39]; dn[3]=cn[38]; dn[4]=cn[4];
    dn[5]=cn[5]; dn[6]=cn[35]; dn[7]=cn[34]; dn[8]=cn[8];
    dn[9]=cn[9]; dn[10]=cn[47]; dn[11]=cn[46]; dn[12]=cn[12];
    dn[13]=cn[13]; dn[14]=cn[43]; dn[15]=cn[42]; dn[16]=cn[16];
    dn[17]=cn[17]; dn[18]=cn[55]; dn[19]=cn[54]; dn[20]=cn[20];
    dn[21]=cn[21]; dn[22]=cn[51]; dn[23]=cn[50]; dn[24]=cn[24];
    dn[25]=cn[25]; dn[26]=cn[63]; dn[27]=cn[62]; dn[28]=cn[28];
    dn[29]=cn[29]; dn[30]=cn[59]; dn[31]=cn[58]; dn[32]=cn[32];
    dn[33]=cn[33]; dn[34]=cn[7]; dn[35]=cn[6]; dn[36]=cn[36];
    dn[37]=cn[37]; dn[38]=cn[3]; dn[39]=cn[2]; dn[40]=cn[40];
    dn[41]=cn[41]; dn[42]=cn[15]; dn[43]=cn[14]; dn[44]=cn[44];
    dn[45]=cn[45]; dn[46]=cn[11]; dn[47]=cn[10]; dn[48]=cn[48];
    dn[49]=cn[49]; dn[50]=cn[23]; dn[51]=cn[22]; dn[52]=cn[52];
    dn[53]=cn[53]; dn[54]=cn[19]; dn[55]=cn[18]; dn[56]=cn[56];
    dn[57]=cn[57]; dn[58]=cn[31]; dn[59]=cn[30]; dn[60]=cn[60];
    dn[61]=cn[61]; dn[62]=cn[27]; dn[63]=cn[26]; dn[64]=cn[64];
}
/**/
/* Transformation: Type 6 = 4 & 5 */
void trans6() {
    short n;
    trans4();
    for(n=1; n<65; n++) { cn[n]=dn[n]; }
    trans5();
}
/**/
/* Mirror Reflection */
void reflect() {
    short n;
    for(n=1; n<65; n++) { dn[n]=cn[n]; }
    if(cn[9]>cn[2]) {
        dn[1]=cn[1]; dn[2]=cn[9]; dn[3]=cn[17]; dn[4]=cn[25];
        dn[5]=cn[33]; dn[6]=cn[41]; dn[7]=cn[49]; dn[8]=cn[57];
        dn[9]=cn[2]; dn[10]=cn[10]; dn[11]=cn[18]; dn[12]=cn[26];
        dn[13]=cn[34]; dn[14]=cn[42]; dn[15]=cn[50]; dn[16]=cn[58];
        dn[17]=cn[3]; dn[18]=cn[11]; dn[19]=cn[19]; dn[20]=cn[27];
        dn[21]=cn[35]; dn[22]=cn[43]; dn[23]=cn[51]; dn[24]=cn[59];
        dn[25]=cn[4]; dn[26]=cn[12]; dn[27]=cn[20]; dn[28]=cn[28];
        dn[29]=cn[36]; dn[30]=cn[44]; dn[31]=cn[52]; dn[32]=cn[60];
        dn[33]=cn[5]; dn[34]=cn[13]; dn[35]=cn[21]; dn[36]=cn[29];
        dn[37]=cn[37]; dn[38]=cn[45]; dn[39]=cn[53]; dn[40]=cn[61];
        dn[41]=cn[6]; dn[42]=cn[14]; dn[43]=cn[22]; dn[44]=cn[30];
    }
}

```

```

    dn[45]=cn[38]; dn[46]=cn[46]; dn[47]=cn[54]; dn[48]=cn[62];
dn[49]=cn[7]; dn[50]=cn[15]; dn[51]=cn[23]; dn[52]=cn[31];
    dn[53]=cn[39]; dn[54]=cn[47]; dn[55]=cn[55]; dn[56]=cn[63];
dn[57]=cn[8]; dn[58]=cn[16]; dn[59]=cn[24]; dn[60]=cn[32];
    dn[61]=cn[40]; dn[62]=cn[48]; dn[63]=cn[56]; dn[64]=cn[64];
}
}
/**/

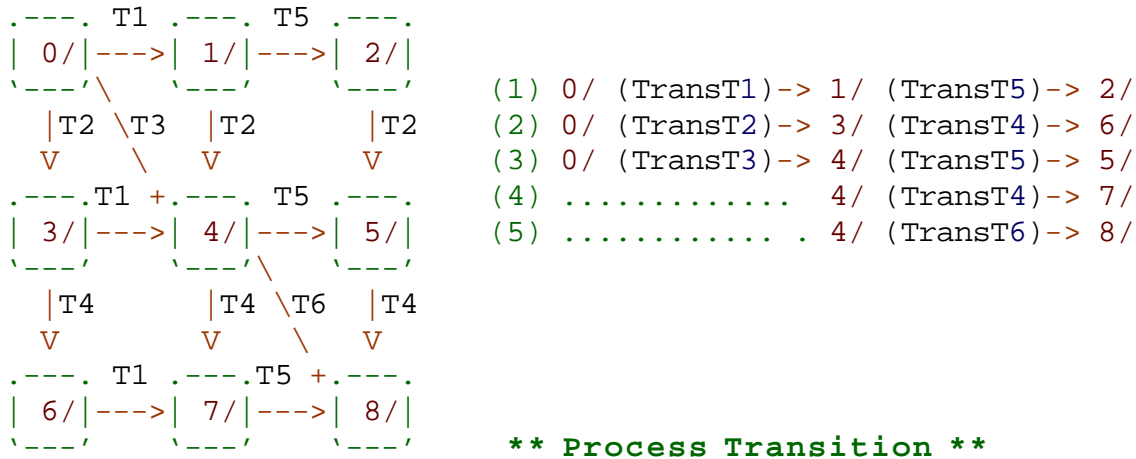
```

The last procedure 'reflect' is taken up for the purpose of normalizing answer forms.

**7. How do you use these 6 types of transformations for our job?**  
 See the diagram of 'Process Transition'.

**\*\* How to Transform the Original into 8 other Solutions \*\***

**Origin**



We now know how to make the original one into 8 other different objects. Repeat the same job of applying to every Fundamental Ten one after another. And you will surely have got all the 90 object solutions including F10 themselves.

The next list demonstrates the result of my recent experiment for making 10 into 90. Please check if the reconstruction list below is really complete and correct.

**\*\*\* Transform the Most Fundamental Ten of \*\*\***  
**\*\* 'Composite & Complete' Magic Squares 8x8 \*\***  
**\*\*\* into the 90 Fundamental Solutions \*\*\***

[No. 1]	0/ 8	1/ 2	2/72
1 63 6 60 14 52 9 55	1 63 5 59 14 52 10 56	1 62 19 48 51 16 33 30	
62 4 57 7 49 15 54 12	62 4 58 8 49 15 53 11	60 7 42 21 10 53 28 39	
19 45 24 42 32 34 27 37	19 45 23 41 32 34 28 38	2 61 20 47 52 15 34 29	
48 18 43 21 35 29 40 26	48 18 44 22 35 29 39 25	59 8 41 22 9 54 27 40	
51 13 56 10 64 2 59 5	51 13 55 9 64 2 60 6	14 49 32 35 64 3 46 17	
16 50 11 53 3 61 8 58	16 50 12 54 3 61 7 57	55 12 37 26 5 58 23 44	
33 31 38 28 46 20 41 23	33 31 37 27 46 20 42 24	13 50 31 36 63 4 45 18	
30 36 25 39 17 47 22 44	30 36 26 40 17 47 21 43	56 11 38 25 6 57 24 43	

	3/ 7		4/ 1		5/71
1 63 6 60 14 52 9 55	1 63 5 59 14 52 10 56	1 62 17 46 51 16 35 32			
62 4 57 7 49 15 54 12	62 4 58 8 49 15 53 11	60 7 44 23 10 53 26 37			
17 47 22 44 30 36 25 39	17 47 21 43 30 36 26 40	2 61 18 45 52 15 36 31			
46 20 41 23 33 31 38 28	46 20 42 24 33 31 37 27	59 8 43 24 9 54 25 38			
51 13 56 10 64 2 59 5	51 13 55 9 64 2 60 6	14 49 30 33 64 3 48 19			
16 50 11 53 3 61 8 58	16 50 12 54 3 61 7 57	55 12 39 28 5 58 21 42			
35 29 40 26 48 18 43 21	35 29 39 25 48 18 44 22	13 50 29 34 63 4 47 20			
32 34 27 37 19 45 24 42	32 34 28 38 19 45 23 41	56 11 40 27 6 57 22 41			

	6/60		7/59		8/89
1 63 6 60 14 52 9 55	1 63 5 59 14 52 10 56	1 60 2 59 14 55 13 56			
48 18 43 21 35 29 40 26	48 18 44 22 35 29 39 25	48 21 47 22 35 26 36 25			
3 61 8 58 16 50 11 53	3 61 7 57 16 50 12 54	3 58 4 57 16 53 15 54			
46 20 41 23 33 31 38 28	46 20 42 24 33 31 37 27	46 23 45 24 33 28 34 27			
51 13 56 10 64 2 59 5	51 13 55 9 64 2 60 6	51 10 52 9 64 5 63 6			
30 36 25 39 17 47 22 44	30 36 26 40 17 47 21 43	30 39 29 40 17 44 18 43			
49 15 54 12 62 4 57 7	49 15 53 11 62 4 58 8	49 12 50 11 62 7 61 8			
32 34 27 37 19 45 24 42	32 34 28 38 19 45 23 41	32 37 31 38 19 42 20 41			

[No. 2]	0/10		1/ 4		2/70
1 63 6 60 22 44 17 47	1 63 5 59 22 44 18 48	1 62 11 56 43 24 33 30			
62 4 57 7 41 23 46 20	62 4 58 8 41 23 45 19	60 7 50 13 18 45 28 39			
11 53 16 50 32 34 27 37	11 53 15 49 32 34 28 38	2 61 12 55 44 23 34 29			
56 10 51 13 35 29 40 26	56 10 52 14 35 29 39 25	59 8 49 14 17 46 27 40			
43 21 48 18 64 2 59 5	43 21 47 17 64 2 60 6	22 41 32 35 64 3 54 9			
24 42 19 45 3 61 8 58	24 42 20 46 3 61 7 57	47 20 37 26 5 58 15 52			
33 31 38 28 54 12 49 15	33 31 37 27 54 12 50 16	21 42 31 36 63 4 53 10			
30 36 25 39 9 55 14 52	30 36 26 40 9 55 13 51	48 19 38 25 6 57 16 51			

	3/ 9		4/ 3		5/68
1 63 6 60 22 44 17 47	1 63 5 59 22 44 18 48	1 62 9 54 43 24 35 32			
62 4 57 7 41 23 46 20	62 4 58 8 41 23 45 19	60 7 52 15 18 45 26 37			
9 55 14 52 30 36 25 39	9 55 13 51 30 36 26 40	2 61 10 53 44 23 36 31			
54 12 49 15 33 31 38 28	54 12 50 16 33 31 37 27	59 8 51 16 17 46 25 38			
43 21 48 18 64 2 59 5	43 21 47 17 64 2 60 6	22 41 30 33 64 3 56 11			
24 42 19 45 3 61 8 58	24 42 20 46 3 61 7 57	47 20 39 28 5 58 13 50			
35 29 40 26 56 10 51 13	35 29 39 25 56 10 52 14	21 42 29 34 63 4 55 12			
32 34 27 37 11 53 16 50	32 34 28 38 11 53 15 49	48 19 40 27 6 57 14 49			

	6/53		7/51		8/85
1 63 6 60 22 44 17 47	1 63 5 59 22 44 18 48	1 60 2 59 22 47 21 48			
56 10 51 13 35 29 40 26	56 10 52 14 35 29 39 25	56 13 55 14 35 26 36 25			
3 61 8 58 24 42 19 45	3 61 7 57 24 42 20 46	3 58 4 57 24 45 23 46			
54 12 49 15 33 31 38 28	54 12 50 16 33 31 37 27	54 15 53 16 33 28 34 27			
43 21 48 18 64 2 59 5	43 21 47 17 64 2 60 6	43 18 44 17 64 5 63 6			
30 36 25 39 9 55 14 52	30 36 26 40 9 55 13 51	30 39 29 40 9 52 10 51			
41 23 46 20 62 4 57 7	41 23 45 19 62 4 58 8	41 20 42 19 62 7 61 8			
32 34 27 37 11 53 16 50	32 34 28 38 11 53 15 49	32 37 31 38 11 50 12 49			

. . . . . (If you want the full list, [please click here.](#))

[No.10]	0/48		1/44		2/74
1 63 4 62 8 58 5 59	1 63 3 61 8 58 6 60	1 62 2 61 8 59 7 60			
56 10 53 11 49 15 52 14	56 10 54 12 49 15 51 13	56 11 55 12 49 14 50 13			
25 39 28 38 32 34 29 35	25 39 27 37 32 34 30 36	25 38 26 37 32 35 31 36			
48 18 45 19 41 23 44 22	48 18 46 20 41 23 43 21	48 19 47 20 41 22 42 21			
57 7 60 6 64 2 61 3	57 7 59 5 64 2 62 4	57 6 58 5 64 3 63 4			
16 50 13 51 9 55 12 54	16 50 14 52 9 55 11 53	16 51 15 52 9 54 10 53			
33 31 36 30 40 26 37 27	33 31 35 29 40 26 38 28	33 30 34 29 40 27 39 28			
24 42 21 43 17 47 20 46	24 42 22 44 17 47 19 45	24 43 23 44 17 46 18 45			

3/47								4/43								5/73							
1	63	4	62	8	58	5	59	1	63	3	61	8	58	6	60	1	62	2	61	8	59	7	60
56	10	53	11	49	15	52	14	56	10	54	12	49	15	51	13	56	11	55	12	49	14	50	13
17	47	20	46	24	42	21	43	17	47	19	45	24	42	22	44	17	46	18	45	24	43	23	44
40	26	37	27	33	31	36	30	40	26	38	28	33	31	35	29	40	27	39	28	33	30	34	29
57	7	60	6	64	2	61	3	57	7	59	5	64	2	62	4	57	6	58	5	64	3	63	4
16	50	13	51	9	55	12	54	16	50	14	52	9	55	11	53	16	51	15	52	9	54	10	53
41	23	44	22	48	18	45	19	41	23	43	21	48	18	46	20	41	22	42	21	48	19	47	20
32	34	29	35	25	39	28	38	32	34	30	36	25	39	27	37	32	35	31	36	25	38	26	37
6/57								7/55								8/81							
1	63	4	62	8	58	5	59	1	63	3	61	8	58	6	60	1	62	2	61	8	59	7	60
48	18	45	19	41	23	44	22	48	18	46	20	41	23	43	21	48	19	47	20	41	22	42	21
9	55	12	54	16	50	13	51	9	55	11	53	16	50	14	52	9	54	10	53	16	51	15	52
40	26	37	27	33	31	36	30	40	26	38	28	33	31	35	29	40	27	39	28	33	30	34	29
57	7	60	6	64	2	61	3	57	7	59	5	64	2	62	4	57	6	58	5	64	3	63	4
24	42	21	43	17	47	20	46	24	42	22	44	17	47	19	45	24	43	23	44	17	46	18	45
49	15	52	14	56	10	53	11	49	15	51	13	56	10	54	12	49	14	50	13	56	11	55	12
32	34	29	35	25	39	28	38	32	34	30	36	25	39	27	37	32	35	31	36	25	38	26	37

[Count = 9 x 10]

You now know we could make F10 into all 90 solutions by our simple transformation process. We can say loudly that those Fundamental 10 are really 'fundamental' of all.

When I have got the diagram of 'Transition', I feel we could make a certain contact to the beautiful structure in the solution set of C&C magic squares of order 8. We have just come in to the real entrance for the fundamental study of magic forms.

**(Revised Edition written on December 24, 2002 by Kanji Setsuda)**

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