

PARALLEL POWER FLOW CALCULATION IN ELECTRIC DISTRIBUTION NETWORKS

Yoshikazu Fukuyama, Yosuke Nakanishi
Fuji Electric Co. R&D., Ltd.
No. 1, Fuji-machi, Hino-city, Tokyo
191 JAPAN
FCN02384@niftyserve.or.jp (Fukuyama)

Hsaio-Dong Chiang
School of Electrical Engineering
Cornell University
Ithaca, NY 14853, USA
chiang@ee.cornell.edu

Abstract: This paper develops a parallel method for fast distribution power flow studies using multi-processors. The method belongs to the class of coarse-grain and exhibits good convergence characteristic over wide range of r/x ratio of distribution lines. The method can be applied to radial distribution systems. It uses only active and reactive power injections at the main feeder and at laterals as state variables; thus reducing the number of state variables. Radial distribution networks are mapped into the tree structure of multi-processor system for allocating each node to processes. Forward/backward sweep approach is realized by communication between the root processor and leaf processors. The proposed method is evaluated on various distribution systems with promising results.

1. INTRODUCTION

Various power flow calculation methods have been developed in the literature. Most of them are developed especially for transmission systems. According to the following facts, however, it is appropriate to develop power flow calculation methods tailored for distribution system analysis.

- * Distribution networks can be numerically ill-conditioned, due to wide ranges of r/x ratio and the inherent radial structure, when using the decoupled method [1] and the Newton-Raphson method [2], which are wisely used in transmission systems.
- * Due to the radial structure, distribution power flow equations are different in nature from transmission power flow equations.

Efficient power flow algorithms for solving radial distribution power flow problem have been developed [3-5]. All these methods are based on the backward/forward sweep which updates and calculates the state variables from the source node of the radial network to the end of the feeders and laterals and vice versa. The Newton-Raphson method is used for updating state variables in [3,4] and it leads faster convergence characteristics. Moreover, the Jacobian matrix of the system equation is investigated and the fast decoupled power flow especially for the distribution system is introduced in [4]. These methods should be applied to various functions of distribution automation such as capacitor placement [3], optimal network configuration [6], and service restoration [7]. Fast distribution power flow method is eagerly required for both off-line and on-line applications.

This paper proposes a parallel method for fast distribution power flow studies using multi-processors. The proposed method seems to have good convergence characteristics and can deal with various r/x ratios of distribution lines. This paper considers:

- * the general pi-model for distribution lines,
- * a general radial network structure,
- * transformers, cogenerators, and switches,

- * a general load model with constant impedance, constant current, and constant complex power.

The proposed parallel method maps a given radial distribution network into a tree structure of multi-processors. The proposed method belongs to the class of coarse-grain [8] that achieves the trade-off between computational speed and hardware cost. The method is implemented on transputer, which is one of the practical multi-processors. The proposed method has been evaluated on test systems with 208, 354, 528 nodes. Transputer can be easily installed on engineering work station (EWS), which is now widely used. Therefore, the results indicate the possibility of using practical multi-processors for fast distribution power flow calculation in actual use.

2. DISTRIBUTION POWER FLOW

2.1 Power flow equations on single feeder

The proposed method uses the network model shown in Fig. 1. State variables of receiving end can be expressed by the state variables of sending end on a single feeder as follows (see Fig. 2):

$$\begin{aligned}
 P_{k+1} &= P_k - P_{loss,k} - P_{Lk+1} \\
 &= P_k - (r_k \frac{P_k^2 + Q_k^2}{|V_k|^2}) - P_{Lk+1} \\
 &= P_k - \frac{r_k}{|V_k|^2} \{P_k^2 + (Q_k + Y_k |V_k|^2)^2\} - P_{Lk+1} \\
 &= f_P(P_k, Q_k, |V_k|^2)
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 Q_{k+1} &= Q_k - Q_{loss,k} - Q_{Lk+1} \\
 &= Q_k - \{x_k \frac{P_k^2 + Q_k^2}{|V_k|^2} - Y_{k1} |V_k|^2 - Y_{k2} |V_{k+1}|^2\} \\
 &\quad - Q_{Lk+1} \\
 &= Q_k - [\frac{x_k}{|V_k|^2} \{P_k^2 + (Q_k + Y_{k1} |V_k|^2)^2\} \\
 &\quad - Y_{k1} |V_k|^2 - Y_{k2} f_{v2}(P_k, Q_k, |V_k|^2)] - Q_{Lk+1} \\
 &= f_Q(P_k, Q_k, |V_k|^2)
 \end{aligned} \tag{2}$$

$$V_{k+1} = V_k - (r_k + jx_k) \frac{(P_k + jQ_k)^*}{V_k^*}$$

$$= f_v(P_k, Q_k, V_k) \quad (3)$$

$$|V_{k+1}|^2 = |V_k|^2 + \frac{r_k^2 + x_k^2}{|V_k|^2} (P_k^2 + Q_k^2) - 2(r_k P_k + x_k Q_k)$$

$$= |V_k|^2 + \frac{r_k^2 + x_k^2}{|V_k|^2} \{P_k^2 + (Q_k + Y_k |V_k|^2)^2\}$$

$$- 2\{r_k P_k + x_k (Q_k + Y_k |V_k|^2)\}$$

$$= f_{v2}(P_k, Q_k, |V_k|^2) \quad (4)$$

The above equation indicates that state variables at the receiving (down-stream) nodes can be calculated successively from the source node to the end of the feeder once state variables at the source node of the feeder are given. In other words, all state variables at each node should not be state variables and only the most up-stream state variables (P_0, Q_0, V_0) should be state variables. If the most up-stream node (the first node of the feeder) is a source node of the system, V_0 should be a constant value because the secondary side voltage of the substation is assumed to be fixed. Power flows P_n and Q_n at the end of the feeder must be zero. These can be expressed as two boundary conditions as follows:

$$V_0 = \bar{V} \quad (5)$$

$$P_n = Q_n = 0 \quad (6)$$

where, \bar{V} : constant voltage at the source node of the system.

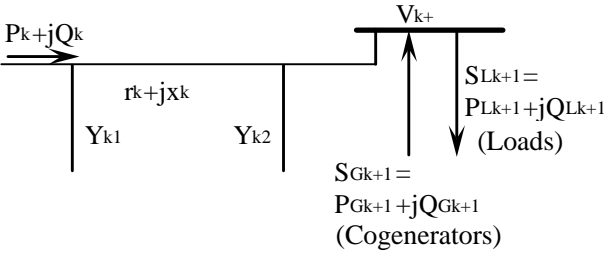


Fig. 1 A network model.

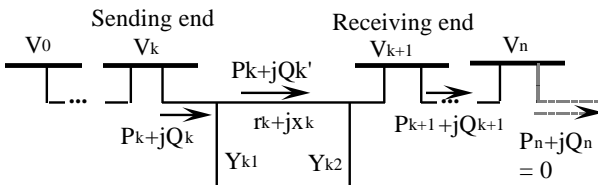


Fig. 2 Single feeder .

2.2 Power Flow equations on general radial network

2.2.1 Levelization of feeders

The concept of level for feeders can be introduced in order to distinguish between the main feeder, its laterals, sub-laterals from the laterals, and so on. Fig. 3 shows the concept of levelization on a general radial network. A feeder at level k is defined as a lateral branching out from a lateral at level $k-1$. The level of the main feeder connected to the secondary side of the substation is 0. A route starting from the substation to an end node is considered as the main feeder if it contains the largest number of nodes. However, due to the issues of load balancing and communication requirements which are important in our proposed parallel method, we select a main feeder with considerations of the number of laterals attached to it; the larger the number of laterals attached to, the better our parallel method will perform. Laterals at level k are identified according to the above rules among laterals that have a node connected to laterals at level $k-1$.

2.2.2 Power flow equations

Considering the above relation between up-stream and down-stream nodes on single feeder, it is obvious that only injection powers to the main feeder and each lateral can be state variables and the rest of the state variables can be calculated from the source node to the end nodes of the laterals successively. The power flow equations for a general radial distribution network can be expressed as follows:

$$P_{00n_{00}} = \hat{P}_{00n_{00}}(Z_{000}, Z_{100}, \dots, Z_{1k_1 0}, |V_{000}|) = 0$$

$$Q_{00n_{00}} = \hat{Q}_{00n_{00}}(Z_{000}, Z_{100}, \dots, Z_{1k_1 0}, |V_{000}|) = 0$$

$$P_{ijn_{00}} = \hat{P}_{ijn_{00}}(Z_{ij0}, Z_{i+100}, \dots, Z_{i+1k_{i+1} 0}, |V_{000}|) = 0$$

$$Q_{ijn_{00}} = \hat{Q}_{ijn_{00}}(Z_{ij0}, Z_{i+100}, \dots, Z_{i+1k_{i+1} 0}, |V_{000}|) = 0$$

$$Z_{ijkij} = (P_{ijk_{ij}}, Q_{ijk_{ij}}), i = 0, \dots, l, j = 0, \dots, i_m, k_{ij} = 0, \dots, n_{ij} \quad (7)$$

where, l : the number of level,
 i_m : the number of laterals at level i ,
 n_{ij} : the number of nodes of lateral j at level i .

The above nonlinear equations can be solved using the very fast decoupled algorithm [4]. The correction of the state variables at each iteration can be realized by successive calculation from laterals at the highest level to the main feeder at the lowest level 0 (backward sweep). The rest of state variables can be calculated by successive calculation from the source node (level 0) to end of the laterals at the highest level (forward sweep; see fig. 3). The calculation repeats the backward and forward sweeps until it converges. A study of the convergence properties and convergence speed associated with the above solution method can be found in [4].

3. PARALLEL DISTRIBUTION POWER FLOW

3.1 Parallel computation

The following observations are useful for our development of parallel distribution power flow method.

- * The backward and forward sweep for feeders at the same level can be calculated independently.
- * Each Sweep for the feeders and the laterals at level k requires the feeder and lateral information at $k \pm 1$ level.

Based on the above observation, a parallel computation can be developed by mapping the radial network into a tree structure for multi-processors. Feeders and laterals are allocated to processors so that the distance from the root processor is equal to the feeder level. According to the mapping, each processor should communicate with only neighboring processors during calculation.

3.2 Load balancing among processors

The proposed method utilizes the following simplified static allocation scheme considering trade-off between computation time and optimality of allocation.

- (Step 1) Sum up the number of nodes in feeders at a certain level and calculate the average number of nodes to be allocated to each processor (N_{ave}).
- (Step 2) Allocate feeders to a processor and sum up total number of nodes for the processors. If the total number of nodes exceeds N_{ave} , finish to allocate feeders to the processor and allocate feeders to the next processor. Continue the procedures until all feeders are allocated.

Fig. 4 summarizes a flow chart of the proposed method.

4. NUMERICAL EXAMPLES

4.1 Simulation conditions

The proposed method is applied to the test systems with 207, 354, and 528 nodes, which are obtained by expanding the original system with 69 nodes in [3]. Each test system has the same three subsystems connected to a substation. Therefore, three subsystems can be calculated independently. Since total number of nodes for each subsystem is equal, load allocation for the processors is similar if four processors including the root processor are utilized.

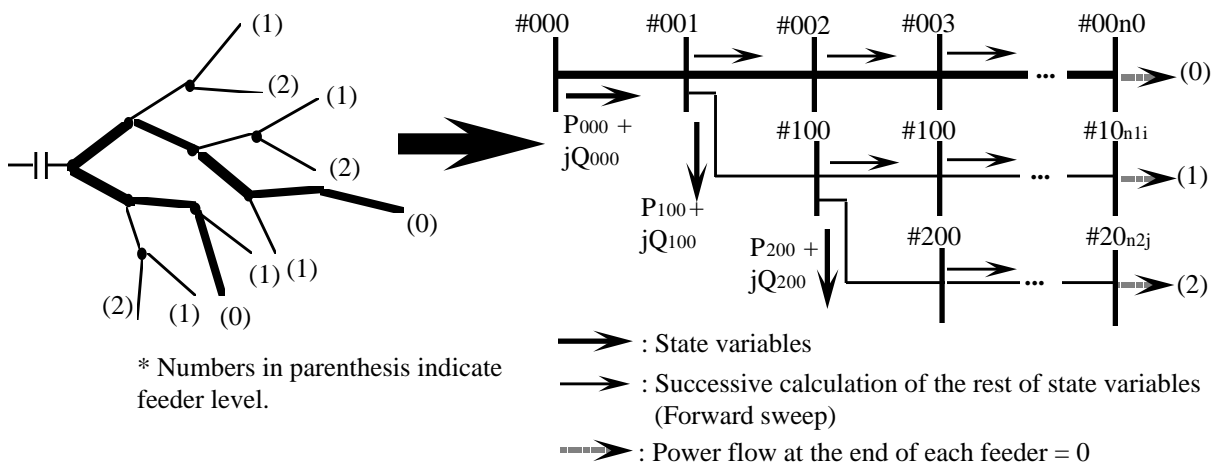


Fig. 3 Concept of levelization and treatment of state variables (forward sweep).

The cases on 1, 4, and 12 processors are compared. Here, one to three transputers were used. Logical processes on transputers are used for the cases with 4 and further number of processors. In such cases, computation time is calculated by combining calculation time and time for communications.

4.2 Results

Fig. 5 shows the calculation time for each test system on various number of processors. The results show that parallel computation is efficient when the number of nodes is increased. For example, the computation time for the system with 528 nodes on 13 processors is about three times faster than that on 1

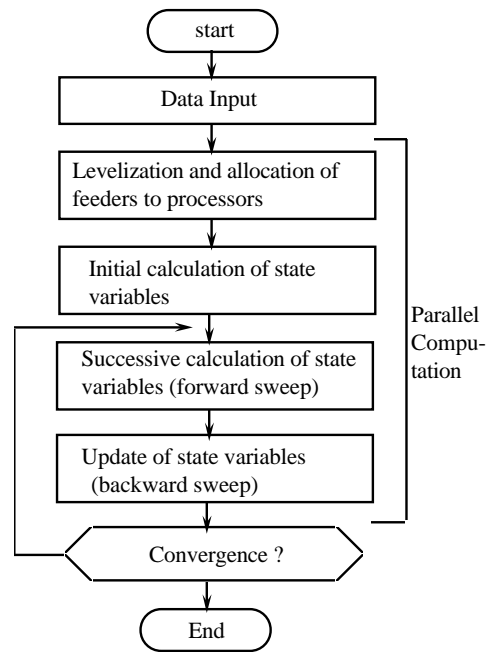


Fig. 4 A flow chart of the proposed method.

processor. However, considering parallel efficiency of the computation, it is obvious that the system with 4 processors is the most efficient.

Fig. 6 shows the computation time required in each step of the parallel algorithm for the case on 4 processors. The result shows the computation time in one processor; indicating that the rate of communication decreases as the number of nodes increases. Therefore, the method is more efficient when the number of nodes is increased.

5. CONCLUSIONS AND FUTURE WORKS

This paper presents a fast distribution power flow method implemented on multi-processors. The results can be summarized as follows:

- * The radial structure of distribution networks is mapped into a tree structure of multi-processors. The proposed method belongs to the class of coarse-grain which achieves the trade-off between computational speed and hardware cost.
- * The method is implemented on transputers that can be easily installed on EWS. Since EWS is now widely used, our results indicate that running multi-processors on EWS independently for distribution power flow analysis may be useful for distribution automation applications.

The parallel power flow method proposed in this paper can be improved further in several ways. Although the method is confined to radial distribution networks, the method can be applied to weakly meshed power networks using the compensation-based technique developed in [5]. In the method, breakpoints are selected in loops and original networks are converted to a radial network. Then, mismatch of power flows at the breakpoints are corrected. Therefore, the simplified allocation scheme for communication and load balancing between processors requires certain modification. Unbalanced loads should be considered in some cases. The proposed method can be applied

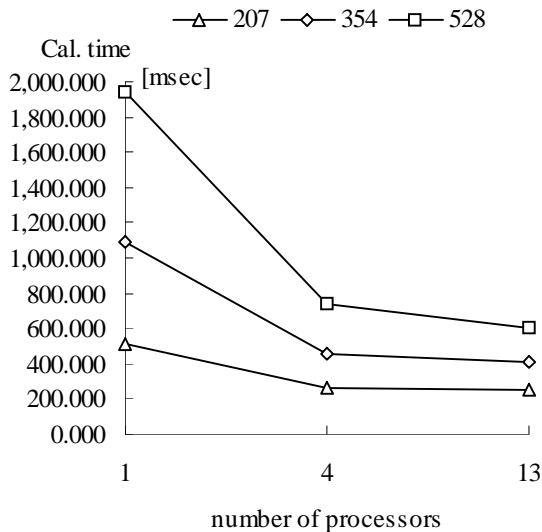


Fig. 5 Calculation time on various number of processors for the test systems with 207, 354, and 528 nodes.

for the three phase network representation with slight modification. Parallel computation must be more efficient for three phase network because it requires more computational efforts.

6. REFERENCES

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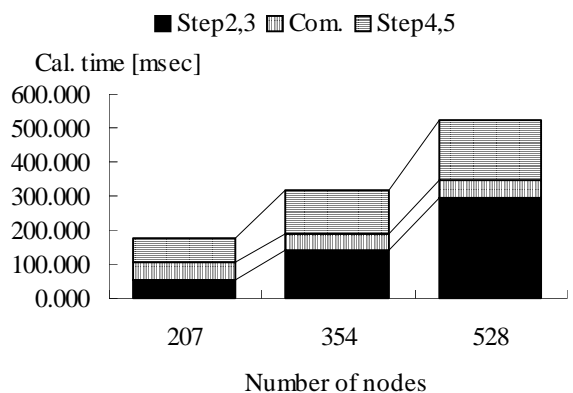


Fig. 6 Calculation time for each step in the case with 4 processors.