Chapter 2 : New Study of Magic Squares 4x4: Kanji Setsuda

Section 4 : 'Composite and Complete' Pan-magic Squares 4x4 and the meaning of Complementary Pairs

#1. Let's study and know about the true meaning of Complementary Pairs for all types of pan-magic squares of order 4 right here.

For that purpose, let's make 'Composite and Complete' pan-magic squares of order 4 at first and study them precisely by our algebraic calculations.

```
* 'Composite & Complete' Pan-magic Squares of Order 4 *
** Definition of Basic Form: **
 14 15 16 13 14 15 16 13 14 15
          .--.--.--.--.
  2  3  4| 1| 2| 3| 4| 1  2  3
         |--+--+--+--|
  6  7  8| 5| 6| 7| 8| 5  6  7
         |--+--+--+--|
 10 11 12| 9|10|11|12| 9 10 11
         |--+--+--+--|
 14 15 16|13|14|15|16|13 14 15
          '--'--'--'--'
  2  3  4  1  2  3  4  1  2  3
** Definition of Basic Conditions: C=34 **
  n1+n2+n3+n4=C      ...rw; | n1+n5+n9+n13=C    ...cl;
** Composite Conditions: C=34 **
  n1+n2+n5+n6=C      ...c1; | n2+n3+n6+n7=C     ...c2;
  n3+n4+n7+n8=C      ...c3; | n4+n1+n8+n5=C     ...c4;
  n5+n6+n9+n10=C     ...c5; | n6+n7+n10+n11=C   ...c6;
  n7+n8+n11+n12=C    ...c7; | n8+n5+n12+n9=C    ...c8;
  n9+n10+n13+n14=C   ...c9; | n10+n11+n14+n15=C ...c10;
  n11+n12+n15+n16=C  ...c11;| n12+n9+n16+n13=C  ...c12;
  n13+n14+n1+n2=C    ...c13;| n14+n15+n2+n3=C   ...c14;
  n15+n16+n3+n4=C    ...c15;| n16+n13+n4+n1=C   ...c16;
** Complete Conditions: CC=17 **
  n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC  ...cc
** List-forming Inequality Conditions for Standard Solutions: **
  n1<n16, n1<n4, n1<n13, and n2>n5;
```

The next list of solutions shows some specimens of our 'Composite and Complete' pan-magic squares 4x4 composed with the Basic Form and Definitions above.

```
** List of Sample Solutions of 'C&C' Pan-magic Squares of Order 4 **
                 1/                 2/                 3/                 4/
 11  8 10  5 11  8   7 12  6  9  7 12  13  8 10  3 13  8   7 14  4  9  7 14
   .--.--.--.--.       .--.--.--.--.      .--.--.--.--.      .--.--.--.--.
 14| 1|15| 4|14| 1  14| 1|15| 4|14| 1  12| 1|15| 6|12| 1  12| 1|15| 6|12| 1
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
  7|12| 6| 9| 7|12  11| 8|10| 5|11| 8   7|14| 4| 9| 7|14  13| 8|10| 3|13| 8
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
  2|13| 3|16| 2|13   2|13| 3|16| 2|13   2|11| 5|16| 2|11   2|11| 5|16| 2|11
   |--+--+--+--|      |--+--+--+--|      |--+--+--+--|      |--+--+--+--|
 11| 8|10| 5|11| 8   7|12| 6| 9| 7|12  13| 8|10| 3|13| 8   7|14| 4| 9| 7|14
   '--'--'--'--'      '--'--'--'--'      '--'--'--'--'      '--'--'--'--'
 14  1 15  4 14  1  14  1 15  4 14  1  12  1 15  6 12  1  12  1 15  6 12  1
```

```
                5/
13 12  6  3 13 12
 8| 1|15|10| 8| 1
11|14| 4| 5|11|14
 2| 7| 9|16| 2| 7
13|12| 6| 3|13|12
 8  1 15 10  8  1
```

```
                6/
11 14  4  5 11 14
 8| 1|15|10| 8| 1
13|12| 6| 3|13|12
 2| 7| 9|16| 2| 7
11|14| 4| 5|11|14
 8  1 15 10  8  1
```

```
                7/
10  8 11  5 10  8
15| 1|14| 4|15| 1
 6|12| 7| 9| 6|12
 3|13| 2|16| 3|13
10| 8|11| 5|10| 8
15  1 14  4 15  1
```

```
                8/
 6 12  7  9  6 12
15| 1|14| 4|15| 1
10| 8|11| 5|10| 8
 3|13| 2|16| 3|13
 6|12| 7| 9| 6|12
15  1 14  4 15  1
```

```
                9/
 6 15  4  9  6 15
12| 1|14| 7|12| 1
13| 8|11| 2|13| 8
 3|10| 5|16| 3|10
 6|15| 4| 9| 6|15
12  1 14  7 12  1
```

```
               10/
10 15  4  5 10 15
 8| 1|14|11| 8| 1
13|12| 7| 2|13|12
 3| 6| 9|16| 3| 6
10|15| 4| 5|10|15
 8  1 14 11  8  1
```

```
               11/
 4 14  7  9  4 14
15| 1|12| 6|15| 1
10| 8|13| 3|10| 8
 5|11| 2|16| 5|11
 4|14| 7| 9| 4|14
15  1 12  6 15  1
```

```
               12/
 4 15  6  9  4 15
14| 1|12| 7|14| 1
11| 8|13| 2|11| 8
 5|10| 3|16| 5|10
 4|15| 6| 9| 4|15
14  1 12  7 14  1
```

```
               13/
12  7  9  6 12  7
13| 2|16| 3|13| 2
 8|11| 5|10| 8|11
 1|14| 4|15| 1|14
12| 7| 9| 6|12| 7
13  2 16  3 13  2
```

```
               14/
 8 11  5 10  8 11
13| 2|16| 3|13| 2
12| 7| 9| 6|12| 7
 1|14| 4|15| 1|14
 8|11| 5|10| 8|11
13  2 16  3 13  2
```

```
               15/
14  7  9  4 14  7
11| 2|16| 5|11| 2
 8|13| 3|10| 8|13
 1|12| 6|15| 1|12
14| 7| 9| 4|14| 7
11  2 16  5 11  2
```

```
               16/
 8 13  3 10  8 13
11| 2|16| 5|11| 2
14| 7| 9| 4|14| 7
 1|12| 6|15| 1|12
 8|13| 3|10| 8|13
11  2 16  5 11  2
```

```
               21/
 5 16  3 10  5 16
11| 2|13| 8|11| 2
14| 7|12| 1|14| 7
 4| 9| 6|15| 4| 9
 5|16| 3|10| 5|16
11  2 13  8 11  2
```

```
               22/
 9 16  3  6  9 16
 7| 2|13|12| 7| 2
14|11| 8| 1|14|11
 4| 5|10|15| 4| 5
 9|16| 3| 6| 9|16
 7  2 13 12  7  2
```

```
               23/
 3 13  8 10  3 13
16| 2|11| 5|16| 2
 9| 7|14| 4| 9| 7
 6|12| 1|15| 6|12
 3|13| 8|10| 3|13
16  2 11  5 16  2
```

```
               24/
 3 16  5 10  3 16
13| 2|11| 8|13| 2
12| 7|14| 1|12| 7
 6| 9| 4|15| 6| 9
 3|16| 5|10| 3|16
13  2 11  8 13  2
```

```
               25/
12  6  9  7 12  6
13| 3|16| 2|13| 3
 8|10| 5|11| 8|10
 1|15| 4|14| 1|15
12| 6| 9| 7|12| 6
13  3 16  2 13  3
```

```
               26/
 8 10  5 11  8 10
13| 3|16| 2|13| 3
12| 6| 9| 7|12| 6
 1|15| 4|14| 1|15
 8|10| 5|11| 8|10
13  3 16  2 13  3
```

```
               27/
15  6  9  4 15  6
10| 3|16| 5|10| 3
 8|13| 2|11| 8|13
 1|12| 7|14| 1|12
15| 6| 9| 4|15| 6
10  3 16  5 10  3
```

```
               28/
 8 13  2 11  8 13
10| 3|16| 5|10| 3
15| 6| 9| 4|15| 6
 1|12| 7|14| 1|12
 8|13| 2|11| 8|13
10  3 16  5 10  3
```

```
                37/                  38/                  39/                  40/
16  5 10  3 16  5    7 14  1 12  7 14    16  9  6  3 16  9    11 14  1  8 11 14
  .--.--.--.--.        .--.--.--.--.        .--.--.--.--.        .--.--.--.--.
 9| 4|15| 6| 9| 4     9| 4|15| 6| 9| 4     5| 4|15|10| 5| 4     5| 4|15|10| 5| 4
  |--+--+--+--|        |--+--+--+--|        |--+--+--+--|        |--+--+--+--|
 7|14| 1|12| 7|14    16| 5|10| 3|16| 5    11|14| 1| 8|11|14    16| 9| 6| 3|16| 9
  |--+--+--+--|        |--+--+--+--|        |--+--+--+--|        |--+--+--+--|
 2|11| 8|13| 2|11     2|11| 8|13| 2|11     2| 7|12|13| 2| 7     2| 7|12|13| 2| 7
  |--+--+--+--|        |--+--+--+--|        |--+--+--+--|        |--+--+--+--|
16| 5|10| 3|16| 5     7|14| 1|12| 7|14    16| 9| 6| 3|16| 9    11|14| 1| 8|11|14
  '--'--'--'--'        '--'--'--'--'        '--'--'--'--'        '--'--'--'--'
 9  4 15  6  9  4     9  4 15  6  9  4     5  4 15 10  5  4     5  4 15 10  5  4
. . . . . .
```

They should certainly make the same solution set with the ones of 'Pan-diagonal' type, 'Complete' or 'Composite and Pan-diagonal'. It has the 384 'Primitive' solutions and the 48 'Standard' ones chosen by the list-forming inequality conditions.

It is called the "miraculous unity" of all types of pan-magic squares of order 4.

Take your careful look at the list of sample solutions above, and you can easily find the number 1 is always accompanied with the same members {8, 12, 14 and 15} on the left, right, upper and lower next. No one else would sit next to the number 1.

How interesting it is! Don't you think so?

What makes it possible? We have to find any reason why they are made so.

#2. This "miraculous unity" means that all of (1) Pan-diagonal type, (2) Complete one, (3) Composite pan-magic one and (4) Composite and Complete one are really equivalent and indicates the only same concept of pan-magic squares of order 4.

All those four types of pan-magic squares 4x4 have the same set of 384 'Primitive' solutions in common and consequently the same set of 48 'Standard' solutions.

And you can easily transform those simultaneous equations algebraically from each set to the others at the early definition stage. Let me demonstrate one of my actual transformations to you, for instance, from the definitions of 'Composite and Complete' type to the ones of ordinary Pan-diagonal type.

```
n1+n2+n3+n4=C  ...rw;  n1+n2+n5+n6=C  ...c1;  ->  n3+n4=n5+n6
n1+n2+n3+n4=C  ...rw;  n3+n4+n7+n8=C  ...c3;  ->  n1+n2=n7+n8
Therefore, n5+n6+n7+n8=n1+n2+n3+n4=C
In the same way, n9+n10+n11+n12=n13+n14+n15+n16=n1+n2+n3+n4=C
n1+n5+n9+n13=C ...cl;  n1+n2+n5+n6=C  ...c1;  ->  n9+n13=n2+n6
n1+n5+n9+n13=C ...cl;  n9+n10+n13+n14=C ...c9;->  n1+n5=n10+n14
Therefore, n2+n6+n10+n14=n1+n5+n9+n13=C
In the same way, n3+n7+n11+n15=n4+n8+n12+n16=n1+n5+n9+n13=C
```

```
14 15 16 13 14 15 16 13 14 15
         .--.--.--.--.
 2  3  4| 1| 2| 3| 4| 1  2  3
         |--+--+--+--|
 6  7  8| 5| 6| 7| 8| 5  6  7
         |--+--+--+--|
10 11 12| 9|10|11|12| 9 10 11
         |--+--+--+--|
14 15 16|13|14|15|16|13 14 15
         '--'--'--'--'
 2  3  4  1  2  3  4  1  2  3
```

Thus each row and each column should add up to the same constant sum C=34.

About pan-diagonals we can calculate the 'Complete Conditions' as follows.

```
n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC  ...cc

n1+n6+n11+n16=(n1+n11)+(n6+n16)=CC+CC=17+17=34=C  ...pd1;
n2+n7+n12+n13=(n2+n12)+(n7+n13)=CC+CC=17+17=34=C  ...pd2;
n3+n8+n9+n14=(n3+n9)+(n8+n14)=CC+CC=17+17=34=C    ...pd3;
n4+n5+n10+n15=(n4+n10)+(n5+n15)=CC+CC=17+17=34=C  ...pd4;
n1+n8+n11+n14=(n1+n11)+(n8+n14)=CC+CC=17+17=34=C  ...pb1;
n2+n5+n12+n15=(n2+n12)+(n5+n15)=CC+CC=17+17=34=C  ...pb2;
n3+n6+n9+n16=(n3+n9)+(n6+n16)=CC+CC=17+17=34=C    ...pb3;
n4+n7+n10+n13=(n4+n10)+(n7+n13)=CC+CC=17+17=34=C  ...pb4;
```

Thus every pan-diagonal should add up to the same constant sum C(=34).

Our 'Composite and Complete' magic squares of order 4 prove to be one of the special types of ordinary pan-diagonal magic squares.

But, as you know in the previous sections, all pan-diagonal MS44 should naturally have the same properties with 'Composite' and 'Complete' pan-magic squares 4x4.

Therefore, we can say both of the two types are logically equivalent and naturally have the same properties in common as well as the same common solution-set.

It means any type of pan-magic squares 4x4: (1) 'Pan-diagonal', (2) 'Complete', (3) 'Composite', and (4) 'Composite and Complete' is logically equivalent to one another.

If you would study one, then you could study all those four at the same time.

#3. Let's study further about our 'Composite and Complete' MS44, especially about their Complementary Pairs. How many pairs do we have to have to make this type?

When M+N=CC(Complementary Constant-Sum), we use the word "Complementary Pairs" for (M, N) just like "(M, N) is a complementary pair of CC".

```
* Algebraic Calculation: [1] *
  n1+n2+n5+n6=C     ...c1; n5+n6+n9+n10=C    ...c5; ->  n1+n2=n9+n10=cp1
  n1+n2+n3+n4=C     ...rw; n3+n4+n7+n8=C     ...c3; ->  n1+n2=n7+n8=cp1
  n7+n8+n11+n12=C   ...c7; n11+n12+n15+n16=C ..c11; ->  n7+n8=n15+n16=cp1
  n3+n4+n7+n8=C     ...c3; n7+n8+n11+n12=C   ...c7; ->  n3+n4=n11+n12=cp2
  n1+n2+n3+n4=C     ...rw; n1+n2+n5+n6=C     ...c1; ->  n3+n4=n5+n6=cp2
  n5+n6+n9+n10=C    ...c5; n9+n10+n13+n14=C  ...c9; ->  n5+n6=n13+n14=cp2
  Therefore  n1+n2=n7+n8=n9+n10=n15+n16=cp1;
             n3+n4=n5+n6=n11+n12=n13+n14=cp2; cp1+cp2=C;
* Algebraic Calculation: [2] *
  n1+n2+n5+n6=C     ...c1; n2+n3+n6+n7=C     ...c2; ->  n1+n5=n3+n7=cp3
  n1+n5+n9+n13=C    ...cl; n9+n10+n13+n14=C  ...c9; ->  n1+n5=n10+n14=cp3
  n10+n11+n14+n15=C ..c10; n11+n12+n15+n16=C ..c11; ->  n10+n14=n12+n16=cp3
  n2+n3+n6+n7=C     ...c2; n3+n4+n7+n8=C     ...c3; ->  n2+n6=n4+n8=cp4
  n1+n2+n5+n6=C     ...c1; n1+n5+n9+n13=C    ...cl; ->  n2+n6=n9+n13=cp4
  n9+n10+n13+n14=C  ...c9; n10+n11+n14+n15=C ..c10; ->  n9+n13=n11+n15=cp4
  Therefore  n1+n5=n3+n7=n10+n14=n12+n16=cp3;
             n2+n6=n4+n8=n9+n13=n11+n15=cp4; cp3+cp4=C;
* Algebraic Calculation: [3] *
  n1+n2+n3+n4=C     ...rw; n2+n3+n6+n7=C     ...c2; ->  n1+n4=n6+n7=cp5
  n6+n7+n10+n11=C   ...c6; n10+n11+n14+n15=C ..c10; ->  n6+n7=n14+n15=cp5
  n4+n1+n8+n5=C     ...c4; n8+n5+n12+n9=C    ...c8; ->  n1+n4=n9+n12=cp5
  n1+n2+n3+n4=C     ...rw; n4+n1+n8+n5=C     ...c4; ->  n2+n3=n5+n8=cp6
  n2+n3+n6+n7=C     ...c2; n6+n7+n10+n11=C   ...c6; ->  n2+n3=n10+n11=cp6
  n8+n5+n12+n9=C    ...c8; n12+n9+n16+n13=C  ...c12;->  n5+n8=n13+n16=cp6
  Therefore  n1+n4=n6+n7=n9+n12=n14+n15=cp5;
```

```
            n2+n3=n5+n8=n10+n11=n13+n16=cp6; cp5+cp6=C;

                .--.--.--.--.
  2   3   4|  1|  2|  3|  4|  1   2   3
           |--+--+--+--|
  6   7   8|  5|  6|  7|  8|  5   6   7
           |--+--+--+--|
 10  11  12|  9|10|11|12|  9  10  11
           |--+--+--+--|
 14  15  16|13|14|15|16|13  14  15
            '--'--'--'--'
```

```
* Algebraic Calculation: [4] *
  n1+n5+n9+n13=C    ...cl; n5+n6+n9+n10=C    ...c5; ->  n1+n13=n6+n10=cp7
  n6+n7+n10+n11=C   ...c6; n7+n8+n11+n12=C   ...c7; ->  n6+n10=n8+n12=cp7
  n13+n14+n1+n2=C   ...c13;n14+n15+n2+n3=C   ...c14;->  n1+n13=n3+n15=cp7
  n1+n5+n9+n13=C    ...cl; n13+n14+n1+n2=C   ...c13;->  n5+n9=n2+n14=cp8
  n5+n6+n9+n10=C    ...c5; n6+n7+n10+n11=C   ...c6; ->  n5+n9=n7+n11=cp8
  n14+n15+n2+n3=C   ...c14;n15+n16+n3+n4=C   ...c15;->  n2+n14=n4+n16=cp8
  Therefore  n1+n13=n6+n10=n3+n15=n8+n12=cp7;
             n5+n9=n2+n14=n7+n11=n4+n16=cp8; cp7+cp8=C;
```

We could find 4 groups of complementary pairs: 2 sets for each group: 4 pairs for each set. Each set has its own constant sum. Two constant sums in each group must add up to the magic constant C(=34).

The last fifth group contains the next 8 pairs whose sum is equal to 17.

```
[5]  n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=17  ...cc
```

Yes. They come from the very 'Complete Conditions' at the first definition stage.

Now let's adopt such a new notation for our complementary pairs to show explicitly where any pair might be located in the 4x4 diagram as: When M+N=CC, we want to write M for N. We mean M+M=CC and (M, M) is a complementary pair of CC.

```
* Five Groups of Complementary Pairs in Our New Notation *
  [1] n1+n2=n7+n8=n9+n10=n15+n16
      =n1+n1=n7+n7=n9+n9=n15+n15=cp1;
      n3+n4=n5+n6=n11+n12=n13+n14
      =n3+n3=n5+n5=n11+n11=n13+n13=cp2;   and   cp1+cp2=C;
  [2] n1+n5=n3+n7=n10+n14=n12+n16
      =n1+n1=n3+n3=n10+n10=n12+n12=cp3;
      n2+n6=n4+n8=n9+n13=n11+n15
      =n2+n2=n4+n4=n9+n9=n11+n11=cp4;     and   cp3+cp4=C;
```

```
         [1]                      [2]                      [3]
 15 13 13 15 15 13        12  9 10 11 12  9        13 13 14 14 13 13
    .-----------.            .-----------.            .-----------.
  3|  1|  1|  3|  3| 1      4|  1|  2|  3|  4| 1      1|  1|  2|  2|  1| 1
   |--+--+--+--|             |--+--+--+--|            |--+--+--+--|
  7|  5|  5|  7|  7| 5      4|  1|  2|  3|  4| 1      5|  5|  6|  6|  5| 5
   |--+--+--+--|             |--+--+--+--|            |--+--+--+--|
 11|  9|  9|11|11|  9      12|  9|10|11|12|  9      9|  9|10|10|  9| 9
   |--+--+--+--|             |--+--+--+--|            |--+--+--+--|
 15|13|13|15|15|13         12|  9|10|11|12|  9     13|13|14|14|13|13
    '-----------'            '-----------'            '-----------'
  3  1  1  3  3  1          4  1  2  3  4  1         1  1  2  2  1  1
```

```
[3] n1+n4=n6+n7=n9+n12=n14+n15
    =n1+n1=n6+n6=n9+n9=n14+n14=cp5;
    n2+n3=n5+n8=n10+n11=n13+n16
    =n2+n2=n5+n5=n10+n10=n13+n13=cp6;  and  cp5+cp6=C;
```

```
         [4]                              [5]
 4  1  2  3  4  1          8  5  6  7  8  5  6  7  8  5
   .-----------.                    .-----------.
 4| 1| 2| 3| 4| 1          2  3  4| 1| 2| 3| 4| 1  2  3
  |--+--+--+--|                    |--+--+--+--|
 8| 5| 6| 7| 8| 5          6  7  8| 5| 6| 7| 8| 5  6  7
  |--+--+--+--|                    |--+--+--+--|
 8| 5| 6| 7| 8| 5          4  1  2| 3| 4| 1| 2| 3  4  1
  |--+--+--+--|                    |--+--+--+--|
 4| 1| 2| 3| 4| 1          8  5  6| 7| 8| 5| 6| 7  8  5
  '-----------'                    '-----------'
 4  1  2  3  4  1          2  3  4  1  2  3  4  1  2  3
```

```
[4] n1+n13=n6+n10=n3+n15=n8+n12
    =n1+n1=n6+n6=n3+n3=n8+n8=cp7;
    n5+n9=n2+n14=n7+n11=n4+n16
    = n5+n5=n2+n2=n7+n7=n4+n4=cp8;     and  cp7+cp8=C;
[5] n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14
    =n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7=n8+n8=17 ...cc;
```

We need to have all these 5 groups of real complementary pairs and we have to make them appropriately located in any 'Composite & Complete' MS44 according to the five diagrams above. We must neither use any number twice or more often in any group of complementary pairs, nor un-use any number there.

We don't yet know the real sums of cp1, cp2, cp3, ... , cp7 and cp8 at the definition stage, but only know about the constant sum (=17) of the fifth group.

#4. How can we have all the real complementary pairs correctly as many as 5x8?

We have to obey such the conditions that we should use any natural number from 1 to 16 strictly once and two constant sums should add up the magic constant 34 in each group: [1] cp1+cp2=34; [2] cp3+cp4=34; [3] cp5+cp6=34; [4] cp7+cp8=34;

Suppose cp1=5 and cp2=29, for instance. How can you make any set of real complementary pairs whose sums are 5 and 29, combining any two natural numbers from 1 to 16? You can make {(1, 4), (16, 13)}, {(2, 3), (15, 14)}, {(3, 2), (14,15)} and {(4, 1), (13, 16)} for your answers. But the last halves of those 8 pairs have the same combinations with the first halves. You can only make the real pairs as many as 4 in all. They are not enough.

Suppose cp1=6 and cp2=28 next. How can you make any real complementary pairs whose sums are 6 and 28, combining any two natural numbers from 1 to 16? You can make {(1, 5), (16, 12)}, {(2, 4), (15, 13)}, {~~(4, 2)~~, ~~(13, 15)~~} and {~~(5, 1)~~, ~~(12, 16)~~} for your answers. You can really make only 4 pairs. They are not enough.

....

The next list shows the result of my calculation.

```
/CC= 5: C1( 1, 4), C2( 2, 3),
\DD=29: D1(16,13), D2(15,14),

/CC= 6: C1( 1, 5), C2( 2, 4),
\DD=28: D1(16,12), D2(15,13),
```

```
/CC= 7: C1( 1, 6), C2( 2, 5), C3( 3, 4),
\DD=27: D1(16,11), D2(15,12), D3(14,13),

/CC= 8: C1( 1, 7), C2( 2, 6), C3( 3, 5),
\DD=26: D1(16,10), D2(15,11), D3(14,12),

/CC= 9: C1( 1, 8), C2( 2, 7), C3( 3, 6), C4( 4, 5),
\DD=25: D1(16, 9), D2(15,10), D3(14,11), D4(13,12)   ... OK!

/CC=10: C1( 1, 9), C2( 3, 7), C3( 4, 6),
\DD=24: D1(16, 8), D2(14,10), D3(13,11),

/CC=11: C1( 1,10), C2( 2, 9), C3( 5, 6),
\DD=23: D1(16, 7), D2(15, 8), D3(12,11),

/CC=12: C1( 1,11), C2( 2,10), C3( 3, 9),
\DD=22: D1(16, 6), D2(15, 7), D3(14, 8),

/CC=13: C1( 1,12), C2( 2,11), C3( 3,10), C4( 4, 9),
\DD=21: D1(16, 5), D2(15, 6), D3(14, 7), D4(13, 8)   ... OK!

/CC=14: C1( 1,13), C2( 2,12), C3( 3,11),
\DD=20: D1(16, 4), D2(15, 5), D3(14, 6),

/CC=15: C1( 1,14), C2( 2,13), C3( 5,10), C4( 6, 9),
\DD=19: D1(16, 3), D2(15, 4), D3(12, 7), D4(11, 8)   ... OK!

/CC=16: C1( 1,15), C2( 3,13), C3( 5,11), C4( 7, 9),
\DD=18: D1(16, 2), D2(14, 4), D3(12, 6), D4(10, 8)   ... OK!

/CC=DD=17: CD1( 1,16), CD2( 2,15), CD3( 3,14), CD4( 4,13),
    CD5( 5,12), CD6( 6,11), CD7( 7,10), CD8( 8, 9)   ... OK!
```

The next computer program would make such a similar list of possible complementary pairs as above for pan-magic squares of order 4.

```
/** Possible Complementary Pairs **/
/**  for Pan-Magic Squares 4x4:  **/
/**  'CmpPairs4.c' Built by Kanji Setsuda **/
/** on May 16, 2007 with MacOSX & Xcode 2 **/
/**/
#include <stdio.h>
/**/
short LSM, CSM;
short ctbl[17], dtbl[17];
short uflg[17];
/**/
int maktbl(short x);
void prans(short x, short y);
/**/
/* main program */
int main(void){
 short cc, n;
  printf("\n* Possible Complementary Pairs");
  printf(" for Magic Squares of Order 4 *\n");
  LSM=34; CSM=17;
  for(cc=5;cc<=CSM;cc++){
    for(n=0;n<CSM;n++){ctbl[n]=0; dtbl[n]=0;}
    for(n=0;n<CSM;n++){uflg[n]=0;}
    n=maktbl(cc);
```

```
    prans(cc,n);
  }
  printf("\n [OK!]\n");
 return 0;
}
/**/
int maktbl(short x){
 short c, d, cn, dn;
 short m,n,p,q;
  cn=0; dn=0; m=0; p=CSM;
  c=x; d=LSM-x;
  do{m++; n=c-m;
    if((m<n)&&(uflg[m]==0)&&(uflg[n]==0)){
      ctbl[cn]=m; cn++; uflg[m]=1;
      ctbl[cn]=n; cn++; uflg[n]=1;}
    p--; q=d-p;
    if((p>q)&&(uflg[p]==0)&&(uflg[q]==0)){
      dtbl[dn]=p; dn++; uflg[p]=1;
      dtbl[dn]=q; dn++; uflg[q]=1;}
  }while((m<n)&&(p>q));
 return cn;
}
/**/
void prans(short x, short y){
 short cc, dd;
 short m,n,c,d;
  cc=x; dd=LSM-x;
  if(cc==dd){printf("\n/CC=DD=%2d:",cc);
    for(m=0;m<y;m=m+2){
      c=ctbl[m]; d=ctbl[m+1];
      if((c>0)&&(d>0)){printf(" CD%d(%2d,%2d),",m/2+1,c,d);}
      if(m%7==6){printf("\n  ");}
    }
    printf("... OK!\n");}
  else{printf("\n/CC=%2d:",cc);
    for(m=0;m<y;m=m+2){
      c=ctbl[m]; d=ctbl[m+1];
      if((c>0)&&(d>0)){printf(" C%d(%2d,%2d),",m/2+1,c,d);}
    }
    printf("\n\\DD=%2d:",dd);
    for(n=0;n<y;n=n+2){
      c=dtbl[n]; d=dtbl[n+1];
      if((c>0)&&(d>0)){printf(" D%d(%2d,%2d),",n/2+1,c,d);}
    }
    if((m==8)&&(n==8)){printf("... OK!");}
    printf("\n");}
}
/**/
```

The next 5 groups of real complementary pairs are all of our correct answers that would satisfy all the conditions we need. I could find only these 5 groups of 8 pairs in all, just as many as we need.

```
/CC= 9: C1( 1, 8), C2( 2, 7), C3( 3, 6), C4( 4, 5),
\DD=25: D1(16, 9), D2(15,10), D3(14,11), D4(13,12)   ... OK!

/CC=13: C1( 1,12), C2( 2,11), C3( 3,10), C4( 4, 9),
\DD=21: D1(16, 5), D2(15, 6), D3(14, 7), D4(13, 8)   ... OK!
```

```
/CC=15: C1( 1,14), C2( 2,13), C3( 5,10), C4( 6, 9),
\DD=19: D1(16, 3), D2(15, 4), D3(12, 7), D4(11, 8)   ... OK!

/CC=16: C1( 1,15), C2( 3,13), C3( 5,11), C4( 7, 9),
\DD=18: D1(16, 2), D2(14, 4), D3(12, 6), D4(10, 8)   ... OK!

/CC=DD=17: CD1( 1,16), CD2( 2,15), CD3( 3,14), CD4( 4,13),
    CD5( 5,12), CD6( 6,11), CD7( 7,10), CD8( 8, 9)   ... OK!
```

#5. How do we have to place all those complementary pairs in the correct positions?

It is those 8 pairs of the fifth group above that we only know how to place. Yes. We have to place them on all pan-diagonals, according to the diagram [5] below.

```
        [1]                    [2]                    [3]

 15 13 13 15 15 13      12  9 10 11 12  9      13 13 14 14 13 13
   .-----------.          .-----------.          .-----------.
  3| 1| 1| 3| 3| 1       4| 1| 2| 3| 4| 1       1| 1| 2| 2| 1| 1
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
  7| 5| 5| 7| 7| 5       4| 1| 2| 3| 4| 1       5| 5| 6| 6| 5| 5
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
 11| 9| 9|11|11| 9      12| 9|10|11|12| 9       9| 9|10|10| 9| 9
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
 15|13|13|15|15|13      12| 9|10|11|12| 9      13|13|14|14|13|13
   '-----------'          '-----------'          '-----------'
  3  1  1  3  3  1       4  1  2  3  4  1       1  1  2  2  1  1
```

```
        [4]                        [5]

  4  1  2  3  4  1      8  5  6  7  8  5  6  7  8  5
   .-----------.                .-----------.
  4| 1| 2| 3| 4| 1      2  3  4| 1| 2| 3| 4| 1  2  3
   |--+--+--+--|                |--+--+--+--|
  8| 5| 6| 7| 8| 5      6  7  8| 5| 6| 7| 8| 5  6  7
   |--+--+--+--|                |--+--+--+--|
  8| 5| 6| 7| 8| 5      4  1  2| 3| 4| 1| 2| 3  4  1
   |--+--+--+--|                |--+--+--+--|
  4| 1| 2| 3| 4| 1      8  5  6| 7| 8| 5| 6| 7  8  5
   '-----------'                '-----------'
  4  1  2  3  4  1      2  3  4  1  2  3  4  1  2  3
```

We have to place the other 4 groups of real complementary pairs [1], [2], [3] and [4] according to the position diagrams as [1], [2], [3] or [4] above.

When n1 takes the numerical value 1, the four diagrams above indicate where 1 should be placed: on n2, n5, n4 or n13, the four adjacent positions next to n1.

'1' means 8, 12, 14 or 15 as you know, when you study the list of possible complementary pairs, since the top pair of each group contains (1, 1).

```
                 1/                     2/                     3/
 11  8 10  5 11  8      13  8 10  3 13  8      13 12  6  3 13 12
   .-----------.          .-----------.          .-----------.
 14| 1|15| 4|14| 1      12| 1|15| 6|12| 1       8| 1|15|10| 8| 1
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
  7|12| 6| 9| 7|12       7|14| 4| 9| 7|14      11|14| 4| 5|11|14
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
  2|13| 3|16| 2|13       2|11| 5|16| 2|11       2| 7| 9|16| 2| 7
   |--+--+--+--|          |--+--+--+--|          |--+--+--+--|
 11| 8|10| 5|11| 8      13| 8|10| 3|13| 8      13|12| 6| 3|13|12
   '-----------'          '-----------'          '-----------'
 14  1 15  4 14  1      12  1 15  6 12  1       8  1 15 10  8  1
```

{(n1,n2),(n1,n4),(n1,n5),(n1,n13)}={(1, 8),(1,12),(1,14),(1,15)}

Watch the actual sample solutions above, please. Do you understand why 1 should be always accompanied with 8, 12, 14 and 15 on the left, right, upper or lower next?

It is because of the inner structure of pan-magic squares 4x4 and also because of the inherent properties of the serial natural numbers from 1 to 16 themselves.

#6. The next problem we must know about is how these 5 groups of complementary pairs should determine the total count of solutions.

```
When n1=1,   {n2, n4, n5, n13}={8, 12, 14, 15}.
```

This is the basic fact we can start with. How many ways can we make these four numbers {8, 12, 14, 15} put on {n2, n4, n5, n13} differently from one another?

It is the problem of permutation: ${}_4P_4$ = 4x3x2x1 = 24

On the other hand the variable n1 can take each value of {1, 2, 3, ... , 15, 16}.

```
When n1=2,   {n2, n4, n5, n13}={7, 11, 13, 16}.
When n1=3,   {n2, n4, n5, n13}={6, 10, 13, 16}.
  ....
When n1=15,  {n2, n4, n5, n13}={1, 4, 6, 10}.
When n1=16,  {n2, n4, n5, n13}={2, 3, 5, 9}.
```

Then, we could count all the possible ways of arrangement as 16x${}_4P_4$ =16x24=384.

This is actually the same with the total count of 'Primitive' solutions of this type.

Let me list out part of the primitive solutions of C&C MS44 as follows.

```
* 'Composite & Complete' Magic Squares of Order 4 *
  * List of the 384 Primitive Solutions *
          1/           2/           3/           4/           5/           6/
  1 15  4 14   1 15  4 14   1 15  6 12   1 15  6 12   1 15 10  8   1 15 10  8
 12  6  9  7   8 10  5 11  14  4  9  7   8 10  3 13  14  4  5 11  12  6  3 13
 13  3 16  2  13  3 16  2  11  5 16  2  11  5 16  2   7  9 16  2   7  9 16  2
  8 10  5 11  12  6  9  7   8 10  3 13  14  4  9  7  12  6  3 13  14  4  5 11
          7/           8/           9/          10/          11/          12/
  1 14  4 15   1 14  4 15   1 14  7 12   1 14  7 12   1 14 11  8   1 14 11  8
 12  7  9  6   8 11  5 10  15  4  9  6   8 11  2 13  15  4  5 10  12  7  2 13
 13  2 16  3  13  2 16  3  10  5 16  3  10  5 16  3   6  9 16  3   6  9 16  3
  8 11  5 10  12  7  9  6   8 11  2 13  15  4  9  6  12  7  2 13  15  4  5 10
         13/          14/          15/          16/          17/          18/
  1 12  6 15   1 12  6 15   1 12  7 14   1 12  7 14   1 12 13  8   1 12 13  8
 14  7  9  4   8 13  3 10  15  6  9  4   8 13  2 11  15  6  3 10  14  7  2 11
 11  2 16  5  11  2 16  5  10  3 16  5  10  3 16  5   4  9 16  5   4  9 16  5
  8 13  3 10  14  7  9  4   8 13  2 11  15  6  9  4  14  7  2 11  15  6  3 10
         19/          20/          21/          22/          23/          24/
  1  8 10 15   1  8 10 15   1  8 11 14   1  8 11 14   1  8 13 12   1  8 13 12
 14 11  5  4  12 13  3  6  15 10  5  4  12 13  2  7  15 10  3  6  14 11  2  7
  7  2 16  9   7  2 16  9   6  3 16  9   6  3 16  9   4  5 16  9   4  5 16  9
 12 13  3  6  14 11  5  4  12 13  2  7  15 10  5  4  14 11  2  7  15 10  3  6
         25/          26/          27/          28/          29/          30/
  2 16  3 13   2 16  3 13   2 16  5 11   2 16  5 11   2 16  9  7   2 16  9  7
 11  5 10  8   7  9  6 12  13  3 10  8   7  9  4 14  13  3  6 12  11  5  4 14
 14  4 15  1  14  4 15  1  12  6 15  1  12  6 15  1   8 10 15  1   8 10 15  1
  7  9  6 12  11  5 10  8   7  9  4 14  13  3 10  8  11  5  4 14  13  3  6 12
         31/          32/          33/          34/          35/          36/
  2 13  3 16   2 13  3 16   2 13  8 11   2 13  8 11   2 13 12  7   2 13 12  7
 11  8 10  5   7 12  6  9  16  3 10  5   7 12  1 14  16  3  6  9  11  8  1 14
 14  1 15  4  14  1 15  4   9  6 15  4   9  6 15  4   5 10 15  4   5 10 15  4
  7 12  6  9  11  8 10  5   7 12  1 14  16  3 10  5  11  8  1 14  16  3  6  9
```

```
          37/          38/          39/          40/          41/          42/
 2 11  5 16   2 11  5 16   2 11  8 13   2 11  8 13   2 11 14  7   2 11 14  7
13  8 10  3   7 14  4  9  16  5 10  3   7 14  1 12  16  5  4  9  13  8  1 12
12  1 15  6  12  1 15  6   9  4 15  6   9  4 15  6   3 10 15  6   3 10 15  6
 7 14  4  9  13  8 10  3   7 14  1 12  16  5 10  3  13  8  1 12  16  5  4  9
          43/          44/          45/          46/          47/          48/
 2  7  9 16   2  7  9 16   2  7 12 13   2  7 12 13   2  7 14 11   2  7 14 11
13 12  6  3  11 14  4  5  16  9  6  3  11 14  1  8  16  9  4  5  13 12  1  8
 8  1 15 10   8  1 15 10   5  4 15 10   5  4 15 10   3  6 15 10   3  6 15 10
11 14  4  5  13 12  6  3  11 14  1  8  16  9  6  3  13 12  1  8  16  9  4  5
          49/          50/          51/          52/          53/          54/
 3 16  2 13   3 16  2 13   3 16  5 10   3 16  5 10   3 16  9  6   3 16  9  6
10  5 11  8   6  9  7 12  13  2 11  8   6  9  4 15  13  2  7 12  10  5  4 15
15  4 14  1  15  4 14  1  12  7 14  1  12  7 14  1   8 11 14  1   8 11 14  1
 6  9  7 12  10  5 11  8   6  9  4 15  13  2 11  8  10  5  4 15  13  2  7 12

          73/          74/          75/          76/          77/          78/
 4 15  1 14   4 15  1 14   4 15  6  9   4 15  6  9   4 15 10  5   4 15 10  5
 9  6 12  7   5 10  8 11  14  1 12  7   5 10  3 16  14  1  8 11   9  6  3 16
16  3 13  2  16  3 13  2  11  8 13  2  11  8 13  2   7 12 13  2   7 12 13  2
 5 10  8 11   9  6 12  7   5 10  3 16  14  1 12  7   9  6  3 16  14  1  8 11

          97/          98/          99/         100/         101/         102/
 5 16  2 11   5 16  2 11   5 16  3 10   5 16  3 10   5 16  9  4   5 16  9  4
10  3 13  8   4  9  7 14  11  2 13  8   4  9  6 15  11  2  7 14  10  3  6 15
15  6 12  1  15  6 12  1  14  7 12  1  14  7 12  1   8 13 12  1   8 13 12  1
 4  9  7 14  10  3 13  8   4  9  6 15  11  2 13  8  10  3  6 15  11  2  7 14

         121/         122/         123/         124/         125/         126/
 6 15  1 12   6 15  1 12   6 15  4  9   6 15  4  9   6 15 10  3   6 15 10  3
 9  4 14  7   3 10  8 13  12  1 14  7   3 10  5 16  12  1  8 13   9  4  5 16
16  5 11  2  16  5 11  2  13  8 11  2  13  8 11  2   7 14 11  2   7 14 11  2
 3 10  8 13   9  4 14  7   3 10  5 16  12  1 14  7   9  4  5 16  12  1  8 13

         145/         146/         147/         148/         149/         150/
 7 14  1 12   7 14  1 12   7 14  4  9   7 14  4  9   7 14 11  2   7 14 11  2
 9  4 15  6   2 11  8 13  12  1 15  6   2 11  5 16  12  1  8 13   9  4  5 16
16  5 10  3  16  5 10  3  13  8 10  3  13  8 10  3   6 15 10  3   6 15 10  3
 2 11  8 13   9  4 15  6   2 11  5 16  12  1 15  6   9  4  5 16  12  1  8 13

         169/         170/         171/         172/         173/         174/
 8 13  2 11   8 13  2 11   8 13  3 10   8 13  3 10   8 13 12  1   8 13 12  1
10  3 16  5   1 12  7 14  11  2 16  5   1 12  6 15  11  2  7 14  10  3  6 15
15  6  9  4  15  6  9  4  14  7  9  4  14  7  9  4   5 16  9  4   5 16  9  4
 1 12  7 14  10  3 16  5   1 12  6 15  11  2 16  5  10  3  6 15  11  2  7 14

         193/         194/         195/         196/         197/         198/
 9 16  2  7   9 16  2  7   9 16  3  6   9 16  3  6   9 16  5  4   9 16  5  4
 6  3 13 12   4  5 11 14   7  2 13 12   4  5 10 15   7  2 11 14   6  3 10 15
15 10  8  1  15 10  8  1  14 11  8  1  14 11  8  1  12 13  8  1  12 13  8  1
 4  5 11 14   6  3 13 12   4  5 10 15   7  2 13 12   6  3 10 15   7  2 11 14

         217/         218/         219/         220/         221/         222/
10 15  1  8  10 15  1  8  10 15  4  5  10 15  4  5  10 15  6  3  10 15  6  3
 5  4 14 11   3  6 12 13   8  1 14 11   3  6  9 16   8  1 12 13   5  4  9 16
16  9  7  2  16  9  7  2  13 12  7  2  13 12  7  2  11 14  7  2  11 14  7  2
 3  6 12 13   5  4 14 11   3  6  9 16   8  1 14 11   5  4  9 16   8  1 12 13
```

```
 . . . . .

          373/          374/          375/          376/          377/          378/
 16  3  6  9   16  3  6  9   16  3 10  5   16  3 10  5   16  3 13  2   16  3 13  2
  5 10 15  4    2 13 12  7    9  6 15  4    2 13  8 11    9  6 12  7    5 10  8 11
 11  8  1 14   11  8  1 14    7 12  1 14    7 12  1 14    4 15  1 14    4 15  1 14
  2 13 12  7    5 10 15  4    2 13  8 11    9  6 15  4    5 10  8 11    9  6 12  7
          379/          380/          381/          382/          383/          384/
 16  2  7  9   16  2  7  9   16  2 11  5   16  2 11  5   16  2 13  3   16  2 13  3
  5 11 14  4    3 13 12  6    9  7 14  4    3 13  8 10    9  7 12  6    5 11  8 10
 10  8  1 15   10  8  1 15    6 12  1 15    6 12  1 15    4 14  1 15    4 14  1 15
  3 13 12  6    5 11 14  4    3 13  8 10    9  7 14  4    5 11  8 10    9  7 12  6
[Count = 384]  OK!
```

Do these two counts 384 mean an accidental coincidence?

No. It is necessary that those 5 groups of complementary pairs should make those primitive solutions as many as 384. $16 \times {}_4P_4 = 384$ must determine them directly.

The total count 48 of 'Standard solutions' is determined by the choice out of the 'Primitive solutions' with the following list-forming inequality conditions.

Let me show you how to get the list of 'Standard solutions' of 'C&C' PMS44.

```
/** 'Composite & Complete' Pan-Magic Squares of Order 4 **/
/** 'CCMS44S.c' built by Kanji Setsuda **/
/** on Dec.18, 2004; May.16, 2007; **/
/** Worked on MacOSX and Xcode 2.2 **/
/*
** Basic Forms: **
 14 15 16 13 14 15 16 13 14 15
           .--.--.--.--.                 .--.--.--.--.
  2  3  4|  1|  2|  3|  4|  1  2  3       |  1|15|  4|14|
           |--+--+--+--|                 |--+--+--+--|
  6  7  8|  5|  6|  7|  8|  5  6  7       |12|  6|  9|  7|
           |--+--+--+--|                 |--+--+--+--|
 10 11 12|  9|10|11|12|  9 10 11         |13|  3|16|  2|
           |--+--+--+--|                 |--+--+--+--|
 14 15 16|13|14|15|16|13 14 15           |  8|10|  5|11|
           '--'--'--'--'                 '--'--'--'--'
  2  3  4  1  2  3  4  1  2  3
** Basic Conditions: **
  n1+n2+n3+n4=C       ...rw1; | n1+n5+n9+n13=C     ...cl1;
** Composite Conditions: **
  n1+n2+n5+n6=C       ...c1;  | n2+n3+n6+n7=C      ...c2;
  n3+n4+n7+n8=C       ...c3;  | n4+n1+n8+n5=C      ...c4;
  n5+n6+n9+n10=C      ...c5;  | n6+n7+n10+n11=C    ...c6;
  n7+n8+n11+n12=C     ...c7;  | n8+n5+n12+n9=C     ...c8;
  n9+n10+n13+n14=C    ...c9;  | n10+n11+n14+n15=C  ...c10;
  n11+n12+n15+n16=C   ...c11; | n12+n9+n16+n13=C   ...c12;
  n13+n14+n1+n2=C     ...c13; | n14+n15+n2+n3=C    ...c14;
  n15+n16+n3+n4=C     ...c15; | n16+n13+n4+n1=C    ...c16;
** Complete Conditions: **
  n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC   ...cc
** List-forming Inequality Conditions: **
  n1<n16, n1<n4, n1<n13, and n2>n5;    ...ic
*/
/**/
#include <stdio.h>
/**/
```

```
short cnt, cnt2;
short LSM, PSM;
short nm[17], uflg[17];
short anm[9][17];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void ansprint(void);
void printsol(short x);
/**/
/* Main Program */
int main(){
 short n;
  printf("\n* 'Composite & Complete' Pan-magic Squares of Order 4 *\n");
  printf("   ** List of the 48 Standard Solutions **\n");
  for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
  LSM=34; PSM=17; cnt=0; cnt2=0;
  stp01();/* Begin the Calculations */
  if(cnt2>0){printsol(cnt2);}
  printf(" [Count = %d]  OK!\n",cnt);
 return 0;
}
/* Begin the Calculations */
/* Search Level: 1 */
/* Set n1 & n11 */
void stp01(){
 short a,b;
  for(a=1;a<17;a++){b=PSM-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      uflg[a]=1; uflg[b]=1;
      nm[1]=a; nm[11]=b;
      stp02();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n2 & n12 */
void stp02(){
 short a,b;
  for(a=16;a>0;a--){b=PSM-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      uflg[a]=1; uflg[b]=1;
      nm[2]=a; nm[12]=b;
      stp03();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set N4(>n1) & n10 */
void stp03(){
 short a,b;
  for(a=16;a>nm[1];a--){b=PSM-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      uflg[a]=1; uflg[b]=1;
      nm[4]=a; nm[10]=b;
      stp04();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n3=LSM-n1-n2-n4 & n9 */
void stp04(){
```

```
 short a,b;
  a=LSM-nm[1]-nm[2]-nm[4];
  if((0<a)&&(a<17)){
    b=LSM-nm[10]-nm[11]-nm[12];
    if(a+b==PSM){
      if((uflg[a]==0)&&(uflg[b]==0)){
        uflg[a]=1; uflg[b]=1;
        nm[3]=a; nm[9]=b;
        stp05();
        uflg[b]=0; uflg[a]=0;}}}
}
/* Search Level: 2 */
/* Set N5(<n2) & n15 */
void stp05(){
 short a,b;
  for(a=nm[2]-1;a>0;a--){b=PSM-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      uflg[a]=1; uflg[b]=1;
      nm[5]=a; nm[15]=b;
      stp06();
      uflg[b]=0; uflg[a]=0;}
  }
}
/* Set n6=LSM-n1-n2-n5 & n16(>n1) */
void stp06(){
 short a,b,c,d;
  a=LSM-nm[1]-nm[2]-nm[5];
  if((0<a)&&(a<17)){
    b=LSM-nm[11]-nm[12]-nm[15];
    if((a+b==PSM)&&(b>nm[1])){
      c=LSM-nm[5]-nm[9]-nm[10];
      d=LSM-nm[15]-nm[3]-nm[4];
      if((a==c)&&(b==d)){
       if((uflg[a]==0)&&(uflg[b]==0)){
        uflg[a]=1; uflg[b]=1;
        nm[6]=a; nm[16]=b;
        stp07();
        uflg[b]=0; uflg[a]=0;}}}}
}
/* Set n7=LSM-n2-n3-n6 & n13(>n1) */
void stp07(){
 short a,b,c,d,e;
  a=LSM-nm[2]-nm[3]-nm[6];
  if((0<a)&&(a<17)){
    b=LSM-nm[12]-nm[9]-nm[16];
    if((a+b==PSM)&&(b>nm[1])){
      c=LSM-nm[6]-nm[10]-nm[11];
      d=LSM-nm[16]-nm[4]-nm[1];
      e=LSM-nm[1]-nm[5]-nm[9];
      if((a==c)&&(b==d)&&(b==e)){
       if((uflg[a]==0)&&(uflg[b]==0)){
        uflg[a]=1; uflg[b]=1;
        nm[7]=a; nm[13]=b;
        stp08();
        uflg[b]=0; uflg[a]=0;}}}}
}
/* Set n8=LSM-n3-n4-n7 & n14 */
void stp08(){
 short a,b,c,d,e,f,g,h;
```

```
  a=LSM-nm[3]-nm[4]-nm[7];
  if((0<a)&&(a<17)){
    b=LSM-nm[9]-nm[10]-nm[13];
    if(a+b==PSM){
      c=LSM-nm[4]-nm[1]-nm[5];
      d=LSM-nm[7]-nm[11]-nm[12];
      e=LSM-nm[5]-nm[12]-nm[9];
      if((a==c)&&(a==d)&&(a==e)){
        f=LSM-nm[10]-nm[11]-nm[15];
        g=LSM-nm[13]-nm[1]-nm[2];
        h=LSM-nm[15]-nm[2]-nm[3];
        if((b==f)&&(b==g)&&(b==h)){
         if((uflg[a]==0)&&(uflg[b]==0)){
          uflg[a]=1; uflg[b]=1;
          nm[8]=a; nm[14]=b;
          ansprint();
          uflg[b]=0; uflg[a]=0;}}}}}
}
/**/
/* Print the Answers */
void ansprint(){
 short n;
  cnt++;
  anm[cnt2][0]=cnt;
  for(n=1;n<17;n++){anm[cnt2][n]=nm[n];}
  cnt2++; if(cnt2==6){printsol(cnt2); cnt2=0;}
}
/* Print the Solutions */
void printsol(short x){
 short l,l4,m,n;
  for(m=0;m<x;m++){printf("%12d/",anm[m][0]);}
  printf("\n");
  for(l=0;l<4;l++){l4=l*4;
    for(m=0;m<x;m++){printf(" ");
      for(n=1;n<5;n++){printf("%3d",anm[m][l4+n]);}
    }
    printf("\n");
  }
}
/**/
```

```
* 'Composite & Complete' Pan-magic Squares of Order 4 *
    ** List of the 48 Standard Solutions **
           1/           2/           3/           4/           5/           6/
   1 15  4 14   1 15  4 14   1 15  6 12   1 15  6 12   1 15 10  8   1 15 10  8
  12  6  9  7   8 10  5 11  14  4  9  7   8 10  3 13  14  4  5 11  12  6  3 13
  13  3 16  2  13  3 16  2  11  5 16  2  11  5 16  2   7  9 16  2   7  9 16  2
   8 10  5 11  12  6  9  7   8 10  3 13  14  4  9  7  12  6  3 13  14  4  5 11
           7/           8/           9/          10/          11/          12/
   1 14  4 15   1 14  4 15   1 14  7 12   1 14 11  8   1 12  6 15   1 12  7 14
  12  7  9  6   8 11  5 10   8 11  2 13  12  7  2 13   8 13  3 10   8 13  2 11
  13  2 16  3  13  2 16  3  10  5 16  3   6  9 16  3  11  2 16  5  10  3 16  5
   8 11  5 10  12  7  9  6  15  4  9  6  15  4  5 10  14  7  9  4  15  6  9  4
          13/          14/          15/          16/          17/          18/
   2 16  3 13   2 16  3 13   2 16  5 11   2 16  5 11   2 16  9  7   2 16  9  7
  11  5 10  8   7  9  6 12  13  3 10  8   7  9  4 14  13  3  6 12  11  5  4 14
  14  4 15  1  14  4 15  1  12  6 15  1  12  6 15  1   8 10 15  1   8 10 15  1
   7  9  6 12  11  5 10  8   7  9  4 14  13  3 10  8  11  5  4 14  13  3  6 12
```

```
          19/          20/          21/          22/          23/          24/
  2 13  3 16   2 13  3 16   2 13  8 11   2 13 12  7   2 11  5 16   2 11  8 13
 11  8 10  5   7 12  6  9   7 12  1 14  11  8  1 14   7 14  4  9   7 14  1 12
 14  1 15  4  14  1 15  4   9  6 15  4   5 10 15  4  12  1 15  6   9  4 15  6
  7 12  6  9  11  8 10  5  16  3 10  5  16  3  6  9  13  8 10  3  16  5 10  3
          25/          26/          27/          28/          29/          30/
  3 16  2 13   3 16  2 13   3 16  5 10   3 16  5 10   3 16  9  6   3 16  9  6
 10  5 11  8   6  9  7 12  13  2 11  8   6  9  4 15  13  2  7 12  10  5  4 15
 15  4 14  1  15  4 14  1  12  7 14  1  12  7 14  1   8 11 14  1   8 11 14  1
  6  9  7 12  10  5 11  8   6  9  4 15  13  2 11  8  10  5  4 15  13  2  7 12
          31/          32/          33/          34/          35/          36/
  3 13  2 16   3 13  2 16   3 13  8 10   3 13 12  6   4 15  1 14   4 15  1 14
 10  8 11  5   6 12  7  9   6 12  1 15  10  8  1 15   9  6 12  7   5 10  8 11
 15  1 14  4  15  1 14  4   9  7 14  4   5 11 14  4  16  3 13  2  16  3 13  2
  6 12  7  9  10  8 11  5  16  2 11  5  16  2  7  9   5 10  8 11   9  6 12  7
          37/          38/          39/          40/          41/          42/
  4 15  6  9   4 15  6  9   4 15 10  5   4 15 10  5   4 14  1 15   4 14  1 15
 14  1 12  7   5 10  3 16  14  1  8 11   9  6  3 16   9  7 12  6   5 11  8 10
 11  8 13  2  11  8 13  2   7 12 13  2   7 12 13  2  16  2 13  3  16  2 13  3
  5 10  3 16  14  1 12  7   9  6  3 16  14  1  8 11   5 11  8 10   9  7 12  6
          43/          44/          45/          46/          47/          48/
  4 14  7  9   4 14 11  5   5 16  2 11   5 16  3 10   6 15  1 12   6 15  4  9
  5 11  2 16   9  7  2 16   4  9  7 14   4  9  6 15   3 10  8 13   3 10  5 16
 10  8 13  3   6 12 13  3  15  6 12  1  14  7 12  1  16  5 11  2  13  8 11  2
 15  1 12  6  15  1  8 10  10  3 13  8  11  2 13  8   9  4 14  7  12  1 14  7
 [Count = 48]  OK!
```

In order to pick up the 3 'Fundamental' solutions out of the 384 'Primitive' ones, we usually take the next conditions: n1=1; n2>n5, n2>n4 and n5>n13

As we know the solution count is 24 when n1 takes the value 1, we could only get the next 3 solutions for the Fundamental: 24÷2÷2÷2=3

```
** The Fundamental Three of 'Composite and Complete' PMS44 **
                    1/                2/                3/
  11  8 10  5 11  8   13  8 10  3 13  8   13 12  6  3 13 12
     .--.--.--.--.       .--.--.--.--.       .--.--.--.--.
  14|  1|15|  4|14|  1  12|  1|15|  6|12|  1   8|  1|15|10|  8|  1
    |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
   7|12|  6|  9|  7|12   7|14|  4|  9|  7|14  11|14|  4|  5|11|14
    |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
   2|13|  3|16|  2|13   2|11|  5|16|  2|11   2|  7|  9|16|  2|  7
    |--+--+--+--|       |--+--+--+--|       |--+--+--+--|
  11|  8|10|  5|11|  8  13|  8|10|  3|13|  8  13|12|  6|  3|13|12
     '--'--'--'--'       '--'--'--'--'       '--'--'--'--'
  14  1 15  4 14  1   12  1 15  6 12  1    8  1 15 10  8  1

 [Count = 3]  OK!
```

Thus we now understand what properties make everything as they are.

It is the inner structure of 'C&C' pan-magic squares of order 4 and also the inherent properties of the serial natural numbers from 1 to 16 that should produce all types of pandiagonal magic squares of order 4 in the same form.

#7. Can we apply this method to any objects of higher orders such as 'C&C' PMS88?

Can we get any meaningful groups of complementary pairs and make them explain

about the special structure of 'C&C' PMS88 and the total solution-count of them?
It seems to be more difficult, for we can get 7 groups of 32 complementary pairs, while we can only have the four adjacent positions next to n1.

** Basic Form for 'C&C' Pan-Magic Squares of Order 8 **

```
 61 62 63 64 57 58 59 60 61 62 63 64 57 58 59 60
             .--.--.--.--.--.--.--.--.
  5  6  7  8| 1| 2| 3| 4| 5| 6| 7| 8| 1  2  3  4
            |--+--+--+--+--+--+--+--|
 13 14 15 16| 9|10|11|12|13|14|15|16| 9 10 11 12
            |--+--+--+--+--+--+--+--|
 21 22 23 24|17|18|19|20|21|22|23|24|17 18 19 20
            |--+--+--+--+--+--+--+--|
 29 30 31 32|25|26|27|28|29|30|31|32|25 26 27 28
            |--+--+--+--+--+--+--+--|
 37 38 39 40|33|34|35|36|37|38|39|40|33 34 35 36
            |--+--+--+--+--+--+--+--|
 45 46 47 48|41|42|43|44|45|46|47|48|41 42 43 44
            |--+--+--+--+--+--+--+--|
 53 54 55 56|49|50|51|52|53|54|55|56|49 50 51 52
            |--+--+--+--+--+--+--+--|
 61 62 63 64|57|58|59|60|61|62|63|64|57 58 59 60
            '--'--'--'--'--'--'--'--'
  5  6  7  8  1  2  3  4  5  6  7  8  1  2  3  4
```

** Composite Conditions: SS=130 **

```
 n1+n2+n9+n10=SS     ...c01 | n22+n23+n30+n31=SS  ...c22 | n43+n44+n51+n52=SS ...c43
 n2+n3+n10+n11=SS    ...c02 | n23+n24+n31+n32=SS  ...c23 | n44+n45+n52+n53=SS ...c44
 n3+n4+n11+n12=SS    ...c03 | n24+n17+n32+n25=SS  ...c24 | n45+n46+n53+n54=SS ...c45
 n4+n5+n12+n13=SS    ...c04 | n25+n26+n33+n34=SS  ...c25 | n46+n47+n54+n55=SS ...c46
 n5+n6+n13+n14=SS    ...c05 | n26+n27+n34+n35=SS  ...c26 | n47+n48+n55+n56=SS ...c47
 n6+n7+n14+n15=SS    ...c06 | n27+n28+n35+n36=SS  ...c27 | n48+n41+n56+n49=SS ...c48
 n7+n8+n15+n16=SS    ...c07 | n28+n29+n36+n37=SS  ...c28 | n49+n50+n57+n58=SS ...c49
 n8+n1+n16+n9=SS     ...c08 | n29+n30+n37+n38=SS  ...c29 | n50+n51+n58+n59=SS ...c50
 n9+n10+n17+n18=SS   ...c09 | n30+n31+n38+n39=SS  ...c30 | n51+n52+n59+n60=SS ...c51
 n10+n11+n18+n19=SS  ...c10 | n31+n32+n39+n40=SS  ...c31 | n52+n53+n60+n61=SS ...c52
 n11+n12+n19+n20=SS  ...c11 | n32+n25+n40+n33=SS  ...c32 | n53+n54+n61+n62=SS ...c53
 n12+n13+n20+n21=SS  ...c12 | n33+n34+n41+n42=SS  ...c33 | n54+n55+n62+n63=SS ...c54
 n13+n14+n21+n22=SS  ...c13 | n34+n35+n42+n43=SS  ...c34 | n55+n56+n63+n64=SS ...c55
 n14+n15+n22+n23=SS  ...c14 | n35+n36+n43+n44=SS  ...c35 | n56+n49+n64+n57=SS ...c56
 n15+n16+n23+n24=SS  ...c15 | n36+n37+n44+n45=SS  ...c36 | n57+n58+n1+n2=SS   ...c57
 n16+n9+n24+n17=SS   ...c16 | n37+n38+n45+n46=SS  ...c37 | n58+n59+n2+n3=SS   ...c58
 n17+n18+n25+n26=SS  ...c17 | n38+n39+n46+n47=SS  ...c38 | n59+n60+n3+n4=SS   ...c59
 n18+n19+n26+n27=SS  ...c18 | n39+n40+n47+n48=SS  ...c39 | n60+n61+n4+n5=SS   ...c60
 n19+n20+n27+n28=SS  ...c19 | n40+n33+n48+n41=SS  ...c40 | n61+n62+n5+n6=SS   ...c61
 n20+n21+n28+n29=SS  ...c20 | n41+n42+n49+n50=SS  ...c41 | n62+n63+n6+n7=SS   ...c62
 n21+n22+n29+n30=SS  ...c21 | n42+n43+n50+n51=SS  ...c42 | n63+n64+n7+n8=SS   ...c63
and  n64+n57+n8+n1=SS     ...c64
```

* List of Possible Complementary Pairs: CC+DD=130 *

```
/CC=31: (1,30), (2,29), (3,28), (4,27), (5,26), (6,25), (7,24), (8,23),
     (9,22), (10,21), (11,20), (12,19), (13,18), (14,17), (15,16),
\DD=99: (64,35), (63,36), (62,37), (61,38), (60,39), (59,40), (58,41), (57,42),
     (56,43), (55,44), (54,45), (53,46), (52,47), (51,48), (50,49),

/CC=32: (1,31), (2,30), (3,29), (4,28), (5,27), (6,26), (7,25), (8,24),
     (9,23), (10,22), (11,21), (12,20), (13,19), (14,18), (15,17),
\DD=98: (64,34), (63,35), (62,36), (61,37), (60,38), (59,39), (58,40), (57,41),
```

```
     (56,42), (55,43), (54,44), (53,45), (52,46), (51,47), (50,48),

/CC=33: (1,32), (2,31), (3,30), (4,29), (5,28), (6,27), (7,26), (8,25),
     (9,24), (10,23), (11,22), (12,21), (13,20), (14,19), (15,18), (16,17),
\DD=97: (64,33), (63,34), (62,35), (61,36), (60,37), (59,38), (58,39), (57,40),
     (56,41), (55,42), (54,43), (53,44), (52,45), (51,46), (50,47), (49,48)  ...OK!

/CC=34: (1,33), (3,31), (4,30), (5,29), (6,28), (7,27), (8,26), (9,25),
     (10,24), (11,23), (12,22), (13,21), (14,20), (15,19), (16,18),
\DD=96: (64,32), (62,34), (61,35), (60,36), (59,37), (58,38), (57,39), (56,40),
     (55,41), (54,42), (53,43), (52,44), (51,45), (50,46), (49,47),

/CC=35: (1,34), (2,33), (5,30), (6,29), (7,28), (8,27), (9,26), (10,25),
     (11,24), (12,23), (13,22), (14,21), (15,20), (16,19), (17,18),
\DD=95: (64,31), (63,32), (60,35), (59,36), (58,37), (57,38), (56,39), (55,40),
     (54,41), (53,42), (52,43), (51,44), (50,45), (49,46), (48,47),


/CC=47: (1,46), (2,45), (3,44), (4,43), (5,42), (6,41), (7,40), (8,39),
     (9,38), (10,37), (11,36), (12,35), (13,34), (14,33),
\DD=83: (64,19), (63,20), (62,21), (61,22), (60,23), (59,24), (58,25), (57,26),
     (56,27), (55,28), (54,29), (53,30), (52,31), (51,32),

/CC=48: (1,47), (2,46), (3,45), (4,44), (5,43), (6,42), (7,41), (8,40),
     (9,39), (10,38), (11,37), (12,36), (13,35), (14,34), (15,33),
\DD=82: (64,18), (63,19), (62,20), (61,21), (60,22), (59,23), (58,24), (57,25),
     (56,26), (55,27), (54,28), (53,29), (52,30), (51,31), (50,32),

/CC=49: (1,48), (2,47), (3,46), (4,45), (5,44), (6,43), (7,42), (8,41),
     (9,40), (10,39), (11,38), (12,37), (13,36), (14,35), (15,34), (16,33),
\DD=81: (64,17), (63,18), (62,19), (61,20), (60,21), (59,22), (58,23), (57,24),
     (56,25), (55,26), (54,27), (53,28), (52,29), (51,30), (50,31), (49,32)  ...OK!

/CC=50: (1,49), (2,48), (3,47), (4,46), (5,45), (6,44), (7,43), (8,42),
     (9,41), (10,40), (11,39), (12,38), (13,37), (14,36), (15,35),
\DD=80: (64,16), (63,17), (62,18), (61,19), (60,20), (59,21), (58,22), (57,23),
     (56,24), (55,25), (54,26), (53,27), (52,28), (51,29), (50,30),

/CC=51: (1,50), (2,49), (3,48), (4,47), (5,46), (6,45), (7,44), (8,43),
     (9,42), (10,41), (11,40), (12,39), (13,38), (14,37),
\DD=79: (64,15), (63,16), (62,17), (61,18), (60,19), (59,20), (58,21), (57,22),
     (56,23), (55,24), (54,25), (53,26), (52,27), (51,28),


/CC=55: (1,54), (2,53), (3,52), (4,51), (5,50), (6,49), (7,48), (8,47),
     (9,46), (10,45), (21,34), (22,33), (25,30), (26,29), (27,28),
\DD=75: (64,11), (63,12), (62,13), (61,14), (60,15), (59,16), (58,17), (57,18),
     (56,19), (55,20), (44,31), (43,32), (40,35), (39,36), (38,37),

/CC=56: (1,55), (2,54), (3,53), (4,52), (5,51), (6,50), (7,49), (8,48),
     (9,47), (19,37), (20,36), (21,35), (22,34), (23,33),
\DD=74: (64,10), (63,11), (62,12), (61,13), (60,14), (59,15), (58,16), (57,17),
     (56,18), (46,28), (45,29), (44,30), (43,31), (42,32),

/CC=57: (1,56), (2,55), (3,54), (4,53), (5,52), (6,51), (7,50), (8,49),
     (17,40), (18,39), (19,38), (20,37), (21,36), (22,35), (23,34), (24,33),
\DD=73: (64,9), (63,10), (62,11), (61,12), (60,13), (59,14), (58,15), (57,16),
     (48,25), (47,26), (46,27), (45,28), (44,29), (43,30), (42,31), (41,32)  ...OK!

/CC=58: (1,57), (2,56), (3,55), (4,54), (5,53), (6,52), (7,51), (15,43),
     (16,42), (17,41), (18,40), (19,39), (20,38), (21,37),
\DD=72: (64,8), (63,9), (62,10), (61,11), (60,12), (59,13), (58,14), (50,22),
     (49,23), (48,24), (47,25), (46,26), (45,27), (44,28),
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/CC=59: (1,58), (2,57), (3,56), (4,55), (5,54), (6,53), (13,46), (14,45),
      (15,44), (16,43), (17,42), (18,41), (25,34), (26,33), (29,30),
\DD=71: (64,7), (63,8), (62,9), (61,10), (60,11), (59,12), (52,19), (51,20),
      (50,21), (49,22), (48,23), (47,24), (40,31), (39,32), (36,35),

/CC=60: (1,59), (2,58), (3,57), (4,56), (5,55), (11,49), (12,48), (13,47),
      (14,46), (15,45), (21,39), (22,38), (23,37), (24,36), (25,35),
\DD=70: (64,6), (63,7), (62,8), (61,9), (60,10), (54,16), (53,17), (52,18),
      (51,19), (50,20), (44,26), (43,27), (42,28), (41,29), (40,30),

/CC=61: (1,60), (2,59), (3,58), (4,57), (9,52), (10,51), (11,50), (12,49),
      (17,44), (18,43), (19,42), (20,41), (25,36), (26,35), (27,34), (28,33),
\DD=69: (64,5), (63,6), (62,7), (61,8), (56,13), (55,14), (54,15), (53,16),
      (48,21), (47,22), (46,23), (45,24), (40,29), (39,30), (38,31), (37,32)  ...OK!

/CC=62: (1,61), (2,60), (3,59), (7,55), (8,54), (9,53), (13,49), (14,48),
      (15,47), (19,43), (20,42), (21,41), (25,37), (26,36), (27,35),
\DD=68: (64,4), (63,5), (62,6), (58,10), (57,11), (56,12), (52,16), (51,17),
      (50,18), (46,22), (45,23), (44,24), (40,28), (39,29), (38,30),

/CC=63: (1,62), (2,61), (5,58), (6,57), (9,54), (10,53), (13,50), (14,49),
      (17,46), (18,45), (21,42), (22,41), (25,38), (26,37), (29,34), (30,33),
\DD=67: (64,3), (63,4), (60,7), (59,8), (56,11), (55,12), (52,15), (51,16),
      (48,19), (47,20), (44,23), (43,24), (40,27), (39,28), (36,31), (35,32)  ...OK!

/CC=64: (1,63), (3,61), (5,59), (7,57), (9,55), (11,53), (13,51), (15,49),
      (17,47), (19,45), (21,43), (23,41), (25,39), (27,37), (29,35), (31,33),
\DD=66: (64,2), (62,4), (60,6), (58,8), (56,10), (54,12), (52,14), (50,16),
      (48,18), (46,20), (44,22), (42,24), (40,26), (38,28), (36,30), (34,32)  ...OK!

/CC=DD=65: (1,64), (2,63), (3,62), (4,61), (5,60), (6,59), (7,58), (8,57),
      (9,56), (10,55), (11,54), (12,53), (13,52), (14,51), (15,50), (16,49),
      (17,48), (18,47), (19,46), (20,45), (21,44), (22,43), (23,42), (24,41),
      (25,40), (26,39), (27,38), (28,37), (29,36), (30,35), (31,34), (32,33)  ...OK!
```

But it seems to be hopeful in the case of 'C&C' Pan-magic Cubes of order 4. See the next diagram. You could find six adjacent positions next to n1, couldn't you? You could also put '<u>n1</u>' (Complementary pair of n1) in n43.

```
* Basic Diagrams for Extended Space of Pan-triagonal Magic Cube 444 *
64            61            62            63            64            61
  52            49            50            51            52            49
    56           |53            54            55            56           |53
16    60      13 |  57      14    58      15    59      16    60      13 |  57
   4----64------ 1----61------ 2----62------ 3----63------ 4----64------ 1----61
     8    52     | 5    49       6    50       7    51     | 8    52     | 5    49
32    12      29 |   9      30    10      31    11      32 |  12      29 |   9
  20    16      17    13------18----14------19----15------20----16      17    13
    24     4     |21   | 1      22     2      23     3     |24   | 4      21     1
48    28      45 |  25 |    46    26      47    27      48 |  28 |    45    25
  36    32      33    29      34    30      35    31      36    32      33    29
    40    20     |37   |17      38    18      39    19     |40   |20      37    17
64    44      61 |  41 |    62    42      63    43      64 |  44 |    61    41
  52    48      49----45------50----46------51----47------52    48      49    45
    56    36     |53   |33      54    34      55    35      56   |36      53    33
16    60      13 |  57 |    14    58      15    59      16    60 |    13    57
   4    64        1    61------ 2----62------ 3----63------ 4----64        1    61
     8    52        5    49       6    50       7    51       8    52        5    49
      12             9            10            11            12             9
        16            13            14            15            16            13
           4             1             2             3             4             1

* Composite Conditions: C=130 *
```

n1+n2+n5+n6=C ...c1a;	n1+n2+n17+n18=C ...c1b;	n1+n5+n17+n21=C ...c1c;
n2+n3+n6+n7=C ...c2a;	n2+n3+n18+n19=C ...c2b;	n2+n6+n18+n22=C ...c2c;
n3+n4+n7+n8=C ...c3a;	n3+n4+n19+n20=C ...c3b;	n3+n7+n19+n23=C ...c3c;
n4+n1+n8+n5=C ...c4a;	n4+n1+n20+n17=C ...c4b;	n4+n8+n20+n24=C ...c4c;
n5+n6+n9+n10=C ...c5a;	n5+n6+n21+n22=C ...c5b;	n5+n9+n21+n25=C ...c5c;
n6+n7+n10+n11=C ...c6a;	n6+n7+n22+n23=C ...c6b;	n6+n10+n22+n26=C ...c6c;
n7+n8+n11+n12=C ...c7a;	n7+n8+n23+n24=C ...c7b;	n7+n11+n23+n27=C ...c7c;
n8+n5+n12+n9=C ...c8a;	n8+n5+n24+n21=C ...c8b;	n8+n12+n24+n28=C ...c8c;
n9+n10+n13+n14=C ...c9a;	n9+n10+n25+n26=C ...c9b;	n9+n13+n25+n29=C ...c9c;
n10+n11+n14+n15=C ...c10a;	n10+n11+n26+n27=C ...c10b;	n10+n14+n26+n30=C ...c10c;
n11+n12+n15+n16=C ...c11a;	n11+n12+n27+n28=C ...c11b;	n11+n15+n27+n31=C ...c11c;
n12+n9+n16+n13=C ...c12a;	n12+n9+n28+n25=C ...c12b;	n12+n16+n28+n32=C ...c12c;
n13+n14+n1+n2=C ...c13a;	n13+n14+n29+n30=C ...c13b;	n13+n1+n29+n17=C ...c13c;
n14+n15+n2+n3=C ...c14a;	n14+n15+n30+n31=C ...c14b;	n14+n2+n30+n18=C ...c14c;
n15+n16+n3+n4=C ...c15a;	n15+n16+n31+n32=C ...c15b;	n15+n3+n31+n19=C ...c15c;
n16+n13+n4+n1=C ...c16a;	n16+n13+n32+n29=C ...c16b;	n16+n4+n32+n20=C ...c16c;
n17+n18+n21+n22=C ...c17a;	n17+n18+n33+n34=C ...c17b;	n17+n21+n33+n37=C ...c17c;
n18+n19+n22+n23=C ...c18a;	n18+n19+n34+n35=C ...c18b;	n18+n22+n34+n38=C ...c18c;
n19+n20+n23+n24=C ...c19a;	n19+n20+n35+n36=C ...c19b;	n19+n23+n35+n39=C ...c19c;
n20+n17+n24+n21=C ...c20a;	n20+n17+n36+n33=C ...c20b;	n20+n24+n36+n40=C ...c20c;
n21+n22+n25+n26=C ...c21a;	n21+n22+n37+n38=C ...c21b;	n21+n25+n37+n41=C ...c21c;
n22+n23+n26+n27=C ...c22a;	n22+n23+n38+n39=C ...c22b;	n22+n26+n38+n42=C ...c22c;
n23+n24+n27+n28=C ...c23a;	n23+n24+n39+n40=C ...c23b;	n23+n27+n39+n43=C ...c23c;
n24+n21+n28+n25=C ...c24a;	n24+n21+n40+n37=C ...c24b;	n24+n28+n40+n44=C ...c24c;
n25+n26+n29+n30=C ...c25a;	n25+n26+n41+n42=C ...c25b;	n25+n29+n41+n45=C ...c25c;
n26+n27+n30+n31=C ...c26a;	n26+n27+n42+n43=C ...c26b;	n26+n30+n42+n46=C ...c26c;
n27+n28+n31+n32=C ...c27a;	n27+n28+n43+n44=C ...c27b;	n27+n31+n43+n47=C ...c27c;
n28+n25+n32+n29=C ...c28a;	n28+n25+n44+n41=C ...c28b;	n28+n32+n44+n48=C ...c28c;
n29+n30+n17+n18=C ...c29a;	n29+n30+n45+n46=C ...c29b;	n29+n17+n45+n33=C ...c29c;
n30+n31+n18+n19=C ...c30a;	n30+n31+n46+n47=C ...c30b;	n30+n18+n46+n34=C ...c30c;
n31+n32+n19+n20=C ...c31a;	n31+n32+n47+n48=C ...c31b;	n31+n19+n47+n35=C ...c31c;
n32+n29+n20+n17=C ...c32a;	n32+n29+n48+n45=C ...c32b;	n32+n20+n48+n36=C ...c32c;
n33+n34+n37+n38=C ...c33a;	n33+n34+n49+n50=C ...c33b;	n33+n37+n49+n53=C ...c33c;
n34+n35+n38+n39=C ...c34a;	n34+n35+n50+n51=C ...c34b;	n34+n38+n50+n54=C ...c34c;
n35+n36+n39+n40=C ...c35a;	n35+n36+n51+n52=C ...c35b;	n35+n39+n51+n55=C ...c35c;
n36+n33+n40+n37=C ...c36a;	n36+n33+n52+n49=C ...c36b;	n36+n40+n52+n56=C ...c36c;
n37+n38+n41+n42=C ...c37a;	n37+n38+n53+n54=C ...c37b;	n37+n41+n53+n57=C ...c37c;
n38+n39+n42+n43=C ...c38a;	n38+n39+n54+n55=C ...c38b;	n38+n42+n54+n58=C ...c38c;
n39+n40+n43+n44=C ...c39a;	n39+n40+n55+n56=C ...c39b;	n39+n43+n55+n59=C ...c39c;
n40+n37+n44+n41=C ...c40a;	n40+n37+n56+n53=C ...c40b;	n40+n44+n56+n60=C ...c40c;
n41+n42+n45+n46=C ...c41a;	n41+n42+n57+n58=C ...c41b;	n41+n45+n57+n61=C ...c41c;
n42+n43+n46+n47=C ...c42a;	n42+n43+n58+n59=C ...c42b;	n42+n46+n58+n62=C ...c42c;
n43+n44+n47+n48=C ...c43a;	n43+n44+n59+n60=C ...c43b;	n43+n47+n59+n63=C ...c43c;
n44+n41+n48+n45=C ...c44a;	n44+n41+n60+n57=C ...c44b;	n44+n48+n60+n64=C ...c44c;
n45+n46+n33+n34=C ...c45a;	n45+n46+n61+n62=C ...c45b;	n45+n33+n61+n49=C ...c45c;
n46+n47+n34+n35=C ...c46a;	n46+n47+n62+n63=C ...c46b;	n46+n34+n62+n50=C ...c46c;
n47+n48+n35+n36=C ...c47a;	n47+n48+n63+n64=C ...c47b;	n47+n35+n63+n51=C ...c47c;
n48+n45+n36+n33=C ...c48a;	n48+n45+n64+n61=C ...c48b;	n48+n36+n64+n52=C ...c48c;
n49+n50+n53+n54=C ...c49a;	n49+n50+n1+n2=C ...c49b;	n49+n53+n1+n5=C ...c49c;
n50+n51+n54+n55=C ...c50a;	n50+n51+n2+n3=C ...c50b;	n50+n54+n2+n6=C ...c50c;
n51+n52+n55+n56=C ...c51a;	n51+n52+n3+n4=C ...c51b;	n51+n55+n3+n7=C ...c51c;
n52+n49+n56+n53=C ...c52a;	n52+n49+n4+n1=C ...c52b;	n52+n56+n4+n8=C ...c52c;
n53+n54+n57+n58=C ...c53a;	n53+n54+n5+n6=C ...c53b;	n53+n57+n5+n9=C ...c53c;
n54+n55+n58+n59=C ...c54a;	n54+n55+n6+n7=C ...c54b;	n54+n58+n6+n10=C ...c54c;
n55+n56+n59+n60=C ...c55a;	n55+n56+n7+n8=C ...c55b;	n55+n59+n7+n11=C ...c55c;
n56+n53+n60+n57=C ...c56a;	n56+n53+n8+n5=C ...c56b;	n56+n60+n8+n12=C ...c56c;
n57+n58+n61+n62=C ...c57a;	n57+n58+n9+n10=C ...c57b;	n57+n61+n9+n13=C ...c57c;
n58+n59+n62+n63=C ...c58a;	n58+n59+n10+n11=C ...c58b;	n58+n62+n10+n14=C ...c58c;
n59+n60+n63+n64=C ...c59a;	n59+n60+n11+n12=C ...c59b;	n59+n63+n11+n15=C ...c59c;
n60+n57+n64+n61=C ...c60a;	n60+n57+n12+n9=C ...c60b;	n60+n64+n12+n16=C ...c60c;
n61+n62+n49+n50=C ...c61a;	n61+n62+n13+n14=C ...c61b;	n61+n49+n13+n1=C ...c61c;
n62+n63+n50+n51=C ...c62a;	n62+n63+n14+n15=C ...c62b;	n62+n50+n14+n2=C ...c62c;
n63+n64+n51+n52=C ...c63a;	n63+n64+n15+n16=C ...c63b;	n63+n51+n15+n3=C ...c63c;
n64+n61+n52+n49=C ...c64a;	n64+n61+n16+n13=C ...c64b;	n64+n52+n16+n4=C ...c64c;

As {n2, n4, n5, n13, n17, n49} are the 'next positions to n1', all ways of putting those 6 sets of 32 complementary pairs in the next positions to n1 differently from one another should be counted as follows: ${}_6P_6$ = 6x5x4x3x2x1= 720

It means the total count of 'Primitive' solutions with n1=1 and n43=64, and we can calculate the total counts of 'Primitive' solutions and 'Standard' ones as follows:

64 x ${}_6P_6$ = 64x720 = 46080; 46080÷48 = 960

They should be the total counts of solutions for 'Composite and Complete' pan-magic cubes of order 4.

I want to include n43 as the seventh member in the group of 'next positions to n1', because we can put any group of 32 pairs freely and equally on all pan-triagonals.

All ways of putting those 7 sets of 32 complementary pairs in the next positions to n1 differently from one another should be counted as follows:

${}_7P_7$ = 7x6x5x4x3x2x1= 5040

It means the total count of 'Primitive' solutions with n1=1, and we should be able to calculate the total counts of 'Primitive' solutions and 'Standard' solutions as follows:

64 x ${}_7P_7$ = 64x5040 = 322560; 322560÷48 = 6720

They should be the total counts of solutions for 'Composite and Pantriagonal' magic cubes of order 4.

Do you think we could predict the truth correctly? We must find those by studying precisely about our object.

Read the old article of mine in Part 1, Chapter 4, Section 4, please.

Written on May 21 and revised on October 29, 2007 with MacOSX and Xcode2

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