Part 4. "New Advanced Study of Magic Squares and Cubes"

Chapter 7. New Method of Composing High-Dimensional Extra-Cubic Objects and their Developed Forms: **Kanji Setsuda**

Section 5. New Method of Composing Two Types of Magic Squares 4^2 Using All View-Forms of Extra-Cubic Object 2^4

I have recently invented a new, unique way of composing special types of magic squares and cubes of order 4, 8, and 9, when I was studying high dimensional extra-cubic objects and their 'basic view-forms' for the developed ones. Whenever I made various tests to this method of composition, I always had good results. I would like to report of this new method precisely in the following two or three sections.

#1. How could I invent this new method of composition?

The idea came up to me when I wondered how many 'basic view-diagrams' of developed ECO we could draw and how we could make our computer draw them all.

Let me take the case of $ECO2^4$ for instance at first.

We now know in the first section: if we use four-time loops of for(...){...} sentences and produce binary numbers, we can draw the first 'basic view-diagram' for $ECO2^4$.

If you change the original program a little in the order of variables {d0,d1,d2,d3}, you could have any different view-forms of the same ECO. You could draw different pictures with n1=1 as many as 24 in all. Because ${}_4P_4$=4x3x2x1=24.

If you put any count-down type of conditions in the for(...){ } sentences, you could have another type of different view-forms of the same ECO. How many different pictures could you draw for that type, then?

We use 4 loops of for(...){...} sentences in all. And we have two cases for each loop: whether we use the count-down condition or not. Therefore we could have different pictures of that type as many as 16 in all, because 2^4=16.

Take your careful look at the next lists.

```
** Print the Basic View-Diagram for Extra-Cubic Object 2^4 **
 BD/d0          /d1           SC/           CC/
   1---- 2      3---- 4        1  2  3  4    1  2  4  3
   |  9--+-10   | 11--+-12     5  6  7  8    5  6  8  7
   5--|- 6  |   7--|- 8  |     9 10 11 12   13 14 16 15
     13----14     15----16    13 14 15 16    9 10 12 11

** Print Type #1 of View-Forms for Developed MS44(SC-type) **
          1/           2/           3/           4/           5/           6/
  1  2  3  4   1  3  2  4   1  2  5  6   1  3  5  7   1  5  2  6   1  5  3  7
  5  6  7  8   5  7  6  8   3  4  7  8   2  4  6  8   3  7  4  8   2  6  4  8
  9 10 11 12   9 11 10 12   9 10 13 14   9 11 13 15   9 13 10 14   9 13 11 15
 13 14 15 16  13 15 14 16  11 12 15 16  10 12 14 16  11 15 12 16  10 14 12 16
          7/           8/           9/          10/          11/          12/
  1  2  3  4   1  3  2  4   1  2  5  6   1  3  5  7   1  5  2  6   1  5  3  7
  9 10 11 12   9 11 10 12   9 10 13 14   9 11 13 15   9 13 10 14   9 13 11 15
  5  6  7  8   5  7  6  8   3  4  7  8   2  4  6  8   3  7  4  8   2  6  4  8
 13 14 15 16  13 15 14 16  11 12 15 16  10 12 14 16  11 15 12 16  10 14 12 16
         13/          14/          15/          16/          17/          18/
  1  2  9 10   1  3  9 11   1  2  9 10   1  3  9 11   1  5  9 13   1  5  9 13
  3  4 11 12   2  4 10 12   5  6 13 14   5  7 13 15   2  6 10 14   3  7 11 15
  5  6 13 14   5  7 13 15   3  4 11 12   2  4 10 12   3  7 11 15   2  6 10 14
  7  8 15 16   6  8 14 16   7  8 15 16   6  8 14 16   4  8 12 16   4  8 12 16
```

```
         19/          20/          21/          22/          23/          24/
 1  9  2 10   1  9  3 11   1  9  2 10   1  9  3 11   1  9  5 13   1  9  5 13
 3 11  4 12   2 10  4 12   5 13  6 14   5 13  7 15   2 10  6 14   3 11  7 15
 5 13  6 14   5 13  7 15   3 11  4 12   2 10  4 12   3 11  7 15   2 10  6 14
 7 15  8 16   6 14  8 16   7 15  8 16   6 14  8 16   4 12  8 16   4 12  8 16
```

```
** Print Type #2 of View-Forms for Developed MS44 **
           1/           2/           3/           4/
  1  2  3  4    2  1  4  3    3  4  1  2    4  3  2  1
  5  6  7  8    6  5  8  7    7  8  5  6    8  7  6  5
  9 10 11 12   10  9 12 11   11 12  9 10   12 11 10  9
 13 14 15 16   14 13 16 15   15 16 13 14   16 15 14 13
           5/           6/           7/           8/
  5  6  7  8    6  5  8  7    7  8  5  6    8  7  6  5
  1  2  3  4    2  1  4  3    3  4  1  2    4  3  2  1
 13 14 15 16   14 13 16 15   15 16 13 14   16 15 14 13
  9 10 11 12   10  9 12 11   11 12  9 10   12 11 10  9
           9/          10/          11/          12/
  9 10 11 12   10  9 12 11   11 12  9 10   12 11 10  9
 13 14 15 16   14 13 16 15   15 16 13 14   16 15 14 13
  1  2  3  4    2  1  4  3    3  4  1  2    4  3  2  1
  5  6  7  8    6  5  8  7    7  8  5  6    8  7  6  5
          13/          14/          15/          16/
 13 14 15 16   14 13 16 15   15 16 13 14   16 15 14 13
  9 10 11 12   10  9 12 11   11 12  9 10   12 11 10  9
  5  6  7  8    6  5  8  7    7  8  5  6    8  7  6  5
  1  2  3  4    2  1  4  3    3  4  1  2    4  3  2  1
```

I drew these pictures and listed them as above in the style of developed SCMS44 here. All of them are different 'view-forms' of the same ECO2^4.

How many different pictures in all can we draw for the view-forms of the developed ECO, then? We want to know the total count of 'primitive' pictures.

As you guess you may combine those 2 sets of view-forms above one by one. We can draw them so many as 384 in all, because 24x16=384.

But why do they count **384**? It is the same count just as the primitive solutions of 'self-complementary' MS44 or various types of 'pan-diagonal' ones, isn't it?

Is it only an accidental agreement? It doesn't look so. What does it really mean?

When I watched those primitive 384 view forms, I noticed it is quite similar to the solution set of 'Prototype Squares' of order 4, which begins with the same form as the regular 4x4 array at the first solution. I remember I once made the list of them long before. The new set of pictures is really just the same with the old solution set.

Then I expected I might compose our object solutions by these Prototype Squares and the '**D**(o it) **A**(fter the) **M**(odel) Transformation', if I just try to do that.

I believed I could surely make those by myself.

For the next step I had to find any appropriate 'Model' solutions and select the best one for my purpose.

I tried to find it out, designed how to compose the object solutions, dictated the first test programs, and carefully examined them many times.

#2. How do you compose them?

Now let's try to use this method actually and compose two types of 2-dimensional magic squares of order 4: 'Self-complementary' and 'Pan-diagonal'.

(1) First of all we are going to draw all possible view-forms of developed ECO2^4. We use 4-time loops of for(...){...} sentences and produce binary numbers.

For the preparation step we calculate two types of modifications and record the

results to the tables beforehand. We use the data afterward by ‘indirect addressing mode’ for the 4-time loops. It is rather a high-leveled technique of programming, though I am afraid of confusing you a little with it.
(2) Next we make each view-form into our object, ‘Self-complementary’ MS44 solution by DAM Transformation at first.
(3) Then we change each SCMS44 solution into ‘Pan-diagonal’ one by the typical method of transformation, which is already familiar to us.
That is all for us to do now. It looks very simple, doesn’t it?

The following diagrams illustrate a representative solution selected for our best ‘Model’ and concept of ‘DAM Transformation’. A half of elements must be changed by the ‘Self-complementary Transformation’ and the rest half be kept unchanged.

```
/*
/* Concept of DAM Transformation */
 BD/d0          /d1           PT/
   1---- 2      3---- 4         1  2  3  4
   |  9--+-10   | 11--+-12      5  6  7  8
   5--|- 6  |   7--|- 8  |      9 10 11 12
     13----14     15----16     13 14 15 16
 MD/d0          /d1           SC/
   1----15     14---- 4         1 15 14  4
   |  8--+-10   | 11--+- 5     12  6  7  9
  12--|- 6  |   7--|- 9  |      8 10 11  5
     13---- 3      2----16     13  3  2 16
*/
/* Self-Complementary Transformation for Half */
  CC=17;
  y[1]=x[1];     y[2]=CC-x[2];    y[3]=CC-x[3];    y[4]=x[4];
  y[5]=CC-x[5];  y[6]=x[6];       y[7]=x[7];       y[8]=CC-x[8];
  y[9]=CC-x[9];  y[10]=x[10];     y[11]=x[11];     y[12]=CC-x[12];
  y[13]=x[13];   y[14]=CC-x[14];  y[15]=CC-x[15];  y[16]=x[16];
*/
```

Let’s compose the Self-complementary Magic Squares of order 4 at first by this method. The next list is a sample program made for our purpose only with a few essential components so that you could understand it easily.

```
/**  New Method of Composing S-C Magic Squares of Order 4^2  **/
/** Using All View-Diagrams of Developed ExCubic Objects 2^4 **/
/** 'SNewMS4.c' by Kanji Setsuda on MacOS X 1.5; Apr.26,'05  **/
/**/
#include <stdio.h>
/**/
short tcnt;
short vc, pc;
short tv[16][4];
short td[24][4];
short nm[17];
short tnm[385][17];
/**/
void mkfv16(void), prmt24(void);
void d24(short x, short y);
void mksol24(void);
void pr6s24(void);
/**/
```

```
/* Main Program */
int main(){
  short m,n;
    printf("\n**  Newest Method of Composing Magic Squares of Order 4^2:  **\n");
    printf("** Using All View-Diagrams of Developed ExCubic Objects 2^4 **\n");
    printf("**  Primitive 384 of 'Self-Complementary' Magic Squares 4^2 **\n");
    vc=0; mkfv16();
    pc=0; prmt24();
    tcnt=pc*vc;
    for(m=0;m<vc;m++){for(n=0;n<pc;n++){d24(m,n);}}
    mksol24();
    pr6s24();
    printf("\n [Counts=%d]\n",tcnt);
    printf(" OK!\n");
  return 0;
}
/**/
/* Sub-Procedures for ECO2^4 */
/**/
void mkfv16(void){
  short d0,d1,d2,d3;
    for(d0=0;d0<2;d0++){
      for(d1=0;d1<2;d1++){
        for(d2=0;d2<2;d2++){
          for(d3=0;d3<2;d3++){
            tv[vc][0]=d0; tv[vc][1]=d1; tv[vc][2]=d2; tv[vc][3]=d3;
            vc++;
    } } } }
}
/**/
void prmt24(void){
  short d0,d1,d2,d3,n;
  short uflg[4];
    for(n=0;n<4;n++){uflg[n]=0;}
    for(d0=0;d0<4;d0++){
     uflg[d0]=1;
      for(d1=0;d1<4;d1++){
       if(uflg[d1]==0){uflg[d1]=1;
        for(d2=0;d2<4;d2++){
         if(uflg[d2]==0){uflg[d2]=1;
          for(d3=0;d3<4;d3++){
           if(uflg[d3]==0){uflg[d3]=1;
            td[pc][0]=d0; td[pc][1]=d1; td[pc][2]=d2; td[pc][3]=d3;
            pc++;
            uflg[d3]=0;
          }}
          uflg[d2]=0;
        }}
        uflg[d1]=0;
      }}
      uflg[d0]=0;
    }
}
/**/
void d24(short x, short y){
  short d0,d1,d2,d3;
  short t0,t1,t2,t3;
  short CC,c,n;
  short s[4], t[4][2];
```

```
  /**/
    CC=1;
    for(n=0;n<4;n++){
      t[n][0]=tv[x][n]; t[n][1]=CC-tv[x][n];}
    /**/
    c=0;
    for(d0=0;d0<2;d0++){
      for(d1=0;d1<2;d1++){
        for(d2=0;d2<2;d2++){
          for(d3=0;d3<2;d3++){c++;
            s[0]=t[0][d0]; s[1]=t[1][d1]; s[2]=t[2][d2]; s[3]=t[3][d3];
            t0=td[y][0]; t1=td[y][1]; t2=td[y][2]; t3=td[y][3];
            tnm[x*pc+y][c]=((s[t0]*2+s[t1])*2+s[t2])*2+s[t3]+1;
    } } } }
}
/**/
void mksol24(void){
 short m,n,CC;
  CC=17;
  for(m=0;m<tcnt;m++){
    nm[1]=tnm[m][1];     nm[2]=CC-tnm[m][2];    nm[3]=CC-tnm[m][3];    nm[4]=tnm[m][4];
    nm[5]=CC-tnm[m][5];  nm[6]=tnm[m][6];       nm[7]=tnm[m][7];       nm[8]=CC-tnm[m][8];
    nm[9]=CC-tnm[m][9];  nm[10]=tnm[m][10];     nm[11]=tnm[m][11];     nm[12]=CC-tnm[m][12];
    nm[13]=tnm[m][13];   nm[14]=CC-tnm[m][14];  nm[15]=CC-tnm[m][15];  nm[16]=tnm[m][16];
    for(n=1;n<17;n++){tnm[m][n]=nm[n];}
  }
}
/**/
/* Print Solutions by 6 pieces */
void pr6s24(void){
  short m,l,l4,n,p;
    for(m=0;m<tcnt;m=m+6){
      printf("%13d/%13d/%13d/%13d/%13d/%13d/\n",m+1,m+2,m+3,m+4,m+5,m+6);
      for(l=0;l<4;l++){l4=l*4;
        for(p=0;p<6;p++){
          printf("  ");
          for(n=0;n<4;n++){
            printf("%3d",tnm[m+p][l4+n+1]);
        } }
        printf("\n");
    } }
}
/**/
```

This program may give you a little different impression from the other old ones.

It is very short, shorter than any other old programs. It runs very fast, say, faster than any other programs I have ever written before.

It has a new, unique style of programming. You could find nothing like any old calculations familiar to you, even nothing like any definitions with many simultaneous equations, either. Nor you could find any kind of check procedures here.

Though it may look peculiar, it will surely give you a normal, reasonable result.

Let me list out the result, but part of the ‘primitive’ solutions of SC-type of MS44.

```
**  Newest Method of Composing Magic Squares of Order 4^2:  **
** Using All View-Diagrams of Developed ExCubic Objects 2^4 **
**  Primitive 384 of 'Self-Complementary' Magic Squares 4^2 **
```

1/

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

2/

1	14	15	4
12	7	6	9
8	11	10	5
13	2	3	16

3/

1	15	12	6
14	4	7	9
8	10	13	3
11	5	2	16

4/

1	14	12	7
15	4	6	9
8	11	13	2
10	5	3	16

5/

1	12	15	6
14	7	4	9
8	13	10	3
11	2	5	16

6/

1	12	14	7
15	6	4	9
8	13	11	2
10	3	5	16

7/

1	15	14	4
8	10	11	5
12	6	7	9
13	3	2	16

8/

1	14	15	4
8	11	10	5
12	7	6	9
13	2	3	16

9/

1	15	12	6
8	10	13	3
14	4	7	9
11	5	2	16

10/

1	14	12	7
8	11	13	2
15	4	6	9
10	5	3	16

11/

1	12	15	6
8	13	10	3
14	7	4	9
11	2	5	16

12/

1	12	14	7
8	13	11	2
15	6	4	9
10	3	5	16

13/

1	15	8	10
14	4	11	5
12	6	13	3
7	9	2	16

14/

1	14	8	11
15	4	10	5
12	7	13	2
6	9	3	16

15/

1	15	8	10
12	6	13	3
14	4	11	5
7	9	2	16

16/

1	14	8	11
12	7	13	2
15	4	10	5
6	9	3	16

17/

1	12	8	13
15	6	10	3
14	7	11	2
4	9	5	16

18/

1	12	8	13
14	7	11	2
15	6	10	3
4	9	5	16

19/

1	8	15	10
14	11	4	5
12	13	6	3
7	2	9	16

20/

1	8	14	11
15	10	4	5
12	13	7	2
6	3	9	16

21/

1	8	15	10
12	13	6	3
14	11	4	5
7	2	9	16

22/

1	8	14	11
12	13	7	2
15	10	4	5
6	3	9	16

23/

1	8	12	13
15	10	6	3
14	11	7	2
4	5	9	16

24/

1	8	12	13
14	11	7	2
15	10	6	3
4	5	9	16

25/

2	16	13	3
11	5	8	10
7	9	12	6
14	4	1	15

26/

3	16	13	2
10	5	8	11
6	9	12	7
15	4	1	14

27/

2	16	11	5
13	3	8	10
7	9	14	4
12	6	1	15

28/

3	16	10	5
13	2	8	11
6	9	15	4
12	7	1	14

29/

5	16	11	2
10	3	8	13
4	9	14	7
15	6	1	12

30/

5	16	10	3
11	2	8	13
4	9	15	6
14	7	1	12

31/

2	16	13	3
7	9	12	6
11	5	8	10
14	4	1	15

32/

3	16	13	2
6	9	12	7
10	5	8	11
15	4	1	14

33/

2	16	11	5
7	9	14	4
13	3	8	10
12	6	1	15

34/

3	16	10	5
6	9	15	4
13	2	8	11
12	7	1	14

35/

5	16	11	2
4	9	14	7
10	3	8	13
15	6	1	12

36/

5	16	10	3
4	9	15	6
11	2	8	13
14	7	1	12

37/

2	16	7	9
13	3	12	6
11	5	14	4
8	10	1	15

38/

3	16	6	9
13	2	12	7
10	5	15	4
8	11	1	14

39/

2	16	7	9
11	5	14	4
13	3	12	6
8	10	1	15

40/

3	16	6	9
10	5	15	4
13	2	12	7
8	11	1	14

41/

5	16	4	9
11	2	14	7
10	3	15	6
8	13	1	12

42/

5	16	4	9
10	3	15	6
11	2	14	7
8	13	1	12

43/

9	16	7	2
6	3	12	13
4	5	14	11
15	10	1	8

44/

9	16	6	3
7	2	12	13
4	5	15	10
14	11	1	8

45/

9	16	7	2
4	5	14	11
6	3	12	13
15	10	1	8

46/

9	16	6	3
4	5	15	10
7	2	12	13
14	11	1	8

47/

9	16	4	5
7	2	14	11
6	3	15	10
12	13	1	8

48/

9	16	4	5
6	3	15	10
7	2	14	11
12	13	1	8

49/

3	13	16	2
10	8	5	11
6	12	9	7
15	1	4	14

50/

2	13	16	3
11	8	5	10
7	12	9	6
14	1	4	15

51/

5	11	16	2
10	8	3	13
4	14	9	7
15	1	6	12

52/

5	10	16	3
11	8	2	13
4	15	9	6
14	1	7	12

53/

2	11	16	5
13	8	3	10
7	14	9	4
12	1	6	15

54/

3	10	16	5
13	8	2	11
6	15	9	4
12	1	7	14

55/

3	13	16	2
6	12	9	7
10	8	5	11
15	1	4	14

56/

2	13	16	3
7	12	9	6
11	8	5	10
14	1	4	15

57/

5	11	16	2
4	14	9	7
10	8	3	13
15	1	6	12

58/

5	10	16	3
4	15	9	6
11	8	2	13
14	1	7	12

59/

2	11	16	5
7	14	9	4
13	8	3	10
12	1	6	15

60/

3	10	16	5
6	15	9	4
13	8	2	11
12	1	7	14

61/

9	7	16	2
6	12	3	13
4	14	5	11
15	1	10	8

62/

9	6	16	3
7	12	2	13
4	15	5	10
14	1	11	8

63/

9	7	16	2
4	14	5	11
6	12	3	13
15	1	10	8

64/

9	6	16	3
4	15	5	10
7	12	2	13
14	1	11	8

65/

9	4	16	5
7	14	2	11
6	15	3	10
12	1	13	8

66/

9	4	16	5
6	15	3	10
7	14	2	11
12	1	13	8

			67/				68/				69/				70/				71/				72/
2	7	16	9	3	6	16	9	2	7	16	9	3	6	16	9	5	4	16	9	5	4	16	9
13	12	3	6	13	12	2	7	11	14	5	4	10	15	5	4	11	14	2	7	10	15	3	6
11	14	5	4	10	15	5	4	13	12	3	6	13	12	2	7	10	15	3	6	11	14	2	7
8	1	10	15	8	1	11	14	8	1	10	15	8	1	11	14	8	1	13	12	8	1	13	12

			73/				74/				75/				76/				77/				78/
4	14	15	1	4	15	14	1	6	12	15	1	7	12	14	1	6	15	12	1	7	14	12	1
9	7	6	12	9	6	7	12	9	7	4	14	9	6	4	15	9	4	7	14	9	4	6	15
5	11	10	8	5	10	11	8	3	13	10	8	2	13	11	8	3	10	13	8	2	11	13	8
16	2	3	13	16	3	2	13	16	2	5	11	16	3	5	10	16	5	2	11	16	5	3	10

			79/				80/				81/				82/				83/				84/
4	14	15	1	4	15	14	1	6	12	15	1	7	12	14	1	6	15	12	1	7	14	12	1
5	11	10	8	5	10	11	8	3	13	10	8	2	13	11	8	3	10	13	8	2	11	13	8
9	7	6	12	9	6	7	12	9	7	4	14	9	6	4	15	9	4	7	14	9	4	6	15
16	2	3	13	16	3	2	13	16	2	5	11	16	3	5	10	16	5	2	11	16	5	3	10

			85/				86/				87/				88/				89/				90/
10	8	15	1	11	8	14	1	10	8	15	1	11	8	14	1	13	8	12	1	13	8	12	1
5	11	4	14	5	10	4	15	3	13	6	12	2	13	7	12	3	10	6	15	2	11	7	14
3	13	6	12	2	13	7	12	5	11	4	14	5	10	4	15	2	11	7	14	3	10	6	15
16	2	9	7	16	3	9	6	16	2	9	7	16	3	9	6	16	5	9	4	16	5	9	4

			91/				92/				93/				94/				95/				96/
10	15	8	1	11	14	8	1	10	15	8	1	11	14	8	1	13	12	8	1	13	12	8	1
5	4	11	14	5	4	10	15	3	6	13	12	2	7	13	12	3	6	10	15	2	7	11	14
3	6	13	12	2	7	13	12	5	4	11	14	5	4	10	15	2	7	11	14	3	6	10	15
16	9	2	7	16	9	3	6	16	9	2	7	16	9	3	6	16	9	5	4	16	9	5	4

			97/				98/				99/				100/				101/				102/
5	11	10	8	5	10	11	8	3	13	10	8	2	13	11	8	3	10	13	8	2	11	13	8
16	2	3	13	16	3	2	13	16	2	5	11	16	3	5	10	16	5	2	11	16	5	3	10
4	14	15	1	4	15	14	1	6	12	15	1	7	12	14	1	6	15	12	1	7	14	12	1
9	7	6	12	9	6	7	12	9	7	4	14	9	6	4	15	9	4	7	14	9	4	6	15

			103/				104/				105/				106/				107/				108/
9	7	6	12	9	6	7	12	9	7	4	14	9	6	4	15	9	4	7	14	9	4	6	15
16	2	3	13	16	3	2	13	16	2	5	11	16	3	5	10	16	5	2	11	16	5	3	10
4	14	15	1	4	15	14	1	6	12	15	1	7	12	14	1	6	15	12	1	7	14	12	1
5	11	10	8	5	10	11	8	3	13	10	8	2	13	11	8	3	10	13	8	2	11	13	8

			109/				110/				111/				112/				113/				114/
3	13	6	12	2	13	7	12	5	11	4	14	5	10	4	15	2	11	7	14	3	10	6	15
16	2	9	7	16	3	9	6	16	2	9	7	16	3	9	6	16	5	9	4	16	5	9	4
10	8	15	1	11	8	14	1	10	8	15	1	11	8	14	1	13	8	12	1	13	8	12	1
5	11	4	14	5	10	4	15	3	13	6	12	2	13	7	12	3	10	6	15	2	11	7	14

			115/				116/				117/				118/				119/				120/
3	6	13	12	2	7	13	12	5	4	11	14	5	4	10	15	2	7	11	14	3	6	10	15
16	9	2	7	16	9	3	6	16	9	2	7	16	9	3	6	16	9	5	4	16	9	5	4
10	15	8	1	11	14	8	1	10	15	8	1	11	14	8	1	13	12	8	1	13	12	8	1
5	4	11	14	5	4	10	15	3	6	13	12	2	7	13	12	3	6	10	15	2	7	11	14

			121/				122/				123/				124/				125/				126/
6	12	9	7	7	12	9	6	4	14	9	7	4	15	9	6	7	14	9	4	6	15	9	4
15	1	4	14	14	1	4	15	15	1	6	12	14	1	7	12	12	1	6	15	12	1	7	14
3	13	16	2	2	13	16	3	5	11	16	2	5	10	16	3	2	11	16	5	3	10	16	5
10	8	5	11	11	8	5	10	10	8	3	13	11	8	2	13	13	8	3	10	13	8	2	11

			127/				128/				129/				130/				131/				132/
10	8	5	11	11	8	5	10	10	8	3	13	11	8	2	13	13	8	3	10	13	8	2	11
15	1	4	14	14	1	4	15	15	1	6	12	14	1	7	12	12	1	6	15	12	1	7	14
3	13	16	2	2	13	16	3	5	11	16	2	5	10	16	3	2	11	16	5	3	10	16	5
6	12	9	7	7	12	9	6	4	14	9	7	4	15	9	6	7	14	9	4	6	15	9	4

133/

4	14	5	11
15	1	10	8
9	7	16	2
6	12	3	13

134/

4	15	5	10
14	1	11	8
9	6	16	3
7	12	2	13

135/

6	12	3	13
15	1	10	8
9	7	16	2
4	14	5	11

136/

7	12	2	13
14	1	11	8
9	6	16	3
4	15	5	10

137/

6	15	3	10
12	1	13	8
9	4	16	5
7	14	2	11

138/

7	14	2	11
12	1	13	8
9	4	16	5
6	15	3	10

139/

11	14	5	4
8	1	10	15
2	7	16	9
13	12	3	6

140/

10	15	5	4
8	1	11	14
3	6	16	9
13	12	2	7

141/

13	12	3	6
8	1	10	15
2	7	16	9
11	14	5	4

142/

13	12	2	7
8	1	11	14
3	6	16	9
10	15	5	4

143/

10	15	3	6
8	1	13	12
5	4	16	9
11	14	2	7

144/

11	14	2	7
8	1	13	12
5	4	16	9
10	15	3	6

145/

7	9	12	6
14	4	1	15
2	16	13	3
11	5	8	10

146/

6	9	12	7
15	4	1	14
3	16	13	2
10	5	8	11

147/

7	9	14	4
12	6	1	15
2	16	11	5
13	3	8	10

148/

6	9	15	4
12	7	1	14
3	16	10	5
13	2	8	11

149/

4	9	14	7
15	6	1	12
5	16	11	2
10	3	8	13

150/

4	9	15	6
14	7	1	12
5	16	10	3
11	2	8	13

151/

11	5	8	10
14	4	1	15
2	16	13	3
7	9	12	6

152/

10	5	8	11
15	4	1	14
3	16	13	2
6	9	12	7

153/

13	3	8	10
12	6	1	15
2	16	11	5
7	9	14	4

154/

13	2	8	11
12	7	1	14
3	16	10	5
6	9	15	4

155/

10	3	8	13
15	6	1	12
5	16	11	2
4	9	14	7

156/

11	2	8	13
14	7	1	12
5	16	10	3
4	9	15	6

157/

11	5	14	4
8	10	1	15
2	16	7	9
13	3	12	6

158/

10	5	15	4
8	11	1	14
3	16	6	9
13	2	12	7

159/

13	3	12	6
8	10	1	15
2	16	7	9
11	5	14	4

160/

13	2	12	7
8	11	1	14
3	16	6	9
10	5	15	4

161/

10	3	15	6
8	13	1	12
5	16	4	9
11	2	14	7

162/

11	2	14	7
8	13	1	12
5	16	4	9
10	3	15	6

163/

4	5	14	11
15	10	1	8
9	16	7	2
6	3	12	13

164/

4	5	15	10
14	11	1	8
9	16	6	3
7	2	12	13

165/

6	3	12	13
15	10	1	8
9	16	7	2
4	5	14	11

166/

7	2	12	13
14	11	1	8
9	16	6	3
4	5	15	10

167/

6	3	15	10
12	13	1	8
9	16	4	5
7	2	14	11

168/

7	2	14	11
12	13	1	8
9	16	4	5
6	3	15	10

169/

8	10	11	5
13	3	2	16
1	15	14	4
12	6	7	9

170/

8	11	10	5
13	2	3	16
1	14	15	4
12	7	6	9

171/

8	10	13	3
11	5	2	16
1	15	12	6
14	4	7	9

172/

8	11	13	2
10	5	3	16
1	14	12	7
15	4	6	9

173/

8	13	10	3
11	2	5	16
1	12	15	6
14	7	4	9

174/

8	13	11	2
10	3	5	16
1	12	14	7
15	6	4	9

175/

12	6	7	9
13	3	2	16
1	15	14	4
8	10	11	5

176/

12	7	6	9
13	2	3	16
1	14	15	4
8	11	10	5

177/

14	4	7	9
11	5	2	16
1	15	12	6
8	10	13	3

178/

15	4	6	9
10	5	3	16
1	14	12	7
8	11	13	2

179/

14	7	4	9
11	2	5	16
1	12	15	6
8	13	10	3

180/

15	6	4	9
10	3	5	16
1	12	14	7
8	13	11	2

181/

12	6	13	3
7	9	2	16
1	15	8	10
14	4	11	5

182/

12	7	13	2
6	9	3	16
1	14	8	11
15	4	10	5

183/

14	4	11	5
7	9	2	16
1	15	8	10
12	6	13	3

184/

15	4	10	5
6	9	3	16
1	14	8	11
12	7	13	2

185/

14	7	11	2
4	9	5	16
1	12	8	13
15	6	10	3

186/

15	6	10	3
4	9	5	16
1	12	8	13
14	7	11	2

187/

12	13	6	3
7	2	9	16
1	8	15	10
14	11	4	5

188/

12	13	7	2
6	3	9	16
1	8	14	11
15	10	4	5

189/

14	11	4	5
7	2	9	16
1	8	15	10
12	13	6	3

190/

15	10	4	5
6	3	9	16
1	8	14	11
12	13	7	2

191/

14	11	7	2
4	5	9	16
1	8	12	13
15	10	6	3

192/

15	10	6	3
4	5	9	16
1	8	12	13
14	11	7	2

193/

9	7	6	12
4	14	15	1
16	2	3	13
5	11	10	8

194/

9	6	7	12
4	15	14	1
16	3	2	13
5	10	11	8

195/

9	7	4	14
6	12	15	1
16	2	5	11
3	13	10	8

196/

9	6	4	15
7	12	14	1
16	3	5	10
2	13	11	8

197/

9	4	7	14
6	15	12	1
16	5	2	11
3	10	13	8

198/

9	4	6	15
7	14	12	1
16	5	3	10
2	11	13	8

. . . (Skip) . . .

```
        361/          362/          363/          364/          365/          366/
 16  2  3 13   16  3  2 13   16  2  5 11   16  3  5 10   16  5  2 11   16  5  3 10
  5 11 10  8    5 10 11  8    3 13 10  8    2 13 11  8    3 10 13  8    2 11 13  8
  9  7  6 12    9  6  7 12    9  7  4 14    9  6  4 15    9  4  7 14    9  4  6 15
  4 14 15  1    4 15 14  1    6 12 15  1    7 12 14  1    6 15 12  1    7 14 12  1
        367/          368/          369/          370/          371/          372/
 16  2  3 13   16  3  2 13   16  2  5 11   16  3  5 10   16  5  2 11   16  5  3 10
  9  7  6 12    9  6  7 12    9  7  4 14    9  6  4 15    9  4  7 14    9  4  6 15
  5 11 10  8    5 10 11  8    3 13 10  8    2 13 11  8    3 10 13  8    2 11 13  8
  4 14 15  1    4 15 14  1    6 12 15  1    7 12 14  1    6 15 12  1    7 14 12  1
        373/          374/          375/          376/          377/          378/
 16  2  9  7   16  3  9  6   16  2  9  7   16  3  9  6   16  5  9  4   16  5  9  4
  3 13  6 12    2 13  7 12    5 11  4 14    5 10  4 15    2 11  7 14    3 10  6 15
  5 11  4 14    5 10  4 15    3 13  6 12    2 13  7 12    3 10  6 15    2 11  7 14
 10  8 15  1   11  8 14  1   10  8 15  1   11  8 14  1   13  8 12  1   13  8 12  1
        379/          380/          381/          382/          383/          384/
 16  9  2  7   16  9  3  6   16  9  2  7   16  9  3  6   16  9  5  4   16  9  5  4
  3  6 13 12    2  7 13 12    5  4 11 14    5  4 10 15    2  7 11 14    3  6 10 15
  5  4 11 14    5  4 10 15    3  6 13 12    2  7 13 12    3  6 10 15    2  7 11 14
 10 15  8  1   11 14  8  1   10 15  8  1   11 14  8  1   13 12  8  1   13 12  8  1

[Counts=384]
```

#3. Discussion #1

The result shows our success.

It is certified that each solution is really a Self-complementary Magic Square of order 4^2. We couldn't really make any other type of solutions. We could make neither duplications nor drop-offs of any solution.

It is amazing that we could make all by the simplest method directly from the basic view-forms of $ECO2^4$, isn't it?

But why could we do that? What makes it possible?

I am moved a lot by that success, but I am also puzzled as much.

#4. Composition of 'Pan-Diagonal' Magic Squares of Order 4^2

Let's compose another type of MS44 next: 'Pandiagonal', 'Complete' or 'Composite & Complete'. We know any type of them has the same, common solution set.

This time we are going to make the 48 'standard' solutions of that type.

What do we have to do, then?

We only have to add a few procedures to the former program.

(1) We have to change S-C type into P-D one by the following program that you might already know well about and be familiar to.

(2) We have to add some 'list-forming inequality conditions' at the final step just before printing procedure such as: n1<n4; n1<n13; n1<n16; and n2>n5;

(3) You may well add anything like a 'sort' program, if necessary.

```
** Type Conversion Program between 'S-C' and 'P-D' **
  1/d0            /d1           SC/           PD/
   1---- 2        3---- 4          1  2  3  4     1  2  4  3
   |  9--+-10     | 11--+-12       5  6  7  8     5  6  8  7
   5--|- 6  |     7--|- 8  |       9 10 11 12    13 14 16 15
      13----14       15----16     13 14 15 16     9 10 12 11
/**/
  d[1]=c[1];     d[2]=c[2];     d[3]=c[4];     d[4]=c[3];
  d[5]=c[5];     d[6]=c[6];     d[7]=c[8];     d[8]=c[7];
  d[9]=c[13];    d[10]=c[14];   d[11]=c[16];   d[12]=c[15];
```

```
  d[13]=c[9];   d[14]=c[10];  d[15]=c[12];  d[16]=c[11];
/**/
```

The following list shows the execution result of our new composition.

```
** Make 48 Standard Solutions of 'Pan-Diagonal' Magic Squares 4^2 **
            1/           2/           3/           4/           5/           6/
  1 15 10  8   1 15  6 12   1 15 10  8   1 15  4 14   1 15  6 12   1 15  4 14
 14  4  5 11  14  4  9  7  12  6  3 13  12  6  9  7   8 10  3 13   8 10  5 11
  7  9 16  2  11  5 16  2   7  9 16  2  13  3 16  2  11  5 16  2  13  3 16  2
 12  6  3 13   8 10  3 13  14  4  5 11   8 10  5 11  14  4  9  7  12  6  9  7
            7/           8/           9/          10/          11/          12/
  1 14 11  8   1 14  4 15   1 14  7 12   1 14  4 15   1 12  7 14   1 12  6 15
 12  7  2 13  12  7  9  6   8 11  2 13   8 11  5 10   8 13  2 11   8 13  3 10
  6  9 16  3  13  2 16  3  10  5 16  3  13  2 16  3  10  3 16  5  11  2 16  5
 15  4  5 10   8 11  5 10  15  4  9  6  12  7  9  6  15  6  9  4  14  7  9  4
           13/          14/          15/          16/          17/          18/
  2 16  9  7   2 16  5 11   2 16  9  7   2 16  3 13   2 16  5 11   2 16  3 13
 13  3  6 12  13  3 10  8  11  5  4 14  11  5 10  8   7  9  4 14   7  9  6 12
  8 10 15  1  12  6 15  1   8 10 15  1  14  4 15  1  12  6 15  1  14  4 15  1
 11  5  4 14   7  9  4 14  13  3  6 12   7  9  6 12  13  3 10  8  11  5 10  8
           19/          20/          21/          22/          23/          24/
  2 13 12  7   2 13  3 16   2 13  8 11   2 13  3 16   2 11  8 13   2 11  5 16
 11  8  1 14  11  8 10  5   7 12  1 14   7 12  6  9   7 14  1 12   7 14  4  9
  5 10 15  4  14  1 15  4   9  6 15  4  14  1 15  4   9  4 15  6  12  1 15  6
 16  3  6  9   7 12  6  9  16  3 10  5  11  8 10  5  16  5 10  3  13  8 10  3
           25/          26/          27/          28/          29/          30/
  3 16  9  6   3 16  5 10   3 16  9  6   3 16  2 13   3 16  5 10   3 16  2 13
 13  2  7 12  13  2 11  8  10  5  4 15  10  5 11  8   6  9  4 15   6  9  7 12
  8 11 14  1  12  7 14  1   8 11 14  1  15  4 14  1  12  7 14  1  15  4 14  1
 10  5  4 15   6  9  4 15  13  2  7 12   6  9  7 12  13  2 11  8  10  5 11  8
           31/          32/          33/          34/          35/          36/
  3 13 12  6   3 13  2 16   3 13  8 10   3 13  2 16   4 15 10  5   4 15  6  9
 10  8  1 15  10  8 11  5   6 12  1 15   6 12  7  9  14  1  8 11  14  1 12  7
  5 11 14  4  15  1 14  4   9  7 14  4  15  1 14  4   7 12 13  2  11  8 13  2
 16  2  7  9   6 12  7  9  16  2 11  5  10  8 11  5   9  6  3 16   5 10  3 16
           37/          38/          39/          40/          41/          42/
  4 15 10  5   4 15  1 14   4 15  6  9   4 15  1 14   4 14 11  5   4 14  1 15
  9  6  3 16   9  6 12  7   5 10  3 16   5 10  8 11   9  7  2 16   9  7 12  6
  7 12 13  2  16  3 13  2  11  8 13  2  16  3 13  2   6 12 13  3  16  2 13  3
 14  1  8 11   5 10  8 11  14  1 12  7   9  6 12  7  15  1  8 10   5 11  8 10
           43/          44/          45/          46/          47/          48/
  4 14  7  9   4 14  1 15   5 16  3 10   5 16  2 11   6 15  4  9   6 15  1 12
  5 11  2 16   5 11  8 10   4  9  6 15   4  9  7 14   3 10  5 16   3 10  8 13
 10  8 13  3  16  2 13  3  14  7 12  1  15  6 12  1  13  8 11  2  16  5 11  2
 15  1 12  6   9  7 12  6  11  2 13  8  10  3 13  8  12  1 14  7   9  4 14  7

** Monitor of Correspondence between Old and New **
 ??: 0
  1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 25: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

 [Count = 48]
```

We have now got the complete set of 48 standard solutions of pandiagonal MS44. We could have got neither duplications nor drop-offs of any solutions.

#5. Discussion #2

It looks like a ‘magic’, doesn’t it? Don’t you really think so?

Why could we make all objects by this unique, simplest method directly from the possible view-forms of developed ECO2⁴?

(1) We did not yet define any simultaneous equations beforehand. We only made possible ‘view-forms’ of developed ECO and transformed them directly into objects. Why aren’t these definitions necessary at all at the beginning? Have we really started without any kinds of premises at all?

(2) What does it mean by the counts 384? We can draw 384 different view-forms in all for the developed ECO, and we know every important type of MS44 has the set of 384 ‘primitive’ solutions. Especially every pandiagonal type has the same, common set of 384 solutions. Does this fact only mean an accidental agreement?

We could not really start anything without premises at all. In fact we used those four-time loops of for(...){...} sentences to make binary numbers by calculations. Yes. Our secret ‘master key’ is just the binary number system.

```
** The Fundamental Three of SC & PD Magic Squares 4^2 **
[1]
 P1/d0          /d1           SC/            PD/
   1---- 2      3---- 4        1  2  3  4     1  2  4  3
   |  9--+-10   | 11--+-12     5  6  7  8     5  6  8  7
   5--|- 6  |   7--|- 8  |     9 10 11 12    13 14 16 15
     13----14     15----16    13 14 15 16     9 10 12 11
 EC/D2i
  0--0   0--0    0--0   0--0    0--0   1--1    0--1   0--1
  | 1--1 | 1--1  | 0--0 | 0--0  | 0--0 | 1--1  | 0--1 | 0--1
  0-|0 | 0-|0 |  1-|1 | 1-|1 |  0-|0 | 1-|1 |  0-|1 | 0-|1 |
    1--1   1--1    1--1   1--1    0--0   1--1    0--1   0--1
   /2^3           /2^2           /2^1           /2^0
 SC/D2i                                    PD/D2i
  0 0 0 0  0 0 0 0  0 0 1 1  0 1 0 1   0 0 0 0  0 0 0 0  0 0 1 1  0 1 1 0
  0 0 0 0  1 1 1 1  0 0 1 1  0 1 0 1   0 0 0 0  1 1 1 1  0 0 1 1  0 1 1 0
  1 1 1 1  0 0 0 0  0 0 1 1  0 1 0 1   1 1 1 1  1 1 1 1  0 0 1 1  0 1 1 0
  1 1 1 1  1 1 1 1  0 0 1 1  0 1 0 1   1 1 1 1  0 0 0 0  0 0 1 1  0 1 1 0
  /2^3     /^2      /^1      /^0       /2^3     /^2      /^1      /^0

 S1/d0          /d1           SC/            PD/
   1----15     14---- 4        1 15 14  4     1 15  4 14
   |  8--+-10   | 11--+- 5    12  6  7  9    12  6  9  7
  12--|- 6  |   7--|- 9  |     8 10 11  5    13  3 16  2
     13---- 3      2----16    13  3  2 16     8 10  5 11
 EC/D2i
  0--1   1--0    0--1   1--0    0--1   0--1    0--0   1--1
  | 0--1 | 1--0  | 1--0 | 0--1  | 1--0 | 1--0  | 1--1 | 0--0
  1-|0 | 0-|1 |  0-|1 | 1-|0 |  1-|0 | 1-|0 |  1-|1 | 0-|0 |
    1--0   0--1    1--0   0--1    0--1   0--1    0--0   1--1
   /2^3           /2^2           /2^1           /2^0
 SC/D2i                                    PD/D2i
  0 1 1 0  0 1 1 0  0 1 0 1  0 0 1 1   0 1 0 1  0 1 0 1  0 1 1 0  0 0 1 1
  1 0 0 1  0 1 1 0  1 0 1 0  1 1 0 0   1 0 1 0  0 1 0 1  1 0 0 1  1 1 0 0
  0 1 1 0  1 0 0 1  1 0 1 0  1 1 0 0   1 0 1 0  1 0 1 0  0 1 1 0  0 0 1 1
  1 0 0 1  1 0 0 1  0 1 0 1  0 0 1 1   0 1 0 1  1 0 1 0  1 0 0 1  1 1 0 0
  /2^3     /^2      /^1      /^0       /2^3     /^2      /^1      /^0

[2]
```

```
P2/d0          /d1          SC/           PD/
  1---- 2      5---- 6        1  2  5  6    1  2  6  5
  |  9--+-10     | 13--+-14     3  4  7  8    3  4  8  7
  3--|- 4  |     7--|- 8  |      9 10 13 14   11 12 16 15
    11----12      15----16     11 12 15 16    9 10 14 13
EC/D2i
 0--0   0--0    0--0   1--1    0--0   0--0    0--1   0--1
 | 1--1 | 1--1  | 0--0 | 1--1  | 0--0 | 0--0  | 0--1 | 0--1
 0-|0 | 0-|0 |  0-|0 | 1-|1 |  1-|1 | 1-|1 |  0-|1 | 0-|1 |
   1--1   1--1     0--0   1--1     1--1   1--1     0--1   0--1
  /2^3           /2^2           /2^1           /2^0
SC/D2i                                PD/D2i
 0 0 0 0  0 0 1 1  0 0 0 0  0 1 0 1   0 0 0 0  0 0 1 1  0 0 0 0  0 1 1 0
 0 0 0 0  0 0 1 1  1 1 1 1  0 1 0 1   0 0 0 0  0 0 1 1  1 1 1 1  0 1 1 0
 1 1 1 1  0 0 1 1  0 0 0 0  0 1 0 1   1 1 1 1  0 0 1 1  1 1 1 1  0 1 1 0
 1 1 1 1  0 0 1 1  1 1 1 1  0 1 0 1   1 1 1 1  0 0 1 1  0 0 0 0  0 1 1 0
 /2^3     /^2      /^1      /^0        /2^3     /^2      /^1      /^0

S2/d0          /d1          SC/           PD/
  1----15     12---- 6        1 15 12  6    1 15  6 12
  |  8--+-10     | 13--+- 3    14  4  7  9   14  4  9  7
 14--|- 4  |     7--|- 9  |      8 10 13  3   11  5 16  2
    11---- 5       2----16     11  5  2 16    8 10  3 13
EC/D2i
 0--1   1--0    0--1   0--1    0--1   1--0    0--0   1--1
 | 0--1 | 1--0  | 1--0 | 1--0  | 1--0 | 0--1  | 1--1 | 0--0
 1-|0 | 0-|1 |  1-|0 | 1-|0 |  0-|1 | 1-|0 |  1-|1 | 0-|0 |
   1--0   0--1     0--1   0--1     1--0   0--1     0--0   1--1
  /2^3           /2^2           /2^1           /2^0
SC/D2i                                PD/D2i
 0 1 1 0  0 1 0 1  0 1 1 0  0 0 1 1   0 1 0 1  0 1 1 0  0 1 0 1  0 0 1 1
 1 0 0 1  1 0 1 0  0 1 1 0  1 1 0 0   1 0 1 0  1 0 0 1  0 1 0 1  1 1 0 0
 0 1 1 0  1 0 1 0  1 0 0 1  1 1 0 0   1 0 1 0  0 1 1 0  1 0 1 0  0 0 1 1
 1 0 0 1  0 1 0 1  1 0 0 1  0 0 1 1   0 1 0 1  1 0 0 1  1 0 1 0  1 1 0 0
 /2^3     /^2      /^1      /^0        /2^3     /^2      /^1      /^0
```

[3]

```
P3/d0          /d1          SC/           PD/
  1---- 2      9----10        1  2  9 10    1  2 10  9
  |  5--+- 6     | 13--+-14     3  4 11 12    3  4 12 11
  3--|- 4  |   11--|-12  |      5  6 13 14    7  8 16 15
     7---- 8      15----16      7  8 15 16    5  6 14 13
EC/D2i
 0--0   1--1    0--0   0--0    0--0   0--0    0--1   0--1
 | 0--0 | 1--1  | 1--1 | 1--1  | 0--0 | 0--0  | 0--1 | 0--1
 0-|0 | 1-|1 |  0-|0 | 0-|0 |  1-|1 | 1-|1 |  0-|1 | 0-|1 |
   0--0   1--1     1--1   1--1     1--1   1--1     0--1   0--1
  /2^3           /2^2           /2^1           /2^0
SC/D2i                                PD/D2i
 0 0 1 1  0 0 0 0  0 0 0 0  0 1 0 1   0 0 1 1  0 0 0 0  0 0 0 0  0 1 1 0
 0 0 1 1  0 0 0 0  1 1 1 1  0 1 0 1   0 0 1 1  0 0 0 0  1 1 1 1  0 1 1 0
 0 0 1 1  1 1 1 1  0 0 0 0  0 1 0 1   0 0 1 1  1 1 1 1  1 1 1 1  0 1 1 0
 0 0 1 1  1 1 1 1  1 1 1 1  0 1 0 1   0 0 1 1  1 1 1 1  0 0 0 0  0 1 1 0
 /2^3     /^2      /^1      /^0        /2^3     /^2      /^1      /^0

S3/d0          /d1          SC/           PD/
  1----15      8----10        1 15  8 10    1 15 10  8
  | 12--+- 6     | 13--+- 3    14  4 11  5   14  4  5 11
 14--|- 4  |   11--|- 5  |     12  6 13  3    7  9 16  2
     7---- 9       2----16      7  9  2 16   12  6  3 13
```

```
EC/D2i
 0--1   0--1    0--1   1--0    0--1   1--0    0--0   1--1
 | 1--0 | 1--0  | 0--1 | 1--0  | 1--0 | 0--1  | 1--1 | 0--0
 1-|0 | 1-|0 |  1-|0 | 0-|1 |  0-|1 | 1-|0 |  1-|1 | 0-|0 |
   0--1   0--1    1--0   0--1    1--0   0--1    0--0   1--1
  /2^3           /2^2           /2^1           /2^0
SC/D2i                                      PD/D2i
 0 1 0 1  0 1 1 0  0 1 1 0  0 0 1 1   0 1 1 0  0 1 0 1  0 1 0 1  0 0 1 1
 1 0 1 0  1 0 0 1  0 1 1 0  1 1 0 0   1 0 0 1  1 0 1 0  0 1 0 1  1 1 0 0
 1 0 1 0  0 1 1 0  1 0 0 1  1 1 0 0   0 1 1 0  1 0 1 0  1 0 1 0  0 0 1 1
 0 1 0 1  1 0 0 1  1 0 0 1  0 0 1 1   1 0 0 1  0 1 0 1  1 0 1 0  1 1 0 0
 /2^3     /^2      /^1      /^0       /2^3     /^2      /^1      /^0
```

Let me show you the next diagrams above with binary decompositions. I put them to both the prototype and corresponding object solutions.

What do they mean?

Do you notice the fact all the solutions are ‘Complete Euler Squares’ of order 4? Every row and column of the solutions has the same number pattern of {0,0,1,1} in any layer of binary decompositions. Each pandiagonal has also the same combination of numbers. Every little square of 2 by 2 in any layer has the same thing, too. It means the mathematical basis of the same ‘magic constant’ for all of them.

Yes. We prepared everything in order to make ‘Complete Euler Squares’ of order 4 for our objects without any consciousness.

We know any ‘CES44’ has the same count of primitive solutions as many as 384.

We actually make the ‘prototype squares’ by drawing all possible view-forms of the same ECO, and make them into our objects, ‘CEulerS44’, by ‘DAM Transformation’. That is the true reason for our success, I believe.

What is the meaning of the fact both view-forms of ECO and the solutions of our object have the same count 384?

I think it means $ECO2^4$ has the only one form in the 4-dimensional world. Only Divine Being might enjoy the original form as it is. But it appears in the 384 different view forms to us. Human beings can only enjoy them individually as they are in this world. Imagine if any object had 1152 solutions while we could draw only 384 pictures for ECO, and we could have guessed there might be 3 original forms of ECO in the 4-dimensional world, and there might be 3 most fundamental solutions.

I am excited most by my own intuition that there might be a great relationship among ‘binary number system’, ‘Complete Euler Squares’, ‘High Dimensional Extra-Cubic Objects’, ‘Prototype Squares’ and ‘DAM Transformation’. For they are all the special topics that I have always studied as hard as I could.

This ‘great relationship’ might be one of the ‘greatest mysteries’ revealed to us by our Divine Being. My philosophical mind could hardly sleep any longer.

(Written in English on MacOS X & Xcode 1.5; May 11, 2005; Kanji Setsuda)

#6. Additional Report

Is it true we can really have the only one best Model for our ‘Do-it-After-the-Model Transformation’?

In our previous experiment we selected the next Model to compose the 384 object solutions of Self-complementary MS44 from the 384 Basic Forms and we assumed such a transformation method as shown again below.

```
/* Concept of 'Do-it-After-the-Model Transformation' */
 BD/d0          /d1          PT/
   1---- 2      3---- 4        1  2  3  4
   |  9--+-10   | 11--+-12     5  6  7  8
   5--|- 6  |   7--|- 8  |     9 10 11 12
     13----14     15----16    13 14 15 16
 MD/d0          /d1          SC/ Selected Model <-
   1----15     14---- 4        1 15 14  4
   |  8--+-10   | 11--+- 5    12  6  7  9
  12--|- 6  |   7--|- 9  |     8 10 11  5
     13---- 3     2----16     13  3  2 16
*/
/* Sample Program for the 'Do-it-After-the-Model Transformation' */
  CC=17;
  y[1]=x[1];    y[2]=CC-x[2];   y[3]=CC-x[3];   y[4]=x[4];
  y[5]=CC-x[5]; y[6]=x[6];      y[7]=x[7];      y[8]=CC-x[8];
  y[9]=CC-x[9]; y[10]=x[10];    y[11]=x[11];    y[12]=CC-x[12];
  y[13]=x[13];  y[14]=CC-x[14]; y[15]=CC-x[15]; y[16]=x[16];
```

I must confess I like this model and this style of transformation best of all, but is it really true that it is the only one best model that can compose the complete set of object solutions? I know a few other solutions can perform well for the model during my study of Prototype squares and 'DAM Transformation' for S-C MS44.

I felt I had to make any additional experiments to examine precisely if it is true.

At first I took the next solution for our temporary model and wrote such a program for its new DAM Transformation as follows.

```
          1/P   8/T Model     | * DAM Transformation *
  1  2  3  4    1 14 15  4    | y[1]=x[1];   y[2]=x[14]; y[3]=x[15]; y[4]=x[4];
  5  6  7  8    8 11 10  5    | y[5]=x[8];   y[6]=x[11]; y[7]=x[10]; y[8]=x[5];
  9 10 11 12   12  7  6  9    | y[9]=x[12];  y[10]=x[7]; y[11]=x[6]; y[12]=x[9];
 13 14 15 16   13  2  3 16    | y[13]=x[13]; y[14]=x[2]; y[15]=x[3]; y[16]=x[16];
```

Please take your kind look at the next list that shows the execution result of our new experiment. I have changed nothing about the Basic Forms.

```
**  Prototype Squares and Self-complementary MS44 Composed  **
         1/P   8/T                2/P   7/T                3/P   14/T
  1  2  3  4   1 14 15  4    1  2  3  4   1 14 15  4    1  2  5  6   1 12 15  6
  5  6  7  8   8 11 10  5    9 10 11 12  12  7  6  9    3  4  7  8   8 13 10  3
  9 10 11 12  12  7  6  9    5  6  7  8   8 11 10  5    9 10 13 14  14  7  4  9
 13 14 15 16  13  2  3 16   13 14 15 16  13  2  3 16   11 12 15 16  11  2  5 16
         4/P   13/T               5/P   20/T               6/P   19/T
  1  2  5  6   1 12 15  6    1  2  9 10   1  8 15 10    1  2  9 10   1  8 15 10
  9 10 13 14  14  7  4  9    3  4 11 12  12 13  6  3    5  6 13 14  14 11  4  5
  3  4  7  8   8 13 10  3    5  6 13 14  14 11  4  5    3  4 11 12  12 13  6  3
 11 12 15 16  11  2  5 16    7  8 15 16   7  2  9 16    7  8 15 16   7  2  9 16
         7/P   2/T                8/P   1/T                9/P   16/T
  1  3  2  4   1 15 14  4    1  3  2  4   1 15 14  4    1  3  5  7   1 12 14  7
  5  7  6  8   8 10 11  5    9 11 10 12  12  6  7  9    2  4  6  8   8 13 11  2
  9 11 10 12  12  6  7  9    5  7  6  8   8 10 11  5    9 11 13 15  15  6  4  9
 13 15 14 16  13  3  2 16   13 15 14 16  13  3  2 16   10 12 14 16  10  3  5 16
```

			10/P	15/T							11/P	22/T							12/P	21/T			
1	3	5	7	1	12	14	7	1	3	9	11	1	8	14	11	1	3	9	11	1	8	14	11
9	11	13	15	15	6	4	9	2	4	10	12	12	13	7	2	5	7	13	15	15	10	4	5
2	4	6	8	8	13	11	2	5	7	13	15	15	10	4	5	2	4	10	12	12	13	7	2
10	12	14	16	10	3	5	16	6	8	14	16	6	3	9	16	6	8	14	16	6	3	9	16
			13/P	4/T							14/P	3/T							15/P	10/T			
1	5	2	6	1	15	12	6	1	5	2	6	1	15	12	6	1	5	3	7	1	14	12	7
3	7	4	8	8	10	13	3	9	13	10	14	14	4	7	9	2	6	4	8	8	11	13	2
9	13	10	14	14	4	7	9	3	7	4	8	8	10	13	3	9	13	11	15	15	4	6	9
11	15	12	16	11	5	2	16	11	15	12	16	11	5	2	16	10	14	12	16	10	5	3	16
			16/P	9/T							17/P	24/T							18/P	23/T			
1	5	3	7	1	14	12	7	1	5	9	13	1	8	12	13	1	5	9	13	1	8	12	13
9	13	11	15	15	4	6	9	2	6	10	14	14	11	7	2	3	7	11	15	15	10	6	3
2	6	4	8	8	11	13	2	3	7	11	15	15	10	6	3	2	6	10	14	14	11	7	2
10	14	12	16	10	5	3	16	4	8	12	16	4	5	9	16	4	8	12	16	4	5	9	16
			19/P	6/T							20/P	5/T							21/P	12/T			
1	9	2	10	1	15	8	10	1	9	2	10	1	15	8	10	1	9	3	11	1	14	8	11
3	11	4	12	12	6	13	3	5	13	6	14	14	4	11	5	2	10	4	12	12	7	13	2
5	13	6	14	14	4	11	5	3	11	4	12	12	6	13	3	5	13	7	15	15	4	10	5
7	15	8	16	7	9	2	16	7	15	8	16	7	9	2	16	6	14	8	16	6	9	3	16
			22/P	11/T							23/P	18/T							24/P	17/T			
1	9	3	11	1	14	8	11	1	9	5	13	1	12	8	13	1	9	5	13	1	12	8	13
5	13	7	15	15	4	10	5	2	10	6	14	14	7	11	2	3	11	7	15	15	6	10	3
2	10	4	12	12	7	13	2	3	11	7	15	15	6	10	3	2	10	6	14	14	7	11	2
6	14	8	16	6	9	3	16	4	12	8	16	4	9	5	16	4	12	8	16	4	9	5	16
			25/P	32/T							26/P	31/T							27/P	38/T			
2	1	4	3	2	13	16	3	2	1	4	3	2	13	16	3	2	1	6	5	2	11	16	5
6	5	8	7	7	12	9	6	10	9	12	11	11	8	5	10	4	3	8	7	7	14	9	4
10	9	12	11	11	8	5	10	6	5	8	7	7	12	9	6	10	9	14	13	13	8	3	10
14	13	16	15	14	1	4	15	14	13	16	15	14	1	4	15	12	11	16	15	12	1	6	15

			190/P	179/T							191/P	186/T							192/P	185/T			
8	16	6	14	8	11	1	14	8	16	7	15	8	10	1	15	8	16	7	15	8	10	1	15
7	15	5	13	13	2	12	7	4	12	3	11	11	5	14	4	6	14	5	13	13	3	12	6
4	12	2	10	10	5	15	4	6	14	5	13	13	3	12	6	4	12	3	11	11	5	14	4
3	11	1	9	3	16	6	9	2	10	1	9	2	16	7	9	2	10	1	9	2	16	7	9
			193/P	200/T							194/P	199/T							195/P	206/T			
9	1	10	2	9	7	16	2	9	1	10	2	9	7	16	2	9	1	11	3	9	6	16	3
11	3	12	4	4	14	5	11	13	5	14	6	6	12	3	13	10	2	12	4	4	15	5	10
13	5	14	6	6	12	3	13	11	3	12	4	4	14	5	11	13	5	15	7	7	12	2	13
15	7	16	8	15	1	10	8	15	7	16	8	15	1	10	8	14	6	16	8	14	1	11	8
			196/P	205/T							197/P	212/T							198/P	211/T			
9	1	11	3	9	6	16	3	9	1	13	5	9	4	16	5	9	1	13	5	9	4	16	5
13	5	15	7	7	12	2	13	10	2	14	6	6	15	3	10	11	3	15	7	7	14	2	11
10	2	12	4	4	15	5	10	11	3	15	7	7	14	2	11	10	2	14	6	6	15	3	10
14	6	16	8	14	1	11	8	12	4	16	8	12	1	13	8	12	4	16	8	12	1	13	8

. . . . . .

			376/P	369/T							377/P	384/T							378/P	383/T			
16	14	12	10	16	5	3	10	16	14	15	13	16	2	3	13	16	14	15	13	16	2	3	13
15	13	11	9	9	4	6	15	8	6	7	5	5	11	10	8	12	10	11	9	9	7	6	12
8	6	4	2	2	11	13	8	12	10	11	9	9	7	6	12	8	6	7	5	5	11	10	8
7	5	3	1	7	14	12	1	4	2	3	1	4	14	15	1	4	2	3	1	4	14	15	1

```
            379/P  366/T                    380/P  365/T                    381/P  372/T
 16 15  8  7   16  9  2  7     16 15  8  7   16  9  2  7     16 15 12 11   16  5  2 11
 12 11  4  3    3  6 13 12     14 13  6  5    5  4 11 14      8  7  4  3    3 10 13  8
 14 13  6  5    5  4 11 14     12 11  4  3    3  6 13 12     14 13 10  9    9  4  7 14
 10  9  2  1   10 15  8  1     10  9  2  1   10 15  8  1      6  5  2  1    6 15 12  1
            382/P  371/T                    383/P  378/T                    384/P  377/T
 16 15 12 11   16  5  2 11     16 15 14 13   16  3  2 13     16 15 14 13   16  3  2 13
 14 13 10  9    9  4  7 14      8  7  6  5    5 10 11  8     12 11 10  9    9  6  7 12
  8  7  4  3    3 10 13  8     12 11 10  9    9  6  7 12      8  7  6  5    5 10 11  8
  6  5  2  1    6 15 12  1      4  3  2  1    4 15 14  1      4  3  2  1    4 15 14  1
```

```
** Monitor of Correspondence between Old and New **
 ??:  0
  1:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 25:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 49:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 73:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
 97:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
121:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
145:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
169:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
193:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
217:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
241:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
265:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
289:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
313:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
337:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
361:  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1

[Count = 384/384] OK!
```

Do you notice it is all right with this new model and new DAM Transformation and they could successfully compose all what we want to have? It was really surprising.

After I made many experiments with other models, I knew every solution of 24 Self-complementary MS44 with n1=1 could perform well for the representative Model and compose the complete set of 384 object solutions, as well as the best one I once supposed. I came to know everything was born equal in its ability and activity.

There are really many beautiful bridges built between the two parallel worlds: of the Prototype squares and of the object squares, or of the sets of Basic Forms and object solutions. It is truly amazing, isn’t it?

It implies there certainly exists the high grade of ‘symmetry’ and ‘concordane’ in the magic square world. I want to find more about what it really is.

(Revised on Jan.27, 2006; Working on MacOS X(10.4.3) & Xcode 2.1 by Kanji Setsuda)

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