

# The Generation of ZIP-V Curves for Tracing Power System Steady State Stationary Behavior Due to Load and Generation Variations

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**Abstract** — The P-V Curve, Q-V Curve, or P-Q-V Curve have been widely used to analyze power system behaviors under varying loading conditions. These curves have been generated under the condition that the constant P-Q load component of a bus (or a collection of buses) varies, with the constant current load and constant impedance load being kept indeed “constant”. As such, the physical meaning of these curves can be easily explained. Motivated by the facts that load models have profound impacts on power system behaviors and that the nonlinear load model, ZIP-model, is popular in modeling nonlinear behaviors of loads, this paper proposes a new class of curves, called ZIP-V curves, to better trace power system steady-state stationary behavior due to load and generation variations. The ZIP-V curves encompass the traditional P-V, Q-V, P-Q-V curves (constant P-Q load), I-V curve (constant current load), Z-V curve (constant impedance load), or generalized curves such as IP-V (constant current and constant power load), ZP-V (constant impedance and constant power load) or IZ-V (constant current and constant impedance load) curve when the values of corresponding components are kept constant. A tool based on the Continuation Power Flow (CPFLOW) method useful for generating the ZIP-V curves is developed and its application to generate ZIP-V curves of a 4561-bus interconnected power system is illustrated.

## 1. INTRODUCTION

The P-V Curve, Q-V Curve, or P-Q-V Curve have been widely used to analyze power system behavior under varying loading conditions. Voltage stability analysis and loadability analysis are examples of the application of these curves in power system analysis. These curves are obtained by Continuation Power Flow (CPFLOW) [1] as the constant P-Q load component of a bus (or a collection of buses) varies, with the constant current load and constant impedance load being kept indeed “constant”. As such, the physical meaning of these curves can be easily explained and these curves serve their purpose well - on the conservative side in predicting several system limits such as voltage stability limit or loadability limit.

Load models are known to have profound impacts on power system behaviors. It is well-recognized that the constant P-Q load model gives unsatisfactory results for power system voltage stability analysis. The nonlinear load model, ZIP-model which is a combination of constant current, constant power and constant impedance, is popular in modeling the nonlinear behaviors of loads. The purpose of this paper is

not to evaluate the assumption of fixing the values of constant current loads and constant impedance loads in generating the P-V Curve, Q-V Curve, or P-Q-V curves; instead its purpose is to show that the assumption can be relaxed and a new class of curves, called ZIP-V curves can be generated. ZIP-V curves represent power system quasi-steady-state behaviors as all the values of constant Z, I, P-Q (ZIP) vary and their variations are uniform. The ZIP-V curve can become an I-V, Z-V, IP-V, ZP-V or IZ-V curve when the values of the corresponding component are kept constant. A new set of parameterized power flow equations is proposed in the paper. A tool based on the CPFLOW method is developed and its application to generate ZIP-V curves for a 4561-bus interconnected power system is illustrated.

## 2. CPFLOW METHOD

The CPFLOW method is used in this paper to obtain a class of ZIP-V curves. We next briefly discuss the fundamental idea of the CPFLOW method.

Let  $P_i \equiv P_{gi} - P_{di}$  and  $Q_i \equiv Q_{gi} - Q_{di}$ . The lowercase  $g$  represents generation and the lowercase  $d$  represents load demand. The set of power flow equations can be represented in compact form as

$$f(x) \equiv \begin{bmatrix} P(x) - P \\ Q(x) - Q \end{bmatrix} = 0, \text{ where } x = (V, \theta) \quad (1)$$

Now one can investigate the steady-state behavior of the power system under slow variation of both loading conditions and real power redispatches. For example, if one needs to trace the power system state from the base-case load-generation condition  $[P_d^0, Q_d^0, P_g^0]$  to a new load-generation condition  $[P_d^1, Q_d^1, P_g^1]$ , then one can parameterize the set of power flow equations as such

$$F(x, \lambda) \equiv f(x) - \lambda b = 0 \quad (2)$$

where the load-generation vector  $b$  is

$$b \equiv \begin{bmatrix} P^1 - P^0 \\ Q^1 - Q^0 \end{bmatrix} \quad (3)$$

It follows that the parameterized power flow equations become the base-case power flow equations when  $\lambda = 0$ ,

$$F(x, 0) = \begin{bmatrix} P(x) - P^0 \\ Q(x) - Q^0 \end{bmatrix} = 0 \quad (4)$$

and when  $\lambda = 1$ , the power system is at the new load-generation condition  $[P_d^1, Q_d^1, P_g^1]$  and can be described by

$$F(x,1) = f(x) - b = \begin{bmatrix} P(x) - P^1 \\ Q(x) - Q^1 \end{bmatrix} = 0 \quad (5)$$

As shown in the above procedure, one can investigate the effects of varying real power generations as well as varying load demands on power system steady-state behaviors by analyzing the set of parameterized power flow equations (2). In fact, one can parameterize any change in PQ loads in conjunction with any change in P generations by selecting an appropriate vector  $b$ . One can thus compute the desired P-V, Q-V, P-Q-V curves based on equation (2) by varying the parameter  $\lambda$ . However this approach fails when the set of parameterized power flow equations approaches its bifurcation point (nose point) due to the numerical ill-condition of equation (2). Special indirect methods have been constructed to overcome this difficulty[1][2][3][4], such as continuation methods. CPFLOW is one of them and overcomes this numerical difficulty as follows:

1. Treat the  $\lambda$  as another state variable

$$X_{n+1} = \lambda \quad (6)$$

2. Introduce a new parameter  $s$ , the mathematical meaning of  $s$  is the arclength on the solution curve, which is defined as follows:

$$\sum_{j=1}^{n+1} (x_j - x_j(s))^2 = (\Delta s)^2 \quad (7)$$

It can be shown that the new set of equations (2) and (7) is numerically well-conditioned, even at the ‘nose’ point. CPFLOW solves these augmented power flow equations to obtain a solution curve passing through the ‘nose’ point without encountering the numerical difficulty of ill-conditioning.

The implementation of CPFLOW is shown in Figure 1. The major loop of CPFLOW consists of two parts: a predictor and a corrector. The predictor finds an approximation for the next solution on the solution curve and the corrector computes the exact solution based on the combined equation set (2) and (7).

### 3. ZIP LOAD VARIATION

Traditionally, when one applies CPFLOW to compute P-V curves, only the constant PQ portion of a load varies, while the constant current portion and constant impedance portion are kept indeed “constant”. As such, the physical meaning of these curves can be easily explained but the load model may not be accurate. Load models are known to have significant impacts on simulating power system behaviors. In order to accurately characterize the voltage behavior of a power system, it is necessary to use accurate load models in the computation of P-V, Q-V curves. For a power system of  $n$  buses, the equation (2) of  $i$ -th bus can be expressed as follows:

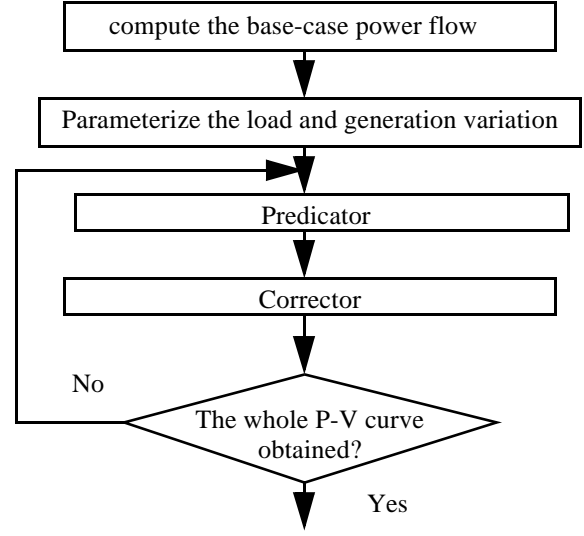


Figure 1 An overview of CPFLOW

$$\begin{cases} P_{0i} - \sum_{j=1}^n v_i v_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) + \lambda (\Delta P_{gi} - \Delta P_{li}) = 0 \\ Q_{0i} - \sum_{j=1}^n v_i v_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) + \lambda (\Delta Q_{gi} - \Delta Q_{li}) = 0 \end{cases} \quad (8)$$

where  $P_{0i}, Q_{0i}$  are the base-case real and reactive power injection at the bus  $i$  respectively,  $v_i$  is voltage magnitude at bus  $i$ .  $G_{ij}, B_{ij}$  are the real part and imaginary part of the network admittance between bus  $i$  and bus  $j$  respectively.  $\delta_{ij}$  is the angle difference between bus  $i$  and bus  $j$ .  $\Delta P_{gi}$  is the proposed real generation variation at bus  $i$  and  $\Delta Q_{gi}$  is the proposed reactive generation variation at bus  $i$ .  $\Delta P_{li}, \Delta Q_{li}$  are the proposed real and reactive load variations at bus  $i$

Without losing generality, equation (8) can be rewritten in the following form:

$$\begin{cases} f_p(\delta, v) + \lambda \Delta G_p - \lambda \Delta L_p = 0 \\ f_q(\delta, v) - \lambda \Delta L_q = 0 \end{cases} \quad (9)$$

where  $f_p(\delta, v), f_q(\delta, v) \in R^n$  are the set of base-case power flow equations.  $\Delta G_p \in R^n$  is a proposed real generation variation.  $\Delta L_p, \Delta L_q \in R^{2n}$  are the proposed real and reactive load variations respectively.

Assuming that the load variation can be expressed as a composition of ZIP (constant impedance, constant current and constant PQ) format and this composition keeps constant,  $\Delta L$  can be expressed as follows:

$$\begin{cases} \Delta L_p = a_{1p} v^2 + a_{2p} v + a_{3p} \\ \Delta L_q = a_{1q} v^2 + a_{2q} v + a_{3q} \end{cases} \quad (10)$$

where:  $a_{1p}, a_{2p}, a_{3p}, a_{1q}, a_{2q}, a_{3q} \in R^{n \times n}$  are diagonal coefficient matrices and respectively represent the constant impedance, constant current and constant PQ portion of the load variation and  $v \in R^n$  is the voltage magnitude vector.

Substituting (10) into (9), we have the following new set of parameterized power flow equations with ZIP variations

$$\begin{cases} f_p(\delta, v) + \lambda \nabla G_p - \lambda(a_{1p}v^2 + a_{2p}v + a_{3p}) = 0 \\ f_q(\delta, v) - \lambda(a_{1q}v^2 + a_{2q}v + a_{3q}) = 0 \end{cases} \quad (11)$$

If only the constant PQ load component varies, then the coefficient matrices  $a_{1p}, a_{1q}$  and  $a_{2p}, a_{2q}$  become zero matrices and the new set of parameterized power flow equations (11) is reduced to the traditional set of parameterized power flow equations (2).

#### 4. CPFLOW WITH ZIP LOAD VARIATION

Combining equation (11) and (7), one gets the following augmented (parameterized) power flow equations:

$$\begin{cases} f(\delta, v) + \lambda \Delta G - \lambda(a_1v^2 + a_2v + a_3) = 0 \\ \sum_{j=1}^{n+1} (x_j - x_j(s))^2 = (\Delta s)^2 \end{cases} \quad (12)$$

where:  $a_1 = \begin{bmatrix} a_{1p} \\ a_{1q} \end{bmatrix}$ ,  $a_2 = \begin{bmatrix} a_{2p} \\ a_{2q} \end{bmatrix}$  and  $a_3 = \begin{bmatrix} a_{3p} \\ a_{3q} \end{bmatrix}$ ,

$$f(\delta, v) = \begin{bmatrix} f_p(\delta, v) \\ f_q(\delta, v) \end{bmatrix}, \quad G = \begin{bmatrix} G_p \\ 0 \end{bmatrix}$$

Solving equations (12) give rise to the desired ZIP-V curves. Next we will show how to modify the original CPFLOW method described in [1] to solve the new set of parameterized power flow equations (12).

#### Predictor

The predictor in CPFLOW is used to find the approximation of the next point in the solution curves. CPFLOW used two predictor approaches: secant and tangent. The secant method, extrapolating the next solution point based on the previous two solution points in the solution curve is not affected by the load model used in the computation. The tangent method, using the tangent at the current point to predict the next solution point, is affected by the load model used.

We define the power flow state variable vector  $X$  as follows:

$$X = [\delta_1, \dots, \delta_n, v_1, \dots, v_n, \lambda]^T \quad (13)$$

Differentiating (13) with respect to the arclength  $s$ , we have:

$$\frac{dX}{ds} = \left[ \frac{d\delta_1}{ds}, \dots, \frac{d\delta_n}{ds}, \frac{dv_1}{ds}, \dots, \frac{dv_n}{ds}, \frac{d\lambda}{ds} \right] \quad (14)$$

where the arclength  $s$  is defined as follows:

$$\Delta X \bullet \langle \Delta X \rangle^T = (\Delta s)^2 \quad (15)$$

Differentiating both side of the equation (12), it follows:

$$D \bullet \frac{dX}{ds} = 0 \quad (16)$$

$$D = \begin{bmatrix} \frac{\partial f}{\partial \delta} & B & C \end{bmatrix} \quad (17)$$

where  $B = \frac{\partial f}{\partial v} - (2a_1V + a_2)\lambda$   $C = \Delta G - (a_1V^2 + a_2V + a_3)$

$\frac{\partial f}{\partial v} = \begin{bmatrix} \frac{\partial f}{\partial v_1} & \dots & \frac{\partial f}{\partial v_n} \end{bmatrix}$  and  $\frac{\partial f}{\partial \delta} = \begin{bmatrix} \frac{\partial f}{\partial \delta_1} & \dots & \frac{\partial f}{\partial \delta_n} \end{bmatrix}$ .  $V \in R^{n \times n}$  is a diagonal matrix of voltage magnitude at each bus.

The following equation is required to ensure that the scalar  $s$  is the arclength on the solution curve

$$\frac{dX}{ds} \bullet \left[ \frac{dX}{ds} \right]^T = 1 \quad (18)$$

The tangent vector along the solution curve  $\frac{dX}{ds}$  is obtained by solving the set of equation (16), (17) and (18).

The predicted vector at the next solution point  $X_{i+1}$  by the tangent method becomes:

$$X_{i+1} = X_i + h \frac{dX_i}{ds} \quad (19)$$

where  $h$  is the step size of arclength  $s$ .

#### Corrector

The corrector solves the augmented equations (12) with the predicted point, say (19) as the initial guess. In principle, any effective numerical procedure for solving a set of non-linear algebraic equations can be used for a corrector. In CPFLOW, we used the Newton-Raphson method in which the system Jacobian matrix plays an important role.

In order to accommodate the ZIP load model in the load variation, we must derive the Jacobian matrix considering ZIP load variation which can be expressed as follows:

$$J = \begin{bmatrix} D \\ E \end{bmatrix} \quad (20)$$

$D$  is given in (17) and

$$E = 2(X - X(s))^T \quad (21)$$

where  $X_i$  is the  $i$ -th solution in the solution curve. Note that the required modifications only appear on the diagonal elements and the arclength  $s$  related elements.

#### 5. NUMERICAL STUDY

We have implemented the ZIP load variation model in the CPFLOW and applied the modified CPFLOW to a real system to generate a family of ZIP-V curves for different load variations and different load compositions.

#### Network Components

Buses: 4561, swing bus: 1, Generator: 1225, Loads: 2754, fixed shunts: 916, switchable shunts: 186, Lines: 7512, Fixed transformer: 755, Fixed phase shifter: 2 ULTC Transformer: 294, ULTC phase shifter: 13, Areas: 22.

## Simulation Results

Due to space limitation, only one numerical simulation is presented. The real power demands at two load buses in area 63 increases 2000 MW, which is supported by the swing bus. The reactive load demands increases proportionally in order to maintain constant power factors. We assume that the load increase is uniformly between these two loads based on their base-case load demands and nominal voltage level (say 1.0). For example, if the base-case load demands for the two buses 37886, 38280 are 10MW and 20MW respectively, a uniform load increase of a total 30 MW adds another 10 MW (at voltage 1.0) to bus 37886 and another 20 MW (voltage 1.0) to bus 38280.

We investigate four different load compositions in the proposed load variation: (I) constant P-Q load; (II) constant current load, (III) constant impedance load and (IV) 20% constant impedance load, 20% constant current load and 60% constant P-Q load. Bus 38280 at 138kv, one of the two load buses that are subject to load variations, is monitored. The generated P-V, I-V, Z-V, and ZIP-V curves for the four different load compositions are shown in Figure 2, Figure 3, Figure 4, and Figure 5, respectively.

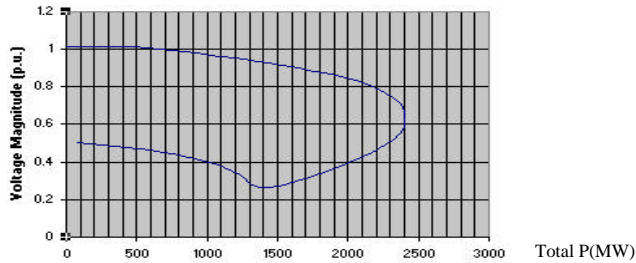


Figure 2 P-V curve at bus 38280 for load composition (i)

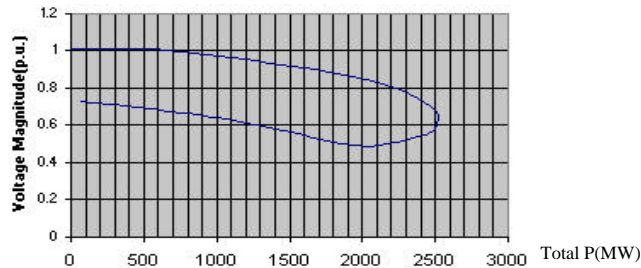


Figure 3 I-V curve at bus 38280 for load composition (ii)

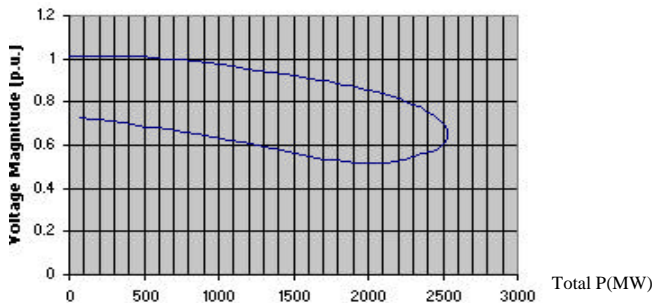


Figure 4 Z-V curve at bus 38280 for load composition (iii)

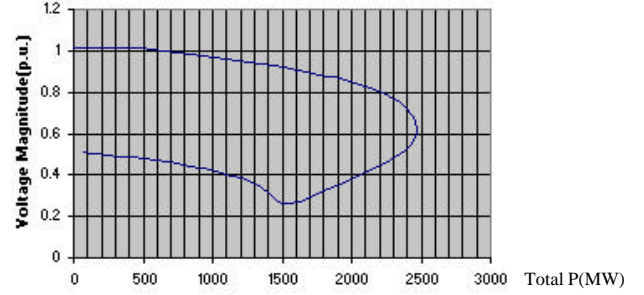


Figure 5 ZIP-V curve at bus 38280 for load composition (iv)

The load margins limited by the nose points for each load composition are listed in Table I.

Table I Load margin to the nose point for different load models.

Load Model	I	II	III	IV
Load Margin	2405	2516	2566	2472
Load margin increase against the constant P-Q load	0%	4.62%	6.7%	2.79%

Among the four different load compositions, the constant P-Q load model has the minimal amount of load margin, which is 2405MW, and the constant impedance load model carries the maximal amount of load margin, which is 2566MW and 6.7% more than that of the constant P-Q load. The constant current loads can carry more load than the constant P-Q load and the constant impedance loads is able to carry most load among the four studied load models, which has been observed in our extensive numerical studies.

This numerical results reconfirm the traditional assumption that the constant P-Q load model serves its purpose well - on the conservative side in predicting several system limits such as voltage stability limit or loadability limit. However the numerical study has demonstrated the significant impact of load model on the load margin of a power system. If one can relax this assumption and take a more accurate load model into consideration, one is able to take an aggressive step to bear more load and achieve more financial benefit. In above example, the constant impedance loads can bear about 6% more load than that of the constant PQ loads.

## 6. CONCLUSION

In this paper, we have developed a tool to generate ZIP-V curves for tracing power system steady state stationary behavior due to both real generation and load variations. We have developed a new set of parameterized power flow equations with ZIP load variations. We have implemented this new function in CPFLOW. The numerical study on a 4561-bus real power system shows that (i) the traditional assumption that only constant P-Q load varies makes the predicted results always on the conservative side; (ii) the above assumption can be relaxed and a new class of curves, called ZIP-V curves, can be generated; and (iii) the load margin based on the ZIP-V curve is generally greater than that based on the traditional P-V curve.

## 7. REFERENCE

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