

Chapter 6: Fundamental Studies of Multi-Dimensional Extra-Cubic Magic Forms and their Decompositions: Kanji Setsuda

Section 3-3: The Most Fundamental Solution and Transformation Process of 'C&C' Magic Cubes of order 4

#1. In the former sections we studied about 6-dimensional extra-cubic magic forms of order 2 and their decompositions. As a result we have obtained 15 fundamental solutions of 'Composite and Complete' Magic Cubes of order 4 and 10 fundamental solutions of 'C&C' Magic Squares of order 8 by dimension-converting from the origin.

We once studied about such inner transformations as we could get 90 more solutions of C&P Magic Cubes of order 4 from the fundamental 15 (See #11 of this article), and in the last section we could get 80 more solutions of C&C Magic Squares of order 8 from the fundamental 10.

No doubt we have succeeded in finding out the true 'fundamental solutions' of all.

But I am not yet satisfied with it. My intellectual request is growing wilder.

Couldn't we find the most 'fundamental solution', say, the only one of all? Couldn't we invent any transformation for the only one Fundamental Solution making into all 15 solutions of 'C&C' Magic Cubes $4 \times 4 \times 4$ and 10 solutions of C&C Magic Squares 8×8 ?

#2. This request arises from the discovery of only one solution of the 6-dimensional extra-cubic magic form of order 2.

In the 6-dimensional world there are 6 axes which meet on the same origin $n_1(0,0,0,0,0,0)$, and there are 6 positions next to n_1 : $n_2(0,0,0,0,0,1)$, $n_3(0,0,0,0,1,0)$, $n_5(0,0,0,1,0,0)$, $n_9(0,0,1,0,0,0)$, $n_{17}(0,1,0,0,0,0)$, and $n_{33}(1,0,0,0,0,0)$.

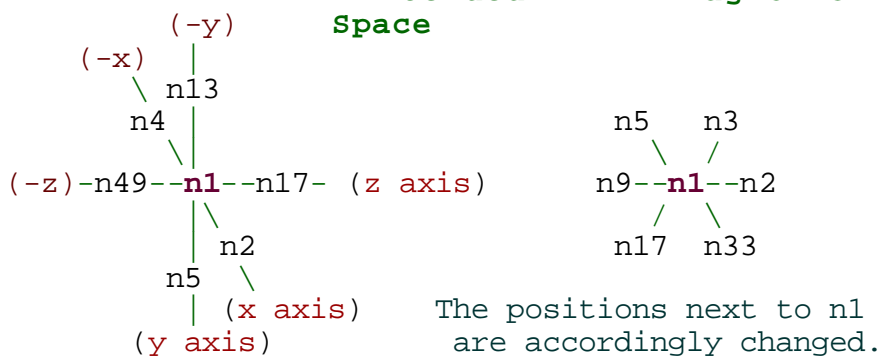
We can see no essential difference among those six positions. We can find neither any rank nor any determined order among them in advance.

In our composite object each one of these six positions could only take any one of the values {63, 62, 60, 56, 48, 32} and nothing else, as we have found before.

There is only one meaning for the fact. I say, I mean it. The six positions { n_2 , n_3 , n_5 , n_9 , n_{17} , n_{33} } next to n_1 could only take those six values {63, 62, 60, 56, 48, 32}. The permutations of both positions and values are essentially meaningless in the 6-dimensional world. They are identified with the representative one after all.

#3. But each solution of C&C Magic Cube of order 4 obtains a different face from others in the 3-dimensional world we live in. When we convert the dimensions from 6 down to 3, there are only 3 axes and the 6 positions next to n_1 must sit coupled on both sides of n_1 .

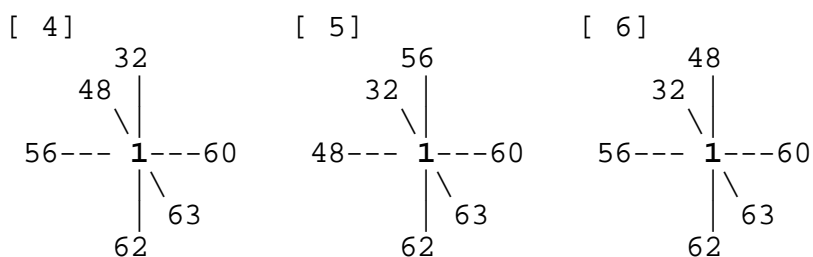
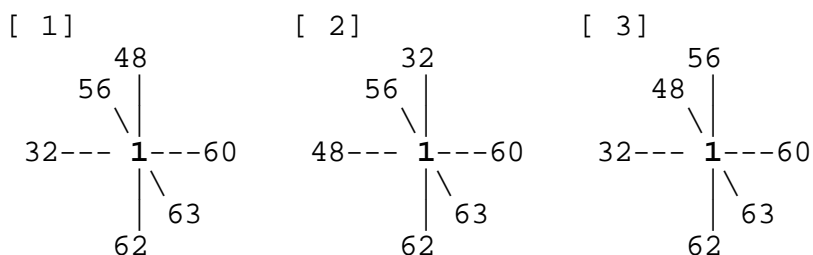
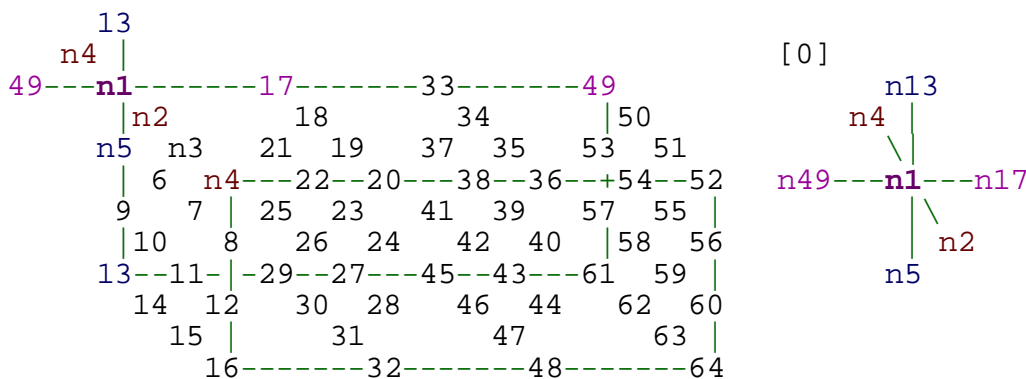
3-dimensional magic cube <-> 6-dimensional
in Extended Space magic form

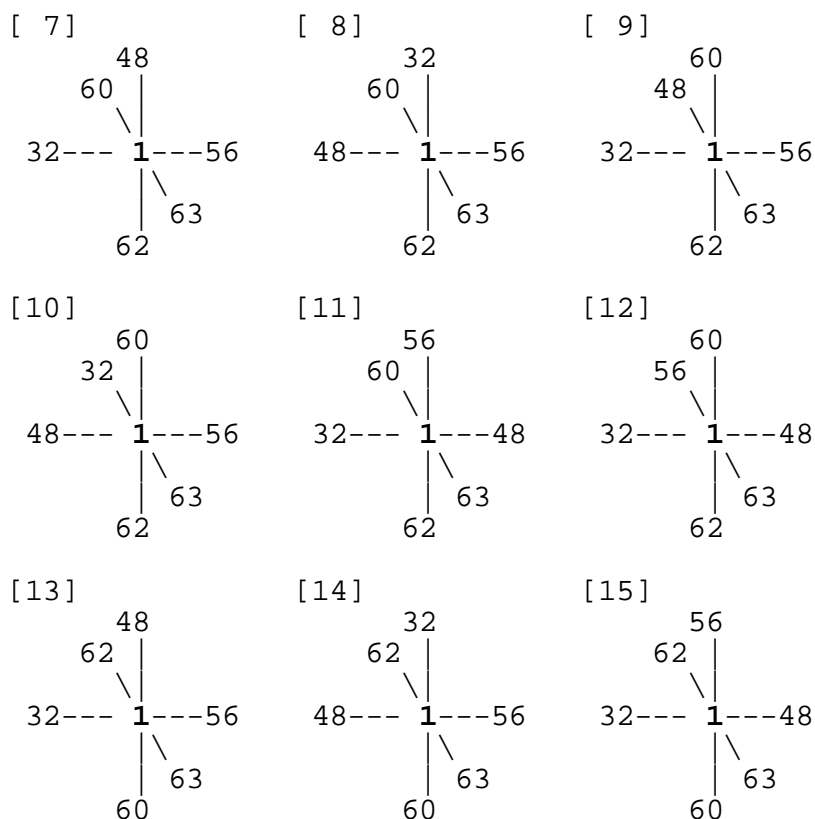


As we can see this visible object, we now notice the different character of each and count them one by one separately.

We know there are 15 fundamental solutions of our magic cubes. It is because there are 15 different ways of making combinations of six positions and six values, and because there is only one way of rearranging all the rest. As a result this count 15 becomes equal to the count of our solutions.

*** Next Numbers Adjacent to n1 ***
** Composite & Complete Magic Cubes 4x4x4: **
*** Standard Type: n1=1; n43=64 ***





[Count = 15]

**** Composite & Complete Magic Cubes 4x4x4: ****
***** Standard Type: n1=1; n43=64 *****

<p>1/ 1-----60-----37-----32 63 6 27 34 62 10 7 51 26 46 35 23 4 56---57--13---40--20--+29--41 19 53 42 16 55 17 14 44 45 11 24 50 9 47 52 22 48--28- -21--33---12--64---49 5 18 38 43 31 54 2 15 59 39 30 3 58 25-----36-----61----- 8</p>	<p>2/ 1-----60-----21-----48 63 6 43 18 62 10 7 51 42 30 19 39 4 56---57--13---24--36--+45--25 35 53 26 16 55 33 14 28 29 11 40 50 9 31 52 38 32--44- -37--17---12--64---49 5 34 22 27 47 54 2 15 59 23 46 3 58 41-----20-----61----- 8</p>	<p>3/ 1-----60-----37-----32 63 6 27 34 62 18 7 43 26 54 35 15 4 48---57--21---40--12--+29--49 11 45 50 24 47 9 22 52 53 19 16 42 17 55 44 14 56--28- -13--33---20--64---41 5 10 38 51 31 46 2 23 59 39 30 3 58 25-----36-----61----- 8</p>
<p>4/ 1-----60-----13-----56 63 6 51 10 62 18 7 43 50 30 11 39 4 48---57--21---16--36--+53--25 35 45 26 24 47 33 22 28 29 19 40 42 17 31 44 38 32--52- -37-- 9---20--64---41 5 34 14 27 55 46 2 23 59 15 54 3 58 49-----12-----61----- 8</p>	<p>5/ 1-----60-----21-----48 63 6 43 18 62 34 7 27 42 54 19 15 4 32---57--37---24--12--+45--49 11 29 50 40 31 9 38 52 53 35 16 26 33 55 28 14 56--44- -13--17---36--64---25 5 10 22 51 47 30 2 39 59 23 46 3 58 41-----20-----61----- 8</p>	<p>6/ 1-----60-----13-----56 63 6 51 10 62 34 7 27 50 46 11 23 4 32---57--37---16--20--+53--41 19 29 42 40 31 17 38 44 45 35 24 26 33 47 28 22 48--52- -21-- 9---36--64---25 5 18 14 43 55 30 2 39 59 15 54 3 58 49-----12-----61----- 8</p>
<p>7/ 1-----56-----41-----32 63 10 23 34 62 6 11 51 22 46 35 27 4 60---53--13---44--20--+29--37 19 57 38 16 59 17 14 40 45 7 28 50 5 47 52 26 48--24- -25--33--- 8--64---49 9 18 42 39 31 58 2 15 55 43 30 3 54 21-----36-----61-----12</p>	<p>8/ 1-----56-----25-----48 63 10 39 18 62 6 11 51 38 30 19 43 4 60---53--13---28--36--+45--21 35 57 22 16 59 33 14 24 29 7 44 50 5 31 52 42 32--40- -41--17--- 8--64---49 9 34 26 23 47 58 2 15 55 27 46 3 54 37-----20-----61-----12</p>	<p>9/ 1-----56-----41-----32 63 10 23 34 62 18 11 39 22 58 35 15 4 48---53--25---44-- 8--+29--49 7 45 50 28 47 5 26 52 57 19 16 38 17 59 40 14 60--24- -13--33---20--64---37 9 6 42 51 31 46 2 27 55 43 30 3 54 21-----36-----61-----12</p>

```

10/          11/          12/
1-----56-----25-----48      1-----48-----49-----32      1-----48-----49-----32
|63      10      39      |18      |63      18      15      |34      |63      18      15      |34
62 34 11 23 38 58 19 15      62 6 19 43 14 54 35 27      62 10 19 39 14 58 35 23
| 4 32---53---41---28-- 8--+45---49      | 4 60---45---21---52---12---+29---37      | 4 56---45---25---52-- 8--+29---41
7 29 | 50 44 31 5 42 52 | 11 57 | 38 24 59 9 22 40 | 7 53 | 42 28 55 5 26 44 |
|57 35 16 22 33 59 |24 14      |53 7 28 42 5 55 |44 26      |57 11 24 38 9 59 |40 22
60--40-|-13--17---36--64---21 9 | 56--16-|-25--33--- 8--64---41 17 | 60--16-|-21--33---12--64---37 17 |
6 26 51 47 30 2 43 55      10 50 39 31 58 2 23 47      6 50 43 31 54 2 27 47
27 | 46      3      54 | 51 | 30      3      46 | 51 | 30      3      46 |
37-----20-----61-----12      13-----36-----61-----20      13-----36-----61-----20

13/          14/          15/
1-----56-----41-----32      1-----56-----25-----48      1-----48-----49-----32
|63      10      23      |34      |63      10      39      |18      |63      18      15      |34
60 4 13 53 20 44 37 29      60 4 13 53 36 28 21 45      60 4 21 45 12 52 37 29
| 6 62---51---11---46--22--+27--35      | 6 62---51---11---30--38---+43--19      | 6 62---43---19---54--14---+27--35
21 57 | 36 16 61 17 12 40 | 37 57 | 20 16 61 33 12 24 | 13 57 | 36 24 61 9 20 40 |
|43 7 30 50 3 47 |54 26      |27 7 46 50 3 31 |54 42      |51 7 30 42 3 55 |46 26
48--24-|-25--33--- 8--64---49 9 | 32--40-|-41--17--- 8--64---49 9 | 56--16-|-25--33--- 8--64---41 17 |
18 42 39 31 58 2 15 55      34 26 23 47 58 2 15 55      10 50 39 31 58 2 23 47
45 | 28      5      52 | 29 | 44      5      52 | 53 | 28      5      44 |
19-----38-----59-----14      35-----22-----59-----14      11-----38-----59-----22

```

[Count = 15]

These 15 solutions shown above are originally taken out of the 6-dimensional extra cubic form of order 2. The list order of solutions is now changed a little bit, so that you could easily find the inner transformations.

#4. I want to invent some kinds of 'inner transformation' process making from one into another. The goal is the only one 'Most Fundamental,' if possible. Could I do it?

Since our 15 solutions are taken out of the same origin, I suppose they must naturally have their common memory of 'brothers and sisters.'

First of all let's take any two similar solutions out of the 15 fundamental list and compare them precisely.

- (1) Find and collect similar pairs with half of all elements are the same and common.
- (2) Collect similar pairs whose 'unmatched patterns' are the same and common.
- (3) Find any simple common transformation method among each group.
- (4) Invent any Transformation Process for making the One into 15 solutions.

```

**** C&C Magic Cubes 4x4x4 ****
*** Find and collect similar pairs ***

```

```

2/0          <-- Difference -->          1/F
*----- *-----21-----48          *----- *-----37-----32
| *      *      43      |18          | *      *      27      |34
* *      * *      42 30 19 39          * *      * *      26 46 35 23
| *      *-- *-- *--24--36--+45--25      | *      *-- *-- *--40--20--+29--41
35 * | 26 *      * 33 * 28 | 19 * | 42 *      * 17 * 44 |
|29 * 40 *      * 31 | * 38 |45 * 24 *      * 47 | * 22
32--44-|-37--17--- *-- *-- * * | 48--28-|-21--33--- *-- *-- * * |
34 22 27 47 * *      * * | 18 38 43 31 * *      * * |
23 | 46      *      * | 39 | 30      *      * |
41-----20----- *----- *          25-----36----- *----- *

```

3/O <-- Difference -->

```

*-----*-----*-----*
| *           *           *           | *
* 18      * 43      * 54      * 15
| * 48--- *--21--- *--12---+ *--49
11 45 | 50 24 47 9 22 52 |
|53 19 16 42 17 55 |44 14
56-- *--|-13-- *--20-- *--41 * |
10 * 51 * 46 * 23 *
* | * * * * |
*-----*-----*-----*

```

1/F

```

*-----*-----*-----*
| *           *           *           | *
* 10      * 51      * 46      * 23
| * 56--- *--13--- *--20---+ *--41
19 53 | 42 16 55 17 14 44 |
|45 11 24 50 9 47 |52 22
48-- *--|-21-- *--12-- *--49 * |
18 * 43 * 54 * 15 *
* | * * * * |
*-----*-----*-----*

```

6/O <-- Difference -->

```

*-----*-----13-----56
| *           *           51           |10
* 34      * 27 50 * 11 *
| * 32--- *--37---16-- *--+53-- *
* 29 | * 40 31 * 38 * |
| * 35      * 26 33 * |28 *
*--52-|- *-- 9--36-- *--25 * |
* 14      * 55 30 * 39 *
15 | 54 * * |
49-----12----- *----- *

```

1/F

```

*-----*-----37-----32
| *           *           27           |34
* 10      * 51 26 * 35 *
| * 56--- *--13---40-- *--+29-- *
* 53 | * 16 55 * 14 * |
| * 11      * 50 9 * |52 *
*--28-|- *--33---12-- *--49 * |
* 38      * 31 54 * 15 *
39 | 30 * * |
25-----36----- *----- *

```

7/O <-- Difference -->

```

*-----56-----41----- *
| *           10           23           | *
* 6 11 * 22 * * 27
| * 60---53-- *--44-- *--+ *--37
* 57 | 38 * 59 * * 40 |
| * 7 28 * 5 * * | * 26
*--24-|-25-- *-- 8-- *-- * 9 |
* 42 39 * 58 * * 55
43 | * * 54 |
21----- *----- *-----12

```

1/F

```

*-----60-----37----- *
| *           6           27           | *
* 10 7 * 26 * * 23
| * 56---57-- *--40-- *--+ *--41
* 53 | 42 * 55 * * 44 |
| * 11 24 * 9 * * | * 22
*--28-|-21-- *--12-- *-- * 5 |
* 38 43 * 54 * * 59
39 | * * 58 |
25----- *----- *----- 8

```

12/O <-- Difference -->

```

*-----48-----49----- *
| *           18           15           | *
* * 19 39 14 58 * *
| * *--45--25--52-- 8--+ *-- *
7 * | * 28 * 5 26 * |
|57 * * 38 * 59 |40 *
60--16-|- *-- *-- *-- *--37 17 |
6 50 * * * * 27 47
51 | * * 46 |
13----- *----- *-----20

```

1/F

```

*-----60-----37----- *
| *           6           27           | *
* * 7 51 26 46 * *
| * *--57--13---40--20--+ *-- *
19 * | * 16 * 17 14 * |
|45 * * 50 * 47 |52 *
48--28-|- *-- *-- *-- *--49 5 |
18 38 * * * * 15 59
39 | * * 58 |
25----- *----- *----- 8

```

4/O <-- Difference -->

```

*-----*-----13-----56
| *           *           51           |10
* 18      * 43 50 * 11 *
| * 48--- *--21---16-- *--+53-- *
* 45 | * 24 47 * 22 * |
| * 19      * 42 17 * |44 *
*--52-|- *-- 9--20-- *--41 * |
* 14      * 55 46 * 23 *
15 | 54 * * |
49-----12----- *----- *

```

2/F

```

*-----*-----21-----48
| *           *           43           |18
* 10      * 51 42 * 19 *
| * 56--- *--13---24-- *--+45-- *
* 53 | * 16 55 * 14 * |
| * 11      * 50 9 * * |52 *
*--44-|- *--17---12-- *--49 * |
* 22      * 47 54 * 15 *
23 | 46 * * |
41-----20----- *----- *

```

5/O <-- Difference -->

```

*-----*-----*-----*
| *      *      *      | *
* 34    * 27    * 54    * 15
| * 32---*--37---*--12---+*--49
11 29 | 50 40 31 9 38 52 |
|53 35 16 26 33 55 |28 14
56--*-|-13--*---36--*---25 * |
10 * 51 * 30 * 39 *
* | * * * * |
*-----*-----*-----*

```

2/F

```

*-----*-----*-----*
| *      *      *      | *
* 10    * 51    * 30    * 39
| * 56---*--13---*--36---+*--25
35 53 | 26 16 55 33 14 28 |
|29 11 40 50 9 31 |52 38
32--*-|-37--*---12--*---49 * |
34 * 27 * 54 * 15 *
* | * * * * |
*-----*-----*-----*

```

8/O <-- Difference -->

```

*-----56-----25-----*
| *      10      39      | *
* 6 11 * 38 * * 43
| * 60---53--*---28--*---+*--21
* 57 | 22 * 59 * * 24 |
| * 7 44 * 5 * * | * 42
*--40-|-41--*--- 8--*--- * 9 |
* 26 23 * 58 * * 55
27 | * * * 54 |
37-----*-----*-----12

```

2/F

```

*-----60-----21-----*
| *      6      43      | *
* 10 7 * 42 * * 39
| * 56---57--*---24--*---+*--25
* 53 | 26 * 55 * * 28 |
| * 11 40 * 9 * * | * 38
*--44-|-37--*---12--*--- * 5 |
* 22 27 * 54 * * 59
23 | * * * 58 |
41-----*-----*----- 8

```

4/O <-- Difference -->

```

*-----*-----13-----56
| *      *      51      |10
* * * * 50 30 11 39
| * *---*--*---16--36---+53--25
35 * | 26 * * 33 * 28 |
|29 * 40 * * 31 | * 38
32--52-|-37-- 9---*--*--- * * |
34 14 27 55 * * * *
15 | 54 * * * |
49-----12-----*-----*

```

3/F

```

*-----*-----37-----32
| *      *      27      |34
* * * * 26 54 35 15
| * *---*--*---40--12---+29--49
11 * | 50 * * 9 * 52 |
|53 * 16 * * 55 | * 14
56--28-|-13--33---*--*--- * * |
10 38 51 31 * * * *
39 | 30 * * * * |
25-----36-----*-----*

```

5/O <-- Difference -->

```

*-----*-----21-----48
| *      *      43      |18
* 34 * 27 42 * 19 *
| * 32---*--37---24--*---+45--*
* 29 | * 40 31 * 38 * |
| * 35 * 26 33 * |28 *
*--44-|-*--17---36--*---25 * |
* 22 * 47 30 * 39 *
23 | 46 * * * |
41-----20-----*-----*

```

3/F

```

*-----*-----37-----32
| *      *      27      |34
* 18 * 43 26 * 35 *
| * 48---*--21---40--*---+29--*
* 45 | * 24 47 * 22 * |
| * 19 * 42 17 * |44 *
*--28-|-*--33---20--*---41 * |
* 38 * 31 46 * 23 *
39 | 30 * * * * |
25-----36-----*-----*

```

9/O <-- Difference -->

```

*-----56-----41-----*
| *      10      23      | *
* * 11 39 22 58 * *
| * *---53--25---44-- 8---+*--*
7 * | * 28 * 5 26 * |
|57 * * 38 * 59 |40 *
60--24-|-*--*---*--*---37 9 |
6 42 * * * * 27 55
43 | * * * 54 |
21-----*-----*-----12

```

3/F

```

*-----60-----37-----*
| *      6      27      | *
* * 7 43 26 54 * *
| * *---57--21---40--12---+*--*
11 * | * 24 * 9 22 * |
|53 * * 42 * 55 |44 *
56--28-|-*--*---*--*---41 5 |
10 38 * * * * 23 59
39 | * * * * 58 |
25-----*-----*----- 8

```

```

11/O          <-- Difference -->          3/F
*-----48-----49----- *          *-----60-----37----- *
| *      18      15      | *          | *      6      27      | *
*  6  19  *  14  *  *  27          *  18  7  *  26  *  *  15
| *  60---45-- *---52-- *---+ *---37          | *  48---57-- *---40-- *---+ *---49
*  57 | 38  *  59  *  *  40 |          *  45 | 50  *  47  *  *  52 |
| *  7  28  *  5  *  | *  26          | *  19  16  *  17  *  | *  14
*--16-|-25-- *--- 8-- *--- *  17 |          *--28-|-13-- *---20-- *--- *  5 |
*  50  39  *  58  *  *  47          *  38  51  *  46  *  *  59
  51 |      *      *      46 |          39 |      *      *      58 |
  13----- *----- *-----20          25----- *----- *----- 8

```

..... (I found 27 similar pairs in all.)

5. What you have to do next is to collect any similar pairs whose 'Unmatched' patterns are the same and to classify them into 6 groups as follows.

```

**** C&C Magic Cubes 4x4x4 ****
* Similar pairs whose 'Unmatched Patterns' are the same *

```

Group of Type 1:

```

3/O          <-- Difference -->          1/F
*----- *----- *----- *          *----- *----- *----- *
| *      *      *      | *          | *      *      *      | *
*  18  *  43  *  54  *  15          *  10  *  51  *  46  *  23
| *  48--- *--21--- *--12---+ *--49          | *  56--- *--13--- *--20---+ *--41
11 45 | 50 24 47  9  22  52 |          19 53 | 42 16 55 17 14 44 |
| 53 19 16 42 17 55 | 44 14          | 45 11 24 50 9 47 | 52 22
56-- *-|-13-- *---20-- *---41  * |          48-- *-|-21-- *---12-- *---49  * |
  10  *  51  *  46  *  23  *          18  *  43  *  54  *  15  *
  *  |      *      *      *      |          *  |      *      *      *      |
  *----- *----- *----- *          *----- *----- *----- *

```

```

5/O          <-- Difference -->          2/F
*----- *----- *----- *          *----- *----- *----- *
| *      *      *      | *          | *      *      *      | *
*  34  *  27  *  54  *  15          *  10  *  51  *  30  *  39
| *  32--- *--37--- *--12---+ *--49          | *  56--- *--13--- *--36---+ *--25
11 29 | 50 40 31  9  38  52 |          35 53 | 26 16 55 33 14 28 |
| 53 35 16 26 33 55 | 28 14          | 29 11 40 50 9 31 | 52 38
56-- *-|-13-- *---36-- *---25  * |          32-- *-|-37-- *---12-- *---49  * |
  10  *  51  *  30  *  39  *          34  *  27  *  54  *  15  *
  *  |      *      *      *      |          *  |      *      *      *      |
  *----- *----- *----- *          *----- *----- *----- *

```

```

6/O          <-- Difference -->          4/F
*----- *----- *----- *          *----- *----- *----- *
| *      *      *      | *          | *      *      *      | *
*  34  *  27  *  46  *  23          *  18  *  43  *  30  *  39
| *  32--- *--37--- *--20---+ *--41          | *  48--- *--21--- *--36---+ *--25
19 29 | 42 40 31 17 38 44 |          35 45 | 26 24 47 33 22 28 |
| 45 35 24 26 33 47 | 28 22          | 29 19 40 42 17 31 | 44 38
48-- *-|-21-- *---36-- *---25  * |          32-- *-|-37-- *---20-- *---41  * |
  18  *  43  *  30  *  39  *          34  *  27  *  46  *  23  *
  *  |      *      *      *      |          *  |      *      *      *      |
  *----- *----- *----- *          *----- *----- *----- *

```

```

9/O          <-- Difference -->          7/F
10/O          <-- Difference -->          8/F
12/O          <-- Difference -->          11/F

```

Group of **Type 2:**

2/0 <-- Difference -->
 ----------21-----48
 | * * * * 43 |18
 * * * * 42 30 19 39
 | * *---*---*---24--36--45--25
 35 * | 26 * * 33 * 28 |
 |29 * 40 * * 31 | * 38
 32--44-|-37--17---*---*---* * |
 34 22 27 47 * * * *
 23 | 46 * * * |
 41-----20-----*-----*

1/F
 ----------37-----32
 | * * * * 27 |34
 * * * * 26 46 35 23
 | * *---*---*---40--20--29--41
 19 * | 42 * * 17 * 44 |
 |45 * 24 * * 47 | * 22
 48--28-|-21--33---*---*---* * |
 18 38 43 31 * * * *
 39 | 30 * * * * |
 25-----36-----*-----*

4/0 <-- Difference -->
 ----------13-----56
 | * * * * 51 |10
 * * * * 50 30 11 39
 | * *---*---*---16--36--53--25
 35 * | 26 * * 33 * 28 |
 |29 * 40 * * 31 | * 38
 32--52-|-37-- 9---*---*---* * |
 34 14 27 55 * * * *
 15 | 54 * * * * |
 49-----12-----*-----*

3/F
 ----------37-----32
 | * * * * 27 |34
 * * * * 26 54 35 15
 | * *---*---*---40--12--29--49
 11 * | 50 * * 9 * 52 |
 |53 * 16 * * 55 | * 14
 56--28-|-13--33---*---*---* * |
 10 38 51 31 * * * *
 39 | 30 * * * * |
 25-----36-----*-----*

6/0 <-- Difference -->
 8/0 <-- Difference -->
 14/0 <-- Difference -->

5/F
 7/F
 13/F

Group of **Type 3:**

6/0 <-- Difference -->
 ----------13-----56
 | * * * * 51 |10
 * 34 * 27 50 * 11 *
 | * 32---*---37---16--*---53--*
 * 29 | * 40 31 * 38 * |
 | * 35 * 26 33 * |28 *
 --52-|---- 9---36--*---25 * |
 * 14 * 55 30 * 39 *
 15 | 54 * * * * |
 49-----12-----*-----*

1/F
 ----------37-----32
 | * * * * 27 |34
 * 10 * 51 26 * 35 *
 | * 56---*---13---40--*---29--*
 * 53 | * 16 55 * 14 * |
 | * 11 * 50 9 * |52 *
 --28-|----33---12--*---49 * |
 * 38 * 31 54 * 15 *
 39 | 30 * * * * |
 25-----36-----*-----*

4/0 <-- Difference -->
 5/0 <-- Difference -->
 10/0 <-- Difference -->

2/F
 3/F
 9/F

Group of **Type 4:**

12/0 <-- Difference -->
 -----48-----49-----
 | * * 18 15 | *
 * * 19 39 14 58 * *
 | * *---45--25--52-- 8--+*---*
 7 * | * 28 * 5 26 * |
 |57 * * 38 * 59 |40 *
 60--16-|-*---*---*---*---37 17 |
 6 50 * * * * 27 47
 51 | * * * * 46 |
 13-----*-----*-----20

1/F
 -----60-----37-----
 | * * 6 27 | *
 * * 7 51 26 46 * *
 | * *---57--13---40--20--+*---*
 19 * | * 16 * 17 14 * |
 |45 * * 50 * 47 |52 *
 48--28-|-*---*---*---*---49 5 |
 18 38 * * * * 15 59
 39 | * * * * 58 |
 25-----*-----*-----8

9/0 <-- Difference -->
 10/0 <-- Difference -->
 11/0 <-- Difference -->
 15/0 <-- Difference -->

3/F
 5/F
 7/F
 13/F


```

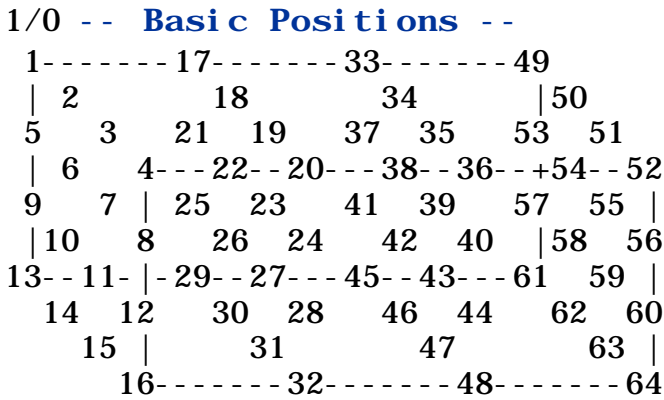
    dn[9]=cn[8];   dn[25]=cn[24]; dn[41]=cn[40];   dn[57]=cn[56];
    dn[10]=cn[7];  dn[26]=cn[23]; dn[42]=cn[39];   dn[58]=cn[55];
    dn[13]=cn[4];  dn[29]=cn[20]; dn[45]=cn[36];   dn[61]=cn[52];
    dn[14]=cn[3];  dn[30]=cn[19]; dn[46]=cn[35];   dn[62]=cn[51];
}
/**/
void trans2(){
    short n;
    for(n=0; n<65; n++){ dn[n]=cn[n]; }
    dn[9]=cn[53];  dn[10]=cn[54]; dn[11]=cn[55];   dn[12]=cn[56];
    dn[13]=cn[49]; dn[14]=cn[50]; dn[15]=cn[51];   dn[16]=cn[52];
    dn[25]=cn[37]; dn[26]=cn[38]; dn[27]=cn[39];   dn[28]=cn[40];
    dn[29]=cn[33]; dn[30]=cn[34]; dn[31]=cn[35];   dn[32]=cn[36];
    dn[33]=cn[29]; dn[34]=cn[30]; dn[35]=cn[31];   dn[36]=cn[32];
    dn[37]=cn[25]; dn[38]=cn[26]; dn[39]=cn[27];   dn[40]=cn[28];
    dn[49]=cn[13]; dn[50]=cn[14]; dn[51]=cn[15];   dn[52]=cn[16];
    dn[53]=cn[9];  dn[54]=cn[10]; dn[55]=cn[11];   dn[56]=cn[12];
}
/**/
void trans3(){
    short n;
    for(n=0; n<65; n++){ dn[n]=cn[n]; }
    dn[3]=cn[50];  dn[7]=cn[54];  dn[11]=cn[58];   dn[15]=cn[62];
    dn[4]=cn[49];  dn[8]=cn[53];  dn[12]=cn[57];   dn[16]=cn[61];
    dn[19]=cn[34]; dn[23]=cn[38]; dn[27]=cn[42];   dn[31]=cn[46];
    dn[20]=cn[33]; dn[24]=cn[37]; dn[28]=cn[41];   dn[32]=cn[45];
    dn[33]=cn[20]; dn[37]=cn[24]; dn[41]=cn[28];   dn[45]=cn[32];
    dn[34]=cn[19]; dn[38]=cn[23]; dn[42]=cn[27];   dn[46]=cn[31];
    dn[49]=cn[4];  dn[53]=cn[8];  dn[57]=cn[12];   dn[61]=cn[16];
    dn[50]=cn[3];  dn[54]=cn[7];  dn[58]=cn[11];   dn[62]=cn[15];
}
/**/
void trans4(){
    short n;
    for(n=0; n<65; n++){ dn[n]=cn[n]; }
    dn[9]=cn[21];  dn[10]=cn[22]; dn[11]=cn[23];   dn[12]=cn[24];
    dn[13]=cn[17]; dn[14]=cn[18]; dn[15]=cn[19];   dn[16]=cn[20];
    dn[17]=cn[13]; dn[18]=cn[14]; dn[19]=cn[15];   dn[20]=cn[16];
    dn[21]=cn[9];  dn[22]=cn[10]; dn[23]=cn[11];   dn[24]=cn[12];
    dn[33]=cn[61]; dn[34]=cn[62]; dn[35]=cn[63];   dn[36]=cn[64];
    dn[37]=cn[57]; dn[38]=cn[58]; dn[39]=cn[59];   dn[40]=cn[60];
    dn[57]=cn[37]; dn[58]=cn[38]; dn[59]=cn[39];   dn[60]=cn[40];
    dn[61]=cn[33]; dn[62]=cn[34]; dn[63]=cn[35];   dn[64]=cn[36];
}
/**/
void trans5(){
    short n;
    for(n=0; n<65; n++){ dn[n]=cn[n]; }
    dn[3]=cn[18];  dn[7]=cn[22];  dn[11]=cn[26];   dn[15]=cn[30];
    dn[4]=cn[17];  dn[8]=cn[21];  dn[12]=cn[25];   dn[16]=cn[29];
    dn[17]=cn[4];  dn[21]=cn[8];  dn[25]=cn[12];   dn[29]=cn[16];
    dn[18]=cn[3];  dn[22]=cn[7];  dn[26]=cn[11];   dn[30]=cn[15];
    dn[33]=cn[52]; dn[37]=cn[56]; dn[41]=cn[60];   dn[45]=cn[64];
}

```

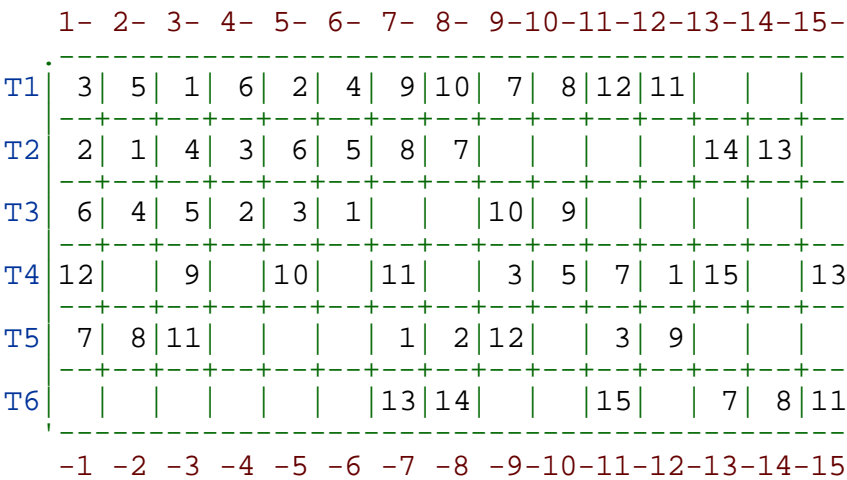
```

    dn[34]=cn[51]; dn[38]=cn[55]; dn[42]=cn[59];   dn[46]=cn[63];
    dn[51]=cn[34]; dn[55]=cn[38]; dn[59]=cn[42];   dn[63]=cn[46];
    dn[52]=cn[33]; dn[56]=cn[37]; dn[60]=cn[41];   dn[64]=cn[45];
}
/**/
void trans6(){
    short n;
    for(n=0; n<65; n++) { dn[n]=cn[n]; }
    dn[3]=cn[6];   dn[19]=cn[22]; dn[35]=cn[38];   dn[51]=cn[54];
    dn[4]=cn[5];   dn[20]=cn[21]; dn[36]=cn[37];   dn[52]=cn[53];
    dn[5]=cn[4];   dn[21]=cn[20]; dn[37]=cn[36];   dn[53]=cn[52];
    dn[6]=cn[3];   dn[22]=cn[19]; dn[38]=cn[35];   dn[54]=cn[51];
    dn[9]=cn[16];  dn[25]=cn[32]; dn[41]=cn[48];   dn[57]=cn[64];
    dn[10]=cn[15]; dn[26]=cn[31]; dn[42]=cn[47];   dn[58]=cn[63];
    dn[15]=cn[10]; dn[31]=cn[26]; dn[47]=cn[42];   dn[63]=cn[58];
    dn[16]=cn[9];  dn[32]=cn[25]; dn[48]=cn[41];   dn[64]=cn[57];
}
/*

```



*** Transformations Diagram ***

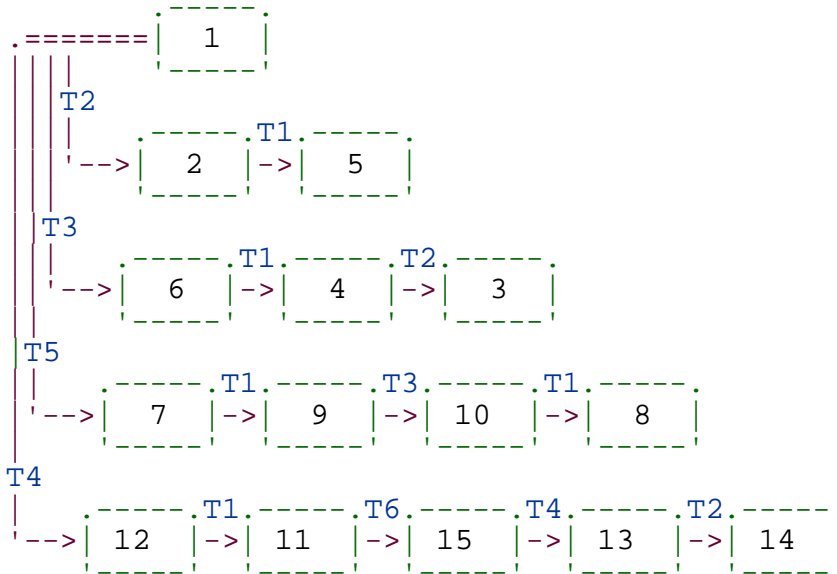


7. How do you use these transformation process for making one into 15?
 I know there are several good ways to make our job, but let me introduce you of my recent invention. I put it in the diagram as follows:

Put No.1 solution into the representataive top position, and transform it by Type 2.
 You will have the answer No.2. Transform the new result by Type 1, and you will have

No.5, then. In order to have No.6, you should transform No.1 by Type 3.

*** Transition Diagram of Transformation Process ***



8. If you have only one solution No.1 for the representative and our new transformation process, you can always make the one into the other 14 solutions. We can now say No.1 is really the “Most Fundamental Solution” for all 15.

But is No.1 the only ‘Most Fundamental Solution’ for all? No, it isn’t.

Anyone of No.2~No.15 could also become the ‘MFS’ for all.

Watch the diagram above carefully, and you will know you can start at any position and can get to everyone of No.1~15 no matter where it might be.

Since any type of 6 transformations is essentially reversible, you can go back and up from any place lower.

Anyone could make all the others by our transformation process.

Therefore we should say anyone of F15 could be the ‘MFS’ as a representative.

But I feel the first six solutions No1~6 make the core of F15, and I should recommend you nominate No.1~6 for the “MFS’ rather than No.7~15.

It is proved that there is only one family in the solution set of C&C Magic Cubes of order 4 and nothing else. It is the most interesting thing, I suppose. I found the same structure really exist even in any solution set of Composite & Pandiagonal Magic Cubes of order 4.

What did I find? I found the same type of ‘Process Transition’ in all.

(Revised on December 26, 2002 by Kanji Setsuda)

*** E-Mail Address: jag12100@nifty.ne.jp