Chapter 3. New Algebraic Study of Magic Squares 5x5

Section 4. Analysis on a 'Classical Composition': **Kanji Setsuda**

#1. You know this 'Classical Composition', don't you?

```
                [Figure 0]
   /Start                    /Goal
   .--------------.          .--------------.
   | 1| 2| 3| 4| 5|          |11|24| 7|20| 3|65
   |--+--+--+--+--|          |--+--+--+--+--|
   | 6| 7| 8| 9|10|          | 4|12|25| 8|16|65
   |--+--+--+--+--|          |--+--+--+--+--|
   |11|12|13|14|15|          |17| 5|13|21| 9|65
   |--+--+--+--+--|          |--+--+--+--+--|
   |16|17|18|19|20|          |10|18| 1|14|22|65
   |--+--+--+--+--|          |--+--+--+--+--|
   |21|22|23|24|25|          |23| 6|19| 2|15|65
   '--------------'          '--------------'
                       65 65 65 65 65 65 65 <-- Sum Check
```

```
[Figure 1] Start
             1
           /   \
          6     2
        /  \  /   \
      11     7     3
     /  \  /  \  /   \
   16    12     8     4
  /  \  /  \  /  \  /   \
21    17    13     9     5  -->
  \  /  \  /  \  /  \  /
   22    18    14    10
     \  /  \  /  \  /
      23    19    15
        \  /  \  /
         24    20
           \  /
            25
```

```
[Figure 2]
             1
           /   \
          6     2
        /  \  /   \
      11-()--7-()--3
     / |\  /  \  / | \
   16 () 12 ()  8 ()  4
  /  \ |/  \  /  \ |/   \
21    17 () 13 ()  9     5  -->
  \  / |\  /  \  / |\  /
   22 () 18 () 14 () 10
     \ |/  \  /  \ |/
      23-()-19-()-15
        \  /  \  /
         24    20
           \  /
            25
```

```
[Figure 3]
             1
           /   \
          6     2
        /  \  /   \
      11-24--7-20--3
     / |           | \
   16 () 12 25  8 ()  4
  /  \ |           |/   \
21    17 () 13 ()  9     5 -->
  \  / |           |\  /
   22 () 18   1 14 () 10
     \ |           |/
      23--6-19--2-15
        \  /  \  /
         24    20
           \  /
            25
```

```
[Figure 4]  Goal
             1
           /   \
          6     2
        /  \  /   \
      11-24--7-20--3
     / |           | \
   16  4 12 25  8 16  4
  /  \ |           |/   \
21    17  5 13 21  9     5
  \  / |           |\  /
   22 10 18   1 14 22 10
     \ |           |/
      23--6-19--2-15
        \  /  \  /
         24    20
           \  /
            25
```

It looks like such an old method that it has been introduced as above for a long time first of all in many books. You must have seen it before, haven't you?

It is a way of composing a self-complementary magic square of order 2n+1 (n=1, 2, 3, 4, 5,...). Let me take an example of order 5. Figure 0 shows the start square and the goal object. How do you move numbers? Which ones don't you have to move? Can you really transform the left square into the right one?

Step 1. This old method directs you to rotate the whole start square by 45 degrees clockwise, and put the result as Figure 1.

Step 2. Choose 3, 11, 23, 15(at middle points of each edge line), and connect them with lines to make a new square frame as Figure 2.

Step 3. Keep the following 13 numbers {3, 7, 11, 8, 12, 9, 13, 17, 14, 18, 15, 19 and 23} unchanged, and do not move them at all. Make the blank positions as Figure 3 for the rest 12 numbers instead.

Step 4. Pick up the 4 tops {1, 5, 21, 25} from outside the new frame and carry them to the opposite side, and put them inside the frame near the central position between (14, 18), (12, 18), (8, 14) and (8, 25).

Step 5. Move the rest 8 numbers outside the frame to the opposite side, and put them on the edge lines by 2 on each. The inner square of order 5 of Figure 4 shows our goal object.

Step 6. Check if every row, every column and every primary diagonal adds up to the magic constant 65. Check if all complementary pairs (1,25), (2,24), (3,23), ... , (11,15), (12,14) are placed symmetrically with respect to 13 on the center.

#2. The Importance of the Basic Form of 5 by 5

Let's make some analyses on this method.

First of all, we have to find some secrets of the start square 5x5.

It is the same with the 'Basic Form' made after the regular array of 5x5. The value of each position is kept equal to the position name of itself.

```
 [Figure 5: Basic Form]                 Pan-diagonals:

  23 24 25 21 22 23 24 25 21 22 23     * 1+ 7+13+19+25=65 ...(1)
          .--------------.               1+10+14+18+22=65 ...(2)
   3  4  5| 1| 2| 3| 4| 5| 1  2  3       2+ 8+14+20+21=65 ...(3)
          |--+--+--+--+--|               2+ 6+15+19+23=65 ...(4)
   8  9 10| 6| 7| 8| 9|10| 6  7  8       3+ 9+15+16+22=65 ...(5)
          |--+--+--+--+--|               3+ 7+11+20+24=65 ...(6)
  13 14 15|11|12|13|14|15|11 12 13       4+10+11+17+23=65 ...(7)
          |--+--+--+--+--|               4+ 8+12+16+25=65 ...(8)
  18 19 20|16|17|18|19|20|16 17 18       5+ 6+12+18+24=65 ...(9)
          |--+--+--+--+--|             * 5+ 9+13+17+21=65 ..(10)
  23 24 25|21|22|23|24|25|21 22 23
          '--------------'             (1) and (10) are
   3  4  5  1  2  3  4  5  1  2  3      'Primary Diagonals'
```

Please check if every pan-diagonal adds up to the magic constant 65. Yes, it does.

And check if all complementary pairs are placed symmetrically with respect to the central 13. Yes, they are.

The Basic Form is always a 'simultaneous' square: both self-complementary and pan-diagonal type, although it is not a 'magic' square at all, since every row and every column does not always add up to the same constant.

If you want any proofs for those properties, you may try to decompose it by positional number system of the base 5.

* Basic Form with /N5i and /D5i *

[BF]	/N5i	/D5i	
1 2 3 4 5	00 01 02 03 04	0 0 0 0 0	0 1 2 3 4
6 7 8 9 10	10 11 12 13 14	1 1 1 1 1	0 1 2 3 4
11 12 13 14 15	20 21 22 23 24	2 2 2 2 2	0 1 2 3 4
16 17 18 19 20	30 31 32 33 34	3 3 3 3 3	0 1 2 3 4
21 22 23 24 25	40 41 42 43 44	4 4 4 4 4	0 1 2 3 4
		H/5^0	L/5^1

Every pan-diagonal always consists of {0, 1, 2, 3, 4} in both higher layer and lower one. Therefore the sum is always calculated in the same way as:

$(0+1+2+3+4) \times 5^1 + (0+1+2+3+4) \times 5^0 = 10 \times 5 + 10 \times 1 = 60$ (Decimal)

It is equivalent to the magic constant 65 in our conventional notation.

As you notice, these properties of the Basic Form are re-used by that classical method of transformation as much as possible.

* Transform it by the Classical Construction *

BD/

4	5	1	2	3	4	5	1	2
9	10	6	7	8	9	10	6	7
14	15	11	12	13	14	15	11	12
19	20	16	17	18	19	20	16	17
24	25	21	22	23	24	25	21	22

CC/

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Trace and find which row the number 1 moved into. The line 10+18+1+14+22. It was (2), one of the pandiagonals of Basic Form.
Where did the number 5 go? Into the pandiagonal 5+ 6+12+18+24=65 ...(9). ...
Where did 2 and 6 go? Into the original pandiagonal 2+ 6+15+19+23=65 ...(4)
This method makes every row and every column for the new object from what was every pan-diagonal of the original start square, and makes it add up to the same constant.
It is the wise rotation of whole square by 45 degrees clockwise that makes it possible.
It is such a magic that it treats old pandiagonals as new rows or columns in elegant manner.
And both 3+8+13+18+23=65 and 11+12+13+14+15=65 in the original are also wisely made into the two primary diagonals of the new object.

The symmetrical positioning of all complementary pairs are kept undestroyed by the straight movement of 12 numbers.

#3. What Potentiality does this Method have?

It is known this method only makes a self-complementary magic square from the Basic Form. But if you suppose it as a transformation technique, what else can you make it do? A wild imagination grew in my mind.
I tried to make some experiments for our Suzuki's Squares, 16 simultaneous magic squares of order 5. They are both self-complementary and pan-diagonal.
Why did I choose Suzuki's Squares? Because they are simultaneous type of magic squares, just the same as Basic Form. We now need both properties of self-complementary and pandiagonal types.

At first, I dictated the transformation rules to my computer program as follows:

```
/**/
/* Classical Construction */
void classicconstr(){
   d[1]=c[11];   d[2]=c[24];   d[3]=c[7];    d[4]=c[20];   d[5]=c[3];
   d[6]=c[4];    d[7]=c[12];   d[8]=c[25];   d[9]=c[8];    d[10]=c[16];
   d[11]=c[17];  d[12]=c[5];   d[13]=c[13];  d[14]=c[21];  d[15]=c[9];
   d[16]=c[10];  d[17]=c[18];  d[18]=c[1];   d[19]=c[14];  d[20]=c[22];
   d[21]=c[23];  d[22]=c[6];   d[23]=c[19];  d[24]=c[2];   d[25]=c[15];
}
/**/
```

Then I applied this procedure to those 16 Suzuki's Squares.
Let me show you the first result of my experiment.

```
** Simultaneous Magic Squares 5*5: Both S-C and P-D Type **
** Transform the first 4 by the 'Classical Construction' **
              1/T0              0/T1              0/T2              0/T3
    1 15 24 18  7   10 11 17  3 24   21 15  4  8 17   20 11  7 23  4
   23 17  6  5 14   18  4 25  6 12    3  7 16 25 14    8 24  5 16 12
   10  4 13 22 16   21  7 13 19  5   20 24 13  2  6    1 17 13  9 25
   12 21 20  9  3   14 20  1 22  8   12  1 10 19 23   14 10 21  2 18
   19  8  2 11 25    2 23  9 15 16    9 18 22 11  5   22  3 19 15  6
              2/T0              0/T1             16/T2              0/T3
    1 23 20 14  7   22  3  9 11 20    5 23 16 12  9   24  3  7 15 16
   15  9  2 21 18   14 16 25  2  8   11  7  4 25 18   12 20 21  4  8
   22 16 13 10  4    5  7 13 19 21   24 20 13  6  2    1  9 13 17 25
    8  5 24 17 11   18 24  1 10 12    8  1 22 19 15   18 22  5  6 14
   19 12  6  3 25    6 15 17 23  4   17 14 10  3 21   10 11 19 23  2
              3/T0              0/T1              0/T2              0/T3
    1 15 22 18  9   10 11 19  3 22   21 15  2  8 19   20 11  9 23  2
   23 19  6  5 12   18  2 25  6 14    3  9 16 25 12    8 22  5 16 14
   10  2 13 24 16   21  9 13 17  5   20 22 13  4  6    1 19 13  7 25
   14 21 20  7  3   12 20  1 24  8   14  1 10 17 23   12 10 21  4 18
   17  8  4 11 25    4 23  7 15 16    7 18 24 11  5   24  3 17 15  6
              4/T0              0/T1             14/T2              0/T3
    1 23 20 12  9   24  3  7 11 20    5 23 16 14  7   22  3  9 15 16
   15  7  4 21 18   12 16 25  4  8   11  9  2 25 18   14 20 21  2  8
   24 16 13 10  2    5  9 13 17 21   22 20 13  6  4    1  7 13 19 25
    8  5 22 19 11   18 22  1 10 14    8  1 24 17 15   18 24  5  6 12
   17 14  6  3 25    6 15 19 23  2   19 12 10  3 21   10 11 17 23  4
```

The list above shows the serial results of repeating the same classical transformation. /T0 is made into /T1, /T1 is made into /T2, and /T2 is made into /T3 by applying the same method to one after another.
But most of the results are against the list-forming inequality conditions: n1<n5<n21 and n1<n25. It may be confusing. For we have to find anything same in any old list of solutions. We have to rewrite each result and get it smart by some simple reflection or rotations, if necessary.
Take your kind look at the next list below.

```
              1/               7/              15/               9/               1/
   1 15 24 18  7    2 14 21 18 10    5 11 22 18  9    4 12 25 18  6    1 15 24 18  7
  23 17  6  5 14   23 20  7  4 11   23 19 10  1 12   23 16  9  2 15   23 17  6  5 14
  10  4 13 22 16    9  1 13 25 17    6  2 13 24 20    7  5 13 21 19   10  4 13 22 16
  12 21 20  9  3   15 22 19  6  3   14 25 16  7  3   11 24 17 10  3   12 21 20  9  3
  19  8  2 11 25   16  8  5 12 24   17  8  4 15 21   20  8  1 14 22   19  8  2 11 25
```

				2/					10/					16/					8/					2/
1	23	20	14	7	4	23	17	15	6	5	23	16	12	9	2	23	19	11	10	1	23	20	14	7
15	9	2	21	18	12	10	1	24	18	11	7	4	25	18	14	6	5	22	18	15	9	2	21	18
22	16	13	10	4	21	19	13	7	5	24	20	13	6	2	25	17	13	9	1	22	16	13	10	4
8	5	24	17	11	8	2	25	16	14	8	1	22	19	15	8	4	21	20	12	8	5	24	17	11
19	12	6	3	25	20	11	9	3	22	17	14	10	3	21	16	15	7	3	24	19	12	6	3	25

				3/					11/					13/					5/					3/
1	15	22	18	9	4	12	21	18	10	5	11	24	18	7	2	14	25	18	6	1	15	22	18	9
23	19	6	5	12	23	20	9	2	11	23	17	10	1	14	23	16	7	4	15	23	19	6	5	12
10	2	13	24	16	7	1	13	25	19	6	4	13	22	20	9	5	13	21	17	10	2	13	24	16
14	21	20	7	3	15	24	17	6	3	12	25	16	9	3	11	22	19	10	3	14	21	20	7	3
17	8	4	11	25	16	8	5	14	22	19	8	2	15	21	20	8	1	12	24	17	8	4	11	25

				4/					6/					14/					12/					4/
1	23	20	12	9	2	23	19	15	6	5	23	16	14	7	4	23	17	11	10	1	23	20	12	9
15	7	4	21	18	14	10	1	22	18	11	9	2	25	18	12	6	5	24	18	15	7	4	21	18
24	16	13	10	2	21	17	13	9	5	22	20	13	6	4	25	19	13	7	1	24	16	13	10	2
8	5	22	19	11	8	4	25	16	12	8	1	24	17	15	8	2	21	20	14	8	5	22	19	11
17	14	6	3	25	20	11	7	3	24	19	12	10	3	21	16	15	9	3	22	17	14	6	3	25

				5/					3/					11/					13/					5/
2	14	25	18	6	1	15	22	18	9	4	12	21	18	10	5	11	24	18	7	2	14	25	18	6
23	16	7	4	15	23	19	6	5	12	23	20	9	2	11	23	17	10	1	14	23	16	7	4	15
9	5	13	21	17	10	2	13	24	16	7	1	13	25	19	6	4	13	22	20	9	5	13	21	17
11	22	19	10	3	14	21	20	7	3	15	24	17	6	3	12	25	16	9	3	11	22	19	10	3
20	8	1	12	24	17	8	4	11	25	16	8	5	14	22	19	8	2	15	21	20	8	1	12	24

				6/					14/					12/					4/					6/
2	23	19	15	6	5	23	16	14	7	4	23	17	11	10	1	23	20	12	9	2	23	19	15	6
14	10	1	22	18	11	9	2	25	18	12	6	5	24	18	15	7	4	21	18	14	10	1	22	18
21	17	13	9	5	22	20	13	6	4	25	19	13	7	1	24	16	13	10	2	21	17	13	9	5
8	4	25	16	12	8	1	24	17	15	8	2	21	20	14	8	5	22	19	11	8	4	25	16	12
20	11	7	3	24	19	12	10	3	21	16	15	9	3	22	17	14	6	3	25	20	11	7	3	24

				7/					15/					9/					1/					7/
2	14	21	18	10	5	11	22	18	9	4	12	25	18	6	1	15	24	18	7	2	14	21	18	10
23	20	7	4	11	23	19	10	1	12	23	16	9	2	15	23	17	6	5	14	23	20	7	4	11
9	1	13	25	17	6	2	13	24	20	7	5	13	21	19	10	4	13	22	16	9	1	13	25	17
15	22	19	6	3	14	25	16	7	3	11	24	17	10	3	12	21	20	9	3	15	22	19	6	3
16	8	5	12	24	17	8	4	15	21	20	8	1	14	22	19	8	2	11	25	16	8	5	12	24

				8/					2/					10/					16/					8/
2	23	19	11	10	1	23	20	14	7	4	23	17	15	6	5	23	16	12	9	2	23	19	11	10
14	6	5	22	18	15	9	2	21	18	12	10	1	24	18	11	7	4	25	18	14	6	5	22	18
25	17	13	9	1	22	16	13	10	4	21	19	13	7	5	24	20	13	6	2	25	17	13	9	1
8	4	21	20	12	8	5	24	17	11	8	2	25	16	14	8	1	22	19	15	8	4	21	20	12
16	15	7	3	24	19	12	6	3	25	20	11	9	3	22	17	14	10	3	21	16	15	7	3	24

				9/					1/					7/					15/					9/
4	12	25	18	6	1	15	24	18	7	2	14	21	18	10	5	11	22	18	9	4	12	25	18	6
23	16	9	2	15	23	17	6	5	14	23	20	7	4	11	23	19	10	1	12	23	16	9	2	15
7	5	13	21	19	10	4	13	22	16	9	1	13	25	17	6	2	13	24	20	7	5	13	21	19
11	24	17	10	3	12	21	20	9	3	15	22	19	6	3	14	25	16	7	3	11	24	17	10	3
20	8	1	14	22	19	8	2	11	25	16	8	5	12	24	17	8	4	15	21	20	8	1	14	22

				10/					16/					8/					2/					10/
4	23	17	15	6	5	23	16	12	9	2	23	19	11	10	1	23	20	14	7	4	23	17	15	6
12	10	1	24	18	11	7	4	25	18	14	6	5	22	18	15	9	2	21	18	12	10	1	24	18
21	19	13	7	5	24	20	13	6	2	25	17	13	9	1	22	16	13	10	4	21	19	13	7	5
8	2	25	16	14	8	1	22	19	15	8	4	21	20	12	8	5	24	17	11	8	2	25	16	14
20	11	9	3	22	17	14	10	3	21	16	15	7	3	24	19	12	6	3	25	20	11	9	3	22

```
           11/              13/               5/               3/              11/
 4 12 21 18 10    5 11 24 18  7    2 14 25 18  6    1 15 22 18  9    4 12 21 18 10
23 20  9  2 11   23 17 10  1 14   23 16  7  4 15   23 19  6  5 12   23 20  9  2 11
 7  1 13 25 19    6  4 13 22 20    9  5 13 21 17   10  2 13 24 16    7  1 13 25 19
15 24 17  6  3   12 25 16  9  3   11 22 19 10  3   14 21 20  7  3   15 24 17  6  3
16  8  5 14 22   19  8  2 15 21   20  8  1 12 24   17  8  4 11 25   16  8  5 14 22
           12/               4/               6/              14/              12/
 4 23 17 11 10    1 23 20 12  9    2 23 19 15  6    5 23 16 14  7    4 23 17 11 10
12  6  5 24 18   15  7  4 21 18   14 10  1 22 18   11  9  2 25 18   12  6  5 24 18
25 19 13  7  1   24 16 13 10  2   21 17 13  9  5   22 20 13  6  4   25 19 13  7  1
 8  2 21 20 14    8  5 22 19 11    8  4 25 16 12    8  1 24 17 15    8  2 21 20 14
16 15  9  3 22   17 14  6  3 25   20 11  7  3 24   19 12 10  3 21   16 15  9  3 22
           13/               5/               3/              11/              13/
 5 11 24 18  7    2 14 25 18  6    1 15 22 18  9    4 12 21 18 10    5 11 24 18  7
23 17 10  1 14   23 16  7  4 15   23 19  6  5 12   23 20  9  2 11   23 17 10  1 14
 6  4 13 22 20    9  5 13 21 17   10  2 13 24 16    7  1 13 25 19    6  4 13 22 20
12 25 16  9  3   11 22 19 10  3   14 21 20  7  3   15 24 17  6  3   12 25 16  9  3
19  8  2 15 21   20  8  1 12 24   17  8  4 11 25   16  8  5 14 22   19  8  2 15 21
           14/              12/               4/               6/              14/
 5 23 16 14  7    4 23 17 11 10    1 23 20 12  9    2 23 19 15  6    5 23 16 14  7
11  9  2 25 18   12  6  5 24 18   15  7  4 21 18   14 10  1 22 18   11  9  2 25 18
22 20 13  6  4   25 19 13  7  1   24 16 13 10  2   21 17 13  9  5   22 20 13  6  4
 8  1 24 17 15    8  2 21 20 14    8  5 22 19 11    8  4 25 16 12    8  1 24 17 15
19 12 10  3 21   16 15  9  3 22   17 14  6  3 25   20 11  7  3 24   19 12 10  3 21
           15/               9/               1/               7/              15/
 5 11 22 18  9    4 12 25 18  6    1 15 24 18  7    2 14 21 18 10    5 11 22 18  9
23 19 10  1 12   23 16  9  2 15   23 17  6  5 14   23 20  7  4 11   23 19 10  1 12
 6  2 13 24 20    7  5 13 21 19   10  4 13 22 16    9  1 13 25 17    6  2 13 24 20
14 25 16  7  3   11 24 17 10  3   12 21 20  9  3   15 22 19  6  3   14 25 16  7  3
17  8  4 15 21   20  8  1 14 22   19  8  2 11 25   16  8  5 12 24   17  8  4 15 21
           16/               8/               2/              10/              16/
 5 23 16 12  9    2 23 19 11 10    1 23 20 14  7    4 23 17 15  6    5 23 16 12  9
11  7  4 25 18   14  6  5 22 18   15  9  2 21 18   12 10  1 24 18   11  7  4 25 18
24 20 13  6  2   25 17 13  9  1   22 16 13 10  4   21 19 13  7  5   24 20 13  6  2
 8  1 22 19 15    8  4 21 20 12    8  5 24 17 11    8  2 25 16 14    8  1 22 19 15
17 14 10  3 21   16 15  7  3 24   19 12  6  3 25   20 11  9  3 22   17 14 10  3 21
```

* Simultaneous Magic Squares 5*5: Both Self-Complementary and Pan-Diagonal *
** Serial Results of Repeating the same Classical Construction 4 times **

You see the smart results of our repeating transformations in the list above. You will probably find anything you want easily in any old solution list.

#4. What could be Found by these Experiments?

These results are really surprising. Each answer could be found in the same original solution set of simultaneous MS55: Both S-C and P-D type.
No.1 is transformed into No.7. No.2 is made into No.10. No.3 into No.11, . . . , No.15 into No.9, and No.16 is made into No.8. … What does this mean?
(1) The original solution set of Suzuki Squares is ‘closed’ with respect to this classical transformation. It never leads us to a broad outer world, but it seems to make us look for the inner structures and classifications.
(2) It is the most surprising that we could find some circular linkages among them:
(No.1 -> No.7 -> No.15 -> No.9 -> No.1),

```
  (No.2 -> No.10 -> No.16 -> No.8 -> No.2),
  (No.3 -> No.11 -> No.13 -> No.5 -> No.3),
  (No.4 -> No.6 -> No.14 -> No.12 -> No.4)
```

Thus we can make those 4 groups of solutions in all such as shown above.
(3) You may find any common properties for each group, or find any 'representative' solutions. You may even discover any family tree relations among them.

#5. 'Complete Euler Squares'

In the previous section we know every Suzuki's Square is always 'Complete Euler Square'. Let's put the Decomposition Layers by the base 5(/D5i) to each solution of each group. What can we find in the /D5i list?
Each solution of the same group has the same layer for higher positions.
What about for lower positions?

```
* Simultaneous Magic Squares 5*5: Both Self-Complementary and Pan-Diagonal *
** Four Groups of Solutions by the Classical Construction with D5i **
[Group 1]
             1/  /D5i       -->             7/  /D5i
  1 15 24 18  7  02431 04321    2 14 21 18 10  02431 13024
 23 17  6  5 14  43102 21043   23 20  7  4 11  43102 24130
 10  4 13 22 16  10243 43210    9  1 13 25 17  10243 30241
 12 21 20  9  3  24310 10432   15 22 19  6  3  24310 41302
 19  8  2 11 25  31024 32104   16  8  5 12 24  31024 02413
                         ^       |
             9/  /D5i    |  <--  v        15/  /D5i
  4 12 25 18  6  02431 31420    5 11 22 18  9  02431 40123
 23 16  9  2 15  43102 20314   23 19 10  1 12  43102 23401
  7  5 13 21 19  10243 14203    6  2 13 24 20  10243 01234
 11 24 17 10  3  24310 03142   14 25 16  7  3  24310 34012
 20  8  1 14 22  31024 42031   17  8  4 15 21  31024 12340

[Group 2]
             2/  /D5i       -->            10/  /D5i
  1 23 20 14  7  04321 02431    4 23 17 15  6  04321 32140
 15  9  2 21 18  21043 43102   12 10  1 24 18  21043 14032
 22 16 13 10  4  43210 10243   21 19 13  7  5  43210 03214
  8  5 24 17 11  10432 24310    8  2 25 16 14  10432 21403
 19 12  6  3 25  32104 31024   20 11  9  3 22  32104 40321

             8/  /D5i       <--            16/  /D5i
  2 23 19 11 10  04321 12304    5 23 16 12  9  04321 42013
 14  6  5 22 18  21043 30412   11  7  4 25 18  21043 01342
 25 17 13  9  1  43210 41230   24 20 13  6  2  43210 34201
  8  4 21 20 12  10432 23041    8  1 22 19 15  10432 20134
 16 15  7  3 24  32104 04123   17 14 10  3 21  32104 13420

[Group 3]
             3/  /D5i       -->            11/  /D5i
  1 15 22 18  9  02431 04123    4 12 21 18 10  02431 31024
 23 19  6  5 12  43102 23041   23 20  9  2 11  43102 24310
 10  2 13 24 16  10243 41230    7  1 13 25 19  10243 10243
 14 21 20  7  3  24310 30412   15 24 17  6  3  24310 43102
 17  8  4 11 25  31024 12304   16  8  5 14 22  31024 02431
```

```
                5/   /D5i        <--             13/   /D5i
  2 14 25 18  6   02431 13420    5 11 24 18  7   02431 40321
 23 16  7  4 15   43102 20134   23 17 10  1 14   43102 21403
  9  5 13 21 17   10243 34201    6  4 13 22 20   10243 03214
 11 22 19 10  3   24310 01342   12 25 16  9  3   24310 14032
 20  8  1 12 24   31024 42013   19  8  2 15 21   31024 32140

[Group 4]
                4/   /D5i        -->              6/   /D5i
  1 23 20 12  9   04321 02413    2 23 19 15  6   04321 12340
 15  7  4 21 18   21043 41302   14 10  1 22 18   21043 34012
 24 16 13 10  2   43210 30241   21 17 13  9  5   43210 01234
  8  5 22 19 11   10432 24130    8  4 25 16 12   10432 23401
 17 14  6  3 25   32104 13024   20 11  7  3 24   32104 40123
                              ^       |
               12/   /D5i     |  <--  v          14/   /D5i
  4 23 17 11 10   04321 32104    5 23 16 14  7   04321 42031
 12  6  5 24 18   21043 10432   11  9  2 25 18   21043 03142
 25 19 13  7  1   43210 43210   22 20 13  6  4   43210 14203
  8  2 21 20 14   10432 21043    8  1 24 17 15   10432 20314
 16 15  9  3 22   32104 04321   19 12 10  3 21   32104 31420

 [Count = 16]
```

Do you notice that you could find any 'rotation' of patterns among lower positions of the same group? Take the G1 for instance, and you can see the anti-clockwise 'rotations' by 90 degrees among 1L -> 7L -> 15L -> 9L. G2 has the clockwise 'rotations' by 90 degrees among 2L -> 10L -> 16L -> 8L. G3 has the anti-clockwise 'rotations' by 90 degrees among 3L -> 11L -> 13L -> 5L. And G4 has the clockwise 'rotations' by 90 degrees among 4L -> 6L -> 14L -> 12L.
If you take the 4th transformation to the last, you can get to the top solution again. This is the true reason for the circular linkage among 4 solutions of the same group.

#5. How many 'Complete Euler Squares' among Self-Complementary type?

By the way, do you know how many 'Complete Euler Squares' exist among Self-Complementary type? I tried to pick up the Euler Squares out of 48544 solutions of self-complementary magic squares 5x5.
(1) I found 384 'Euler Squares(in broader sense)' among 48544 pieces.
(2) I found only 32 'Complete Euler Squares' in all. Each of them has the {0,1,2,3,4} pattern also in the two primary diagonals, not only in every row and every column.
You may be surprised at this report that I could find so few Euler Squares.
I myself was also surprised to know there are only 32 Complete Euler Squares.

See the next list below with decompositions by positional number system of base 5:

```
** Self-Complementary Magic Squares 5*5 **
**   Find the 'Complete Euler Squares'   **

 1/ 3314           /D5i                |  2/ 3692           /D5i
  1 23 20 14  7    0 4 3 2 1  0 2 4 3 1 |   1 18 24 15  7    0 3 4 2 1  0 2 3 4 1
 15  9  2 21 18    2 1 0 4 3  4 3 1 0 2 |  12  9 20 21  3    2 1 3 4 0  1 3 4 0 2
 22 16 13 10  4    4 3 2 1 0  1 0 2 4 3 |  10 22 13  4 16    1 4 2 0 3  4 1 2 3 0
  8  5 24 17 11    1 0 4 3 2  2 4 3 1 0 |  23  5  6 17 14    4 0 1 3 2  2 4 0 1 3
 19 12  6  3 25    3 2 1 0 4  3 1 0 2 4 |  19 11  2  8 25    3 2 0 1 4  3 0 1 2 4
```

```
3/ 3845         /D5i
 1 15 24 18  7  0 2 4 3 1  0 4 3 2 1
23 17  6  5 14  4 3 1 0 2  2 1 0 4 3
10  4 13 22 16  1 0 2 4 3  4 3 2 1 0
12 21 20  9  3  2 4 3 1 0  1 0 4 3 2
19  8  2 11 25  3 1 0 2 4  3 2 1 0 4
4/ 3853         /D5i
 1 14 20 23  7  0 2 3 4 1  0 3 4 2 1
 8 17 24  5 11  1 3 4 0 2  2 1 3 4 0
22 10 13 16  4  4 1 2 3 0  1 4 2 0 3
15 21  2  9 18  2 4 0 1 3  4 0 1 3 2
19  3  6 12 25  3 0 1 2 4  3 2 0 1 4
5/ 4804         /D5i
 1 23 20 12  9  0 4 3 2 1  0 2 4 1 3
15  7  4 21 18  2 1 0 4 3  4 1 3 0 2
24 16 13 10  2  4 3 2 1 0  3 0 2 4 1
 8  5 22 19 11  1 0 4 3 2  2 4 1 3 0
17 14  6  3 25  3 2 1 0 4  1 3 0 2 4
6/ 5297         /D5i
 1 18 22 15  9  0 3 4 2 1  0 2 1 4 3
14  7 20 21  3  2 1 3 4 0  3 1 4 0 2
10 24 13  2 16  1 4 2 0 3  4 3 2 1 0
23  5  6 19 12  4 0 1 3 2  2 4 0 3 1
17 11  4  8 25  3 2 0 1 4  1 0 3 2 4
7/ 5403         /D5i
 1 15 22 18  9  0 2 4 3 1  0 4 1 2 3
23 19  6  5 12  4 3 1 0 2  2 3 0 4 1
10  2 13 24 16  1 0 2 4 3  4 1 2 3 0
14 21 20  7  3  2 4 3 1 0  3 0 4 1 2
17  8  4 11 25  3 1 0 2 4  1 2 3 0 4
8/ 5527         /D5i
 1 12 20 23  9  0 2 3 4 1  0 1 4 2 3
 8 19 22  5 11  1 3 4 0 2  2 3 1 4 0
24 10 13 16  2  4 1 2 3 0  3 4 2 0 1
15 21  4  7 18  2 4 0 1 3  4 0 3 1 2
17  3  6 14 25  3 0 1 2 4  1 2 0 3 4
9/ 9786         /D5i
 2 23 19 15  6  0 4 3 2 1  1 2 3 4 0
14 10  1 22 18  2 1 0 4 3  3 4 0 1 2
21 17 13  9  5  4 3 2 1 0  0 1 2 3 4
 8  4 25 16 12  1 0 4 3 2  2 3 4 0 1
20 11  7  3 24  3 2 1 0 4  4 0 1 2 3
10/10157        /D5i
 2 18 25 14  6  0 3 4 2 1  1 2 4 3 0
11 10 19 22  3  2 1 3 4 0  0 4 3 1 2
 9 21 13  5 17  1 4 2 0 3  3 0 2 4 1
23  4  7 16 15  4 0 1 3 2  2 3 1 0 4
20 12  1  8 24  3 2 0 1 4  4 1 0 2 3
11/10312        /D5i
 2 15 19 23  6  0 2 3 4 1  1 4 3 2 0
 8 16 25  4 12  1 3 4 0 2  2 0 4 3 1
21  9 13 17  5  4 1 2 3 0  0 3 2 1 4
14 22  1 10 18  2 4 0 1 3  3 1 0 4 2
20  3  7 11 24  3 0 1 2 4  4 2 1 0 3
```

```
12/10389        /D5i
 2 14 25 18  6  0 2 4 3 1  1 3 4 2 0
23 16  7  4 15  4 3 1 0 2  2 0 1 3 4
 9  5 13 21 17  1 0 2 4 3  3 4 2 0 1
11 22 19 10  3  2 4 3 1 0  0 1 3 4 2
20  8  1 12 24  3 1 0 2 4  4 2 0 1 3
13/12658        /D5i
 2 23 19 11 10  0 4 3 2 1  1 2 3 0 4
14  6  5 22 18  2 1 0 4 3  3 0 4 1 2
25 17 13  9  1  4 3 2 1 0  4 1 2 3 0
 8  4 21 20 12  1 0 4 3 2  2 3 0 4 1
16 15  7  3 24  3 2 1 0 4  0 4 1 2 3
14/12975        /D5i
 2 18 21 14 10  0 3 4 2 1  1 2 0 3 4
15  6 19 22  3  2 1 3 4 0  4 0 3 1 2
 9 25 13  1 17  1 4 2 0 3  3 4 2 0 1
23  4  7 20 11  4 0 1 3 2  2 3 1 4 0
16 12  5  8 24  3 2 0 1 4  0 1 4 2 3
15/13071        /D5i
 2 14 21 18 10  0 2 4 3 1  1 3 0 2 4
23 20  7  4 11  4 3 1 0 2  2 4 1 3 0
 9  1 13 25 17  1 0 2 4 3  3 0 2 4 1
15 22 19  6  3  2 4 3 1 0  4 1 3 0 2
16  8  5 12 24  3 1 0 2 4  0 2 4 1 3
16/13145        /D5i
 2 11 19 23 10  0 2 3 4 1  1 0 3 2 4
 8 20 21  4 12  1 3 4 0 2  2 4 0 3 1
25  9 13 17  1  4 1 2 3 0  4 3 2 1 0
14 22  5  6 18  2 4 0 1 3  3 1 4 0 2
16  3  7 15 24  3 0 1 2 4  0 2 1 4 3
17/22629        /D5i
 4 23 17 15  6  0 4 3 2 1  3 2 1 4 0
12 10  1 24 18  2 1 0 4 3  1 4 0 3 2
21 19 13  7  5  4 3 2 1 0  0 3 2 1 4
 8  2 25 16 14  1 0 4 3 2  2 1 4 0 3
20 11  9  3 22  3 2 1 0 4  4 0 3 2 1
18/22935        /D5i
 4 18 25 12  6  0 3 4 2 1  3 2 4 1 0
11 10 17 24  3  2 1 3 4 0  0 4 1 3 2
 7 21 13  5 19  1 4 2 0 3  1 0 2 4 3
23  2  9 16 15  4 0 1 3 2  2 1 3 0 4
20 14  1  8 22  3 2 0 1 4  4 3 0 2 1
19/23006        /D5i
 4 15 17 23  6  0 2 3 4 1  3 4 1 2 0
 8 16 25  2 14  1 3 4 0 2  2 0 4 1 3
21  7 13 19  5  4 1 2 3 0  0 1 2 3 4
12 24  1 10 18  2 4 0 1 3  1 3 0 4 2
20  3  9 11 22  3 0 1 2 4  4 2 3 0 1
20/23126        /D5i
 4 12 25 18  6  0 2 4 3 1  3 1 4 2 0
23 16  9  2 15  4 3 1 0 2  2 0 3 1 4
 7  5 13 21 19  1 0 2 4 3  1 4 2 0 3
11 24 17 10  3  2 4 3 1 0  0 3 1 4 2
20  8  1 14 22  3 1 0 2 4  4 2 0 3 1
```

```
21/25584          /D5i
 4 23 17 11 10    0 4 3 2 1   3 2 1 0 4
12  6  5 24 18    2 1 0 4 3   1 0 4 3 2
25 19 13  7  1    4 3 2 1 0   4 3 2 1 0
 8  2 21 20 14    1 0 4 3 2   2 1 0 4 3
16 15  9  3 22    3 2 1 0 4   0 4 3 2 1
22/25936          /D5i
 4 18 21 12 10    0 3 4 2 1   3 2 0 1 4
15  6 17 24  3    2 1 3 4 0   4 0 1 3 2
 7 25 13  1 19    1 4 2 0 3   1 4 2 0 3
23  2  9 20 11    4 0 1 3 2   2 1 3 4 0
16 14  5  8 22    3 2 0 1 4   0 3 4 2 1
23/26051          /D5i
 4 12 21 18 10    0 2 4 3 1   3 1 0 2 4
23 20  9  2 11    4 3 1 0 2   2 4 3 1 0
 7  1 13 25 19    1 0 2 4 3   1 0 2 4 3
15 24 17  6  3    2 4 3 1 0   4 3 1 0 2
16  8  5 14 22    3 1 0 2 4   0 2 4 3 1
24/26069          /D5i
 4 11 17 23 10    0 2 3 4 1   3 0 1 2 4
 8 20 21  2 14    1 3 4 0 2   2 4 0 1 3
25  7 13 19  1    4 1 2 3 0   4 1 2 3 0
12 24  5  6 18    2 4 0 1 3   1 3 4 0 2
16  3  9 15 22    3 0 1 2 4   0 2 3 4 1
25/28638          /D5i
 5 23 16 14  7    0 4 3 2 1   4 2 0 3 1
11  9  2 25 18    2 1 0 4 3   0 3 1 4 2
22 20 13  6  4    4 3 2 1 0   1 4 2 0 3
 8  1 24 17 15    1 0 4 3 2   2 0 3 1 4
19 12 10  3 21    3 2 1 0 4   3 1 4 2 0
26/29068          /D5i
 5 18 24 11  7    0 3 4 2 1   4 2 3 0 1
12  9 16 25  3    2 1 3 4 0   1 3 0 4 2
 6 22 13  4 20    1 4 2 0 3   0 1 2 3 4
23  1 10 17 14    4 0 1 3 2   2 0 4 1 3
19 15  2  8 21    3 2 0 1 4   3 4 1 2 0
```

```
27/29265          /D5i
 5 14 16 23  7    0 2 3 4 1   4 3 0 2 1
 8 17 24  1 15    1 3 4 0 2   2 1 3 0 4
22  6 13 20  4    4 1 2 3 0   1 0 2 4 3
11 25  2  9 18    2 4 0 1 3   0 4 1 3 2
19  3 10 12 21    3 0 1 2 4   3 2 4 1 0
28/29389          /D5i
 5 11 24 18  7    0 2 4 3 1   4 0 3 2 1
23 17 10  1 14    4 3 1 0 2   2 1 4 0 3
 6  4 13 22 20    1 0 2 4 3   0 3 2 1 4
12 25 16  9  3    2 4 3 1 0   1 4 0 3 2
19  8  2 15 21    3 1 0 2 4   3 2 1 4 0
29/30410          /D5i
 5 23 16 12  9    0 4 3 2 1   4 2 0 1 3
11  7  4 25 18    2 1 0 4 3   0 1 3 4 2
24 20 13  6  2    4 3 2 1 0   3 4 2 0 1
 8  1 22 19 15    1 0 4 3 2   2 0 1 3 4
17 14 10  3 21    3 2 1 0 4   1 3 4 2 0
30/30690          /D5i
 5 18 22 11  9    0 3 4 2 1   4 2 1 0 3
14  7 16 25  3    2 1 3 4 0   3 1 0 4 2
 6 24 13  2 20    1 4 2 0 3   0 3 2 1 4
23  1 10 19 12    4 0 1 3 2   2 0 4 3 1
17 15  4  8 21    3 2 0 1 4   1 4 3 2 0
31/30895          /D5i
 5 12 16 23  9    0 2 3 4 1   4 1 0 2 3
 8 19 22  1 15    1 3 4 0 2   2 3 1 0 4
24  6 13 20  2    4 1 2 3 0   3 0 2 4 1
11 25  4  7 18    2 4 0 1 3   0 4 3 1 2
17  3 10 14 21    3 0 1 2 4   1 2 4 3 0
32/30969          /D5i
 5 11 22 18  9    0 2 4 3 1   4 0 1 2 3
23 19 10  1 12    4 3 1 0 2   2 3 4 0 1
 6  2 13 24 20    1 0 2 4 3   0 1 2 3 4
14 25 16  7  3    2 4 3 1 0   3 4 0 1 2
17  8  4 15 21    3 1 0 2 4   1 2 3 4 0
```

```
[Count = 32/48544]
```

They consist of 16 simultaneous type of Suzuki Squares and 16 self-complementary squares transformed from the very Suzuki Squares. Nothing else could be found at all. The next diagrams show how to transform Suzuki Squares into special S-C type by exchanging the 2nd row/column and 4th row/column in both directions.

```
0/Original          /Transformation
  1  2  3  4  5      1  4  3  2  5
  6  7  8  9 10     16 19 18 17 20
 11 12 13 14 15     11 14 13 12 15
 16 17 18 19 20      6  9  8  7 10
 21 22 23 24 25     21 24 23 22 25
```

How about 3600 solutions of pan-diagonal magic squares of order 5?
I once found they are all Complete Euler Squares.
It seems Complete Euler Squares in general should be related close to pan-diagonal magic squares of order 5 rather than any self-complementary type.

#6. What is the most ‘Fundamental Solution’?

I invented the way how to make one solution into all the other 15 Simultaneous Suzuki Squares by the combination of 4 types of different transformations.
Of course, I use what I could know here by our 'Classical Composition' for the next four circular linkages among solutions.

```
(No.1 -> No.7  -> No.15 -> No.9  -> No.1),
(No.2 -> No.10 -> No.16 -> No.8  -> No.2),
(No.3 -> No.11 -> No.13 -> No.5  -> No.3),
(No.4 -> No.6  -> No.14 -> No.12 -> No.4)
```

As you guess, I wanted to know how to make four tops of those groups above. When we know how to build No.1, 2, 3 and 4, we can automatically make all the other 12 pieces.

```
1/                2/                3/                4/
 1 15 24 18  7     1 23 20 14  7     1 15 22 18  9     1 23 20 12  9
23 17  6  5 14    15  9  2 21 18    23 19  6  5 12    15  7  4 21 18
10  4 13 22 16    22 16 13 10  4    10  2 13 24 16    24 16 13 10  2
12 21 20  9  3     8  5 24 17 11    14 21 20  7  3     8  5 22 19 11
19  8  2 11 25    19 12  6  3 25    17  8  4 11 25    17 14  6  3 25
```

I tried to find some effective inter-transformations among No.1 ~ No.4.
I soon began to compare one with the others carefully and know which part I may move and which part I may not move as shown in the list below.

```
***  Simultaneous Suzuki Squares 5x5  ***
* Compare Any Two and Find Anything Common between them *

  3/                 /Common          1/Original
    1 15 22 18  9     1 15  # 18  #     1 15 24 18  7
   23 19  6  5 12    23  #  6  5  #    23 17  6  5 14
   10  2 13 24 16    10  # 13  # 16    10  4 13 22 16
   14 21 20  7  3     # 21 20  #  3    12 21 20  9  3
   17  8  4 11 25     #  8  # 11 25    19  8  2 11 25
  4/                 /Common          1/Reflected
    1 23 20 12  9     1 23  # 12  #     1 23 10 12 19
   15  7  4 21 18    15  #  4 21  #    15 17  4 21  8
   24 16 13 10  2    24  # 13  #  2    24  6 13 20  2
    8  5 22 19 11     #  5 22  # 11    18  5 22  9 11
   17 14  6  3 25     # 14  #  3 25     7 14 16  3 25
  4/                 /Common          2/Original
    1 23 20 12  9     1 23 20  #  #     1 23 20 14  7
   15  7  4 21 18    15  #  # 21 18    15  9  2 21 18
   24 16 13 10  2     # 16 13 10  #    22 16 13 10  4
    8  5 22 19 11     8  5  #  # 11     8  5 24 17 11
   17 14  6  3 25     #  #  6  3 25    19 12  6  3 25
  3/                 /Common          2/Reflected
    1 15 22 18  9     1 15 22  #  #     1 15 22  8 19
   23 19  6  5 12    23  #  #  5 12    23  9 16  5 12
   10  2 13 24 16     #  2 13 24  #    20  2 13 24  6
   14 21 20  7  3    14 21  #  #  3    14 21 10 17  3
   17  8  4 11 25     #  #  4 11 25     7 18  4 11 25
```

```
1/                /Common           3/Original
  1 15 24 18  7    1 15  # 18  #     1 15 22 18  9
 23 17  6  5 14   23  #  6  5  #    23 19  6  5 12
 10  4 13 22 16   10  # 13  # 16    10  2 13 24 16
 12 21 20  9  3    # 21 20  #  3    14 21 20  7  3
 19  8  2 11 25    #  8  # 11 25    17  8  4 11 25
2/                /Common           3/Reflected
  1 23 20 14  7    1 23  # 14  #     1 23 10 14 17
 15  9  2 21 18   15  #  2 21  #    15 19  2 21  8
 22 16 13 10  4   22  # 13  #  4    22  6 13 20  4
  8  5 24 17 11    #  5 24  # 11    18  5 24  7 11
 19 12  6  3 25    # 12  #  3 25     9 12 16  3 25
2/                /Common           4/Original
  1 23 20 14  7    1 23 20  #  #     1 23 20 12  9
 15  9  2 21 18   15  #  # 21 18    15  7  4 21 18
 22 16 13 10  4    # 16 13 10  #    24 16 13 10  2
  8  5 24 17 11    8  5  #  # 11     8  5 22 19 11
 19 12  6  3 25    #  #  6  3 25    17 14  6  3 25
1/                /Common           4/Reflected
  1 15 24 18  7    1 15 24  #  #     1 15 24  8 17
 23 17  6  5 14   23  #  #  5 14    23  7 16  5 14
 10  4 13 22 16    #  4 13 22  #    20  4 13 22  6
 12 21 20  9  3   12 21  #  #  3    12 21 10 19  3
 19  8  2 11 25    #  #  2 11 25     9 18  2 11 25
```

I finally discovered these three transformations dictated as below were effective. They could make all what I have wanted. Though I skip to explain more about how I found them, I combined these three procedures wisely in the two ways for our purpose.
I made No.3 and 4 from No.1 and then made No.2 from No.4.

```
/**/
/* transformation */
void trnsf(){
 d[1]=c[1];   d[2]=c[2];   d[3]=c[14]; d[4]=c[4];   d[5]=c[19];
 d[6]=c[6];   d[7]=c[21];  d[8]=c[8];  d[9]=c[9];   d[10]=c[16];
 d[11]=c[11]; d[12]=c[23]; d[13]=c[13]; d[14]=c[3];  d[15]=c[15];
 d[16]=c[10]; d[17]=c[17]; d[18]=c[18]; d[19]=c[5];  d[20]=c[20];
 d[21]=c[7];  d[22]=c[22]; d[23]=c[12]; d[24]=c[24]; d[25]=c[25];
}
/**/
/* transformation-2 */
void trnsf2(){
 d[1]=c[1];   d[2]=c[2];   d[3]=c[3];   d[4]=c[22]; d[5]=c[7];
 d[6]=c[6];   d[7]=c[5];   d[8]=c[15];  d[9]=c[9];   d[10]=c[10];
 d[11]=c[18]; d[12]=c[12]; d[13]=c[13]; d[14]=c[14]; d[15]=c[8];
 d[16]=c[16]; d[17]=c[17]; d[18]=c[11]; d[19]=c[21]; d[20]=c[20];
 d[21]=c[19]; d[22]=c[4];  d[23]=c[23]; d[24]=c[24]; d[25]=c[25];
}
/**/
/* reflection */
void reflct(){
 d[1]=c[1];  d[2]=c[6];   d[3]=c[11];  d[4]=c[16];  d[5]=c[21];
 d[6]=c[2];  d[7]=c[7];   d[8]=c[12];  d[9]=c[17];  d[10]=c[22];
 d[11]=c[3]; d[12]=c[8];  d[13]=c[13]; d[14]=c[18]; d[15]=c[23];
 d[16]=c[4]; d[17]=c[9];  d[18]=c[14]; d[19]=c[19]; d[20]=c[24];
 d[21]=c[5]; d[22]=c[10]; d[23]=c[15]; d[24]=c[20]; d[25]=c[25];
}
/**/
```

For the rest 12 pieces I took ‘Classical Composition’ fully as our transformation method. I also used Reflection Procedure dictated as above, when I needed.
The next long list shows the result of my effort.

```
*** Compose Simultaneous Magic Squares 5*5 ***
** Both Self-Complementary and Pan-Diagonal **
** By Transforming Each Fundamental into 16 **

 F1/                3/                4/                2/
   1 15 24 18  7     1 15 22 18  9     1 23 20 12  9     1 23 20 14  7
  23 17  6  5 14    23 19  6  5 12    15  7  4 21 18    15  9  2 21 18
  10  4 13 22 16    10  2 13 24 16    24 16 13 10  2    22 16 13 10  4
  12 21 20  9  3    14 21 20  7  3     8  5 22 19 11     8  5 24 17 11
  19  8  2 11 25    17  8  4 11 25    17 14  6  3 25    19 12  6  3 25
  7/               11/                6/               10/
   2 14 21 18 10     4 12 21 18 10     2 23 19 15  6     4 23 17 15  6
  23 20  7  4 11    23 20  9  2 11    14 10  1 22 18    12 10  1 24 18
   9  1 13 25 17     7  1 13 25 19    21 17 13  9  5    21 19 13  7  5
  15 22 19  6  3    15 24 17  6  3     8  4 25 16 12     8  2 25 16 14
  16  8  5 12 24    16  8  5 14 22    20 11  7  3 24    20 11  9  3 22
 15/               13/               14/               16/
   5 11 22 18  9     5 11 24 18  7     5 23 16 14  7     5 23 16 12  9
  23 19 10  1 12    23 17 10  1 14    11  9  2 25 18    11  7  4 25 18
   6  2 13 24 20     6  4 13 22 20    22 20 13  6  4    24 20 13  6  2
  14 25 16  7  3    12 25 16  9  3     8  1 24 17 15     8  1 22 19 15
  17  8  4 15 21    19  8  2 15 21    19 12 10  3 21    17 14 10  3 21
  9/                5/               12/                8/
   4 12 25 18  6     2 14 25 18  6     4 23 17 11 10     2 23 19 11 10
  23 16  9  2 15    23 16  7  4 15    12  6  5 24 18    14  6  5 22 18
   7  5 13 21 19     9  5 13 21 17    25 19 13  7  1    25 17 13  9  1
  11 24 17 10  3    11 22 19 10  3     8  2 21 20 14     8  4 21 20 12
  20  8  1 14 22    20  8  1 12 24    16 15  9  3 22    16 15  7  3 24

 F2/                4/                3/                1/
   1 23 20 14  7     1 23 20 12  9     1 15 22 18  9     1 15 24 18  7
  15  9  2 21 18    15  7  4 21 18    23 19  6  5 12    23 17  6  5 14
  22 16 13 10  4    24 16 13 10  2    10  2 13 24 16    10  4 13 22 16
   8  5 24 17 11     8  5 22 19 11    14 21 20  7  3    12 21 20  9  3
  19 12  6  3 25    17 14  6  3 25    17  8  4 11 25    19  8  2 11 25
 10/                6/               11/                7/
   4 23 17 15  6     2 23 19 15  6     4 12 21 18 10     2 14 21 18 10
  12 10  1 24 18    14 10  1 22 18    23 20  9  2 11    23 20  7  4 11
  21 19 13  7  5    21 17 13  9  5     7  1 13 25 19     9  1 13 25 17
   8  2 25 16 14     8  4 25 16 12    15 24 17  6  3    15 22 19  6  3
  20 11  9  3 22    20 11  7  3 24    16  8  5 14 22    16  8  5 12 24
 16/               14/               13/               15/
   5 23 16 12  9     5 23 16 14  7     5 11 24 18  7     5 11 22 18  9
  11  7  4 25 18    11  9  2 25 18    23 17 10  1 14    23 19 10  1 12
  24 20 13  6  2    22 20 13  6  4     6  4 13 22 20     6  2 13 24 20
   8  1 22 19 15     8  1 24 17 15    12 25 16  9  3    14 25 16  7  3
  17 14 10  3 21    19 12 10  3 21    19  8  2 15 21    17  8  4 15 21
```

8/

2	23	19	11	10
14	6	5	22	18
25	17	13	9	1
8	4	21	20	12
16	15	7	3	24

12/

4	23	17	11	10
12	6	5	24	18
25	19	13	7	1
8	2	21	20	14
16	15	9	3	22

5/

2	14	25	18	6
23	16	7	4	15
9	5	13	21	17
11	22	19	10	3
20	8	1	12	24

9/

4	12	25	18	6
23	16	9	2	15
7	5	13	21	19
11	24	17	10	3
20	8	1	14	22

F3/

1	15	22	18	9
23	19	6	5	12
10	2	13	24	16
14	21	20	7	3
17	8	4	11	25

1/

1	15	24	18	7
23	17	6	5	14
10	4	13	22	16
12	21	20	9	3
19	8	2	11	25

2/

1	23	20	14	7
15	9	2	21	18
22	16	13	10	4
8	5	24	17	11
19	12	6	3	25

4/

1	23	20	12	9
15	7	4	21	18
24	16	13	10	2
8	5	22	19	11
17	14	6	3	25

11/

4	12	21	18	10
23	20	9	2	11
7	1	13	25	19
15	24	17	6	3
16	8	5	14	22

7/

2	14	21	18	10
23	20	7	4	11
9	1	13	25	17
15	22	19	6	3
16	8	5	12	24

10/

4	23	17	15	6
12	10	1	24	18
21	19	13	7	5
8	2	25	16	14
20	11	9	3	22

6/

2	23	19	15	6
14	10	1	22	18
21	17	13	9	5
8	4	25	16	12
20	11	7	3	24

13/

5	11	24	18	7
23	17	10	1	14
6	4	13	22	20
12	25	16	9	3
19	8	2	15	21

15/

5	11	22	18	9
23	19	10	1	12
6	2	13	24	20
14	25	16	7	3
17	8	4	15	21

16/

5	23	16	12	9
11	7	4	25	18
24	20	13	6	2
8	1	22	19	15
17	14	10	3	21

14/

5	23	16	14	7
11	9	2	25	18
22	20	13	6	4
8	1	24	17	15
19	12	10	3	21

5/

2	14	25	18	6
23	16	7	4	15
9	5	13	21	17
11	22	19	10	3
20	8	1	12	24

9/

4	12	25	18	6
23	16	9	2	15
7	5	13	21	19
11	24	17	10	3
20	8	1	14	22

8/

2	23	19	11	10
14	6	5	22	18
25	17	13	9	1
8	4	21	20	12
16	15	7	3	24

12/

4	23	17	11	10
12	6	5	24	18
25	19	13	7	1
8	2	21	20	14
16	15	9	3	22

F4/

1	23	20	12	9
15	7	4	21	18
24	16	13	10	2
8	5	22	19	11
17	14	6	3	25

2/

1	23	20	14	7
15	9	2	21	18
22	16	13	10	4
8	5	24	17	11
19	12	6	3	25

1/

1	15	24	18	7
23	17	6	5	14
10	4	13	22	16
12	21	20	9	3
19	8	2	11	25

3/

1	15	22	18	9
23	19	6	5	12
10	2	13	24	16
14	21	20	7	3
17	8	4	11	25

6/

2	23	19	15	6
14	10	1	22	18
21	17	13	9	5
8	4	25	16	12
20	11	7	3	24

10/

4	23	17	15	6
12	10	1	24	18
21	19	13	7	5
8	2	25	16	14
20	11	9	3	22

7/

2	14	21	18	10
23	20	7	4	11
9	1	13	25	17
15	22	19	6	3
16	8	5	12	24

11/

4	12	21	18	10
23	20	9	2	11
7	1	13	25	19
15	24	17	6	3
16	8	5	14	22

14/

5	23	16	14	7
11	9	2	25	18
22	20	13	6	4
8	1	24	17	15
19	12	10	3	21

16/

5	23	16	12	9
11	7	4	25	18
24	20	13	6	2
8	1	22	19	15
17	14	10	3	21

15/

5	11	22	18	9
23	19	10	1	12
6	2	13	24	20
14	25	16	7	3
17	8	4	15	21

13/

5	11	24	18	7
23	17	10	1	14
6	4	13	22	20
12	25	16	9	3
19	8	2	15	21

```
 12/                 8/                  9/                  5/
  4 23 17 11 10       2 23 19 11 10       4 12 25 18  6       2 14 25 18  6
 12  6  5 24 18      14  6  5 22 18      23 16  9  2 15      23 16  7  4 15
 25 19 13  7  1      25 17 13  9  1       7  5 13 21 19       9  5 13 21 17
  8  2 21 20 14       8  4 21 20 12      11 24 17 10  3      11 22 19 10  3
 16 15  9  3 22      16 15  7  3 24      20  8  1 14 22      20  8  1 12 24
```

To my surprise, we could make all of them whatever we might select and nominate for the tops of 4 groups.
Any one of 4 solutions with n1=1 will do all right, and is enough to be nominated for ‘the Most Fundamental Solution’.
All Simultaneous Suzuki Squares are created to be essentially free and equal.

But I would like to select and nominate No.4 for ‘the Most Fundamental Solution’, since it can be composed from the regular array of 5 by 5 in the smartest way.
Study the next diagrams, if you want to know how to compose it by yourself.

(Written on December 6, 2002; Revised on May 31, 2005;
Worked on MacOS X(10.3.5) and Xcode 1.5 by Kanji Setsuda)

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#7. How to Make Basic Form into Goal Object

```
 /Basic              /Common             /Goal
  1  2  3  4  5       1  #  #  #  #       1 23 20 12  9
  6  7  8  9 10       #  7  #  #  #      15  7  4 21 18
 11 12 13 14 15       #  # 13  #  #      24 16 13 10  2
 16 17 18 19 20       #  #  # 19  #       8  5 22 19 11
 21 22 23 24 25       #  #  #  # 25      17 14  6  3 25
```

```
** How to Make a Simultaneous ‘Suzuki Square’ 5x5 Step by Step **

 1/Start                                2/Place  1  2  3  4  5

  0 25  0  0  0  0 25  0  0  0            3 25  0  0  0  3 25  0  0  0
        .--------------.                        .--------------.
  0  0| 1| 0| 0| 0| 0| 1  0  0            0  0| 1| 0| 0| 0| 0| 1  0  0
      |--+--+--+--+--|                        |--+--+--+--+--|
  0  0| 0| 7| 0| 0| 0| 0  7  0            0  0| 0| 7| 4| 0| 0| 0  7  4
      |--+--+--+--+--|                        |--+--+--+--+--|
  0  0| 0| 0|13| 0| 0| 0  0 13            0  2| 0| 0|13| 0| 2| 0  0 13
      |--+--+--+--+--|                        |--+--+--+--+--|
 19  0| 0| 0| 0|19| 0| 0  0  0           19  0| 0| 5| 0|19| 0| 0  5  0
      |--+--+--+--+--|                        |--+--+--+--+--|
  0 25| 0| 0| 0| 0|25| 0  0  0            3 25| 0| 0| 0| 3|25| 0  0  0
        '--------------'                        '--------------'
  0  0  1  0  0  0  0  1  0  0            0  0  1  0  0  0  0  1  0  0
```

```
3/Place  6  7  8  9 10

 3 25  0  0  6  3 25  0  0  6
     .--------------.
 0  9| 1| 0| 0| 0| 9| 1  0  0
     |--+--+--+--+--|
 0  0| 0| 7| 4| 0| 0| 0  7  4
     |--+--+--+--+--|
10  2| 0| 0|13|10| 2| 0  0 13
     |--+--+--+--+--|
19  0| 8| 5| 0|19| 0| 8  5  0
     |--+--+--+--+--|
 3 25| 0| 0| 6| 3|25| 0  0  6
     '--------------'
 0  9  1  0  0  0  9  1  0  0
```

```
4/Place 11 12 13 14 15

 3 25  0 14  6  3 25  0 14  6
     .--------------.
12  9| 1| 0| 0|12| 9| 1  0  0
     |--+--+--+--+--|
 0  0|15| 7| 4| 0| 0|15  7  4
     |--+--+--+--+--|
10  2| 0| 0|13|10| 2| 0  0 13
     |--+--+--+--+--|
19 11| 8| 5| 0|19|11| 8  5  0
     |--+--+--+--+--|
 3 25| 0|14| 6| 3|25| 0 14  6
     '--------------'
12  9  1  0  0 12  9  1  0  0
```

```
5/Place 16 17 18 19 20

 3 25 17 14  6  3 25 17 14  6
     .--------------.
12  9| 1| 0|20|12| 9| 1  0 20
     |--+--+--+--+--|
 0 18|15| 7| 4| 0|18|15  7  4
     |--+--+--+--+--|
10  2| 0|16|13|10| 2| 0 16 13
     |--+--+--+--+--|
19 11| 8| 5| 0|19|11| 8  5  0
     |--+--+--+--+--|
 3 25|17|14| 6| 3|25|17 14  6
     '--------------'
12  9  1  0 20 12  9  1  0 20
```

```
6/Place 21 22 23 24 25

 3 25 17 14  6  3 25 17 14  6
     .--------------.
12  9| 1|23|20|12| 9| 1 23 20
     |--+--+--+--+--|
21 18|15| 7| 4|21|18|15  7  4
     |--+--+--+--+--|
10  2|24|16|13|10| 2|24 16 13
     |--+--+--+--+--|
19 11| 8| 5|22|19|11| 8  5 22
     |--+--+--+--+--|
 3 25|17|14| 6| 3|25|17 14  6
     '--------------'
12  9  1 23 20 12  9  1 23 20

           [Goal]
```