Chapter 3. New Algebraic Study of Magic Squares of Order 5

Section 3. Study of Simultaneous Magic Squares 5x5: Both Self-Complementary and Pan-diagonal: **Kanji Setsuda**

#1. Simultaneous MS55: Both Self-Complementary & Pan-Diagonal

Let's study now the ‘Simultaneous’ magic squares that has both of the properties of self-complementary and of pan-diagonal type at the same time.
I like to start our discussion simply with the next diagram and equations as shown below.

[Figure 1: Simultaneous Magic Squares 5x5:
 Both Self-Complementary and Pan-Diagonal]

```
23 24 25 21 22 23 24 25 21 22 23
         .--------------.
 3  4  5| 1| 2| 3| 4| 5| 1  2  3
        |--+--+--+--+--|
 8  9 10| 6| 7| 8| 9|10| 6  7  8
        |--+--+--+--+--|
13 14 15|11|12|13|14|15|11 12 13
        |--+--+--+--+--|
18 19 20|16|17|18|19|20|16 17 18
        |--+--+--+--+--|
23 24 25|21|22|23|24|25|21 22 23
        '--------------'
 3  4  5  1  2  3  4  5  1  2  3
```

```
**  Basic Equations: **                   **  Pan-Diagonal Conditions: **
  n1+n2+n3+n4+n5=K        ....(b1) |   n1+n7+n13+n19+n25=K    ....(p1)
  n6+n7+n8+n9+n10=K       ....(b2) |   n1+n10+n14+n18+n22=K   ....(p2)
  n11+n12+n13+n14+n15=K   ....(b3) |   n2+n8+n14+n20+n21=K    ....(p3)
  n16+n17+n18+n19+n20=K   ....(b4) |   n2+n6+n15+n19+n23=K    ....(p4)
  n21+n22+n23+n24+n25=K   ....(b5) |   n3+n9+n15+n16+n22=K    ....(p5)
  n1+n6+n11+n16+n21=K     ....(b6) |   n3+n7+n11+n20+n24=K    ....(p6)
  n2+n7+n12+n17+n22=K     ....(b7) |   n4+n10+n11+n17+n23=K   ....(p7)
  n3+n8+n13+n18+n23=K     ....(b8) |   n4+n8+n12+n16+n25=K    ....(p8)
  n4+n9+n14+n19+n24=K     ....(b9) |   n5+n6+n12+n18+n24=K    ....(p9)
  n5+n10+n15+n20+n25=K    ...(b10) |   n5+n9+n13+n17+n21=K    ...(p10)
```

```
**  Self-Complementary Conditions: **
  n1+n25=n2+n24=n3+n23=n4+n22=n5+n21=
  n6+n20=n7+n19=n8+n18=n9+n17=n10+n16=
  n11+n15=n12+n14=n13+n13=C   ...(sc)
```

The new properties we found and proved in the pan-diagonal type could be accepted here.

```
  n1+n2+n5+n6+n21=65; n1+n7+n10+n22+n25=65; n1+n3+n4+n11+n16=65;
  n1+n13+n14+n18+n19=65; n1+n2+n6=n20+n24+n25   ...(22) etc.
```

The next notation could also be defined and accepted.

[Figure 2: Complementary Notations]

n1+$\underline{\text{n1}}$=n2+$\underline{\text{n2}}$=n3+$\underline{\text{n3}}$=n4+$\underline{\text{n4}}$=n5+$\underline{\text{n5}}$=n6+$\underline{\text{n6}}$
=n7+$\underline{\text{n7}}$=n8+$\underline{\text{n8}}$=n9+$\underline{\text{n9}}$=n10+$\underline{\text{n10}}$=n11+$\underline{\text{n11}}$
=n12+$\underline{\text{n12}}$=C=26 (n13=$\underline{\text{n13}}$=13) ...(sc)'

```
      [Classic]                 [New]
  1---2---3---4---5     1---2---3---4---5
  |   |   |   |   |     |   |   |   |   |
  6---7---8---9--10     6---7---8---9--10
  |   |   |   |   |     |   |   |   |   |
 11--12--13--14--15    11--12--13--12--11
  |   |   |   |   |     |   |   |   |   |
 16--17--18--19--20    10---9---8---7---6
  |   |   |   |   |     |   |   |   |   |
 21--22--23--24--25     5---4---3---2---1
```

n1+ n2+ n3+ n4+ n5=K ....(1)
n6+ n7+ n8+ n9+n10=K ....(2)
n11+n12+n13+$\underline{n12}$+$\underline{n11}$=K ....(3)
$\underline{n10}$+ $\underline{n9}$+ $\underline{n8}$+ $\underline{n7}$+ $\underline{n6}$=K ....(4)
$\underline{n5}$+ $\underline{n4}$+ $\underline{n3}$+ $\underline{n2}$+ $\underline{n1}$=K ....(5)
n1+ n6+n11+$\underline{n10}$+ $\underline{n5}$=K ....(6)
n2+ n7+n12+ $\underline{n9}$+ $\underline{n4}$=K ....(7)
n3+ n8+n13+ $\underline{n8}$+ $\underline{n3}$=K ....(8)
n4+ n9+$\underline{n12}$+ $\underline{n7}$+ $\underline{n2}$=K ....(9)
n5+n10+$\underline{n11}$+ $\underline{n6}$+ $\underline{n1}$=K ...(10)
n1+ n7+n13+ $\underline{n7}$+ $\underline{n1}$=K ...(11)
n5+ n9+n13+ $\underline{n9}$+ $\underline{n5}$=K ...(20)
(K=65)

$\underline{n1}$=n25; $\underline{n2}$=n24; $\underline{n3}$=n23; $\underline{n4}$=n22; $\underline{n5}$=n21;
$\underline{n6}$=n20; $\underline{n7}$=n19; $\underline{n8}$=n18; $\underline{n9}$=n17; $\underline{n10}$=n16;
$\underline{n11}$=n15; $\underline{n12}$=n14; $\underline{n13}$=n13=13; ...(21)'

We can rewrite (22) with this notation and make them shorter:

n1+n2+n5+n6+n21=65; | n1+n7+n10+n22+n25=65;
n1+n2+n6+(n5+$\underline{n5}$)=65; | n7+n10+n22+(n1+$\underline{n1}$)=65;
n1+n2+n6+26=65; | n7+n10+n22+26=65;
n1+n2+n6=39 | n7+n10+n22=39

Three numbers combined into a triangle on every top of the 4 corners add up to the same sum. Put the name to each triangle:

n1+n2+n6=P; n20+n24+n25=Q; n4+n5+n10=R; n16+n21+n22=S
It is known that (P=Q); (R=S)
Q and S could be rewritten as: Q=$\underline{n6}$+$\underline{n2}$+$\underline{n1}$; S=$\underline{n10}$+$\underline{n5}$+$\underline{n4}$
P+Q=n1+n2+n6+$\underline{n6}$+$\underline{n2}$+$\underline{n1}$=(n1+$\underline{n1}$)+(n2+$\underline{n2}$)+(n6+$\underline{n6}$)=26+26+26=78
R+S=n4+n5+n10+$\underline{n10}$+$\underline{n5}$+$\underline{n4}$=(n4+$\underline{n4}$)+(n5+$\underline{n5}$)+(n10+$\underline{n10}$)=26x3=78
Therefore P=Q=39; R=S=39
n1+n2+n6=39; n20+n24+n25=39; n4+n5+n10=39; n16+n21+n22=39 ...(23)

#2. How many solutions could we find?

Add the least inequality conditions to the basic: n1<n5<n21, n1<n25
You must not give too many and too strict conditions. Or you might miss any solutions you could have got and spoil your investigation.
Calculate and count your solutions, and you will find 16 solutions in all.

[Figure 3: Simultaneous Magic Squares 5x5:
Both Self-Complementary and Pan-Diagonal Type]

```
           1/            2/            3/            4/
 1 15 24 18  7   1 23 20 14  7   1 15 22 18  9   1 23 20 12  9
23 17  6  5 14  15  9  2 21 18  23 19  6  5 12  15  7  4 21 18
10  4 13 22 16  22 16 13 10  4  10  2 13 24 16  24 16 13 10  2
12 21 20  9  3   8  5 24 17 11  14 21 20  7  3   8  5 22 19 11
19  8  2 11 25  19 12  6  3 25  17  8  4 11 25  17 14  6  3 25
           5/            6/            7/            8/
 2 14 25 18  6   2 23 19 15  6   2 14 21 18 10   2 23 19 11 10
23 16  7  4 15  14 10  1 22 18  23 20  7  4 11  14  6  5 22 18
 9  5 13 21 17  21 17 13  9  5   9  1 13 25 17  25 17 13  9  1
11 22 19 10  3   8  4 25 16 12  15 22 19  6  3   8  4 21 20 12
20  8  1 12 24  20 11  7  3 24  16  8  5 12 24  16 15  7  3 24
```

```
              9/            10/            11/            12/
 4 12 25 18  6   4 23 17 15  6   4 12 21 18 10   4 23 17 11 10
23 16  9  2 15  12 10  1 24 18  23 20  9  2 11  12  6  5 24 18
 7  5 13 21 19  21 19 13  7  5   7  1 13 25 19  25 19 13  7  1
11 24 17 10  3   8  2 25 16 14  15 24 17  6  3   8  2 21 20 14
20  8  1 14 22  20 11  9  3 22  16  8  5 14 22  16 15  9  3 22
             13/            14/            15/            16/
 5 11 24 18  7   5 23 16 14  7   5 11 22 18  9   5 23 16 12  9
23 17 10  1 14  11  9  2 25 18  23 19 10  1 12  11  7  4 25 18
 6  4 13 22 20  22 20 13  6  4   6  2 13 24 20  24 20 13  6  2
12 25 16  9  3   8  1 24 17 15  14 25 16  7  3   8  1 22 19 15
19  8  2 15 21  19 12 10  3 21  17  8  4 15 21  17 14 10  3 21
```

[Count = 16]

These solutions are so rare that they might almost look like precious jewels.
They are essentially the same with the ones Prof. Mutsumi Suzuki discovered before.
He presented the perfect list in his page. I just verified it for giving my highest praise to his great achievements.
For many years before we had never imagined that such a simultaneous type of magic squares could exist.
In the case of order 4, pan-diagonal magic squares are all 'complete'. It means that all complementary pairs of 17 are placed on pan-diagonals and are not placed elsewhere.
They could be all transformed into self-complementary type. Between those two types does certainly exist the one-to-one correspondence, although they are totally independent by themselves.
Prof. M. Suzuki predicted that the simultaneous type of both self-complementary and pan-diagonal could exist in the case of order 5, 7, 9, 11, ...
I just found some examples of order 7 and 9.

#3. 'Complete Euler Squares'

One of the most precious properties of these 16 jewels is revealed when you put them in the new notation by Positional Number System of the Base 5.
Look at the next list of 16 solutions.

Each contains a classical notation at first, and a new one by the 5th increment, and finally the list of separated layers for higher and lower positions decomposed by positional numbers of the base 5.

[Figure 4: Solution List Decomposed by 5th Increment]

```
           No.1/   /Notation by 5i   /Decomposition by 5i
  1 15 24 18  7    00 24 43 32 11    0 2 4 3 1   0 4 3 2 1
 23 17  6  5 14    42 31 10 04 23    4 3 1 0 2   2 1 0 4 3
 10  4 13 22 16    14 03 22 41 30    1 0 2 4 3   4 3 2 1 0
 12 21 20  9  3    21 40 34 13 02    2 4 3 1 0   1 0 4 3 2
 19  8  2 11 25    33 12 01 20 44    3 1 0 2 4   3 2 1 0 4
/Classical Notation                   Higher 5^1   Lower 5^0
```

```
**  Simultaneous Magic Squares of Order 5:  **
** Both Self-Complementary and Pan-Diagonal **
**  List of 16 Solutions with /N5i and /D5i **
```

```
[ 1]                 /N5i                /D5i
   1 23 20 14  7     00 42 34 23 11      0 4 3 2 1   0 2 4 3 1
  15  9  2 21 18     24 13 01 40 32      2 1 0 4 3   4 3 1 0 2
  22 16 13 10  4     41 30 22 14 03      4 3 2 1 0   1 0 2 4 3
   8  5 24 17 11     12 04 43 31 20      1 0 4 3 2   2 4 3 1 0
  19 12  6  3 25     33 21 10 02 44      3 2 1 0 4   3 1 0 2 4
[ 2]                 /N5i                /D5i
   1 15 24 18  7     00 24 43 32 11      0 2 4 3 1   0 4 3 2 1
  23 17  6  5 14     42 31 10 04 23      4 3 1 0 2   2 1 0 4 3
  10  4 13 22 16     14 03 22 41 30      1 0 2 4 3   4 3 2 1 0
  12 21 20  9  3     21 40 34 13 02      2 4 3 1 0   1 0 4 3 2
  19  8  2 11 25     33 12 01 20 44      3 1 0 2 4   3 2 1 0 4
[ 3]                 /N5i                /D5i
   1 23 20 12  9     00 42 34 21 13      0 4 3 2 1   0 2 4 1 3
  15  7  4 21 18     24 11 03 40 32      2 1 0 4 3   4 1 3 0 2
  24 16 13 10  2     43 30 22 14 01      4 3 2 1 0   3 0 2 4 1
   8  5 22 19 11     12 04 41 33 20      1 0 4 3 2   2 4 1 3 0
  17 14  6  3 25     31 23 10 02 44      3 2 1 0 4   1 3 0 2 4
[ 4]                 /N5i                /D5i
   1 15 22 18  9     00 24 41 32 13      0 2 4 3 1   0 4 1 2 3
  23 19  6  5 12     42 33 10 04 21      4 3 1 0 2   2 3 0 4 1
  10  2 13 24 16     14 01 22 43 30      1 0 2 4 3   4 1 2 3 0
  14 21 20  7  3     23 40 34 11 02      2 4 3 1 0   3 0 4 1 2
  17  8  4 11 25     31 12 03 20 44      3 1 0 2 4   1 2 3 0 4
[ 5]                 /N5i                /D5i
   2 23 19 15  6     01 42 33 24 10      0 4 3 2 1   1 2 3 4 0
  14 10  1 22 18     23 14 00 41 32      2 1 0 4 3   3 4 0 1 2
  21 17 13  9  5     40 31 22 13 04      4 3 2 1 0   0 1 2 3 4
   8  4 25 16 12     12 03 44 30 21      1 0 4 3 2   2 3 4 0 1
  20 11  7  3 24     34 20 11 02 43      3 2 1 0 4   4 0 1 2 3
[ 6]                 /N5i                /D5i
   2 14 25 18  6     01 23 44 32 10      0 2 4 3 1   1 3 4 2 0
  23 16  7  4 15     42 30 11 03 24      4 3 1 0 2   2 0 1 3 4
   9  5 13 21 17     13 04 22 40 31      1 0 2 4 3   3 4 2 0 1
  11 22 19 10  3     20 41 33 14 02      2 4 3 1 0   0 1 3 4 2
  20  8  1 12 24     34 12 00 21 43      3 1 0 2 4   4 2 0 1 3
[ 7]                 /N5i                /D5i
   2 23 19 11 10     01 42 33 20 14      0 4 3 2 1   1 2 3 0 4
  14  6  5 22 18     23 10 04 41 32      2 1 0 4 3   3 0 4 1 2
  25 17 13  9  1     44 31 22 13 00      4 3 2 1 0   4 1 2 3 0
   8  4 21 20 12     12 03 40 34 21      1 0 4 3 2   2 3 0 4 1
  16 15  7  3 24     30 24 11 02 43      3 2 1 0 4   0 4 1 2 3
[ 8]                 /N5i                /D5i
   2 14 21 18 10     01 23 40 32 14      0 2 4 3 1   1 3 0 2 4
  23 20  7  4 11     42 34 11 03 20      4 3 1 0 2   2 4 1 3 0
   9  1 13 25 17     13 00 22 44 31      1 0 2 4 3   3 0 2 4 1
  15 22 19  6  3     24 41 33 10 02      2 4 3 1 0   4 1 3 0 2
  16  8  5 12 24     30 12 04 21 43      3 1 0 2 4   0 2 4 1 3
```

```
[ 9]                  /N5i              /D5i
   4 23 17 15  6      03 42 31 24 10    0 4 3 2 1  3 2 1 4 0
  12 10  1 24 18      21 14 00 43 32    2 1 0 4 3  1 4 0 3 2
  21 19 13  7  5      40 33 22 11 04    4 3 2 1 0  0 3 2 1 4
   8  2 25 16 14      12 01 44 30 23    1 0 4 3 2  2 1 4 0 3
  20 11  9  3 22      34 20 13 02 41    3 2 1 0 4  4 0 3 2 1
[10]                  /N5i              /D5i
   4 12 25 18  6      03 21 44 32 10    0 2 4 3 1  3 1 4 2 0
  23 16  9  2 15      42 30 13 01 24    4 3 1 0 2  2 0 3 1 4
   7  5 13 21 19      11 04 22 40 33    1 0 2 4 3  1 4 2 0 3
  11 24 17 10  3      20 43 31 14 02    2 4 3 1 0  0 3 1 4 2
  20  8  1 14 22      34 12 00 23 41    3 1 0 2 4  4 2 0 3 1
[11]                  /N5i              /D5i
   4 23 17 11 10      03 42 31 20 14    0 4 3 2 1  3 2 1 0 4
  12  6  5 24 18      21 10 04 43 32    2 1 0 4 3  1 0 4 3 2
  25 19 13  7  1      44 33 22 11 00    4 3 2 1 0  4 3 2 1 0
   8  2 21 20 14      12 01 40 34 23    1 0 4 3 2  2 1 0 4 3
  16 15  9  3 22      30 24 13 02 41    3 2 1 0 4  0 4 3 2 1
[12]                  /N5i              /D5i
   4 12 21 18 10      03 21 40 32 14    0 2 4 3 1  3 1 0 2 4
  23 20  9  2 11      42 34 13 01 20    4 3 1 0 2  2 4 3 1 0
   7  1 13 25 19      11 00 22 44 33    1 0 2 4 3  1 0 2 4 3
  15 24 17  6  3      24 43 31 10 02    2 4 3 1 0  4 3 1 0 2
  16  8  5 14 22      30 12 04 23 41    3 1 0 2 4  0 2 4 3 1
[13]                  /N5i              /D5i
   5 23 16 14  7      04 42 30 23 11    0 4 3 2 1  4 2 0 3 1
  11  9  2 25 18      20 13 01 44 32    2 1 0 4 3  0 3 1 4 2
  22 20 13  6  4      41 34 22 10 03    4 3 2 1 0  1 4 2 0 3
   8  1 24 17 15      12 00 43 31 24    1 0 4 3 2  2 0 3 1 4
  19 12 10  3 21      33 21 14 02 40    3 2 1 0 4  3 1 4 2 0
[14]                  /N5i              /D5i
   5 11 24 18  7      04 20 43 32 11    0 2 4 3 1  4 0 3 2 1
  23 17 10  1 14      42 31 14 00 23    4 3 1 0 2  2 1 4 0 3
   6  4 13 22 20      10 03 22 41 34    1 0 2 4 3  0 3 2 1 4
  12 25 16  9  3      21 44 30 13 02    2 4 3 1 0  1 4 0 3 2
  19  8  2 15 21      33 12 01 24 40    3 1 0 2 4  3 2 1 4 0
[15]                  /N5i              /D5i
   5 23 16 12  9      04 42 30 21 13    0 4 3 2 1  4 2 0 1 3
  11  7  4 25 18      20 11 03 44 32    2 1 0 4 3  0 1 3 4 2
  24 20 13  6  2      43 34 22 10 01    4 3 2 1 0  3 4 2 0 1
   8  1 22 19 15      12 00 41 33 24    1 0 4 3 2  2 0 1 3 4
  17 14 10  3 21      31 23 14 02 40    3 2 1 0 4  1 3 4 2 0
[16]                  /N5i              /D5i
   5 11 22 18  9      04 20 41 32 13    0 2 4 3 1  4 0 1 2 3
  23 19 10  1 12      42 33 14 00 21    4 3 1 0 2  2 3 4 0 1
   6  2 13 24 20      10 01 22 43 34    1 0 2 4 3  0 1 2 3 4
  14 25 16  7  3      23 44 30 11 02    2 4 3 1 0  3 4 0 1 2
  17  8  4 15 21      31 12 03 24 40    3 1 0 2 4  1 2 3 4 0
[Count = 16]
```

Watch every solution especially in the separated layers of decomposition, and you may find each row, each column or even each pandiagonal contains {0, 1, 2, 3, 4} strictly once on any plane and no other combinations of the members.
The sum of each row, column and pandiagonal is calculated in such a way as:

$(0+1+2+3+4) \times 5^1 + (0+1+2+3+4) \times 5^0 = 10 \times 5 + 10 \times 1 = 60$(Decimal)

This sum 60 is logically equivalent to the magic constant 65 in our classical notation.
It is the only way how to calculate the sum of each. It shows each sum is always equal to the same constant. Once you notice this characteristic structure, you don't have to calculate anything more. You may well just see it, because you know the answer.

Prof. M. Suzuki taught me the name “Euler Square” for this structure. He said that the legendary greatest mathematician, Leonhard Euler(1707-1783) did not mean even about pan-diagonals.
The reality sometimes surpasses our expectation. All pandiagonals of these 16 jewels have this property ‘Greco-Latinian structure’ of Euler Square.
I would like to call this type “Complete Euler Square” as a natural consequence.

Even 144 pan-diagonal squares of order 5 are all 'Complete Euler Squares.'
Both types of ‘Suzuki's Squares’ are all 'Complete Euler Squares.' It is really amazing.

(Written on August 29, 2001; Revised on May 27, 2005;
Worked on MacOS X(10.3.5) and Xcode 1.5 by Kanji Setsuda)

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```
/**  Simultaneous Magic Squares of Order 5:  **/
/** Both Self-Complementary and Pan-Diagonal **/
/** 'SPMS5DPC.c' Built by Kanji Setsuda **/
/**  on MacOS Xcode 1.5;  Feb.11, 2005  **/
/**/
/* Using Library */
#include <stdio.h>
/**/
/* Global Variables */
short int cntr;
short LSM, CC, NC3;
short nm[26], uflg[26];
/**/
/* Sub-Routines */
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void);
void ansprint(void);
/**/
/* Main Program */
main(){
 short n;
  printf("\n**  Simultaneous Magic Squares of Order 5:  **\n");
  printf("** Both Self-Complementary and Pan-Diagonal **\n");
  printf("**  List of 16 Solutions with /N5i and /D5i **\n");
  for(n=0;n<26;n++){nm[n]=0; uflg[n]=0;}
  LSM=65; CC=26; NC3=LSM-CC; cntr=0;
  nm[13]=13; uflg[13]=1;  /* Set n13=13 */
  stp01();/* Begin the Calculations */
```

```
  uflg[13]=0;
  printf(" [Count = %d]\n",cntr);
  printf(" OK!\n");
 return 0;
}
/* Begin the Calculations */
/* Set n1 & n25 & n1<n25 */
void stp01(){
 short a, b;
  for(a=1;a<13;a++){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[1]=a; nm[25]=b;
      uflg[a]=1; uflg[b]=1;
      stp02();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n5 & n21 & (n1<n5<n21) */
void stp02(){
 short a, b;
  for(a=nm[1]+1;a<13;a++){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[5]=a; nm[21]=b;
      uflg[a]=1; uflg[b]=1;
      stp03();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n2 & n24 */
void stp03(){
 short a, b;
  for(a=25;a>0;a--){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[2]=a; nm[24]=b;
      uflg[a]=1; uflg[b]=1;
      stp04();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n6=39-n1-n2 & n20 */
void stp04(){
 short a, b;
  a=NC3-nm[1]-nm[2];
  b=NC3-nm[25]-nm[24];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[6]=a; nm[20]=b;
      uflg[a]=1; uflg[b]=1;
      stp05();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n4 & n22 */
void stp05(){
 short a, b;
  for(a=25;a>0;a--){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[4]=a; nm[22]=b;
      uflg[a]=1; uflg[b]=1;
      stp06();
```

```
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n3=LSM-n1-n2-n4-n5 & n23 */
void stp06(){
 short a, b;
  a=LSM-nm[1]-nm[2]-nm[4]-nm[5];
  b=LSM-nm[25]-nm[24]-nm[22]-nm[21];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[3]=a; nm[23]=b;
      uflg[a]=1; uflg[b]=1;
      stp07();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n10=39-n4-n5 & n16 */
void stp07(){
 short a, b;
  a=NC3-nm[4]-nm[5];
  b=NC3-nm[22]-nm[21];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[10]=a; nm[16]=b;
      uflg[a]=1; uflg[b]=1;
      stp08();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n11=LSM-n1-n6-n16-n21 & n15 */
void stp08(){
 short a, b;
  a=LSM-nm[1]-nm[6]-nm[16]-nm[21];
  b=LSM-nm[25]-nm[20]-nm[10]-nm[5];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[11]=a; nm[15]=b;
      uflg[a]=1; uflg[b]=1;
      stp09();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n7=LSM-n3-n11-n20-n24 & n19 */
void stp09(){
 short a, b;
  a=LSM-nm[3]-nm[11]-nm[20]-nm[24];
  b=LSM-nm[23]-nm[15]-nm[6]-nm[2];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[7]=a; nm[19]=b;
      uflg[a]=1; uflg[b]=1;
      stp10();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n9=LSM-n3-n15-n16-n22 & n17 */
void stp10(){
 short a, b;
  a=LSM-nm[3]-nm[15]-nm[16]-nm[22];
  b=LSM-nm[23]-nm[11]-nm[10]-nm[4];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[9]=a; nm[17]=b;
```

```
      uflg[a]=1; uflg[b]=1;
      stp11();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n8=LSM-n6-n7-n9-n10 & n18 */
void stp11(){
 short a, b;
  a=LSM-nm[6]-nm[7]-nm[9]-nm[10];
  b=LSM-nm[20]-nm[19]-nm[17]-nm[16];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[8]=a; nm[18]=b;
      uflg[a]=1; uflg[b]=1;
      stp12();
      uflg[b]=0; uflg[a]=0;}}
}
/* Set n12=LSM-n2-n7-n17-n22 & n14 */
void stp12(){
 short a, b;
  a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
  b=LSM-nm[24]-nm[19]-nm[9]-nm[4];
  if((0<a)&&(a<26)&&(a+b==CC)){
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[12]=a; nm[14]=b;
      uflg[a]=1; uflg[b]=1;
      stp13();
      uflg[b]=0; uflg[a]=0;}}
}
/* Checks n11+n12+n13+n14+n15=65 & Others */
void stp13(){
 short sm1, sm2;
  sm1=nm[11]+nm[12]+nm[13]+nm[14]+nm[15];
  sm2=nm[3]+nm[8]+nm[13]+nm[18]+nm[23];
  if((sm1==LSM)&&(sm2==LSM)){stp14();}
}
/* Checks n1+n10+n14+n18+n22=65 & Others */
void stp14(){
 short sm1, sm2, sm3;
  sm1=nm[1]+nm[10]+nm[14]+nm[18]+nm[22];
  sm2=nm[1]+nm[7]+nm[13]+nm[19]+nm[25];
  sm3=nm[2]+nm[8]+nm[14]+nm[20]+nm[21];
  if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){stp15();}
}
/* Checks n5+n9+n13+n17+n21=65 & Others */
void stp15(){
 short sm1, sm2, sm3;
  sm1=nm[4]+nm[8]+nm[12]+nm[16]+nm[25];
  sm2=nm[5]+nm[9]+nm[13]+nm[17]+nm[21];
  sm3=nm[5]+nm[6]+nm[12]+nm[18]+nm[24];
  if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){ansprint();}
}
/**/
/* Decompose and Print */
void ansprint(){
 short n,m,m5,d;
  cntr++;
  printf(" [%2d]               /N5i             /D5i\n",cntr);
  for(m=0;m<5;m++){m5=m*5;
    printf("  ");
```

```
    for(n=1;n<6;n++){printf("%3d",nm[m5+n]);}
    printf("  ");
    for(n=1;n<6;n++){d=nm[m5+n]-1; printf(" %d%d",d/5,d%5);};
    printf("  ");
    for(n=1;n<6;n++){d=nm[m5+n]-1; printf(" %d",d/5);};
    printf(" ");
    for(n=1;n<6;n++){d=nm[m5+n]-1; printf(" %d",d%5);};
    printf(" \n");
  }
}
/**/
/*
 [ 0]                 /N5i              /D5i
    1  2  3  4  5   00 01 02 03 04   0 0 0 0 0  0 1 2 3 4
    6  7  8  9 10   10 11 12 13 14   1 1 1 1 1  0 1 2 3 4
   11 12 13 14 15   20 21 22 23 24   2 2 2 2 2  0 1 2 3 4
   16 17 18 19 20   30 31 32 33 34   3 3 3 3 3  0 1 2 3 4
   21 22 23 24 25   40 41 42 43 44   4 4 4 4 4  0 1 2 3 4
*/
```