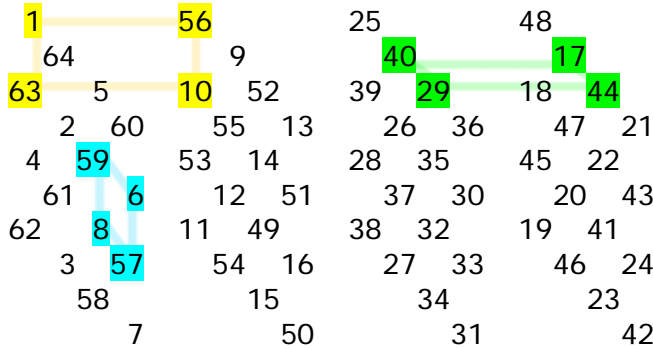


Part 1: "Basic Studies of Magic Squares and Cubes": Kanji Setsuda
 Chapter 4: "Advanced Algebraic Study of Magic Cubes"
 Section 4: 'Composite' Pan-Magic Cubes of Order 4

#1. What is a 'Composite' Pan-Magic Cube like?

Let me take a sample solution shown below for instance.

[Figure 1: Example of 'Composite' Pan-magic Cube]



$1+63+56+10=130$; $59+6+8+57=130$; $40+17+29+44=130$;

Every four adjacent numbers making a little square 2 x 2 on any plane add up to the magic constant 130. Of course, the sum of any next four numbers on any plane combined into a little square 2 x 2 is equal to 130.

We call such a square or a cube as 'Composite' type.

As long as it is a magic cube, every row, every column and every pillar add up to the same constant. And all four primary triagonals have the same constant sums 130 of four numbers on each.

$1+64+5+60=130$; $1+63+4+62=130$; $1+56+25+48=130$;
 $1+55+32+42=130$; $60+14+37+19=130$; $62+12+35+21=130$;
 $7+49+26+48=130$;

#2. Let's Make the 'Composite' Pan-Magic Cubes.

Look at the example cube again. Check if every pantriagonal has the same sum.

$64+14+33+19=130$; $5+51+28+46=130$; $60+10+37+23=130$;
 $1+51+32+46=130$; $63+12+34+21=130$;

Every pantriagonal seems to have the same construction: $33+97=130$

$(64+33)+(14+19)=130$; $(5+28)+(51+46)=130$; $(60+37)+(10+23)=130$;
 $(1+32)+(51+46)=130$; $(63+34)+(12+21)=130$;

Yes. It is not only a 'Composite' magic cube 4x4x4, but also a 'Pan-triagonal' magic cube 4x4x4 at the same time.

Let's make this type of pan-magic cubes of order 4 now.

#3. Basic Form and Basic Conditions

What conditions should we assume for the first definition stage?

[Composite Conditions: $K=130$]

$n1+ n5+ n2+ n6=K$;	$n1+n17+ n2+n18=K$;	$n1+n17+ n5+n21=K$;
$n2+ n6+ n3+ n7=K$;	$n2+n18+ n3+n19=K$;	$n5+n21+ n9+n25=K$;
$n3+ n7+ n4+ n8=K$;	$n3+n19+ n4+n20=K$;	$n9+n25+n13+n29=K$;
$n5+ n9+ n6+n10=K$;	$n5+n21+ n6+n22=K$;	$n2+n18+ n6+n22=K$;
$n6+n10+ n7+n11=K$;	$n6+n22+ n7+n23=K$;	$n6+n22+n10+n26=K$;

$n_7+n_{11}+n_8+n_{12}=K;$	$n_7+n_{23}+n_8+n_{24}=K;$	$n_{10}+n_{26}+n_{14}+n_{30}=K;$
$n_9+n_{13}+n_{10}+n_{14}=K;$	$n_9+n_{25}+n_{10}+n_{26}=K;$	$n_3+n_{19}+n_7+n_{23}=K;$
$n_{10}+n_{14}+n_{11}+n_{15}=K;$	$n_{10}+n_{26}+n_{11}+n_{27}=K;$	$n_7+n_{23}+n_{11}+n_{27}=K;$
$n_{11}+n_{15}+n_{12}+n_{16}=K;$	$n_{11}+n_{27}+n_{12}+n_{63}=K;$	$n_{11}+n_{27}+n_{15}+n_{31}=K;$
$n_{17}+n_{21}+n_{18}+n_{22}=K;$	$n_{13}+n_{29}+n_{14}+n_{30}=K;$	$n_4+n_{20}+n_8+n_{24}=K;$
$n_{18}+n_{22}+n_{19}+n_{23}=K;$	$n_{14}+n_{30}+n_{15}+n_{31}=K;$	$n_8+n_{24}+n_{12}+n_{28}=K;$
.....
$n_{58}+n_{62}+n_{59}+n_{63}=K;$	$n_{46}+n_{62}+n_{47}+n_{63}=K;$	$n_{40}+n_{56}+n_{44}+n_{60}=K;$
$n_{59}+n_{63}+n_{60}+n_{64}=K;$	$n_{47}+n_{63}+n_{48}+n_{64}=K;$	$n_{44}+n_{60}+n_{48}+n_{64}=K;$

[Figure 2: Basic Form for 'Composite' Pan-Magic Cube 4x4x4]

1-----17-----33-----49			
2	18	34	50
5 3	21 19	37 35	53 51
6 4	---22---20---	38--36---	54--52
9 7	25 23	41 39	57 55
10 8	26 24	42 40	58 56
13--11-	---29--27-	---45--43-	---61 59
14 12	30 28	46 44	62 60
15	31	47	63
16-----32-----48-----64			

[Additional Basic Conditions: K=130]

$n_1+n_2+n_3+n_4=K; n_1+n_5+n_9+n_{13}=K; n_1+n_{17}+n_{33}+n_{49}=K;$
 $n_1+n_{22}+n_{43}+n_{64}=K$

#4. Let's find 'Complementary Pairs' as many as possible.

We use so many 'complementary pairs' for making our cube. I want to define this word explicitly right here.

When $m+n=C$, we say "(m, n) is a complementary pair of C."

When we say "(m, n) is a complementary pair of C", we mean

$m+n=C; m=C-n; n=C-m; \underline{m}=C-m=n; \text{ or } \underline{n}=C-n=m;$

Take some of those equations above and compare and calculate:

(1) $n_1+n_5+n_2+n_6=K; n_2+n_6+n_3+n_7=K; n_3+n_7+n_4+n_8=K;$
 $n_1+n_5=n_3+n_7=P_1; n_2+n_6=n_4+n_8=Q_1; P_1+Q_1=K$

Therefore $(n_4+n_8)+(n_1+n_5)=Q_1+P_1=K \dots (c1)$

(2) $n_5+n_9+n_6+n_{10}=K; n_6+n_{10}+n_7+n_{11}=K; n_7+n_{11}+n_8+n_{12}=K;$
 $n_5+n_9=n_7+n_{11}=P_2; n_6+n_{10}=n_8+n_{12}=Q_2; P_2+Q_2=K;$

Therefore $(n_8+n_{12})+(n_5+n_9)=Q_2+P_2=K \dots (c2)$

(3) $n_9+n_{13}+n_{10}+n_{14}=K; n_{10}+n_{14}+n_{11}+n_{15}=K; n_{11}+n_{15}+n_{12}+n_{16}=K;$
 $n_9+n_{13}=n_{11}+n_{15}=P_3; n_{10}+n_{14}=n_{12}+n_{16}=Q_3; P_3+Q_3=K;$

Therefore $(n_{12}+n_{16})+(n_9+n_{13})=Q_3+P_3=K \dots (c3)$

But $(n_1+n_5)+(n_9+n_{13})=K; \text{ Therefore } P_3=Q_1; Q_3=P_1$

These conclusions (c1), (c2) and (c3) mean that you can move the whole pillar $n_1+n_5+n_9+n_{13}$ far to the opposite side and put it in front of the pillar $n_4+n_8+n_{12}+n_{16}$, and you can see the composite conditions kept still true even then.

In the same way you can get such conclusions as:

$n_{17}+n_{21}=n_{19}+n_{23}=P_4; n_{18}+n_{22}=n_{20}+n_{24}=Q_4; P_4+Q_4=K;$

But $n_1+n_5+n_{17}+n_{21}=K; n_2+n_6+n_{18}+n_{22}=K; \text{ Therefore } P_4=Q_1; Q_4=P_1$

$n_{21}+n_{25}=n_{23}+n_{27}=P_5=Q_2; n_{22}+n_{26}=n_{24}+n_{28}=Q_5=P_2; (P_5+Q_5=K)$

$n_{25}+n_{29}=n_{27}+n_{31}=P_6=P_1; n_{26}+n_{30}=n_{28}+n_{32}=Q_6=Q_1; (P_6+Q_6=K)$

$$n_{20}+n_{24}+n_{17}+n_{21}=K; \quad n_{24}+n_{28}+n_{21}+n_{25}=K; \quad n_{28}+n_{32}+n_{25}+n_{29}=K;$$

You can move $n_{17}+n_{21}+n_{25}+n_{29}$ far to the opposite side and put it in front of the pillar $n_{20}+n_{24}+n_{28}+n_{32}$.

.....

These properties are quite similar to any pan-magic type.

Each plane in the left-right direction has the two kinds of complementary pairs of P1 and Q1:

$$\begin{aligned} n_1+n_5=n_3+n_7=n_{10}+n_{14}=n_{12}+n_{16}=n_{18}+n_{22}=n_{20}+n_{24}=n_{25}+n_{29}=n_{27}+n_{31}=n_{33}+n_{37}= \\ n_{35}+n_{39}=n_{42}+n_{46}=n_{44}+n_{48}=n_{50}+n_{54}=n_{52}+n_{56}=n_{57}+n_{61}=n_{59}+n_{63}=P_1; \\ n_2+n_6=n_4+n_8=n_9+n_{13}=n_{11}+n_{15}=n_{17}+n_{21}=n_{19}+n_{23}=n_{26}+n_{30}=n_{28}+n_{32}=n_{34}+n_{38}=n_{36}+n_{40} \\ =n_{41}+n_{45}=n_{43}+n_{47}=n_{49}+n_{53}=n_{51}+n_{55}=n_{58}+n_{62}=n_{60}+n_{64}=Q_1 \quad (P_1+Q_1=K) \quad \dots \quad (cc1) \end{aligned}$$

The same thing could be concluded on every plane in the up-down and back-front directions:

$$\begin{aligned} n_1+n_2=n_7+n_8=n_9+n_{10}=n_{15}+n_{16}=n_{19}+n_{20}=n_{21}+n_{22}=n_{27}+n_{28}=n_{29}+n_{30}=n_{33}+n_{34}= \\ n_{39}+n_{40}=n_{41}+n_{42}=n_{47}+n_{48}=n_{51}+n_{52}=n_{53}+n_{54}=n_{59}+n_{60}=n_{61}+n_{62}=R_1; \\ n_3+n_4=n_5+n_6=n_{11}+n_{12}=n_{13}+n_{14}=n_{17}+n_{18}=n_{23}+n_{24}=n_{25}+n_{26}=n_{31}+n_{32}=n_{35}+n_{36}= \\ n_{37}+n_{38}=n_{43}+n_{44}=n_{45}+n_{46}=n_{49}+n_{50}=n_{55}+n_{56}=n_{57}+n_{58}=n_{63}+n_{64}=S_1 \\ (R_1+S_1=K) \quad \dots \quad (cc2) \\ n_1+n_{17}=n_3+n_{19}=n_6+n_{22}=n_8+n_{24}=n_9+n_{25}=n_{11}+n_{27}=n_{14}+n_{30}=n_{16}+n_{32}=n_{34}+n_{50}=n_{36}+n_{52} \\ =n_{37}+n_{53}=n_{39}+n_{55}=n_{42}+n_{58}=n_{44}+n_{60}=n_{45}+n_{61}=n_{47}+n_{63}=U_1; \\ n_2+n_{18}=n_4+n_{20}=n_5+n_{21}=n_7+n_{23}=n_{10}+n_{26}=n_{12}+n_{28}=n_{13}+n_{29}=n_{15}+n_{31}=n_{33}+n_{49}= \\ n_{35}+n_{51}=n_{38}+n_{54}=n_{40}+n_{56}=n_{41}+n_{57}=n_{43}+n_{59}=n_{46}+n_{62}=n_{48}+n_{64}=V_1 \\ (U_1+V_1=K) \quad \dots \quad (cc3) \end{aligned}$$

(cc1)~(cc3) mean 6 sets of 16 pairs of complementary numbers could be classified into 3 groups of complementary pairs (P1, Q1), (R1, S1) and (U1, V1).

#5. Rows, Columns and Pillars

Does every row, every column and every pillar add up to the magic constant?

$$\begin{aligned} (1) \quad n_5+n_6+n_7+n_8=(n_5+n_6)+(n_7+n_8)=S_1+R_1=K; \\ n_9+n_{10}+n_{11}+n_{12}=R_1+S_1=K; \quad n_{13}+n_{14}+n_{15}+n_{16}=S_1+R_1=K; \\ n_{17}+n_{18}+n_{19}+n_{20}=S_1+R_1=K; \quad n_{21}+n_{22}+n_{23}+n_{24}=R_1+S_1=K; \quad \dots \\ (2) \quad n_2+n_6+n_{10}+n_{14}=(n_2+n_6)+(n_{10}+n_{14})=Q_1+P_1=K; \\ n_3+n_7+n_{11}+n_{15}=P_1+Q_1=K; \quad n_4+n_8+n_{12}+n_{16}=Q_1+P_1=K; \\ n_{17}+n_{21}+n_{25}+n_{29}=Q_1+P_1=K; \quad n_{18}+n_{22}+n_{26}+n_{30}=P_1+Q_1=K; \quad \dots \\ (3) \quad n_2+n_{18}+n_{34}+n_{50}=V_1+U_1=K; \quad n_3+n_{19}+n_{35}+n_{51}=U_1+V_1=K; \\ n_4+n_{20}+n_{36}+n_{52}=V_1+U_1=K; \quad n_5+n_{21}+n_{37}+n_{53}=V_1+U_1=K; \\ n_6+n_{22}+n_{38}+n_{54}=U_1+V_1=K; \quad \dots \end{aligned}$$

The sum of each row, each column and each pillar is equal to the same constant.

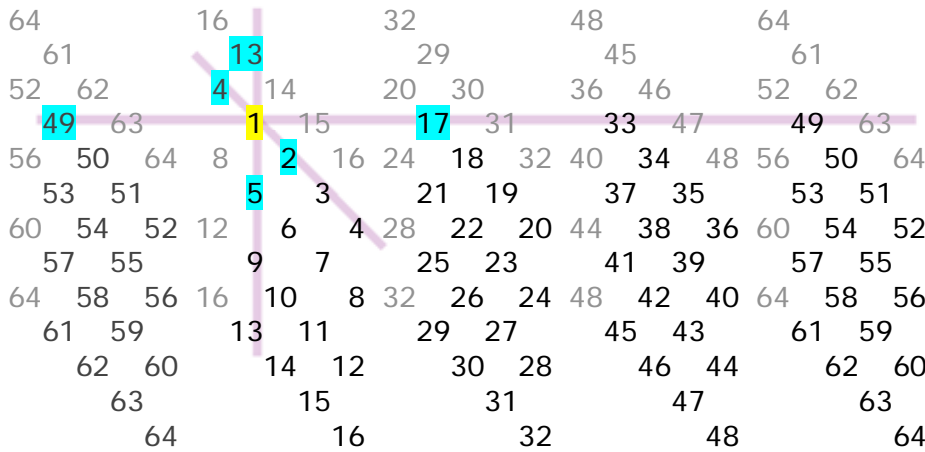
#6. Make the 'Extended Space'

Next let's make the basic form wider into 'Extended Space' just like the case of pan-magic type, as shown below.

You will be able to find any of $\{n_4, n_{13}$ and $n_{49}\}$ could be the next numbers adjacent to n_1 just like $\{n_2, n_5$ and $n_{17}\}$.

$$\begin{aligned} n_1+n_4=n_6+n_7=n_9+n_{12}=n_{14}+n_{15}=n_{18}+n_{19}=n_{21}+n_{24}=n_{26}+n_{27}=n_{29}+n_{32}=n_{33}+n_{36}=n_{38}+n_{39} \\ =n_{41}+n_{44}=n_{46}+n_{48}=n_{50}+n_{51}=n_{53}+n_{56}=n_{58}+n_{59}=n_{61}+n_{64}=PP; \\ n_2+n_3=n_5+n_8=n_{10}+n_{11}=n_{13}+n_{16}=n_{17}+n_{20}=n_{22}+n_{23}=n_{25}+n_{28}=n_{30}+n_{31}=n_{34}+n_{35}=n_{37}+n_{40} \\ =n_{42}+n_{43}=n_{45}+n_{48}=n_{49}+n_{52}=n_{54}+n_{55}=n_{57}+n_{60}=n_{62}+n_{63}=QQ \quad (PP+QQ=K) \quad \dots \quad (cc4) \end{aligned}$$

[Figure 3: Extended Space for 'Pan-Triagonal' Magic Cube]



$$\begin{aligned}
 n_1+n_{13}=n_3+n_{15}=n_6+n_{10}=n_8+n_{12}=n_{18}+n_{30}=n_{20}+n_{32}=n_{21}+n_{25}=n_{23}+n_{27}=n_{33}+n_{45}=n_{35}+n_{47} \\
 =n_{38}+n_{42}=n_{40}+n_{44}=n_{50}+n_{62}=n_{52}+n_{64}=n_{53}+n_{57}=n_{59}+n_{63}=RR; \\
 n_2+n_{14}=n_4+n_{16}=n_5+n_9=n_7+n_{11}=n_{17}+n_{29}=n_{19}+n_{31}=n_{22}+n_{26}=n_{24}+n_{28}=n_{34}+n_{46}=n_{36}+n_{48} \\
 =n_{37}+n_{41}=n_{39}+n_{43}=n_{49}+n_{61}=n_{51}+n_{63}=n_{54}+n_{58}=n_{56}+n_{60}=SS \quad (RR+SS=K) \quad \dots (cc5) \\
 n_1+n_{49}=n_3+n_{51}=n_6+n_{54}+n_8+n_{56}=n_9+n_{57}=n_{11}+n_{59}=n_{14}+n_{62}=n_{16}+n_{64}=n_{18}+n_{34}=n_{20}+n_{36} \\
 =n_{21}+n_{37}=n_{23}+n_{39}=n_{26}+n_{42}+n_{28}+n_{44}=n_{29}+n_{45}=n_{31}+n_{47}=UU; \\
 n_2+n_{50}=n_4+n_{52}=n_5+n_{53}=n_7+n_{55}=n_{10}+n_{58}=n_{12}+n_{60}=n_{13}+n_{61}+n_{15}+n_{63}=n_{17}+n_{33}=n_{19}+n_{35} \\
 =n_{22}+n_{38}=n_{24}+n_{40}=n_{25}+n_{41}=n_{27}+n_{43}=n_{30}+n_{46}=n_{32}+n_{48}=VV \quad (UU+VV=K) \quad \dots (cc6)
 \end{aligned}$$

Another 3 groups of 2 x 16 sets of complementary pairs are needed.

After all we need all those 6 groups of 2 x 16 sets of complementary pairs. Each group has the representative pair such as: (n₁+n₂, n₁+n₄, n₁+n₅, n₁+n₁₃, n₁+n₁₇ and n₁+n₄₉).

#7. Sums of Pan-Triagonals

Let's study about pan-triagonals of our 'Composite' cube now. We assume n₁+n₂₂+n₄₃+n₆₄=K at the first definition stage.

$$\begin{aligned}
 n_1+n_2=n_{21}+n_{22}=n_{41}+n_{42}=n_{61}+n_{62}=R_1; \\
 n_3+n_4=n_{23}+n_{24}=n_{43}+n_{44}=n_{63}+n_{64}=S_1 \quad (R_1+S_1=K); \\
 n_1+n_4=n_{21}+n_{24}=n_{41}+n_{44}=n_{61}+n_{64}=PP; \\
 n_2+n_3=n_{22}+n_{23}=n_{42}+n_{43}=n_{62}+n_{63}=QQ \quad (PP+QQ=K);
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad (n_1+n_{22}+n_{43}+n_{64})+(n_4+n_{23}+n_{42}+n_{61}) &= n_1+n_4+n_{22}+n_{23}+n_{42}+n_{43}+n_{61}+n_{64} \\
 &\quad \underbrace{K}_{\text{K(Basic Condition)}} \quad \underbrace{PP} \quad \underbrace{QQ} \quad \underbrace{QQ} \quad \underbrace{PP} \\
 K+(n_4+n_{23}+n_{42}+n_{61}) &= (PP+QQ)+(PP+QQ) \\
 &\quad \underbrace{K} \quad \underbrace{K}
 \end{aligned}$$

Therefore n₄+n₂₃+n₄₂+n₆₁=K ... (Pd4)

How about n₁+n₂₄+n₄₃+n₆₂?

$$\begin{aligned}
 (2) \quad (n_1+n_{24}+n_{43}+n_{62})+(n_4+n_{23}+n_{42}+n_{61}) &= n_1+n_4+n_{23}+n_{24}+n_{42}+n_{43}+n_{61}+n_{62} \\
 &\quad \underbrace{Pd4(=K)} \quad \underbrace{PP} \quad \underbrace{S1} \quad \underbrace{QQ} \quad \underbrace{R1} \\
 (n_1+n_{24}+n_{43}+n_{62})+K &= (PP+QQ)+(R_1+S_1) \\
 &\quad \underbrace{K} \quad \underbrace{K}
 \end{aligned}$$

Therefore n₁+n₂₄+n₄₃+n₆₂=K ... (Pd12)

How about n₂+n₂₃+n₄₄+n₆₁?

$$\begin{aligned}
 (3) \quad (n_2+n_{23}+n_{44}+n_{61})+(n_1+n_{22}+n_{43}+n_{64}) &= n_1+n_2+n_{22}+n_{23}+n_{43}+n_{44}+n_{61}+n_{64} \\
 &\quad \underbrace{K} \quad \underbrace{R1} \quad \underbrace{QQ} \quad \underbrace{S1} \quad \underbrace{PP}
 \end{aligned}$$

$$(n_2+n_{23}+n_{44}+n_{61})+K=(R_1+S_1)+(PP+QQ)$$

$$\text{Therefore } n_2+n_{23}+n_{44}+n_{61}=K \dots (\text{Pd2})$$

How about $n_2+n_{21}+n_{44}+n_{63}$?

$$(4) (n_2+n_{21}+n_{44}+n_{63})+(n_1+n_{22}+n_{43}+n_{64})=n_1+n_2+n_{21}+n_{22}+n_{43}+n_{44}+n_{63}+n_{64}$$

$$(n_2+n_{21}+n_{44}+n_{63})+K=(R_1+S_1)+(R_1+S_1)$$

$$\text{Therefore } n_2+n_{21}+n_{44}+n_{63}=K \dots (\text{Pd22})$$

How about $n_3+n_{24}+n_{41}+n_{62}$?

$$(5) (n_3+n_{24}+n_{41}+n_{62})+(n_4+n_{23}+n_{42}+n_{61})=n_3+n_4+n_{23}+n_{24}+n_{41}+n_{42}+n_{61}+n_{62}$$

$$(n_3+n_{24}+n_{41}+n_{62})+K=(R_1+S_1)+(R_1+S_1)$$

$$\text{Therefore } n_3+n_{24}+n_{41}+n_{62}=K \dots (\text{Pd3})$$

.....

In the same way the sums of all 64 pantriagonals prove to be equal to the same constant 130.

Our 'Composite' cube is a special type of 'Pan-triagonal' ones.

As a natural consequence the next complementary pairs are also needed to make it pan-magic type (Read the previous section about Pan-magic Cube 4x4x4).

$$n_1+n_{43}=n_3+n_{41}=n_6+n_{48}=n_8+n_{46}=n_9+n_{35}=n_{11}+n_{33}=n_{14}+n_{40}=n_{16}+n_{38}=n_{18}+n_{60}$$

$$=n_{20}+n_{58}=n_{21}+n_{63}=n_{23}+n_{61}=n_{26}+n_{52}=n_{28}+n_{50}=n_{29}+n_{55}=n_{31}+n_{53}=CC$$

$$n_2+n_{44}=n_4+n_{42}=n_5+n_{47}=n_7+n_{45}=n_{10}+n_{36}=n_{12}+n_{34}=n_{13}+n_{39}=n_{15}+n_{37}=n_{17}+n_{59}$$

$$=n_{19}+n_{57}=n_{22}+n_{64}=n_{24}+n_{62}=n_{25}+n_{51}=n_{27}+n_{49}=n_{30}+n_{56}=n_{32}+n_{54}=DD$$

$$(CC+DD=K) \dots (\text{cc7})$$

Our Composite cube needs 7 groups of 2 x 16 sets of complementary pairs after all. But could we make so many complementary pairs with the natural numbers from 1 to 64, using each strictly once?

Yes. When we studied about Pan-magic type before, we have found 7 classes of 2 x 16 complementary pairs. We can use all those classes, each of which begins with such a representative pair as:

$$(1, 32), (1, 48), (1, 56), (1, 60), (1, 62), (1, 63) \text{ and } (1, 64)$$

We can only use all those pairs for making our Composite pan-magic cube and we can only put any of the values {32, 48, 56, 60, 62, 63 and 64} to any of the next positions adjacent to n_1 : { $n_2, n_4, n_5, n_{13}, n_{17}$ and n_{49} } beside n_{43} .

We can neither put them anywhere else, and nor put anything else there.

#8. Making our Composite Pan-magic Cube 444

Take all the conditions defined at #3: Composite conditions and four additional basic conditions. Add some inequality conditions to them such as:

$$n_1=1; n_2>n_5>n_{17}; n_2>n_4; n_5>n_{13}, n_{17}>n_{49}$$

I want to reduce any simple reflections and rotations of the same pattern and get only the 'fundamental solutions.'

According to our consideration we must only think of the way how to combine any value of {32, 48, 56, 60, 62, 63 or 64} with any position of { $n_2, n_4, n_5, n_{13}, n_{17}, n_{49}$ or n_{43} }.

First of all we decide the value of n_4 .

- (1) When $n_4=64$, put $\{32, 48, 56, 60, 62, 63\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (2) When $n_4=63$, put $\{32, 48, 56, 60, 62, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (3) When $n_4=62$, put $\{32, 48, 56, 60, 63, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (4) When $n_4=60$, put $\{32, 48, 56, 62, 63, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (5) When $n_4=56$, put $\{32, 48, 60, 62, 63, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (6) When $n_4=48$, put $\{32, 56, 60, 62, 63, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.
- (7) When $n_4=32$, put $\{48, 56, 60, 62, 63, 64\}$ to $\{n_2, n_4, n_5, n_{13}, n_{17}, n_{49}\}$ one by one.

But in every case we must obey $n_2 > n_5 > n_{17}$; $n_2 > n_4$; $n_5 > n_{13}$; and $n_{17} > n_{49}$;

#9. Beautiful Solutions of our Magic Cube

The next list shows all the 105 fundamental solutions.

*** 'Composite' Pan-Magic Cubes of Order 4 ***

** List of Fundamental Solutions with $n_1=1$; $n_4=64$ **

<p>1/64</p> <pre> 1-----48-----49-----32 63 18 15 34 60 4 21 45 12 52 37 29 6 62---43--19---54--14---27--35 13 57 36 24 61 9 20 40 51 7 30 42 3 55 46 26 56--16- -25--33--- 8--64--41 17 10 50 39 31 58 2 23 47 53 28 5 44 11-----38-----59-----22 </pre>	<p>2/64</p> <pre> 1-----56-----41-----32 63 10 23 34 60 4 13 53 20 44 37 29 6 62---51--11---46--22---27--35 21 57 36 16 61 17 12 40 43 7 30 50 3 47 54 26 48--24- -25--33--- 8--64--49 9 18 42 39 31 58 2 15 55 45 28 5 52 19-----38-----59-----14 </pre>	<p>3/64</p> <pre> 1-----56-----25-----48 63 10 39 18 60 4 13 53 36 28 21 45 6 62---51--11---30--38---43--19 37 57 20 16 61 33 12 24 27 7 46 50 3 31 54 42 32--40- -41--17--- 8--64--49 9 34 26 23 47 58 2 15 55 29 44 5 52 35-----22-----59-----14 </pre>
<p>4/64</p> <pre> 1-----48-----49-----32 63 18 15 34 62 6 19 43 14 54 35 27 4 60---45--21---52--12---29--37 11 57 38 24 59 9 22 40 53 7 28 42 5 55 44 26 56--16- -25--33--- 8--64--41 17 10 50 39 31 58 2 23 47 51 30 3 46 13-----36-----61-----20 </pre>	<p>5/64</p> <pre> 1-----56-----41-----32 63 10 23 34 62 6 11 51 22 46 35 27 4 60---53--13---44--20---29--37 19 57 38 16 59 17 14 40 45 7 28 50 5 47 52 26 48--24- -25--33--- 8--64--49 9 18 42 39 31 58 2 15 55 43 30 3 54 21-----36-----61-----12 </pre>	<p>6/64</p> <pre> 1-----56-----25-----48 63 10 39 18 62 6 11 51 38 30 19 43 4 60---53--13---28--36---45--21 35 57 22 16 59 33 14 24 29 7 44 50 5 31 52 42 32--40- -41--17--- 8--64--49 9 34 26 23 47 58 2 15 55 27 46 3 54 37-----20-----61-----12 </pre>
<p>7/64</p> <pre> 1-----48-----49-----32 63 18 15 34 62 10 19 39 14 58 35 23 4 56---45--25---52-- 8---29--41 7 53 42 28 55 5 26 44 57 11 24 38 9 59 40 22 60--16- -21--33---12--64--37 17 6 50 43 31 54 2 27 47 51 30 3 46 13-----36-----61-----20 </pre>	<p>8/64</p> <pre> 1-----60-----37-----32 63 6 27 34 62 10 7 51 26 46 35 23 4 56---57--13---40--20---29--41 19 53 42 16 55 17 14 44 45 11 24 50 9 47 52 22 48--28- -21--33---12--64--49 5 18 38 43 31 54 2 15 59 39 30 3 58 25-----36-----61----- 8 </pre>	<p>9/64</p> <pre> 1-----60-----21-----48 63 6 43 18 62 10 7 51 42 30 19 39 4 56---57--13---24--36---45--25 35 53 26 16 55 33 14 28 29 11 40 50 9 31 52 38 32--44- -37--17---12--64--49 5 34 22 27 47 54 2 15 59 23 46 3 58 41-----20-----61----- 8 </pre>
<p>10/64</p> <pre> 1-----56-----41-----32 63 10 23 34 62 18 11 39 22 58 35 15 4 48---53--25---44-- 8---29--49 7 45 50 28 47 5 26 52 57 19 16 38 17 59 40 14 60--24- -13--33---20--64--37 9 6 42 51 31 46 2 27 55 43 30 3 54 21-----36-----61-----12 </pre>	<p>11/64</p> <pre> 1-----60-----37-----32 63 6 27 34 62 18 7 43 26 54 35 15 4 48---57--21---40--12---29--49 11 45 50 24 47 9 22 52 53 19 16 42 17 55 44 14 56--28- -13--33---20--64--41 5 10 38 51 31 46 2 23 59 39 30 3 58 25-----36-----61----- 8 </pre>	<p>12/64</p> <pre> 1-----60-----13-----56 63 6 51 10 62 18 7 43 50 30 11 39 4 48---57--21---16--36---53--25 35 45 26 24 47 33 22 28 29 19 40 42 17 31 44 38 32--52- -37-- 9---20--64--41 5 34 14 27 55 46 2 23 59 15 54 3 58 49-----12-----61----- 8 </pre>

13/64

```

1-----56-----25-----48
|63      10      39      |18
62 34  11 23  38 58  19 15
| 4 32---53---41---28-- 8---45---49
7 29 | 50 44  31 5  42 52 |
|57 35  16 22  33 59 |24 14
60--40-|-13--17---36--64---21  9 |
 6 26  51 47  30 2  43 55
27 |      46      3      54 |
 37-----20-----61-----12

```

14/64

```

1-----60-----21-----48
|63      6      43      |18
62 34  7 27  42 54  19 15
| 4 32---57---37---24--12---45---49
11 29 | 50 40  31 9  38 52 |
|53 35  16 26  33 55 |28 14
56--44-|-13--17---36--64---25  5 |
10 22  51 47  30 2  39 59
23 |      46      3      58 |
 41-----20-----61----- 8

```

15/64

```

1-----60-----13-----56
|63      6      51      |10
62 34  7 27  50 46  11 23
| 4 32---57---37---16--20---53---41
19 29 | 42 40  31 17  38 44 |
|45 35  24 26  33 47 |28 22
48--52-|-21-- 9---36--64---25  5 |
18 14  43 55  30 2  39 59
15 |      54      3      58 |
 49-----12-----61----- 8

```

[Count = 15]

** List of Fundamental Solutions with n1=1; n43=63 **

1/63

```

1-----48-----49-----32
|64      17      16      |33
60 3  21 46  12 51  37 30
| 5 62---44---19---53---14---28---35
13 58 | 36 23  61 10  20 39 |
|52 7  29 42  4 55 |45 26
56--15-|-25--34--- 8--63---41 18 |
 9 50  40 31  57 2  24 47
54 |      27      6      43 |
 11-----38-----59-----22

```

2/63

```

1-----56-----41-----32
|64      9      24      |33
60 3  13 54  20 43  37 30
| 5 62---52---11---45---22---28---35
21 58 | 36 15  61 18  12 39 |
|44 7  29 50  4 47 |53 26
48--23-|-25--34--- 8--63---49 10 |
17 42  40 31  57 2  16 55
46 |      27      6      51 |
 19-----38-----59-----14

```

3/63

```

1-----56-----25-----48
|64      9      40      |17
60 3  13 54  36 27  21 46
| 5 62---52---11---29---38---44---19
37 58 | 20 15  61 34  12 23 |
|28 7  45 50  4 31 |53 42
32--39-|-41--18--- 8--63---49 10 |
33 26  24 47  57 2  16 55
30 |      43      6      51 |
 35-----22-----59-----14

```

4/63

```

1-----48-----49-----32
|64      17      16      |33
62 5  19 44  14 53  35 28
| 3 60---46---21---51---12---30---37
11 58 | 38 23  59 10  22 39 |
|54 7  27 42  6 55 |43 26
56--15-|-25--34--- 8--63---41 18 |
 9 50  40 31  57 2  24 47
52 |      29      4      45 |
 13-----36-----61-----20

```

5/63

```

1-----56-----41-----32
|64      9      24      |33
62 5  11 52  22 45  35 28
| 3 60---54---13---43---20---30---37
19 58 | 38 15  59 18  14 39 |
|46 7  27 50  6 47 |51 26
48--23-|-25--34--- 8--63---49 10 |
17 42  40 31  57 2  16 55
44 |      29      4      53 |
 21-----36-----61-----12

```

6/63

```

1-----56-----25-----48
|64      9      40      |17
62 5  11 52  38 29  19 44
| 3 60---54---13---27---36---46---21
35 58 | 22 15  59 34  14 23 |
|30 7  43 50  6 31 |51 42
32--39-|-41--18--- 8--63---49 10 |
33 26  24 47  57 2  16 55
28 |      45      4      53 |
 37-----20-----61-----12

```

7/63

```

1-----48-----49-----32
|64      17      16      |33
62 9  19 40  14 57  35 24
| 3 56---46---25---51-- 8---30---41
7 54 | 42 27  55 6  26 43 |
|58 11  23 38  10 59 |39 22
60--15-|-21--34---12--63---37 18 |
 5 50  44 31  53 2  28 47
52 |      29      4      45 |
 13-----36-----61-----20

```

8/63

```

1-----60-----37-----32
|64      5      28      |33
62 9  7 52  26 45  35 24
| 3 56---58---13---39---20---30---41
19 54 | 42 15  55 18  14 43 |
|46 11  23 50  10 47 |51 22
48--27-|-21--34---12--63---49 6 |
17 38  44 31  53 2  16 59
40 |      29      4      57 |
 25-----36-----61----- 8

```

9/63

```

1-----60-----21-----48
|64      5      44      |17
62 9  7 52  42 29  19 40
| 3 56---58---13---23---36---46---25
35 54 | 26 15  55 34  14 27 |
|30 11  39 50  10 31 |51 38
32--43-|-37--18---12--63---49 6 |
33 22  28 47  53 2  16 59
24 |      45      4      57 |
 41-----20-----61----- 8

```

10/63

```

1-----56-----41-----32
|64      9      24      |33
62 17  11 40  22 57  35 16
| 3 48---54---25---43-- 8---30---49
7 46 | 50 27  47 6  26 51 |
|58 19  15 38  18 59 |39 14
60--23-|-13--34---20--63---37 10 |
 5 42  52 31  45 2  28 55
44 |      29      4      53 |
 21-----36-----61-----12

```

11/63

```

1-----60-----37-----32
|64      5      28      |33
62 17  7 44  26 53  35 16
| 3 48---58---21---39---12---30---49
11 46 | 50 23  47 10  22 51 |
|54 19  15 42  18 55 |43 14
56--27-|-13--34---20--63---41 6 |
 9 38  52 31  45 2  24 59
40 |      29      4      57 |
 25-----36-----61----- 8

```

12/63

```

1-----60-----13-----56
|64      5      52      | 9
62 17  7 44  50 29  11 40
| 3 48---58---21---15---36---54---25
35 46 | 26 23  47 34  22 27 |
|30 19  39 42  18 31 |43 38
32--51-|-37--10---20--63---41 6 |
33 14  28 55  45 2  24 59
16 |      53      4      57 |
 49-----12-----61----- 8

```

13/63

```

1-----56-----25-----48
|64      9      40      |17
62 33  11 24  38 57  19 16
| 3 32---54---41---27-- 8---46---49
7 30 | 50 43  31 6  42 51 |
|58 35  15 22  34 59 |23 14
60--39-|-13--18---36--63---21 10 |
 5 26  52 47  29 2  44 55
28 |      45      4      53 |
 37-----20-----61-----12

```

14/63

```

1-----60-----21-----48
|64      5      44      |17
62 33  7 28  42 53  19 16
| 3 32---58---37---23--12---46---49
11 30 | 50 39  31 10  38 51 |
|54 35  15 26  34 55 |27 14
56--43-|-13--18---36--63---25  6 |
 9 22  52 47  29 2  40 59
24 |      45      4      57 |
 41-----20-----61----- 8

```

15/63

```

1-----60-----13-----56
|64      5      52      | 9
62 33  7 28  50 45  11 24
| 3 32---58---37---15--20---54---41
19 30 | 42 39  31 18  38 43 |
|46 35  23 26  34 47 |27 22
48--51-|-21--10---36--63---25  6 |
17 14  44 55  29 2  40 59
16 |      53      4      57 |
 49-----12-----61----- 8

```

[Count = 15]

**** List of Fundamental Solutions with n1=1; n43=62 ****

<p>1/62</p> <pre> 1-----48-----49-----32 64 17 16 33 60 2 21 47 12 50 37 31 5 63---44--18---53--15---28--34 13 59 36 22 61 11 20 38 52 6 29 43 4 54 45 27 56--14- -25--35--- 8--62---41 19 9 51 40 30 57 3 24 46 55 26 7 42 10-----39-----58-----23 </pre>	<p>2/62</p> <pre> 1-----56-----41-----32 64 9 24 33 60 2 13 55 20 42 37 31 5 63---52--10---45--23---28--34 21 59 36 14 61 19 12 38 44 6 29 51 4 46 53 27 48--22- -25--35--- 8--62---49 11 17 43 40 30 57 3 16 54 47 26 7 50 18-----39-----58-----15 </pre>	<p>3/62</p> <pre> 1-----56-----25-----48 64 9 40 17 60 2 13 55 36 26 21 47 5 63---52--10---29--39---44--18 37 59 20 14 61 35 12 22 28 6 45 51 4 30 53 43 32--38- -41--19--- 8--62---49 11 33 27 24 46 57 3 16 54 31 42 7 50 34-----23-----58-----15 </pre>
<p>4/62</p> <pre> 1-----48-----49-----32 64 17 16 33 63 5 18 44 15 53 34 28 2 60---47--21---50--12---31--37 10 59 39 22 58 11 23 38 55 6 26 43 7 54 42 27 56--14- -25--35--- 8--62---41 19 9 51 40 30 57 3 24 46 52 29 4 45 13-----36-----61-----20 </pre>	<p>5/62</p> <pre> 1-----56-----41-----32 64 9 24 33 63 5 10 52 23 45 34 28 2 60---55--13---42--20---31--37 18 59 39 14 58 19 15 38 47 6 26 51 7 46 50 27 48--22- -25--35--- 8--62---49 11 17 43 40 30 57 3 16 54 44 29 4 53 21-----36-----61-----12 </pre>	<p>6/62</p> <pre> 1-----56-----25-----48 64 9 40 17 63 5 10 52 39 29 18 44 2 60---55--13---26--36---47--21 34 59 23 14 58 35 15 22 31 6 42 51 7 30 50 43 32--38- -41--19--- 8--62---49 11 33 27 24 46 57 3 16 54 28 45 4 53 37-----20-----61-----12 </pre>
<p>7/62</p> <pre> 1-----48-----49-----32 64 17 16 33 63 9 18 40 15 57 34 24 2 56---47--25---50-- 8---31--41 6 55 43 26 54 7 27 42 59 10 22 39 11 58 38 23 60--14- -21--35---12--62---37 19 5 51 44 30 53 3 28 46 52 29 4 45 13-----36-----61-----20 </pre>	<p>8/62</p> <pre> 1-----60-----37-----32 64 5 28 33 63 9 6 52 27 45 34 24 2 56---59--13---38--20---31--41 18 55 43 14 54 19 15 42 47 10 22 51 11 46 50 23 48--26- -21--35---12--62---49 7 17 39 44 30 53 3 16 58 40 29 4 57 25-----36-----61----- 8 </pre>	<p>9/62</p> <pre> 1-----60-----21-----48 64 5 44 17 63 9 6 52 43 29 18 40 2 56---59--13---22--36---47--25 34 55 27 14 54 35 15 26 31 10 38 51 11 30 50 39 32--42- -37--19---12--62---49 7 33 23 28 46 53 3 16 58 24 45 4 57 41-----20-----61----- 8 </pre>
<p>10/62</p> <pre> 1-----56-----41-----32 64 9 24 33 63 17 10 40 23 57 34 16 2 48---55--25---42-- 8---31--49 6 47 51 26 46 7 27 50 59 18 14 39 19 58 38 15 60--22- -13--35---20--62---37 11 5 43 52 30 45 3 28 54 44 29 4 53 21-----36-----61-----12 </pre>	<p>11/62</p> <pre> 1-----60-----37-----32 64 5 28 33 63 17 6 44 27 53 34 16 2 48---59--21---38--12---31--49 10 47 51 22 46 11 23 50 55 18 14 43 19 54 42 15 56--26- -13--35---20--62---41 7 9 39 52 30 45 3 24 58 40 29 4 57 25-----36-----61----- 8 </pre>	<p>12/62</p> <pre> 1-----60-----13-----56 64 5 52 9 63 17 6 44 51 29 10 40 2 48---59--21---14--36---55--25 34 47 27 22 46 35 23 26 31 18 38 43 19 30 42 39 32--50- -37--11---20--62---41 7 33 15 28 54 45 3 24 58 16 53 4 57 49-----12-----61----- 8 </pre>
<p>13/62</p> <pre> 1-----56-----25-----48 64 9 40 17 63 33 10 24 39 57 18 16 2 32---55--41---26-- 8---47--49 6 31 51 42 30 7 43 50 59 34 14 23 35 58 22 15 60--38- -13--19---36--62---21 11 5 27 52 46 29 3 44 54 28 45 4 53 37-----20-----61-----12 </pre>	<p>14/62</p> <pre> 1-----60-----21-----48 64 5 44 17 63 33 6 28 43 53 18 16 2 32---59--37---22--12---47--49 10 31 51 38 30 11 39 50 55 34 14 27 35 54 26 15 56--42- -13--19---36--62---25 7 9 23 52 46 29 3 40 58 24 45 4 57 41-----20-----61----- 8 </pre>	<p>15/62</p> <pre> 1-----60-----13-----56 64 5 52 9 63 33 6 28 51 45 10 24 2 32---59--37---14--20---55--41 18 31 43 38 30 19 39 42 47 34 22 27 35 46 26 23 48--50- -21--11---36--62---25 7 17 15 44 54 29 3 40 58 16 53 4 57 49-----12-----61----- 8 </pre>

[Count = 15]

**** List of Fundamental Solutions with n1=1; n43=60 ****

<p>1/60</p> <pre> 1-----48-----49-----32 64 17 16 33 62 2 19 47 14 50 35 31 3 63---46--18---51--15---30--34 11 61 38 20 59 13 22 36 54 4 27 45 6 52 43 29 56--12- -25--37--- 8--60---41 21 9 53 40 28 57 5 24 44 55 26 7 42 10-----39-----58-----23 </pre>	<p>2/60</p> <pre> 1-----56-----41-----32 64 9 24 33 62 2 11 55 22 42 35 31 3 63---54--10---43--23---30--34 19 61 38 12 59 21 14 36 46 4 27 53 6 44 51 29 48--20- -25--37--- 8--60---49 13 17 45 40 28 57 5 16 52 47 26 7 50 18-----39-----58-----15 </pre>	<p>3/60</p> <pre> 1-----56-----25-----48 64 9 40 17 62 2 11 55 38 26 19 47 3 63---54--10---27--39---46--18 35 61 22 12 59 37 14 20 30 4 43 53 6 28 51 45 32--36- -41--21--- 8--60---49 13 33 29 24 44 57 5 16 52 31 42 7 50 34-----23-----58-----15 </pre>
---	---	---

4/60

```

1-----48-----49-----32
|64      17      16      |33
63 3 18 46 15 51 34 30
| 2 62---47---19---50---14---31---35
10 61 | 39 20 58 13 23 36 |
|55 4 26 45 7 52 |42 29
56--12-|-25--37---8--60--41 21 |
 9 53 40 28 57 5 24 44
 54 | 27 6 43 |
 11-----38-----59-----22

```

5/60

```

1-----56-----41-----32
|64      9      24      |33
63 3 10 54 23 43 34 30
| 2 62---55---11---42---22---31---35
18 61 | 39 12 58 21 15 36 |
|47 4 26 53 7 44 |50 29
48--20-|-25--37---8--60--49 13 |
 17 45 40 28 57 5 16 52
 46 | 27 6 51 |
 19-----38-----59-----14

```

6/60

```

1-----56-----25-----48
|64      9      40      |17
63 3 10 54 39 27 18 46
| 2 62---55---11---26---38---47---19
34 61 | 23 12 58 37 15 20 |
|31 4 42 53 7 28 |50 45
32--36-|-41--21---8--60--49 13 |
 33 29 24 44 57 5 16 52
 30 | 43 6 51 |
 35-----22-----59-----14

```

7/60

```

1-----48-----49-----32
|64      17      16      |33
63 9 18 40 15 57 34 24
| 2 56---47---25---50---8---31---41
4 55 | 45 26 52 7 29 42 |
|61 10 20 39 13 58 |36 23
62--12-|-19--37---14--60---35 21 |
 3 53 46 28 51 5 30 44
 54 | 27 6 43 |
 11-----38-----59-----22

```

8/60

```

1-----62-----35-----32
|64      3      30      |33
63 9 4 54 29 43 34 24
| 2 56---61---11---36---22---31---41
18 55 | 45 12 52 21 15 42 |
|47 10 20 53 13 44 |50 23
48--26-|-19--37---14--60---49 7 |
 17 39 46 28 51 5 16 58
 40 | 27 6 57 |
 25-----38-----59-----8

```

9/60

```

1-----62-----19-----48
|64      3      46      |17
63 9 4 54 45 27 18 40
| 2 56---61---11---20---38---47---25
34 55 | 29 12 52 37 15 26 |
|31 10 36 53 13 28 |50 39
32--42-|-35--21---14--60---49 7 |
 33 23 30 44 51 5 16 58
 24 | 43 6 57 |
 41-----22-----59-----8

```

10/60

```

1-----56-----41-----32
|64      9      24      |33
63 17 10 40 23 57 34 16
| 2 48---55---25---42---8---31---49
4 47 | 53 26 44 7 29 50 |
|61 18 12 39 21 58 |36 15
62--20-|-11--37---22--60---35 13 |
 3 45 54 28 43 5 30 52
 46 | 27 6 51 |
 19-----38-----59-----14

```

11/60

```

1-----62-----35-----32
|64      3      30      |33
63 17 4 46 29 51 34 16
| 2 48---61---19---36---14---31---49
10 47 | 53 20 44 13 23 50 |
|55 18 12 45 21 52 |42 15
56--26-|-11--37---22--60---41 7 |
 9 39 54 28 43 5 24 58
 40 | 27 6 57 |
 25-----38-----59-----8

```

12/60

```

1-----62-----11-----56
|64      3      54      |9
63 17 4 46 53 27 10 40
| 2 48---61---19---12---38---55---25
34 47 | 29 20 44 37 23 26 |
|31 18 36 45 21 28 |42 39
32--50-|-35--13---22--60---41 7 |
 33 15 30 52 43 5 24 58
 16 | 51 6 57 |
 49-----14-----59-----8

```

13/60

```

1-----56-----25-----48
|64      9      40      |17
63 33 10 24 39 57 18 16
| 2 32---55---41---26---8---47---49
4 31 | 53 42 28 7 45 50 |
|61 34 12 23 37 58 |20 15
62--36-|-11--21---38--60---19 13 |
 3 29 54 44 27 5 46 52
 30 | 43 6 51 |
 35-----22-----59-----14

```

14/60

```

1-----62-----19-----48
|64      3      46      |17
63 33 4 30 45 51 18 16
| 2 32---61---35---20---14---47---49
10 31 | 53 36 28 13 39 50 |
|55 34 12 29 37 52 |26 15
56--42-|-11--21---38--60---25 7 |
 9 23 54 44 27 5 40 58
 24 | 43 6 57 |
 41-----22-----59-----8

```

15/60

```

1-----62-----11-----56
|64      3      54      |9
63 33 4 30 53 43 10 24
| 2 32---61---35---12---22---55---41
18 31 | 45 36 28 21 39 42 |
|47 34 20 29 37 44 |26 23
48--50-|-19--13---38--60---25 7 |
 17 15 46 52 27 5 40 58
 16 | 51 6 57 |
 49-----14-----59-----8

```

[Count = 15]

** List of Fundamental Solutions with n1=1; n43=56 **

1/56

```

1-----48-----49-----32
|64      17      16      |33
62 2 19 47 14 50 35 31
| 3 63---46---18---51---15---30---34
7 61 | 42 20 55 13 26 36 |
|58 4 23 45 10 52 |39 29
60--8-|-21--41---12--56---37 25 |
 5 57 44 24 53 9 28 40
 59 | 22 11 38 |
 6-----43-----54-----27

```

2/56

```

1-----60-----37-----32
|64      5      28      |33
62 2 7 59 26 38 35 31
| 3 63---58---6---39---27---30---34
19 61 | 42 8 55 25 14 36 |
|46 4 23 57 10 40 |51 29
48--20-|-21--41---12--56---49 13 |
 17 45 44 24 53 9 16 52
 47 | 22 11 50 |
 18-----43-----54-----15

```

3/56

```

1-----60-----21-----48
|64      5      44      |17
62 2 7 59 42 22 19 47
| 3 63---58---6---23---43---46---18
35 61 | 26 8 55 41 14 20 |
|30 4 39 57 10 24 |51 45
32--36-|-37--25---12--56---49 13 |
 33 29 28 40 53 9 16 52
 31 | 38 11 50 |
 34-----27-----54-----15

```

4/56

```

1-----48-----49-----32
|64      17      16      |33
63 3 18 46 15 51 34 30
| 2 62---47---19---50---14---31---35
6 61 | 43 20 54 13 27 36 |
|59 4 22 45 11 52 |38 29
60--8-|-21--41---12--56---37 25 |
 5 57 44 24 53 9 28 40
 58 | 23 10 39 |
 7-----42-----55-----26

```

5/56

```

1-----60-----37-----32
|64      5      28      |33
63 3 6 58 27 39 34 30
| 2 62---59---7---38---26---31---35
18 61 | 43 8 54 25 15 36 |
|47 4 22 57 11 40 |50 29
48--20-|-21--41---12--56---49 13 |
 17 45 44 24 53 9 16 52
 46 | 23 10 51 |
 19-----42-----55-----14

```

6/56

```

1-----60-----21-----48
|64      5      44      |17
63 3 6 58 43 23 18 46
| 2 62---59---7---22---42---47---19
34 61 | 27 8 54 41 15 20 |
|31 4 38 57 11 24 |50 45
32--36-|-37--25---12--56---49 13 |
 33 29 28 40 53 9 16 52
 30 | 39 10 51 |
 35-----26-----55-----14

```

7/56

```

1-----48-----49-----32
|64      17      16      |33
63 5 18 44 15 53 34 28
| 2 60---47---21---50---12---31---37
4 59 | 45 22 52 11 29 38 |
|61 6 20 43 13 54 |36 27
62-- 8-|-19-41---14-56---35 25 |
3 57 46 24 51 9 30 40
58 | 23 10 39 |
7-----42-----55-----26

```

8/56

```

1-----62-----35-----32
|64      3      30      |33
63 5 4 58 29 39 34 28
| 2 60---61--- 7---36---26---31---37
18 59 | 45 8 52 25 15 38 |
|47 6 20 57 13 40 |50 27
48--22-|-19-41---14-56---49 11 |
17 43 46 24 51 9 16 54
44 | 23 10 53 |
21-----42-----55-----12

```

9/56

```

1-----62-----19-----48
|64      3      46      |17
63 5 4 58 45 23 18 44
| 2 60---61--- 7---20---42---47---21
34 59 | 29 8 52 41 15 22 |
|31 6 36 57 13 24 |50 43
32--38-|-35--25---14-56---49 11 |
33 27 30 40 51 9 16 54
28 | 39 10 53 |
37-----26-----55-----12

```

10/56

```

1-----60-----37-----32
|64      5      28      |33
63 17 6 44 27 53 34 16
| 2 48---59---21---38---12---31---49
4 47 | 57 22 40 11 29 50 |
|61 18 8 43 25 54 |36 15
62--20-|- 7-41---26-56---35 13 |
3 45 58 24 39 9 30 52
46 | 23 10 51 |
19-----42-----55-----14

```

11/56

```

1-----62-----35-----32
|64      3      30      |33
63 17 4 46 29 51 34 16
| 2 48---61---19---36---14---31---49
6 47 | 57 20 40 13 27 50 |
|59 18 8 45 25 52 |38 15
60--22-|- 7-41---26-56---37 11 |
5 43 58 24 39 9 28 54
44 | 23 10 53 |
21-----42-----55-----12

```

12/56

```

1-----62----- 7-----60
|64      3      58      | 5
63 17 4 46 57 23 6 44
| 2 48---61---19--- 8-42---59---21
34 47 | 29 20 40 41 27 22 |
|31 18 36 45 25 24 |38 43
32--50-|-35--13---26-56---37 11 |
33 15 30 52 39 9 28 54
16 | 51 10 53 |
49-----14-----55-----12

```

13/56

```

1-----60-----21-----48
|64      5      44      |17
63 33 6 28 43 53 18 16
| 2 32---59---37---22---12---47---49
4 31 | 57 38 24 11 45 50 |
|61 34 8 27 41 54 |20 15
62--36-|- 7-25---42-56---19 13 |
3 29 58 40 23 9 46 52
30 | 39 10 51 |
35-----26-----55-----14

```

14/56

```

1-----62-----19-----48
|64      3      46      |17
63 33 4 30 45 51 18 16
| 2 32---61---35---20---14---47---49
6 31 | 57 36 24 13 43 50 |
|59 34 8 29 41 52 |22 15
60--38-|- 7-25---42-56---21 11 |
5 27 58 40 23 9 44 54
28 | 39 10 53 |
37-----26-----55-----12

```

15/56

```

1-----62----- 7-----60
|64      3      58      | 5
63 33 4 30 57 39 6 28
| 2 32---61---35--- 8-26---59---37
18 31 | 45 36 24 25 43 38 |
|47 34 20 29 41 40 |22 27
48--50-|-19--13---42-56---21 11 |
17 15 46 52 23 9 44 54
16 | 51 10 53 |
49-----14-----55-----12

```

[Count = 15]

** List of Fundamental Solutions with n1=1; n43=48 **

1/48

```

1-----56-----41-----32
|64      9      24      |33
62 2 11 55 22 42 35 31
| 3 63---54---10---43---23---30---34
7 61 | 50 12 47 21 26 36 |
|58 4 15 53 18 44 |39 29
60-- 8-|-13-49---20-48---37 25 |
5 57 52 16 45 17 28 40
59 | 14 19 38 |
6-----51-----46-----27

```

2/48

```

1-----60-----37-----32
|64      5      28      |33
62 2 7 59 26 38 35 31
| 3 63---58--- 6---39---27---30---34
11 61 | 50 8 47 25 22 36 |
|54 4 15 57 18 40 |43 29
56--12-|-13-49---20-48---41 21 |
5 53 52 16 45 17 24 44
55 | 14 19 42 |
10-----51-----46-----23

```

3/48

```

1-----60-----13-----56
|64      5      52      | 9
62 2 7 59 50 14 11 55
| 3 63---58--- 6---15---51---54---10
35 61 | 26 8 47 49 22 12 |
|30 4 39 57 18 16 |43 53
32--36-|-37--25---20-48---41 21 |
33 29 28 40 45 17 24 44
31 | 38 19 42 |
34-----27-----46-----23

```

4/48

```

1-----56-----41-----32
|64      9      24      |33
63 3 10 54 23 43 34 30
| 2 62---55---11---42---22---31---35
6 61 | 51 12 46 21 27 36 |
|59 4 14 53 19 44 |38 29
60-- 8-|-13-49---20-48---37 25 |
5 57 52 16 45 17 28 40
58 | 15 18 39 |
7-----50-----47-----26

```

5/48

```

1-----60-----37-----32
|64      5      28      |33
63 3 6 58 27 39 34 30
| 2 62---59--- 7---38---26---31---35
10 61 | 51 8 46 25 23 36 |
|55 4 14 57 19 40 |42 29
56--12-|-13-49---20-48---41 21 |
9 53 52 16 45 17 24 44
54 | 15 18 43 |
11-----50-----47-----22

```

6/48

```

1-----60-----13-----56
|64      5      52      | 9
63 3 6 58 51 15 10 54
| 2 62---59--- 7---14---50---55---11
34 61 | 27 8 46 49 23 12 |
|31 4 38 57 19 16 |42 53
32--36-|-37--25---20-48---41 21 |
33 29 28 40 45 17 24 44
30 | 39 18 43 |
35-----26-----47-----22

```

7/48

```

1-----56-----41-----32
|64      9      24      |33
63 5 10 52 23 45 34 28
| 2 60---55---13---42---20---31---37
4 59 | 53 14 44 19 29 38 |
|61 6 12 51 21 46 |36 27
62-- 8-|-11-49---22-48---35 25 |
3 57 54 16 43 17 30 40
58 | 15 18 39 |
7-----50-----47-----26

```

8/48

```

1-----62-----35-----32
|64      3      30      |33
63 5 4 58 29 39 34 28
| 2 60---61--- 7---36---26---31---37
10 59 | 53 8 44 25 23 38 |
|55 6 12 57 21 40 |42 27
56--14-|-11-49---22-48---41 19 |
9 51 54 16 43 17 24 46
52 | 15 18 45 |
13-----50-----47-----20

```

9/48

```

1-----62-----11-----56
|64      3      54      | 9
63 5 4 58 53 15 10 52
| 2 60---61--- 7---12---50---55---13
34 59 | 29 8 44 49 23 14 |
|31 6 36 57 21 16 |42 51
32--38-|-35--25---22-48---41 19 |
33 27 30 40 43 17 24 46
28 | 39 18 45 |
37-----26-----47-----20

```

10/48

```

1-----60-----37-----32
|64      5      28      |33
63 9   6 52 27 45 34 24
| 2 56---59---13---38---20---31---41
4 55 | 57 14 40 19 29 42 |
|61 10 8 51 25 46 |36 23
62--12-|- 7--49---26--48---35 21 |
3 53 58 16 39 17 30 44
54 | 15 18 43 |
11-----50-----47-----22

```

11/48

```

1-----62-----35-----32
|64      3      30      |33
63 9   4 54 29 43 34 24
| 2 56---61---11---36---22---31---41
6 55 | 57 12 40 21 27 42 |
|59 10 8 53 25 44 |38 23
60--14-|- 7--49---26--48---37 19 |
5 51 58 16 39 17 28 46
52 | 15 18 45 |
13-----50-----47-----20

```

12/48

```

1-----62-----7-----60
|64      3      58      | 5
63 9   4 54 57 15 6 52
| 2 56---61---11--- 8--50---59---13
34 55 | 29 12 40 49 27 14 |
|31 10 36 53 25 16 |38 51
32--42-|-35--21---26--48---37 19 |
33 23 30 44 39 17 28 46
24 | 43 18 45 |
41-----22-----47-----20

```

13/48

```

1-----60-----13-----56
|64      5      52      | 9
63 33 6 28 51 45 10 24
| 2 32---59---37---14--20---55---41
4 31 | 57 38 16 19 53 42 |
|61 34 8 27 49 46 |12 23
62--36-|- 7--25---50--48---11 21 |
3 29 58 40 15 17 54 44
30 | 39 18 43 |
35-----26-----47-----22

```

14/48

```

1-----62-----11-----56
|64      3      54      | 9
63 33 4 30 53 43 10 24
| 2 32---61---35---12--22---55---41
6 31 | 57 36 16 21 51 42 |
|59 34 8 29 49 44 |14 23
60--38-|- 7--25---50--48---13 19 |
5 27 58 40 15 17 52 46
28 | 39 18 45 |
37-----26-----47-----20

```

15/48

```

1-----62-----7-----60
|64      3      58      | 5
63 33 4 30 57 39 6 28
| 2 32---61---35--- 8--26---59---37
10 31 | 53 36 16 25 51 38 |
|55 34 12 29 49 40 |14 27
56--42-|-11--21---50--48---13 19 |
9 23 54 44 15 17 52 46
24 | 43 18 45 |
41-----22-----47-----20

```

[Count = 15]

** List of Fundamental Solutions with n1=1; n43=32 **

1/32

```

1-----56-----25-----48
|64      9      40      |17
62 2 11 55 38 26 19 47
| 3 63---54--10--27--39---46--18
7 61 | 50 12 31 37 42 20 |
|58 4 15 53 34 28 |23 45
60-- 8-|-13--49---36--32---21 41 |
5 57 52 16 29 33 44 24
59 | 14 35 22 |
6-----51-----30-----43

```

2/32

```

1-----60-----21-----48
|64      5      44      |17
62 2 7 59 42 22 19 47
| 3 63---58-- 6--23--43---46--18
11 61 | 50 8 31 41 38 20 |
|54 4 15 57 34 24 |27 45
56--12-|-13--49---36--32---25 37 |
9 53 52 16 29 33 40 28
55 | 14 35 26 |
10-----51-----30-----39

```

3/32

```

1-----60-----13-----56
|64      5      52      | 9
62 2 7 59 50 14 11 55
| 3 63---58-- 6--15--51---54--10
19 61 | 42 8 31 49 38 12 |
|46 4 23 57 34 16 |27 53
48--20-|-21--41---36--32---25 37 |
17 45 44 24 29 33 40 28
47 | 22 35 26 |
18-----43-----30-----39

```

4/32

```

1-----56-----25-----48
|64      9      40      |17
63 3 10 54 39 27 18 46
| 2 62---55--11--26--38---47--19
6 61 | 51 12 30 37 43 20 |
|59 4 14 53 35 28 |22 45
60-- 8-|-13--49---36--32---21 41 |
5 57 52 16 29 33 44 24
58 | 15 34 23 |
7-----50-----31-----42

```

5/32

```

1-----60-----21-----48
|64      5      44      |17
63 3 6 58 43 23 18 46
| 2 62---59-- 7--22--42---47--19
10 61 | 51 8 30 41 39 20 |
|55 4 14 57 35 24 |26 45
56--12-|-13--49---36--32---25 37 |
9 53 52 16 29 33 40 28
54 | 15 34 27 |
11-----50-----31-----38

```

6/32

```

1-----60-----13-----56
|64      5      52      | 9
63 3 6 58 51 15 10 54
| 2 62---59-- 7--14--50---55--11
18 61 | 43 8 30 49 39 12 |
|47 4 22 57 35 16 |26 53
48--20-|-21--41---36--32---25 37 |
17 45 44 24 29 33 40 28
46 | 23 34 27 |
19-----42-----31-----38

```

7/32

```

1-----56-----25-----48
|64      9      40      |17
63 5 10 52 39 29 18 44
| 2 60---55--13--26--36---47--21
4 59 | 53 14 28 35 45 22 |
|61 6 12 51 37 30 |20 43
62-- 8-|-11--49---38--32---19 41 |
3 57 54 16 27 33 46 24
58 | 15 34 23 |
7-----50-----31-----42

```

8/32

```

1-----62-----19-----48
|64      3      46      |17
63 5 4 58 45 23 18 44
| 2 60---61-- 7--20--42---47--21
10 59 | 53 8 28 41 39 22 |
|55 6 12 57 37 24 |26 43
56--14-|-11--49---38--32---25 35 |
9 51 54 16 27 33 40 30
52 | 15 34 29 |
13-----50-----31-----36

```

9/32

```

1-----62-----11-----56
|64      3      54      | 9
63 5 4 58 53 15 10 52
| 2 60---61-- 7--12--50---55--13
18 59 | 45 8 28 49 39 14 |
|47 6 20 57 37 16 |26 51
48--22-|-19--41---38--32---25 35 |
17 43 46 24 27 33 40 30
44 | 23 34 29 |
21-----42-----31-----36

```

10/32

```

1-----60-----21-----48
|64      5      44      |17
63 9 6 52 43 29 18 40
| 2 56---59--13--22--36---47--25
4 55 | 57 14 24 35 45 26 |
|61 10 8 51 41 30 |20 39
62--12-|- 7--49---42--32---19 37 |
3 53 58 16 23 33 46 28
54 | 15 34 27 |
11-----50-----31-----38

```

11/32

```

1-----62-----19-----48
|64      3      46      |17
63 9 4 54 45 27 18 40
| 2 56---61--11--20--38---47--25
6 55 | 57 12 24 37 43 26 |
|59 10 8 53 41 28 |22 39
60--14-|- 7--49---42--32---21 35 |
5 51 58 16 23 33 44 30
52 | 15 34 29 |
13-----50-----31-----36

```

12/32

```

1-----62-----7-----60
|64      3      58      | 5
63 9 4 54 57 15 6 52
| 2 56---61--11-- 8--50---59--13
18 55 | 45 12 24 49 43 14 |
|47 10 20 53 41 16 |22 51
48--26-|-19--37---42--32---21 35 |
17 39 46 28 23 33 44 30
40 | 27 34 29 |
25-----38-----31-----36

```

13/32

1	60	13	56
64	5	52	9
63	17	6	44
2	48	59	21
4	47	57	22
61	18	8	43
62	20	7	41
3	45	58	24
46	23	34	27
19	42	31	38

14/32

1	62	11	56
64	3	54	9
63	17	4	46
2	48	61	19
6	47	57	20
59	18	8	45
60	22	7	41
5	43	58	24
44	23	34	29
21	42	31	36

15/32

1	62	7	60
64	3	58	5
63	17	4	46
2	48	61	19
10	47	53	20
55	18	12	45
56	26	11	37
9	39	54	28
40	27	34	29
25	38	31	36

[Count = 15] OK!

We have got 7 x 15 = 105 solutions. Everything looks quite beautiful.

#10. Why are there Solutions as many as 7 x 15 (= 105)?

First of all we must decide the value of n43 out of {64, 63, 62, 60, 56, 48, 32}.

--> 7C1 = 7

Next we must decide the values of (n2, n4), (n5, n13), (n17, n49) on 3 axes out of the rest 6.

--> 6C2 x 4C2 x 2C2 = 15 x 6 x 1 = 90

But n2>n4; n5>n13; n17>n49 --> 2C2 = 1; 90/1 = 90

And n2>n5>n17 --> 3P3 = 3 x 2 x 1 = 6; 90/6 = 15

As a result 7 x 15 = 105.

This means the combination of 7 values of 7 positions determines everything. There is only one way of rearranging the rest. It is amazing, isn't it?

When we put n1=1 and n43=64, then CC=DD=65; all complementary pairs of 65 stay only on its pantriagonals and don't go elsewhere. This type could be transformed into the self-complementary type by some simple rules.

This means our cubes with n1=1 and n43=64 are of 'Complete' type. I would like to call them 'Composite & Complete' magic cubes as usual.

All 105 cubes including any solutions with n43<64 may well be simply called 'Composite Pan-magic' type.

We know Composite Pan-magic cubes 444 have the set of 6720 standard solutions, while 'Composite & Complete' magic cubes 444 have 960 standard solutions in all.

CPMC444 with n1=1 have 840 standard solutions, while CCMC444 with both n1=1 and n43=64 have 120 standard solutions. Each of the other 6 types of CPMC444 with n1=1 has equally the same count of 120 standard solutions. 840=7 x 120

#11. How to make 90 CPMC444 from CCMC444 fundamental 15?

Let's study about the internal relations among 105 solutions.

I want to show you some structural relationships between two corresponding solutions at first.

I rearranged the 'directions' of solutions for the purpose so that you might easily find some common patterns between them.

Look at the next examples shown below.

Between the first two examples there are two common planes: I=I; II=IV

There are also some exchanges of 2x2x2 blocks on the rest two planes.

We can invent the way how to transform one into another, can't we?

I want to propose you some such ways of transformation:

[Transformation Type 1]

1. Copy the plane I of the original and put it onto new I, keeping all data on it unchanged.
2. Copy the plane II of the original and put it onto new IV, keeping all data on it unchanged.
3. Take four blocks of 2x2x2 between Plane III and IV. Transform them by exchanging positions as 'X-crossing' and put them onto new II and III.

[Transformation Type 2]

1. Copy the plane I of original and put it onto new I, keeping all data on it unchanged.
2. Copy the plane IV of original and put it onto new IV, keeping all data on it unchanged.
3. Take four blocks of 2x2x2 between Plane II and III. Transform them by exchanging positions as 'X crossing' and put them in reverse(left-right) onto new II and III.

*** Internal Transformations: Type: $n_{43}=64 \leftrightarrow n_{43}=32 \text{ \& } 48$ ***

1/64

1	48	49	32
63	18	15	34
60	4	21	45
12	52	37	29
6	62	43	19
54	14	27	35
13	57	36	24
61	9	20	40
51	7	30	42
3	55	46	26
56	16	25	33
8	64	41	17
10	50	39	31
58	2	23	47
53	28	5	44
11	38	59	22
I	II	III	IV

1/32

1	64	17	48
63	2	47	18
60	4	5	61
44	20	21	45
6	62	59	3
22	46	43	19
13	57	52	8
29	41	36	24
51	7	14	58
35	23	30	42
56	16	9	49
40	32	25	33
10	50	55	15
26	34	39	31
53	12	37	28
11	54	27	38
I	II	III	IV

[Transform: 1/n₄₃=64 --> 1/n₄₃=32]

Copy I and put it to new I unchanged. Copy II and put it to new IV unchanged. Take III, IV and transform them into new II, III. Exchange four blocks of 2x2x2 corresponding to each other.

1/64

1	48	49	32
63	18	15	34
60	4	21	45
12	52	37	29
6	62	43	19
54	14	27	35
13	57	36	24
61	9	20	40
51	7	30	42
3	55	46	26
56	16	25	33
8	64	41	17
10	50	39	31
58	2	23	47
53	28	5	44
11	38	59	22
I	II	III	IV

1/48

1	64	33	32
63	2	31	34
60	4	5	61
28	36	37	29
6	62	59	3
38	30	27	35
13	57	52	8
45	25	20	40
51	7	14	58
19	39	46	26
56	16	9	49
24	48	41	17
10	50	55	15
42	18	23	47
53	12	21	44
11	54	43	22
I	II	III	IV

[Transform: 1/n₄₃=64 --> 1/n₄₃=48]

Copy I and put it to new I unchanged. Copy IV and put it to new IV unchanged. Take II, III and transform them into new II, III. Exchange four blocks of 2x2x2 with each other and put them in reverse left side to right.

When you assume these transformations, you can make two different 'pan-triangular' cubes (with $n_{43} < 64$) from one 'complete' cube (with $n_{43} = 64$).

[Ori gi nal]	[Transformati on-1]	[Transformati on-2]
1-----17-----33-----49 2-----18-----34-----50 5 3 21 19 37 35 53 51 6 4---22--20---38--36---54--52 9 7 25 23 41 39 57 55 10 8 26 24 42 40 58 56 13--11- -29--27---45--43---61 59 14 12 30 28 46 44 62 60 15 31-----47-----63 16-----32-----48-----64	1-----43-----59-----17 2-----44-----60-----18 5 3 47 41 63 57 21 19 6 4---48--42---64--58---22--20 9 7 35 45 51 61 25 23 10 8 36 46 52 62 26 24 13--11- -39--33---55--49---29 27 14 12 40 34 56 50 30 28 15 37-----53-----31 16-----38-----54-----32	1-----43-----27-----49 2-----44-----28-----50 5 3 47 41 31 25 53 51 6 4---48--42---32--26---54--52 9 7 35 45 19 29 57 55 10 8 36 46 20 30 58 56 13--11- -39--33---23--17---61 59 14 12 40 34 24 18 62 60 15 37-----21-----63 16-----38-----22-----64

How many pantriagonal cubes in all could we make from one?

Because you can apply those rules above to all three directions (left-right, up-down, back-front), you could make 2x3 different pantriagonal cubes from one.

As a result you could make 6x15 Pantriagonal cubes from 15 Complete cubes.

1/64	1/32	1/48
1-----48-----49-----32 63 18 15 34 60 4 21 45 12 52 37 29 6 62---43--19---54--14---27--35 13 57 36 24 61 9 20 40 51 7 30 42 3 55 46 26 56--16- -25--33---8--64---41 17 10 50 39 31 58 2 23 47 53 28-----5-----44 11-----38-----59-----22	1-----64-----17-----48 63 2 47 18 60 4 5 61 44 20 21 45 6 62---59--3---22--46---43--19 13 57 52 8 29 41 36 24 51 7 14 58 35 23 30 42 56--16- -9--49---40--32---25 33 10 50 55 15 26 34 39 31 53 12-----37-----28 11-----54-----27-----38	1-----64-----33-----32 63 2 31 34 60 4 5 61 28 36 37 29 6 62---59--3---38--30---27--35 13 57 52 8 45 25 20 40 51 7 14 58 19 39 46 26 56--16- -9--49---24--48---41 17 10 50 55 15 42 18 23 47 53 12-----21-----44 11-----54-----43-----22

6/64	3/56	3/60
1-----60-----13-----56 63 6 51 10 48 4 21 57 36 16 25 53 18 62---43--7---30--50---39--11 49 45 12 24 61 33 8 28 15 19 54 42 3 31 58 38 32--52- -37--9---20--64---41 5 34 14 27 55 46 2 23 59 29 40-----17-----44 35-----26-----47-----22	1-----64-----5-----60 63 2 59 6 48 4 17 61 44 8 21 57 18 62---47--3---22--58---43--7 49 45 16 20 53 41 12 24 15 19 50 46 11 23 54 42 32--52- -33--13---28--56---37 9 34 14 31 51 38 10 27 55 29 36-----25-----40 35-----30-----39-----26	1-----64-----9-----56 63 2 55 10 48 4 17 61 40 12 25 53 18 62---47--3---26--54---39--11 49 45 16 20 57 37 8 28 15 19 50 46 7 27 58 38 32--52- -33--13---24--60---41 5 34 14 31 51 42 6 23 59 29 36-----21-----44 35-----30-----43-----22

43/64	13/62	13/63
1-----63-----4-----62 60 6 57 7 48 13 18 51 45 16 19 50 21 56---43--10---24--53---42--11 49 36 15 30 52 33 14 31 12 25 54 39 9 28 55 38 32--61- -34--3---29--64---35 2 37 8 27 58 40 5 26 59 20 46-----17-----47 41-----23-----44-----22	1-----64-----2-----63 60 5 59 6 48 13 17 52 47 14 18 51 21 56---44--9---22--55---43--10 49 36 16 29 50 35 15 30 12 25 53 40 11 26 54 39 32--61- -33--4---31--62---34 3 37 8 28 57 38 7 27 58 20 45-----19-----46 41-----24-----42-----23	1-----64-----3-----62 60 5 58 7 48 13 17 52 46 15 19 50 21 56---44--9---23--54---42--11 49 36 16 29 51 34 14 31 12 25 53 40 10 27 55 38 32--61- -33--4---30--63---35 2 37 8 28 57 39 6 26 59 20 45-----18-----47 41-----24-----43-----22

The next list shows the 45 'C&C' cubes and 2 different 'C&P' cubes transformed from each of the former.

I rearranged one Complete solution in three directions for three Complete solutions in one direction, so that you could easily find all transformations in a simple manner.

After all we could have made all 90 pantriagonal cubes from 15 complete cubes.

You can say that these 15 'Composite & Complete' magic cubes 4x4x4 are the real 'fundamentals' of all Composite Pan-magic cubes 4x4x4.

** Make Composite Pan-Magic Cubes 4*4*4 **
 ** by Converting from n43=64 to n43=32-63 **
 *** List of the Fundamental Solutions ***

1/64

1-----48-----49-----32
 |63 18 15 |34
 60 4 21 45 12 52 37 29
 | 6 62---43--19--54--14---+27--35
 13 57 | 36 24 61 9 20 40 |
 |51 7 30 42 3 55 |46 26
 56--16-|-25--33---8--64---41 17 |
 10 50 39 31 58 2 23 47
 53 | 28 5 44 |
 11-----38-----59-----22

1/32

1-----64-----17-----48
 |63 2 47 |18
 60 4 5 61 44 20 21 45
 | 6 62---59--3---22--46---43--19
 13 57 | 52 8 29 41 36 24 |
 |51 7 14 58 35 23 |30 42
 56--16-|-9--49---40--32---25 33 |
 10 50 55 15 26 34 39 31
 53 | 12 37 28 |
 11-----54-----27-----38

1/48

1-----64-----33-----32
 |63 2 31 |34
 60 4 5 61 28 36 37 29
 | 6 62---59--3---38--30---+27--35
 13 57 | 52 8 45 25 20 40 |
 |51 7 14 58 19 39 |46 26
 56--16-|-9--49---24--48---41 17 |
 10 50 55 15 42 18 23 47
 53 | 12 21 44 |
 11-----54-----43-----22

2/64

1-----56-----41-----32
 |63 10 23 |34
 60 4 13 53 20 44 37 29
 | 6 62---51--11---46--22---+27--35
 21 57 | 36 16 61 17 12 40 |
 |43 7 30 50 3 47 |54 26
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 39 | 26 7 58 |
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1-----64----- 5-----60
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1-----64----- 3-----62
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 | 6 56--59-- 9-- 8--54--57--11
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10/56

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26/64

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27/64

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30/64

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31/64

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32/64

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 26 | 40 9 55 |
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13/48

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33/64

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36/64

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 41-----24-----42-----23

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37/64

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38/64

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39/64

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40/64

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41/64

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 14 | 52 5 59 |
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1-----64-----2-----63
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 48--55--|17--10--47--56--18 9 |
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 14 | 51 13 52 |
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11/63

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42/64

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43/64

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44/64

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14/62

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15/62

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12 | 53 11 54 |
49-----16-----50-----15

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15/63

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49-----16-----51-----14

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[Count: n43=64/45, 63/15, 62/15, 60/15, 56/15, 48/15, 32/15] OK!

(Originally Written by Kanji Setsuda on Aug. 8, 2002;
Re-calculated on Mar. 1, 2007 with MacOSX and Xcode2.2)

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