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Practical Equipment Models for Fast Distribution Power Flow Considering Interconnection of Distributed Generators

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Abstract: This paper presents distribution system equipment models for fast distribution load flow calculation considering interconnection of distributed generators. Recently, the number of distributed generators introduced into distribution systems has been increasing, and detailed system analysis using load flow calculation has been eagerly awaited. This paper develops practical equipment models such as various distributed generators, voltage control equipment, and loads. The feasibility of the developed models is verified and demonstrated on practical distribution system models with promising results.

Keywords: Distribution System Equipment, Distribution Plow Flow Calculation, Distributed Generation

I. INTRODUCTION

Conventionally, it is assumed that electric power in distribution systems always flows from substations to the end of feeders in planning and operation. However, introduction of distribution generators under de-regulated environment causes reverse power flow and complicated voltage profiles in the distribution systems. Therefore, it is required to modify planning and operation methods in distribution systems.

Since voltage profiles are directly related to power quality and customers' satisfaction, it is of primary concern in power utilities. Conventionally, voltage regulation is realized by LDC functions of transformers in substations, voltage regulators with LDC functions in distribution systems (called Step Voltage Regulator, SVR, in Japan), and so on. Since conventional voltage regulators cannot handle the reverse power flow and steep voltage fluctuation, new voltage regulation equipment is required. Considering these backgrounds, fast power flow calculation considering various newly introduced equipment models in distribution systems is required.

Various fast backward forward sweep type methods have been developed for distribution power flow [1-7]. Considering introduction of distributed generators and new voltage regulation equipment, practical models for these newly introduced equipment should be developed.

This paper presents distribution system equipment models such as distribution generator and various voltage regulator equipment models for fast distribution power flow. The proposed models are verified using practical distribution models with promising results.

II. FAST DISTRIBUTION POWER FLOW

Power flow equations on single feeder

The proposed method uses the network model shown in Fig. 1. Fig. 2 shows a single feeder distribution model. State variables of receiving end can be expressed by the state variables of sending end on a single feeder as follows:

$$\begin{aligned} P_{k+1} &= P_k - P_{loss,k} - P_{Lk+1} \\ &= P_k - (r_k \frac{P_k^2 + Q'^2_k}{|V_k|^2}) - P_{Lk+1} \\ &= P_k - \frac{r_k}{|V_k|^2}\{P_k^2 + (Q_k + Y_k|V_k|^2)^2\} - P_{Lk+1} \\ &= f_P(P_k, Q_k, |V_k|^2) \end{aligned} \quad (1)$$

$$\begin{aligned} Q_{k+1} &= Q_k - Q_{loss,k} - Q_{Lk+1} \\ &= Q_k - \{x_k \frac{P_k^2 + Q'^2_k}{|V_k|^2} - Y_{k1}|V_k|^2 - Y_{k2}|V_{k+1}|^2\} - Q_{Lk+1} \\ &= Q_k - [\frac{x_k}{|V_k|^2}\{P_k^2 + (Q_k + Y_{k1}|V_k|^2)^2\} \\ &\quad - Y_{k1}|V_k|^2 - Y_{k2} f_{v2}(P_k, Q_k, |V_k|^2)] - Q_{Lk+1} \\ &= f_Q(P_k, Q_k, |V_k|^2) \end{aligned} \quad (2)$$

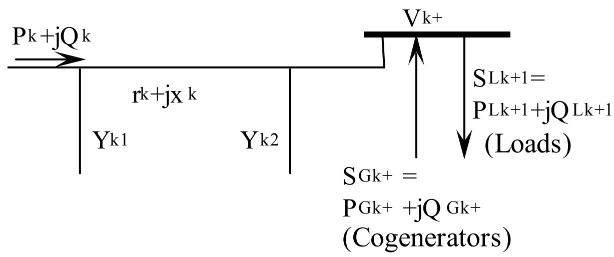


Fig. 1 A distribution network model.

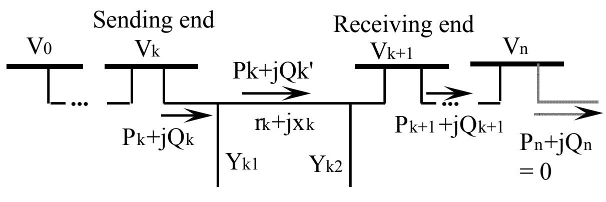


Fig. 2 A single feeder.

$$
\begin{aligned}
V_{k+1} &= V_k - (r_k + jx_k)\frac{(P_k + jQ'_k)^*}{V_k^*} \\
&= f_v(P_k, Q_k, V_k)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
|V_{k+1}|^2 &= |V_k|^2 + \frac{r_k^2 + x_k^2}{|V_k|^2}(P_k^2 + Q'^2_k) - 2(r_k P_k + x_k Q'_k) \\
&= |V_k|^2 + \frac{r_k^2 + x_k^2}{|V_k|^2}\{P_k^2 + (Q_k + Y_k|V_k|^2)^2\} \\
&\quad - 2\{r_k P_k + x_k(Q_k + Y_k|V_k|^2)\} \\
&= f_{v2}(P_k, Q_k, |V_k|^2)
\end{aligned} \tag{4}
$$

The above equation indicates that state variables at the receiving (down-stream) nodes can be calculated successively from the source node to the end of the feeder once state variables at the source node of the feeder are given. In other words, all state variables at each node should not be state variables and only the most up-stream state variables (P_0, Q_0, V_0) should be state variables. If the most up-stream node (the first node of the feeder) is a source node of the system, V_0 should be a constant value because the secondary side voltage of the substation is assumed to be fixed by LDC function of transformers. Power flows P_n and Q_n at the end of the feeder must be zero. The relations can be expressed as two boundary conditions as follows:

$$V_0 = \overline{V} \tag{5}$$

$$P_n = Q_n = 0 \tag{6}$$

where, $\overline{V}$: constant voltage at the source node of the system.

Power Flow equations on general radial network

(1) Levelization of feeders

The concept of level for feeders can be introduced in order to distinguish between the main feeder, its laterals, sub-laterals from the laterals, and so on. Fig. 3 shows the concept of levelization on a general radial network. A feeder at level k is defined as a lateral branching out from a lateral at level k-1. The level of the main feeder connected to the secondary side of the substation is 0. A route starting from the substation to an end node may be considered as the main feeder if it contains the largest number of nodes. Laterals at level k are identified according to the above rules among laterals that have a node connected to laterals at level k-1.

(2) Power flow equations

Considering the above relation between up-stream and down-stream nodes on single feeder, it is obvious that only injection powers to the main feeder and each lateral can be state variables and the rest of the state variables can be calculated from the source node to the end nodes of the laterals successively. The power flow equations for a general radial distribution network can be expressed as follows:

$$
\begin{aligned}
P_{00n_{00}} &= \hat{P}_{00n_{00}}(Z_{000}, Z_{100}, \ldots, Z_{1k_1 0}, |V_{000}|) = 0 \\
Q_{00n_{00}} &= \hat{Q}_{00n_{00}}(Z_{000}, Z_{100}, \ldots, Z_{1k_1 0}, |V_{000}|) = 0 \\
P_{ijn_{00}} &= \hat{P}_{ijn_{00}}(Z_{ij0}, Z_{i+100}, \ldots, Z_{i+1k_{i+1}0}, |V_{000}|) = 0 \\
Q_{ijn_{00}} &= \hat{Q}_{ijn_{00}}(Z_{ij0}, Z_{i+100}, \ldots, Z_{i+1k_{i+1}0}, |V_{000}|) = 0 \\
Z_{ijkij} &= (P_{ijk_{ij}}, Q_{ijk_{ij}}), \\
& i = 0, \ldots, l,\ j = 0, \ldots i_m,\ k_{ij} = 0, \ldots, n_{ij}
\end{aligned} \tag{7}
$$

where, l : the number of level,

i_m : the number of laterals at level i,

n_{ij}: : the number of nodes of lateral j at level i.

The above nonlinear equations can be solved using the very fast decoupled algorithm [2][6]. The correction of the state variables at each iteration can be realized by successive calculation from laterals at the highest level to the main feeder at the lowest level 0 (backward sweep). The rest of state variables can be calculated by successive calculation from the source node (level 0) to end of the laterals at the

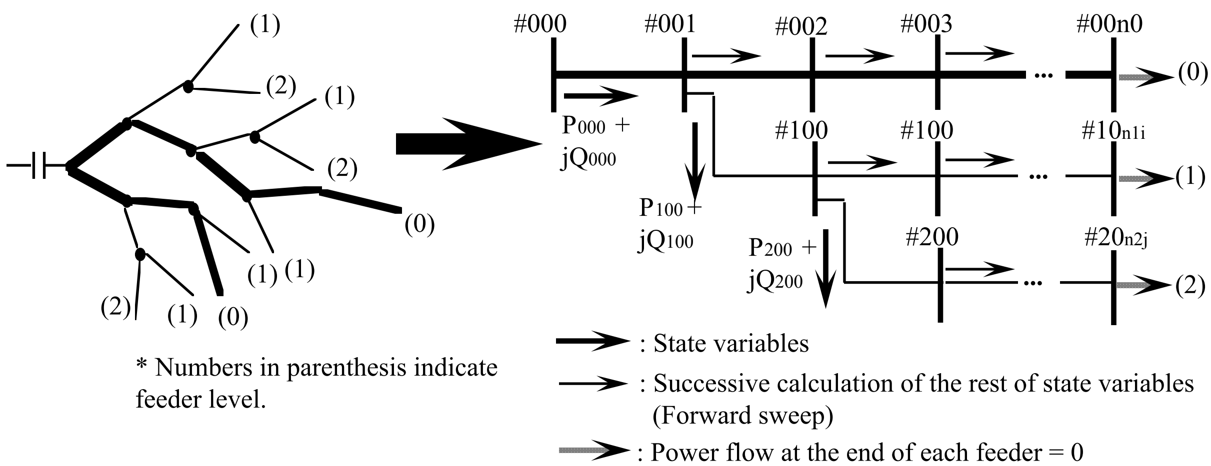


Fig. 3 Concept of levelization and treatment of state variables (forward sweep).

highest level (forward sweep; see fig. 3). The calculation repeats the backward and forward sweeps until it converges. A study of the convergence properties and convergence speed associated with the above solution method can be found in [2].

III. DISTRIBUTION SYSTEM MODELS

Distribution Generators

(1) Synchronous Generators

If a generator is operated using AVR to regulate the terminal voltage at the specified voltage, the model is expressed by PV specified node with fictitious node and impedance. Fictitious branch is utilized to maintain specified voltage by feeding reactive power to the PV specified node. Using the fictitious branch and node, PV specified node can be treated as PQ specified node in calculation procedures. The reactive power specified value can be calculated using voltage specified values and voltage value at fictitious node as follows:

$$Q_{spec} = \left|V_i^t\right| \cdot \left(\frac{V_{fic,i}^t - \left|V_i^t\right|}{X_{fic,i}} \right) \tag{8}$$

where, Q_{spec}: fictitious reactive power specified value,
V_i^t: calculated voltage value in node i at iteration t,
$V_{fic,i}^t$: calculated voltage value in fictitious node of node i at iteration t,
$X_{fic,i}$: fictitious branch impedance of node i.

(2) Inverter connected generator

photovoltaic (PV) power generators and sometimes wind power generators are connected using inverters. In such a case, the generators can be models as inverters with limited output values. Current control type can be treated as PI specified node where active power output and injection current to the power systems are specified. Corresponding PQ specified values can be calculated by calculated voltage and a specified active power output value. Voltage control type can be treated as the above-mentioned PV specified node like synchronous generators. However, available injection current is limited and the type is switched to current control type if the injection current reaches its limited value. When active power output of the generator and injection current are specified, corresponding reactive power specified value can be calculated as follows:

$$Q_{spec.} = \pm\sqrt{|I|^2 (e^2 + f^2) - P_{spec}^2} \tag{9}$$

where, e, f : calculated voltage vector (V=e+jf),
I : specified current value of distributed generator,
P_{spec} : active power specified value,
Q_{spec} : reactive power specified value.
One specific solution can be determined using power factor of the generator.

(3) Induction generator (IG)

Steady state characteristic of induction generator can be expressed using equivalent circuit with slip. Reactive power output of IG can be expressed approximately with a function of voltage to specified active power output. Namely, it can be modeled with ZIP characteristics like load models. State of each parts of IG in fig. 4 can be expressed as follows.

V_1 : primary side terminal voltage,
E : primary side voltage,
E_1 : primary side induced voltage,
E_2' : secondary induced voltage at stationary state converted into primary side value,
I_1 : primary side current,
I_2' : secondary side current converted into primary side value,
I_0 : exciting current,
Z_1 : primary side impedance,
Z_2' : secondary side impedance converted into primary side value.
Y_0 : equals jb_0, excitation admittance.

Using the above states, three phase primary side injection active power (P_1), reactive power (Q_1), and power factor (pf_1) can be calculated. These values are output values for induction generator and input values for induction motor. Supposing E_1=E_2=E and considering the equivalent circuit of fig.4, the following equation can be obtained:

$$\dot{I}_2' = \frac{s\dot{E}_2'}{\dot{Z}_2'} = \frac{s\dot{E}}{r_2' + jsx_2'} \tag{10}$$

$$\dot{I}_1 = \dot{I}_0 + \dot{I}_2' = \dot{E}\left(\dot{Y}_0 + \frac{s}{\dot{Z}_2'}\right) = \dot{E}\left(-jb_0 + \frac{s}{r_2' + jsx_2'}\right) \tag{11}$$

$$\dot{V}_1 = \dot{E} + \dot{I}_1\dot{Z}_1 = \dot{E}\left(1 + \dot{Z}_1\dot{Y}_0 + s\frac{\dot{Z}_1}{\dot{Z}_2'}\right) \tag{12}$$

E can be obtained from (12), and by substituting it into (11), the following equation can be obtained:

$$\dot{I}_1 = \frac{c + jd}{a + jb}\dot{V}_1 = \frac{(ac + bd) + j(ad - bc)}{a^2 + b^2}\dot{V}_1 \tag{13}$$

where,

$$a = (r_1 + r_1 x_2' b_0)s + (r_2' + x_1 r_2' b_0) \tag{14}$$

$$b = (x_2' + x_1 x_2' b_0 + x_1)s + (-r_1 r_2' b_0) \tag{15}$$

$$c = (1 + x_2' b_0)s \tag{16}$$

$$d = -r_2' b_0 \tag{17}$$

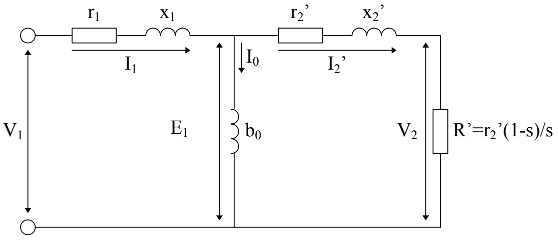


Fig.4 A Steady state model of induction machine.

According to the above relations, P_1, Q_1, and pf_1 can be expressed as follows:

$$P_1 = 3\left[\dot{V}_1 \dot{I}_1^*\right]_{real} = 3\left[\dot{V}_1 \frac{(ac+bd)-j(ad-bc)}{a^2+b^2}\dot{V}_1^*\right]_{real}$$

$$= 3\frac{ac+bd}{a^2+b^2}V_1^2[W] \quad (18)$$

$$Q_1 = 3\left[\dot{V}_1 \dot{I}_1^*\right]_{imaginary}$$

$$= 3\left[\dot{V}_1 \frac{(ac+bd)-j(ad-bc)}{a^2+b^2}\dot{V}_1^*\right]_{imaginary}$$

$$= -3\frac{ad-bc}{a^2+b^2}V_1^2[\mathrm{var}] \quad (19)$$

$$pf_1 = \frac{P_1}{\sqrt{P_1^2+Q_1^2}} \quad (20)$$

When an induction machine is operated at the constant output, active power in (18) can be fixed as constant and reactive power characteristic to voltage can be obtained using (19). Here it is assumed that the characteristic can be expressed with second-order function of voltage.

$$P_1 = Const$$

$$Q_1 = f(V_1, S)$$

$$= f(V_1, S(P_1, V_1)) = f(V_1, P_1)$$

$$\approx a_2(P_1) * V^2 + a_1(P_1) * V + a_0(P_1) \quad (21)$$

excitation susceptance b_0 in fig.4 can be moved to the left side of the equivalent circuit at the armature terminal and b_0 can be removed from (18)(19). Then, the following equation can be obtained.

$$P_1 = \frac{s(r_2+r_1 s)V_1^2}{(r_2+r_1 s)^2+s^2(x_1+x_2)^2} \quad (22)$$

$$Q_1 = \frac{s^2 V_1^2(x_1+x_2)}{(r_2+r_1 s)^2+s^2(x_1+x_2)^2} \quad (23)$$

Slip s can be removed from these equations and an equation regarding Q_1 can be solved as follows.

$$P_1^2(x_1+x_2) = Q_1(V_1^2 - Q_1 x_1 - Q_1 x_2) \quad (24)$$

$$Q_1 = \frac{V_1^2 - \sqrt{V_1^4 - 4P_1^2(x_1+x_2)^2}}{2(x_1+x_2)} \quad (25)$$

$$Q_1 = \frac{V_1^2 + \sqrt{V_1^4 - 4P_1^2(x_1+x_2)^2}}{2(x_1+x_2)} \quad (26)$$

The solution (26) is unstable and the only solution (25) can be utilized. Eq. (25) can be expanded using Taylor's expansion around V_1=1.0 and the following equation can be obtained:

$$Q_1 \approx a_2(P_1) * V^2 + a_1(P_1) * V + a_0(P_1) \quad (27)$$

where,

$$a_2(P_1) = \frac{1}{2(x_1+x_2)} + \frac{1}{(x_1+x_2)(1-4P_1^2(x_1+x_2)^2)^{3/2}} - \frac{3}{2(x_1+x_2)\sqrt{1-4P_1^2(x_1+x_2)^2}} + b_0$$

$$a_1(P_1) = -\frac{2}{(x_1+x_2)(1-4P_1^2(x_1+x_2)^2)^{3/2}} + \frac{2}{\sqrt{1-4P_1^2(x_1+x_2)^2}}$$

$$a_0(P_1) = \frac{1}{(x_1+x_2)(1-4P_1^2(x_1+x_2)^2)^{3/2}} - \frac{1}{2(x_1+x_2)\sqrt{1-4P_1^2(x_1+x_2)^2}} - \frac{\sqrt{1-4P_1^2(x_1+x_2)^2}}{2(x_1+x_2)}$$

Voltage regulation equipment

(1) Transformer and SVR with LDC function

Transformers installed in s/s and SVR installed in each feeder regulate network voltage with the LDC function. The equipment modify a tap position when difference between a reference voltage and calculated voltage considering voltage drop from sending end to a certain point is larger than a dead band value.

(2) FACTS type equipment

Parallel compensation equipment such as SVC can be modeled as follows. One PV specified node is added beyond the SVC installed node. Branch impedance between the PV specified node and the SVC installed node is set to sloop impedance. The node is changed to PQ specified node when the output of the compensation equipment reaches its limit value.

Series compensation equipment and a series part of UPFC can be modeled as variable impedance. Active power loss is compensated from the parallel equipment so that the injection and output active power is coincided.

UPFC can be modeled using both parallel and series compensation equipment.

Load

(a) Constant power load

$$S_{Lk} = \overline{P}_{Lk} + j\overline{Q}_{Lk} \quad (28)$$

,where $\overline{P}_{Lk}, \overline{Q}_{Lk}$: constant values of active and reactive powers.

(b) Constant current load

$$S_{Lk} = V_k \times \overline{I}_{LK}^*$$

$$= |V_k|(\overline{a}_{Lk} + j\overline{b}_{Lk}) \quad (29)$$

, where $\overline{a}_{Lk}, \overline{b}_{Lk}$: constant values of active and reactive load currents.

(c) Constant impedance load

$$S_{LK} = V_k \times (\frac{V_k}{Z_{LK}})^*$$

$$= \frac{|V_k|^2}{\bar{Z}_{LK}} = \frac{|V_k|^2}{\bar{r}_{LK}^2 + \bar{x}_{LK}^2}(\bar{r}_{LK} + j\bar{x}_{LK}) \qquad (30)$$

, where $\bar{r}_{Lk}, \bar{x}_{Lk}$: constant values of active and reactive load impedance.

Combination of the above load types (ZIP load) can be realized by weighted sum of the each load values.

IV. NUMERICAL EXAMPLES

Simulation Conditions

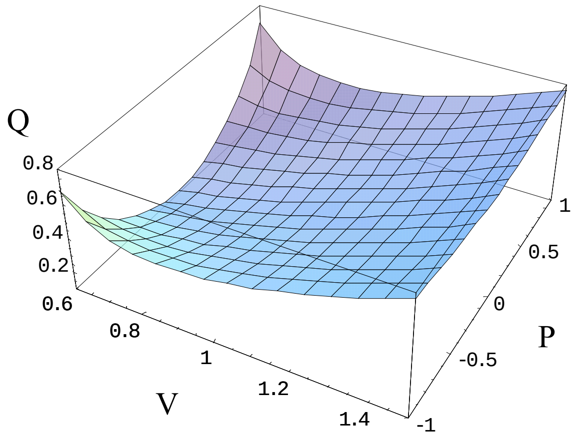
The following numerical simulations are shown as examples.

(1) Induction generator characteristics (Case No.1)

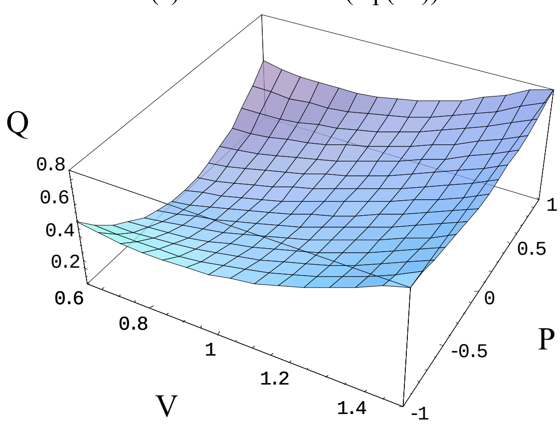
(2) Voltage calculation with SVR, SVC, and DG (Case No.2).

(1) Case No.1

As an example of DG, the detailed (eq. (24) and (25)) and simplified ZIP (eq. (24) and (27)) induction generator models are compared.



(a) detailed model(eq.(22))



(b) simplified model(eq.(25))

Fig. 6 Comparison of steady state characteristics of induction machine models.

(2) Case No.2

Fig. 5 shows a typical 6.6 kV model system with three SVR, one SVC, and one DG. Total line length is 15.38 [km] and total loads are 375 [kW] (heavy load condition). The system has 40 nodes. Voltage profile is calculated when the DG suddenly change output from 0 [kW] to full output assuming sudden output change of wind generators. Therefore, tap position of SVRs are fixed at the simulation. Only the SVC can compensates voltage profile. Practically, SVRs can change tap positions after their time setting values.

Simulation Results

(1) Case No.1

Fig. 6 shows the comparison of P-V-Q output characteristics between detailed and simplified induction generator models with typical machine parameters. Reactive power output change to active power changes is almost same and the results indicate the appropriateness of the proposed ZIP model for induction generators.

(2) Case No.2

Fig. 7 shows voltage profile of case No.2. According to the heavy loading condition, each SVR raise up voltage without DG output (initial conditions). When DG output full power output, SVR tap positions are kept fixed and only SVC decreases voltage in order to keep the feeder end voltage within allowable ranges.

VI. CONCLUSIONS

This paper presents various distribution model considering introduction of distributed generators. All of

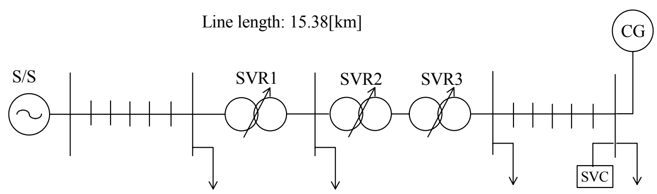


Fig.5 A typical 6.6kV model system.

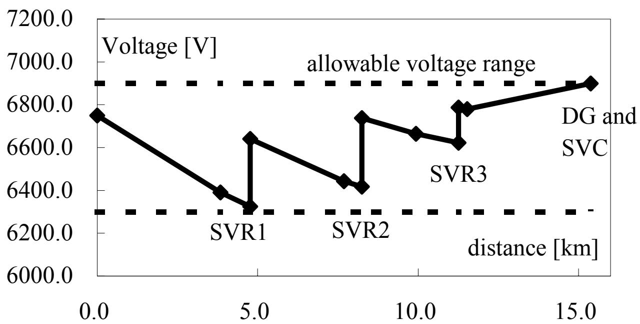


Fig. 7 Voltage profile of case No.2.

equipment models are modeled so as to easily be implemented in backward-forward type fast distribution power flow calculation. The simulation results indicate the appropriateness of the proposed models for simulation considering introduction of distribution generators.

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BIOGRAPHIES

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